SYSTEMATICAL ANALYSIS OF (n,α) REACTION CROSS SECTIONS FOR 6-20 MeV NEUTRONS

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1) Historical preview

2) Statistical model formulae

3) Systematics of (n,α) cross sections and the comparison of theoretical and experimental (n,α) cross sections

- nuclear energy applications
  - estimate helium production
  - nuclear heating
  - transmutations in the structural materials of fission and fusion reactors
- basic nuclear physics problems
  - information on the nuclear reaction mechanisms

1963-1973

- V. N. Levkovsky,

  "Empirical regularities in the (n, p) cross sections at 14-15-MeV neutron energies," Zh. Eksp.Teor. Fiz., 45, No. 2(8), 305 (1963).
- V. N. Levkovsky,

  "The (n, p) and (n, α) cross sections at 14-15 MeV, " Yad. Fiz., 18, No. 4, 705 (1973).

isotopic effect

$$\sigma_{n\alpha} = C\pi r_0^2 \left(1 + A^{1/3}\right)^2 exp\left[-\frac{K(N-Z)}{A}\right]$$

Isotopic effect
in the (n,p) and (n,α) cross sections neutron energy of 14-15 MeV

R.A.Forrest, Report AERE-R 12419, Harwell Laboratory. December, 1986

S.Ait-Tahar, J.Phys.G: Nuclear Physics, v.13, N7, 1987, p.121

K.Gul, INDC (PAK)-009, 1995, IAEA, Vienna

Y.Kasugai *et al*., in: JAERI-conf. 95-008, INDC (JPN)-173/U, March, 1995, p.181

A.Yu.Konobeyev *et al*., Nucl. Instr. Meth. B, v.108, N7, 1996, p.233

A.D.Majdeddin *et al*., INDC (HUN)-031, July, 1997, IAEA, Vienna

Kh.T.Osman, INDC (SUD)-003, December, 1998, IAEA, Vienna

F.I.Habbani, Kh.T.Osman, Appl. Rad. Isot., v.54, 2001, p.283

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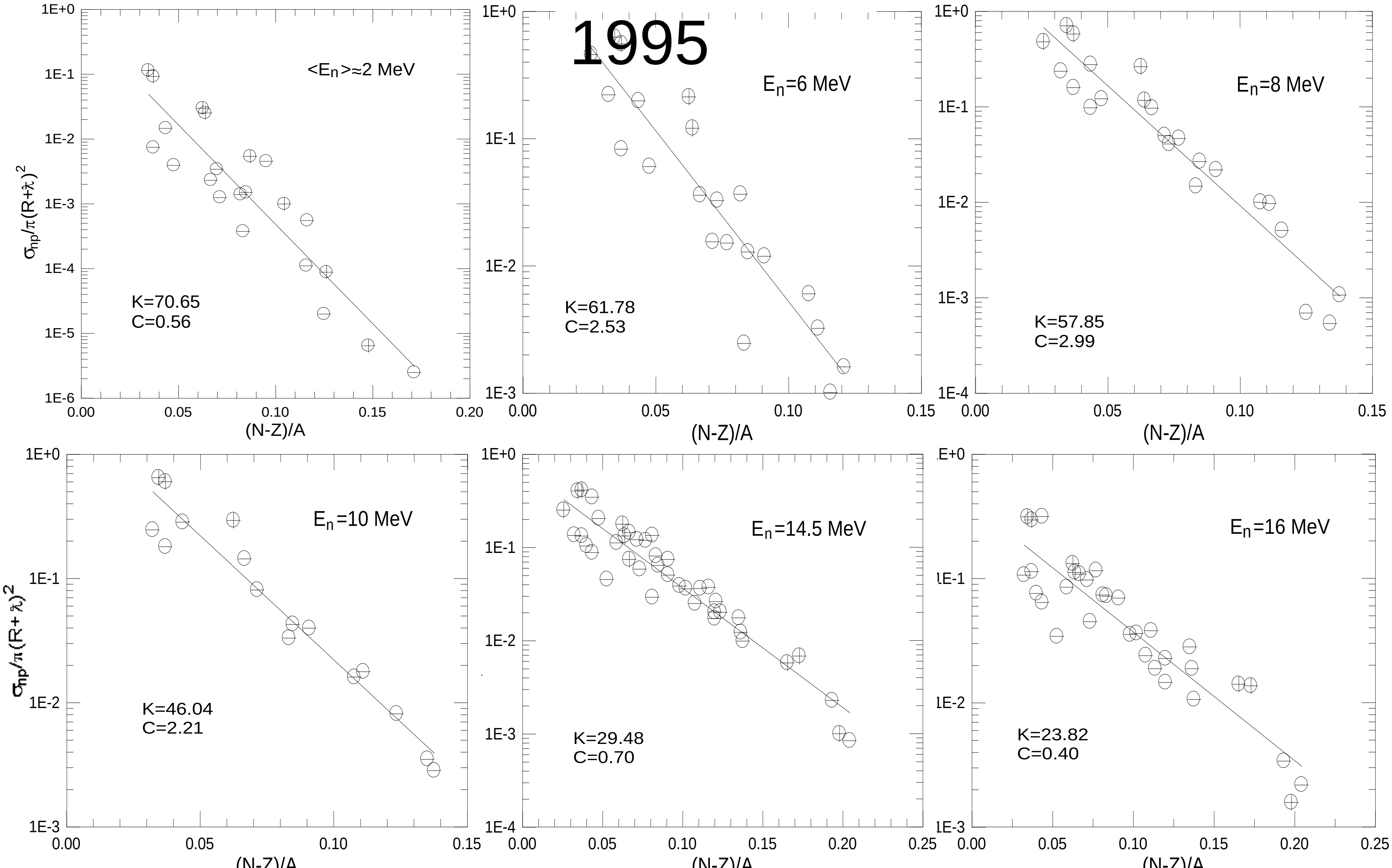
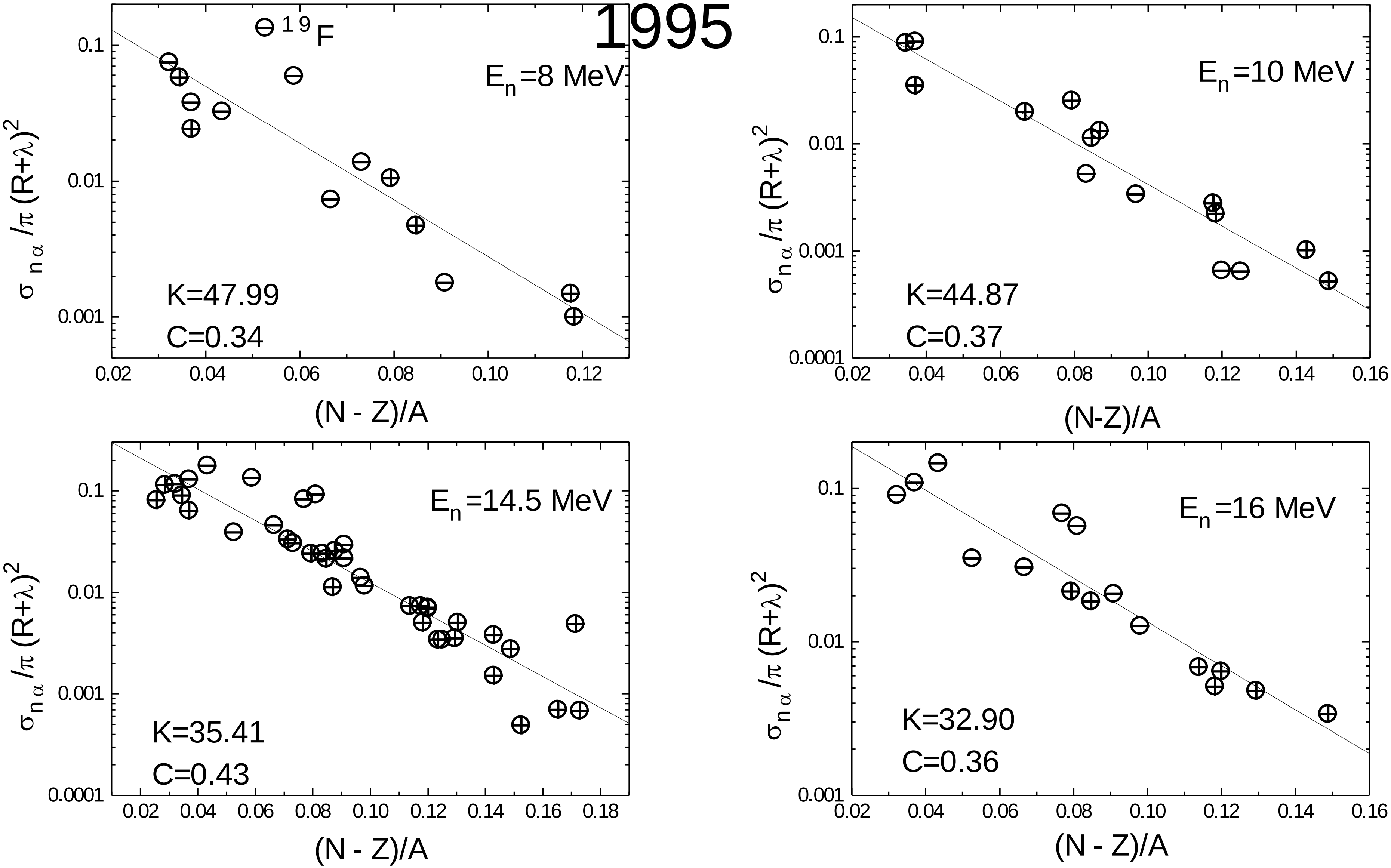
1993-1995



The dependence of reduced (n,p) cross section on the asymmetry parameter (N-Z)/A

1993-1995



The dependence of reduced (n,α) cross sections
on the relative neutron excess (N-Z)/A in the target nucleus

Statistical Model Formulae

$$\sigma(n,\alpha) = \sigma_c(n) \cdot G(\alpha)$$

compound nucleus formation cross section

$$\sigma_c(n) = \pi\left(R + \lambda\!\!\!^{-}_n\right)^2$$

R -target nucleus radius

$\lambda\!\!\!^{-}_n = \lambda/2\pi$ - wavelength of the incident neutron

α-decay probability of the compound nucleus

$$G(\alpha) = \frac{\Gamma_\alpha}{\Gamma} = \frac{\Gamma_\alpha}{\sum_i \Gamma_i}$$

Γ_α and Γ - the alpha and total level widths

V. F. Weisskopf and D. H. Ewing

On the Yield of Nuclear Reactions with Heavy Elements

Phys. Rev. **57**, 472 (1940)– Published March 15, 1940

$$\Gamma_\alpha = \frac{2S_\alpha + 1}{\pi^2 \hbar^2 \rho_c(E_c)} M_\alpha \int_{V_\alpha}^{E_\alpha^{max}} E_\alpha \sigma_{inv}(E_\alpha) \rho_y(U_\alpha) dE_\alpha$$

S_α - spin

M_α - mass

E_α - energy

V_α - Coulomb potential of α-particle

$\rho_c(E_c)$, $\rho_y(U_\alpha)$ — level densities of the compound, residual nuclei

U_α - excitation energy of the residual nuclei

$\sigma_{inv}(E_\alpha)$ - inverse reaction cross section, determined in semiclassical model:

$$\sigma_{inv}(E_\alpha) = \begin{cases} \pi R^2 \left(1 - \dfrac{V_\alpha}{E_\alpha}\right) & \text{for } E_\alpha > V_\alpha \\ 0 & \text{for } E_\alpha < V_\alpha \end{cases}$$

$$\Gamma_\alpha = \frac{2S_\alpha + 1}{\pi\hbar^2} M_\alpha R^2 \int_{V_\alpha}^{E_\alpha^{max}} E_\alpha \left(1 - \frac{V_\alpha}{E_\alpha}\right) e^{-\frac{B_\alpha + \delta_\alpha + E_\alpha}{\theta}} dE_\alpha$$

B_α - binding energy of α-particle for daughter nucleus
δ_α - odd-even effect parameter for Weizsacker's form
$\theta = kT$ - thermodynamical temperature
k - Boltzmann constant

$$\Gamma_i = \frac{2S_i + 1}{\pi\hbar^2} M_i R^2 \int_{V_i}^{E_i^{max}} E_i \left(1 - \frac{V_i}{E_i}\right) e^{-\frac{B_i + \delta_i + E_i}{\theta}} dE_i$$

neglect γ-emission

$$\sigma(n,\alpha) = \frac{\Gamma_\alpha}{\sum_i \Gamma_i} = \sigma_c(n) \frac{(2S_\alpha + 1) M_\alpha e^{-\frac{B_\alpha + \delta_\alpha + V_\alpha}{\theta}} \left\{1 - \frac{W_{n\alpha}}{\theta} e^{-\frac{W_{n\alpha}}{\theta}} - e^{-\frac{W_{n\alpha}}{\theta}}\right\}}{\sum_i (2S_i + 1) M_i e^{-\frac{B_i + \delta_i + V_i}{\theta}} \left\{1 - \frac{W_{ni}}{\theta} e^{-\frac{W_{ni}}{\theta}} - e^{-\frac{W_{ni}}{\theta}}\right\}}$$

where $W_{n\alpha} = E_n + Q_{n\alpha} - V_\alpha$ and $W_{ni} = E_n + Q_{ni} - V_i$

$$\sigma(n,\alpha)=\sigma_c(n)\frac{(2S_\alpha+1)M_\alpha e^{-\frac{B_\alpha+\delta_\alpha+V_\alpha}{\theta}}\left\{1-\frac{W_{n\alpha}}{\theta}e^{-\frac{W_{n\alpha}}{\theta}}-e^{-\frac{W_{n\alpha}}{\theta}}\right\}}{\sum_i(2S_i+1)M_i e^{-\frac{B_i+\delta_i+V_i}{\theta}}\left\{1-\frac{W_{ni}}{\theta}e^{-\frac{W_{ni}}{\theta}}-e^{-\frac{W_{ni}}{\theta}}\right\}}$$

$\Gamma \approx \Gamma_n$ odd-even effect parameters were neglected

$$\sigma(n,\alpha)=\sigma_c(n)\frac{2S_\alpha+1}{2S_n+1}\frac{M_\alpha}{M_n}e^{\frac{Q_{n\alpha}-V_\alpha}{\Theta}}\left\{\frac{1-\frac{W_{n\alpha}}{\Theta}e^{-\frac{W_{n\alpha}}{\Theta}}-e^{-\frac{W_{n\alpha}}{\Theta}}}{1-\frac{E_n}{\Theta}e^{-\frac{E_n}{\Theta}}-e^{-\frac{E_n}{\Theta}}}\right\}$$

$(E_n+Q_{n\alpha}-V_\alpha)>>\theta$ and $(E_n+Q_{ni}-V_i)>>\theta$

$$\sigma(n,\alpha)=\sigma_c(n)\frac{(2S_\alpha+1)}{(2S_n+1)}\frac{M_\alpha}{M_n}e^{\frac{Q_{n\alpha}-V_\alpha}{\Theta}}$$

P.Cuzzocrea *et al.*,
Nuovo Cimento.A, v.4, N2, 1971, p.2

$$\sigma_c(n)=\pi(R+\lambda\!\!\!^{-}_n)^2 \quad \Rightarrow \sigma(n,\alpha)=2\pi(R+\lambda\!\!\!^{-}_n)^2 e^{\frac{Q_{n\alpha}-V_\alpha}{\theta}}$$

$$\sigma(n,\alpha) = 2\pi\left(R + \lambda\!\!\!^{-}_n\right)^2 e^{\frac{Q_{n\alpha} - V_\alpha}{\theta}}$$

Coulomb potential

$$V_\alpha = 2.058\,\frac{Z-2}{(A-3)^{1/3} + 4^{1/3}}\ MeV$$

D.G.Gardner, Yu-Wen Yu,
Nucl. Phys., v.60, N1, 1964, p.49

Weizsacker's formula

$$Q_{n\alpha} = -3\alpha + \beta\left(A^{2/3} - (A-3)^{2/3}\right) + \gamma\left(\frac{Z^2}{A^{1/3}} - \frac{(Z-2)^2}{(A-3)^{1/3}}\right) +$$

$$+\,\xi\left(\frac{(A-2Z)^2}{A} - \frac{(A-2Z+1)^2}{(A-3)}\right) \pm \left(\frac{\delta_f}{(A-3)^{3/4}} - \frac{\delta_i}{A^{3/4}}\right) + \varepsilon_\alpha$$

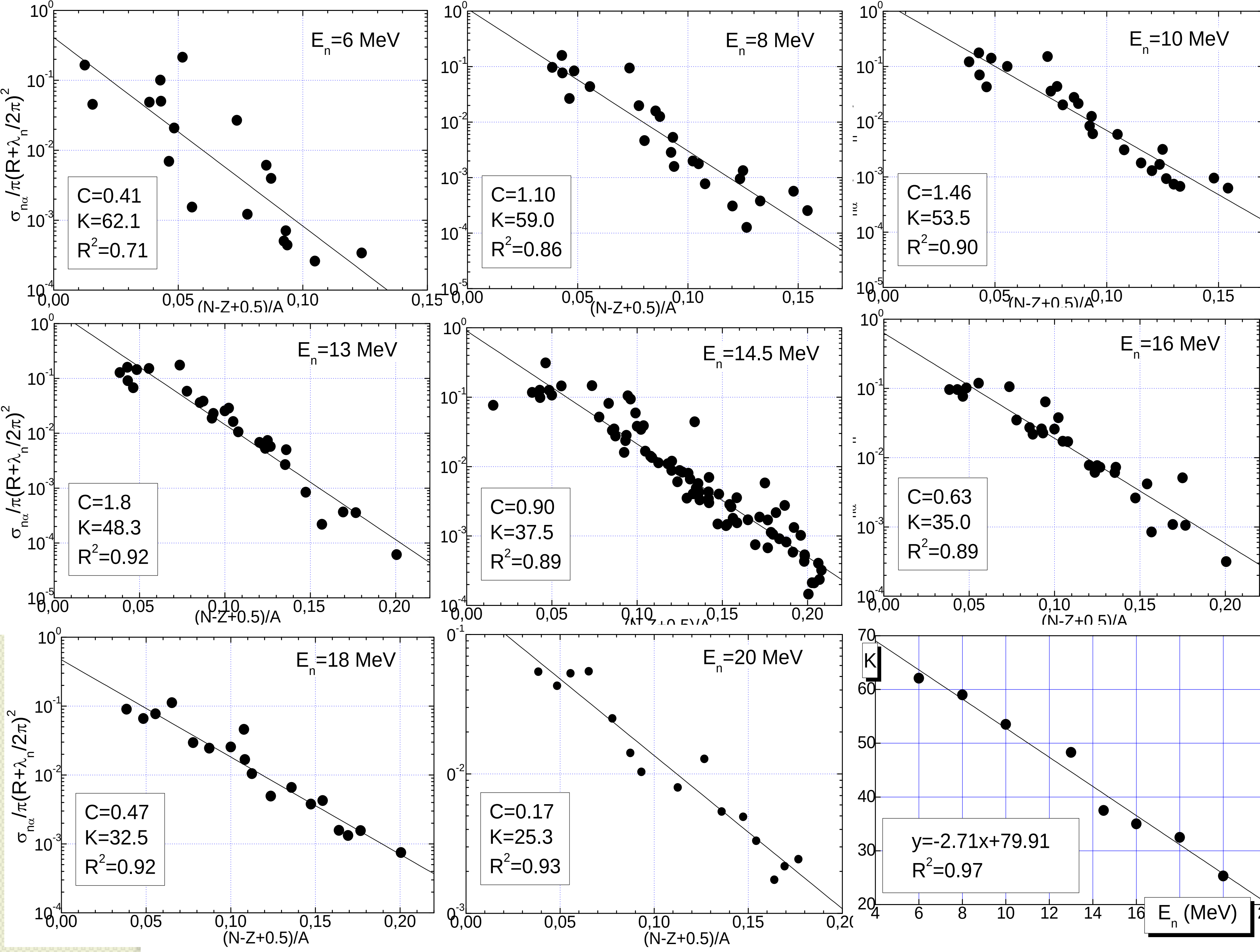
ε_α=28.2 MeV α=15.7 MeV β=17.8 MeV γ=0.71 MeV ξ=23.7 MeV $|\delta|$=34 MeV (or 0)

neglect odd-even effect parameter $\Delta=\delta_f-\delta_i$

$$\Rightarrow \sigma(n,\alpha) = C\pi\left(R + \lambda\!\!\!^{-}_n\right)^2 e^{-K\frac{N-Z+0.5}{A}}$$

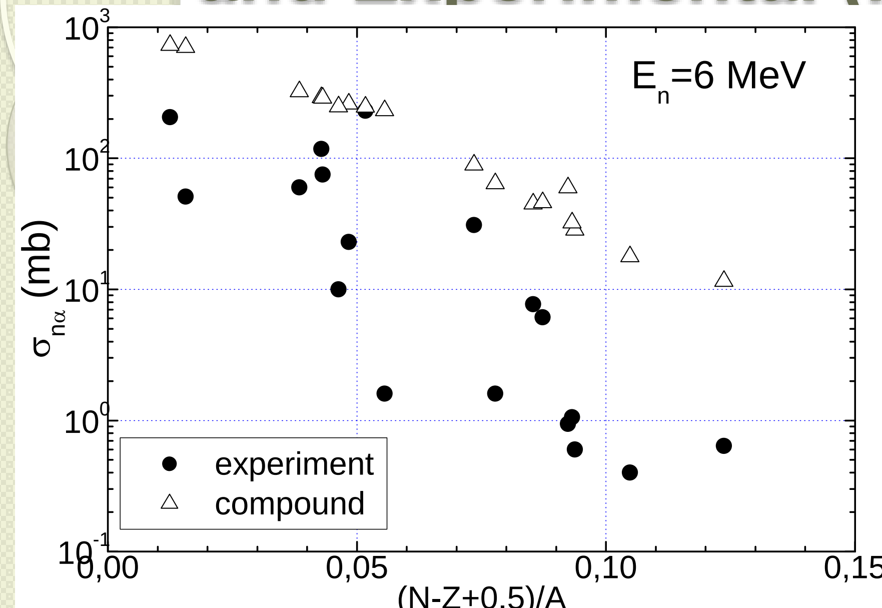
$$C = 2\exp\frac{1}{\theta}\left(-3\alpha + \beta\left[A^{2/3} - (A-3)^{2/3}\right] + \gamma\left(\frac{Z^2}{A^{1/3}} - \frac{(Z-2)^2}{(A-3)^{1/3}}\right) + \varepsilon_\alpha - V_\alpha\right) \qquad K = \frac{2\xi}{\theta}$$

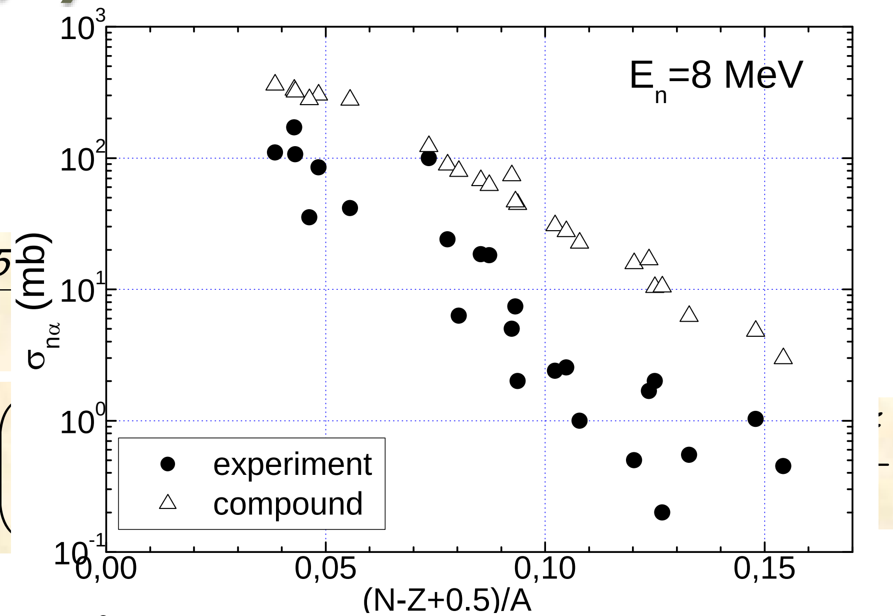
$$\theta = \sqrt{\frac{U_\alpha^{\max}}{a}} = \sqrt{\frac{13.5(E_n + Q_{n\alpha})}{A-3}}$$

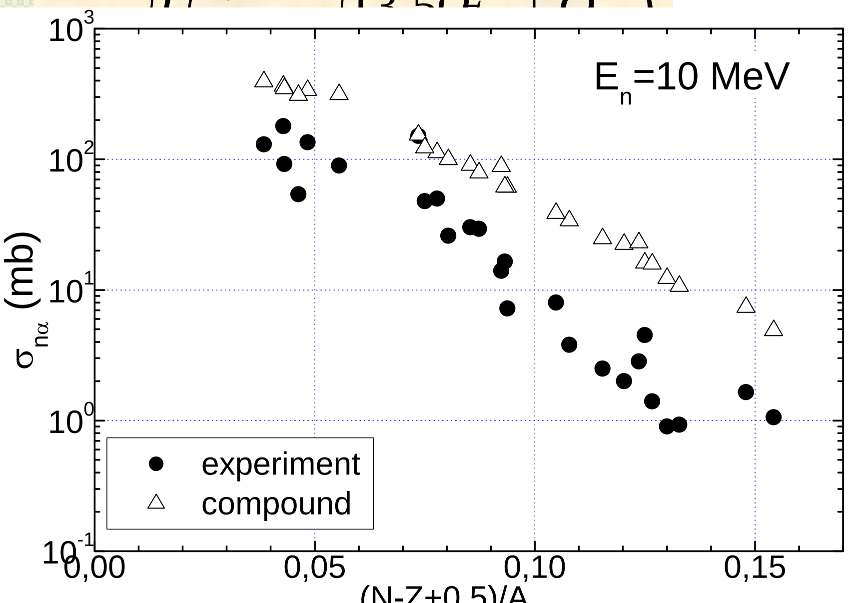


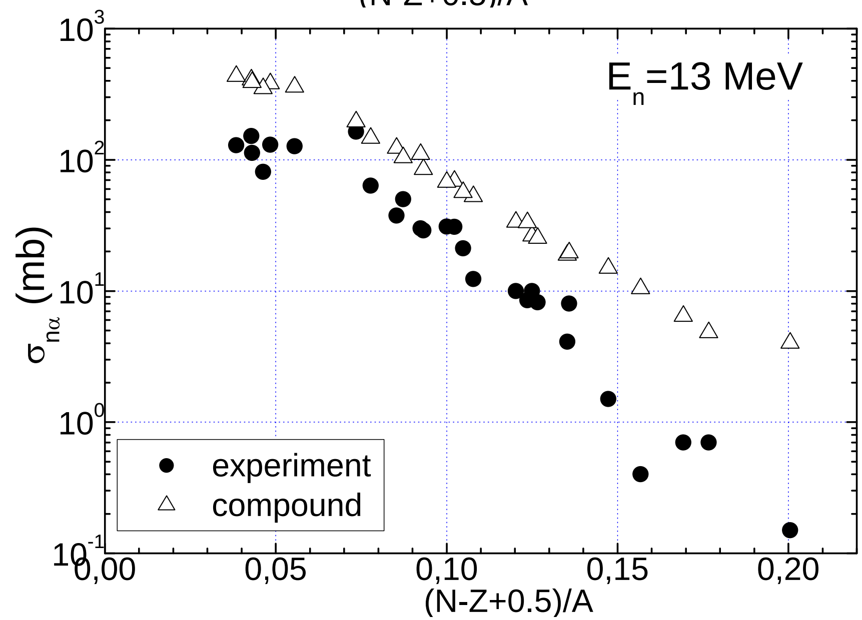
E_n=6 MeV
C=0.41
K=62.1
R^2=0.71
E_n=8 MeV
C=1.10
K=59.0
R^2=0.86
E_n=10 MeV
C=1.46
K=53.5
R^2=0.90
E_n=13 MeV
C=1.8
K=48.3
R^2=0.92
E_n=14.5 MeV
C=0.90
K=37.5
R^2=0.89
E_n=16 MeV
C=0.63
K=35.0
R^2=0.89
E_n=18 MeV
C=0.47
K=32.5
R^2=0.92
E_n=20 MeV
C=0.17
K=25.3
R^2=0.93
K
y=-2.71x+79.91
R^2=0.97
E_n (MeV)
σ_nα/π(R+λ_n/2π)^2
(N-Z+0.5)/A

The Comparison of Theoretical and Experimental (n,α) Cross Sections

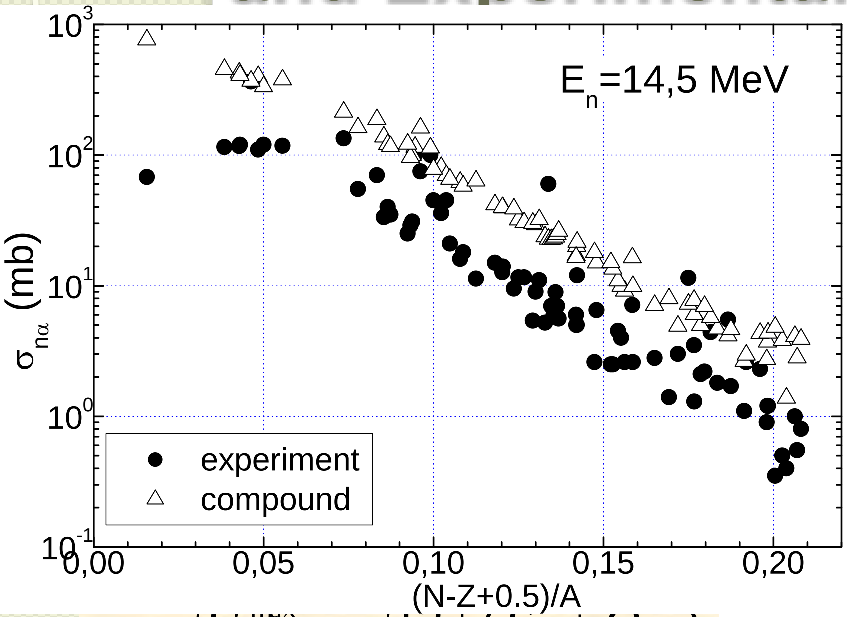


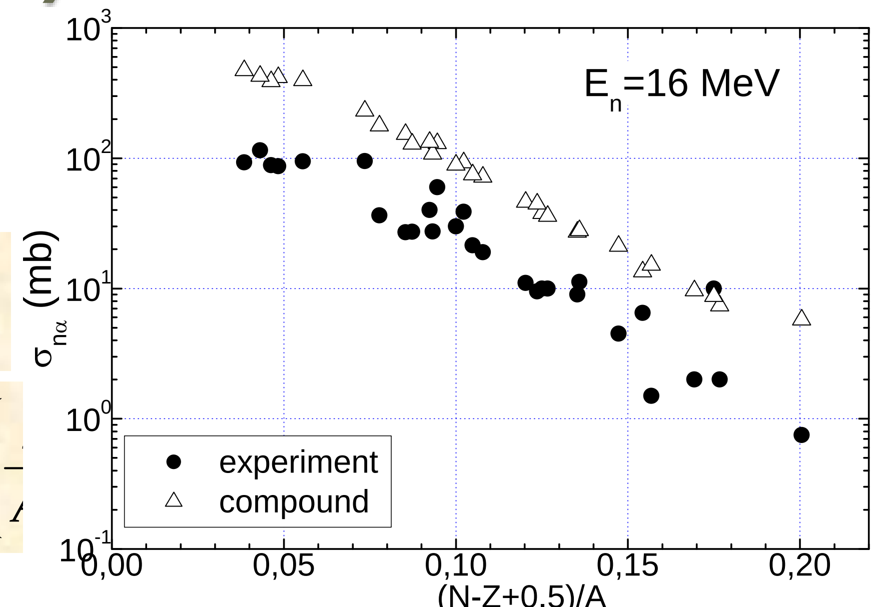


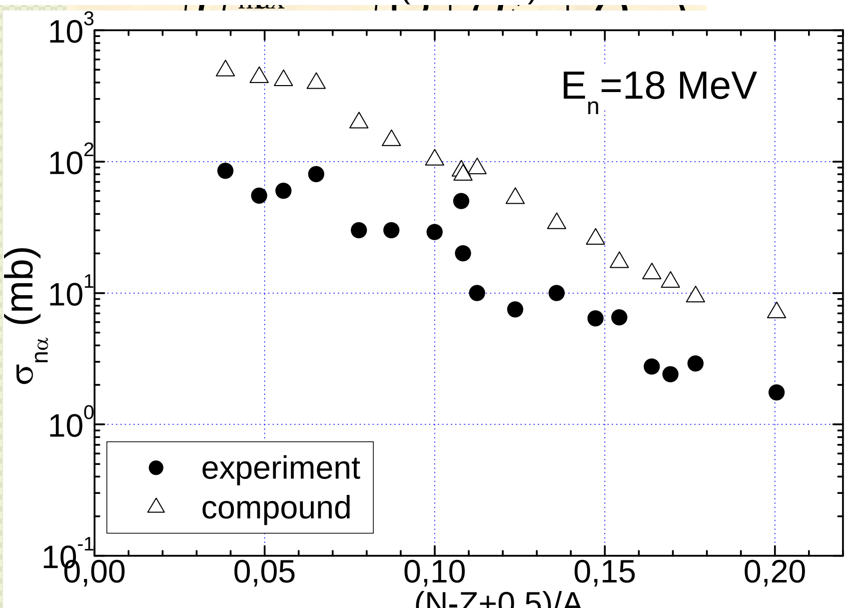


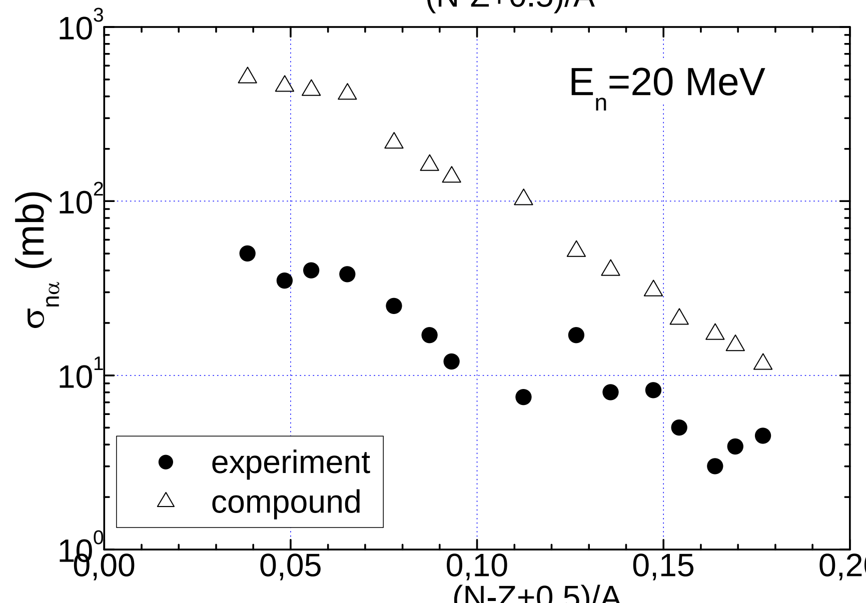


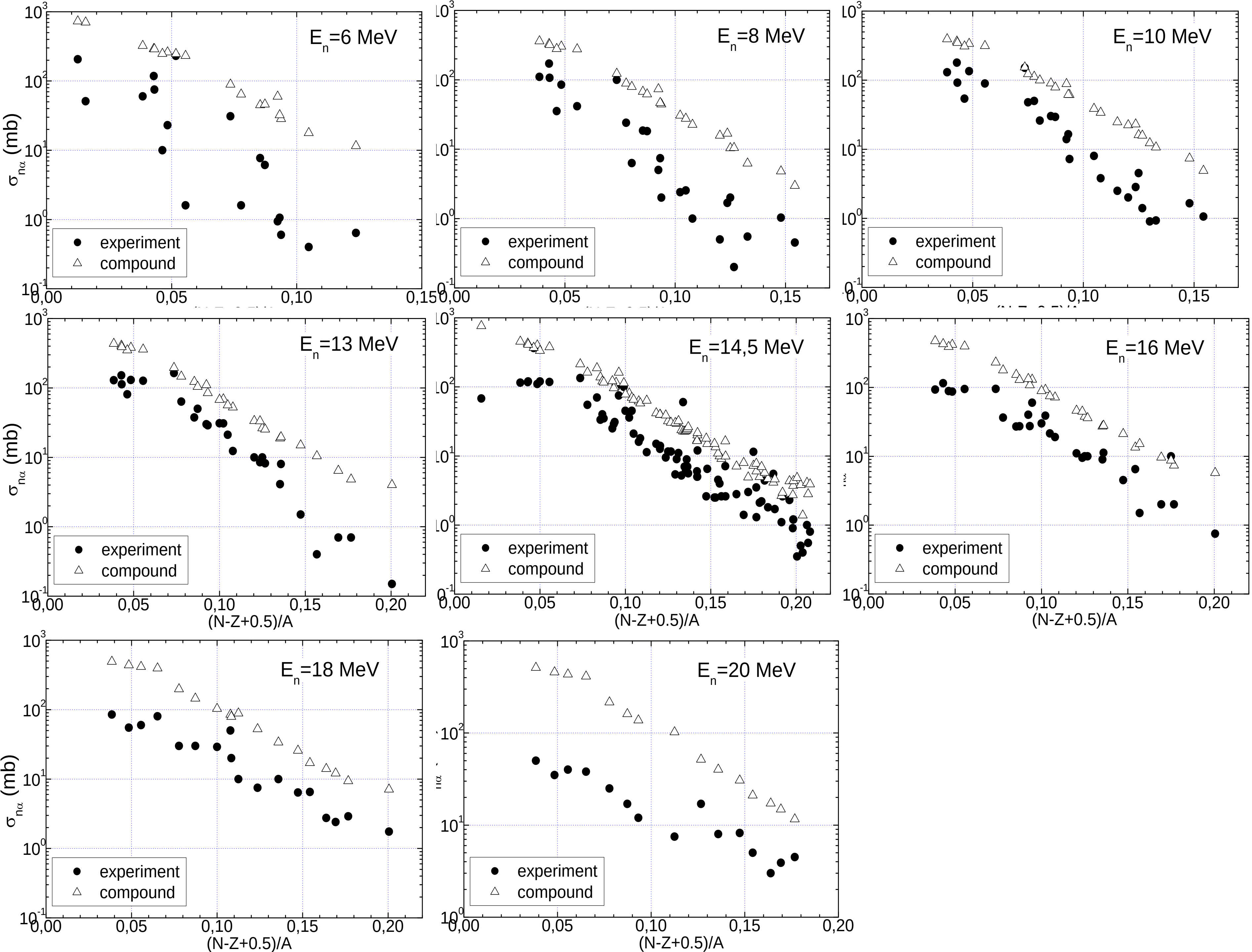
The Comparison of Theoretical and Experimental (n,α) Cross Sections










E_n=6 MeV
E_n=8 MeV
E_n=10 MeV
E_n=13 MeV
E_n=14,5 MeV
E_n=16 MeV
E_n=18 MeV
E_n=20 MeV
experiment
compound
σ_nα (mb)
(N-Z+0.5)/A
0,00
0,05
0,10
0,15
0,20

The (n,α) Cross Section and α-clusterization Factor

G.Khuukhenkhuu, M.Odsuren, Fast Neutron Induced Reaction Cross Sections, 2010, “Munkhiin Useg Group” Co.Ltd.Press, Ulaanbaatar /in

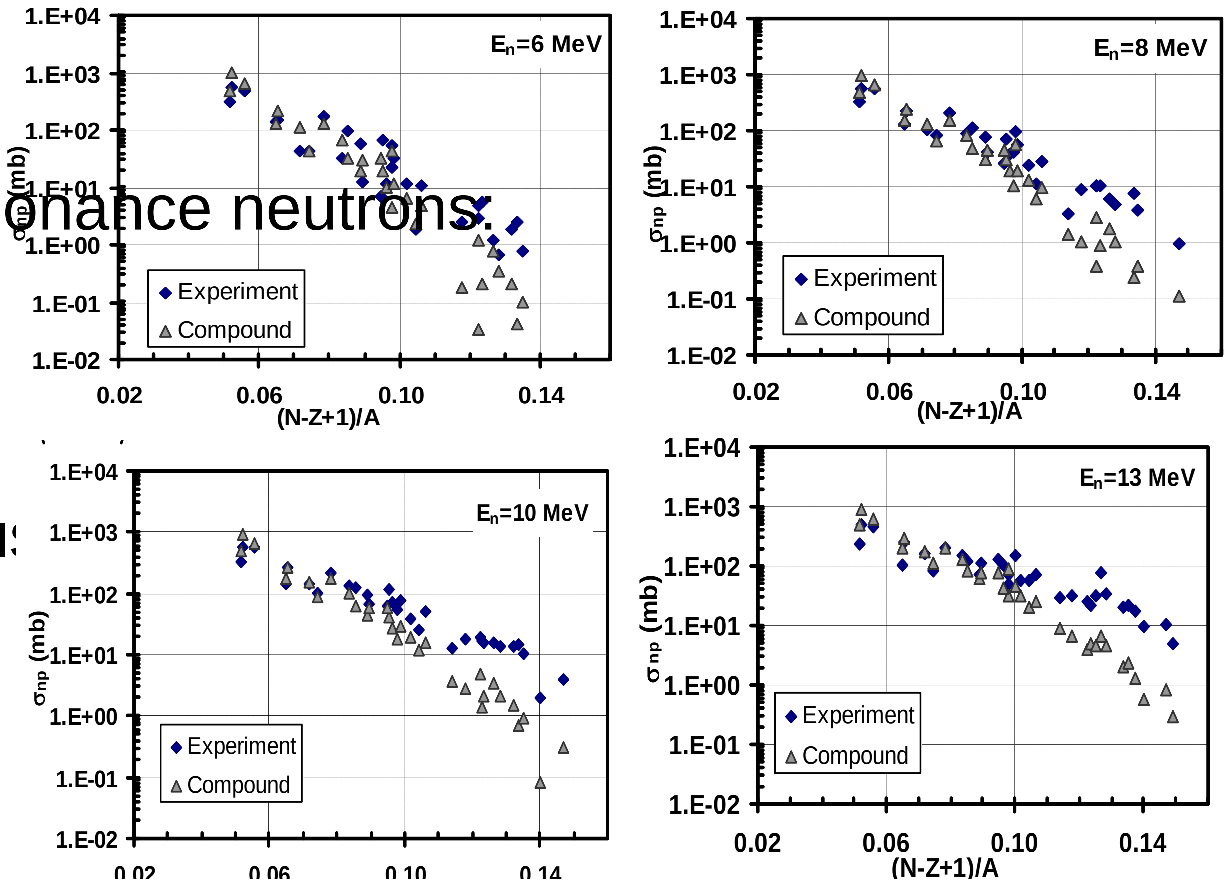
$$\sigma(n,x) = C\pi(R + \lambdabar_n)^2 e^{-K\frac{N-Z+0.5}{A}}$$

$\gamma_n^2 \approx$ … , 1937, p.69

for resonance neutrons:

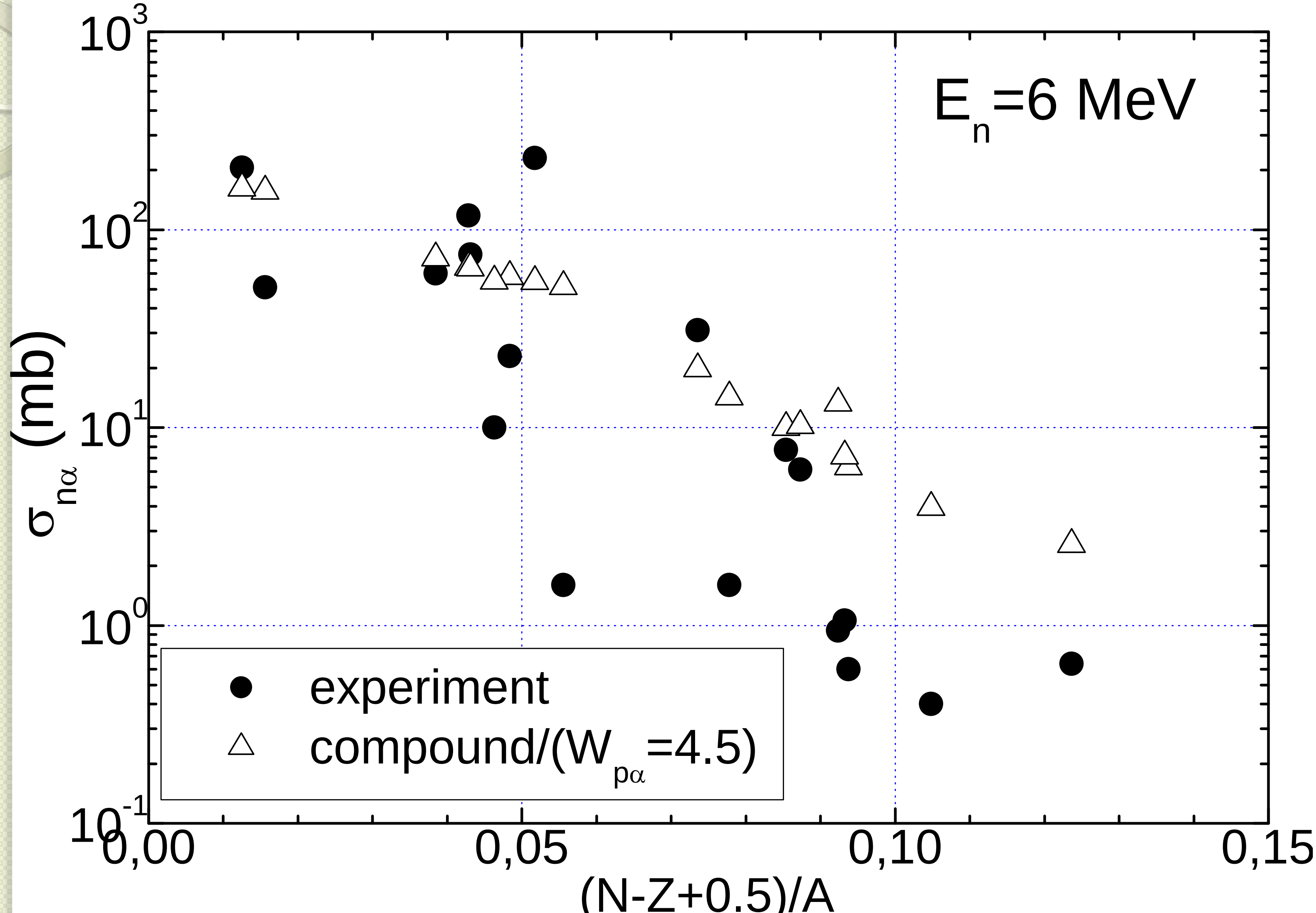
$W_{n\alpha}$ = … 1971, p.913

If we us…



Calculated by statistical model values and experimental data of (n,p) cross section

The (n,α) cross sections calculated by statistical model and experimental data



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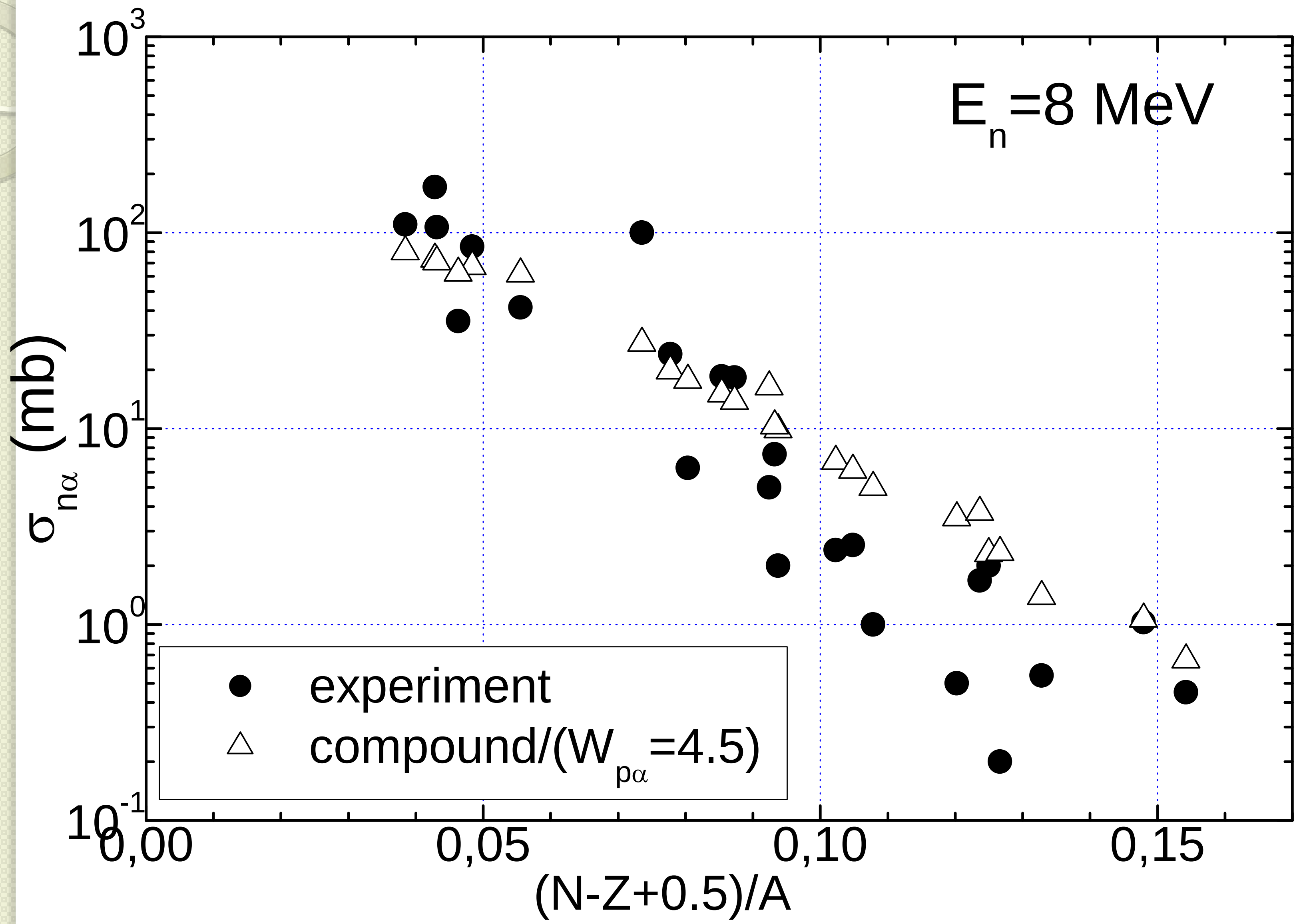

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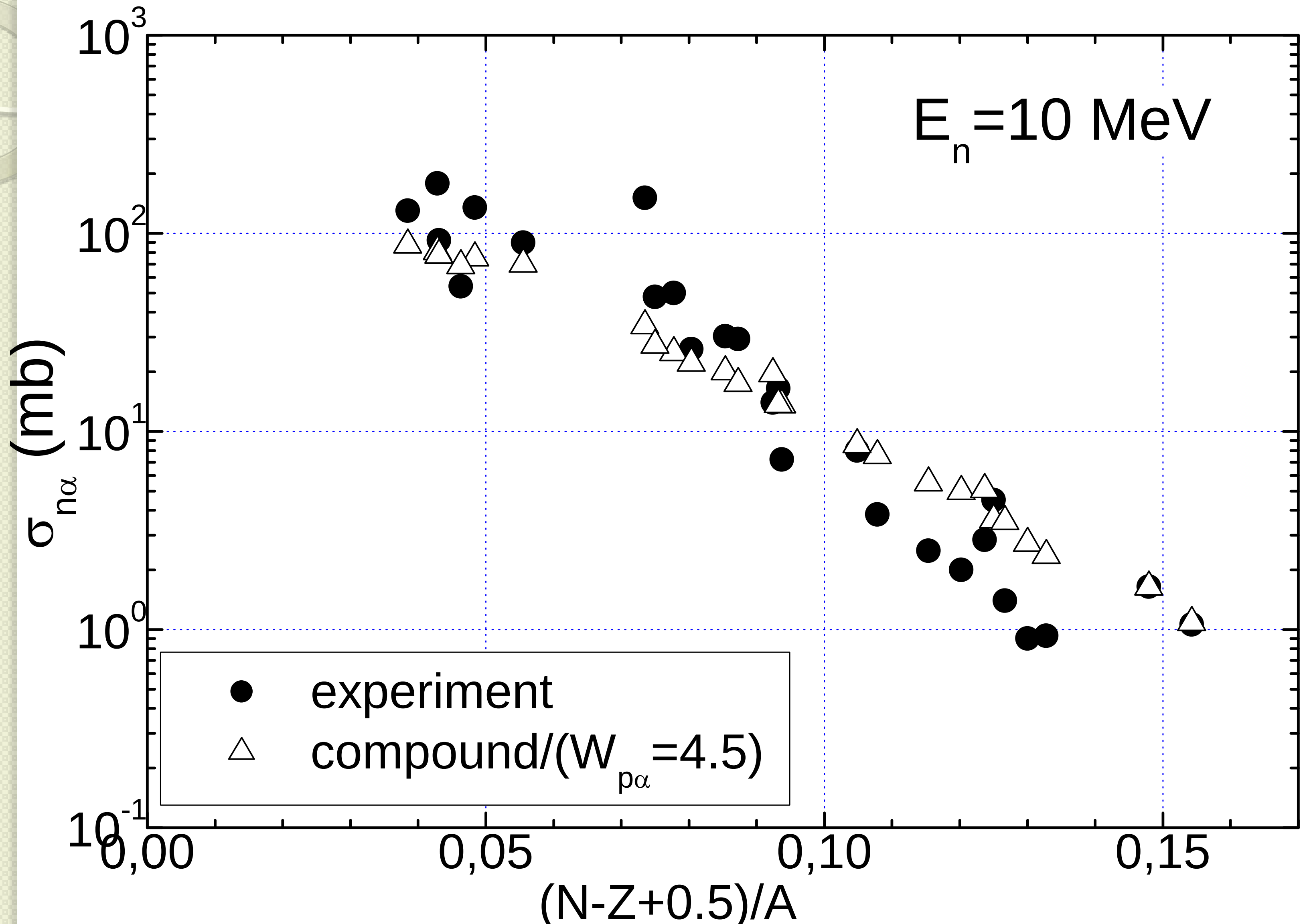
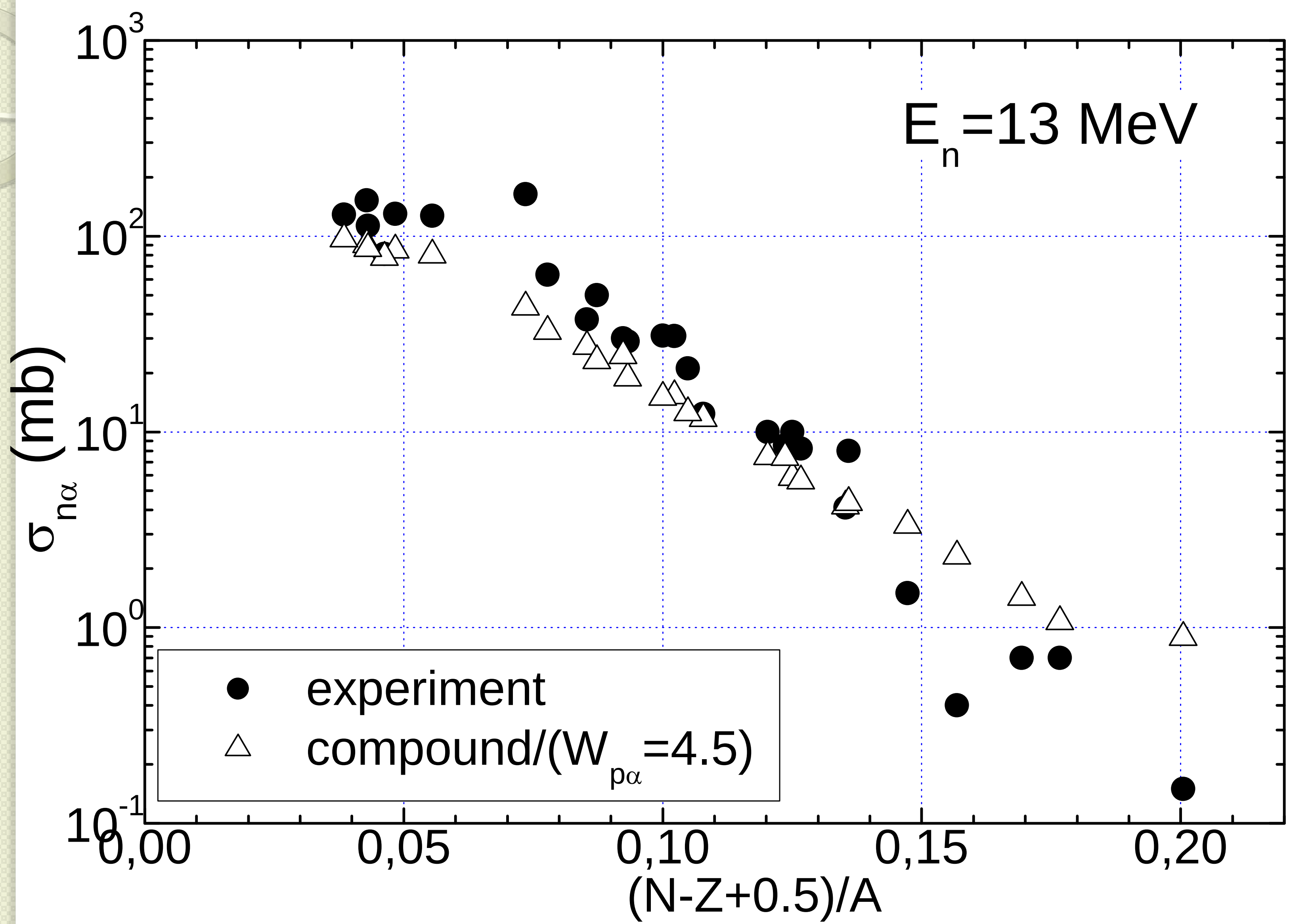


The (n,α) cross sections calculated by statistical model and experimental data



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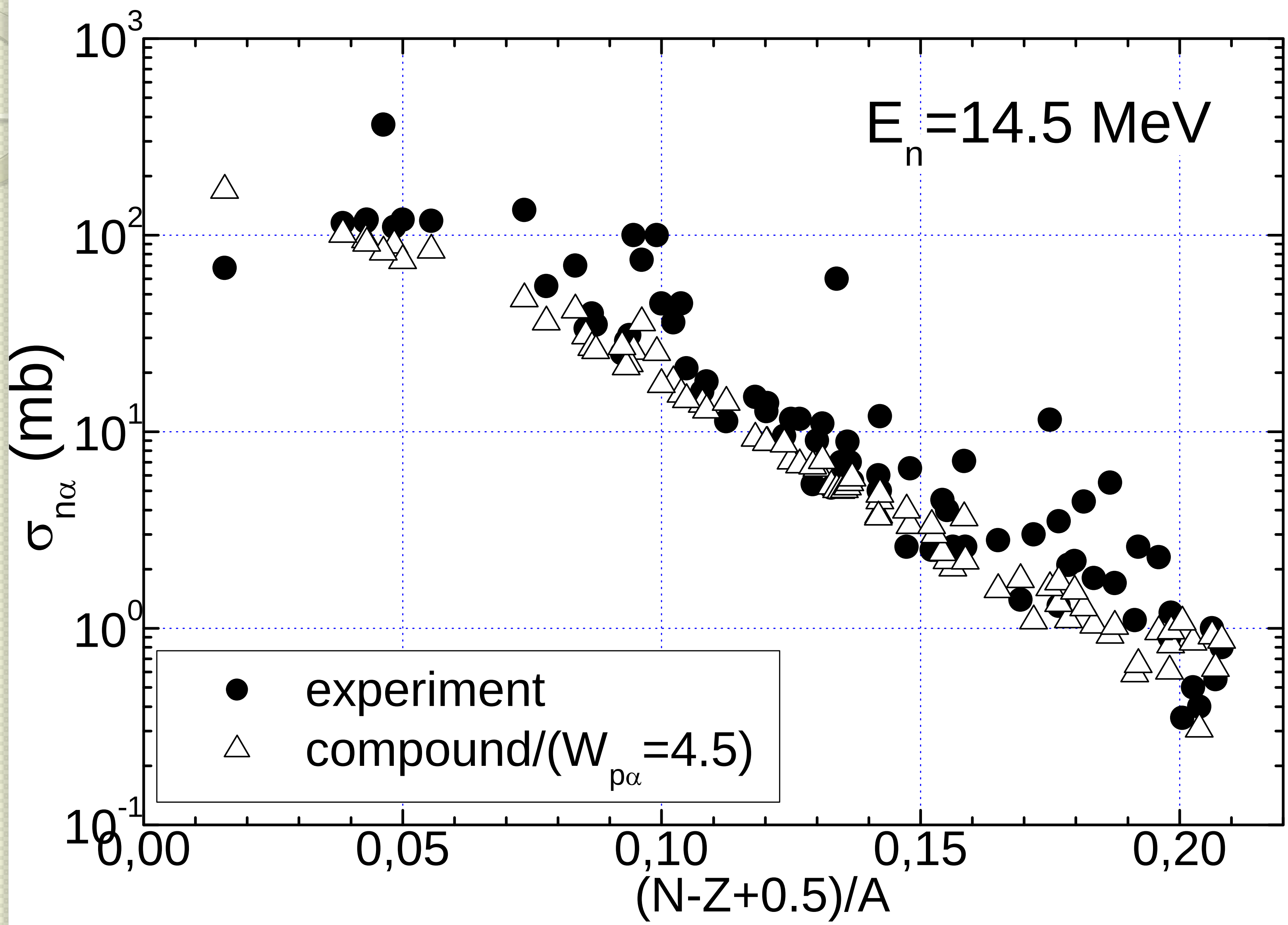
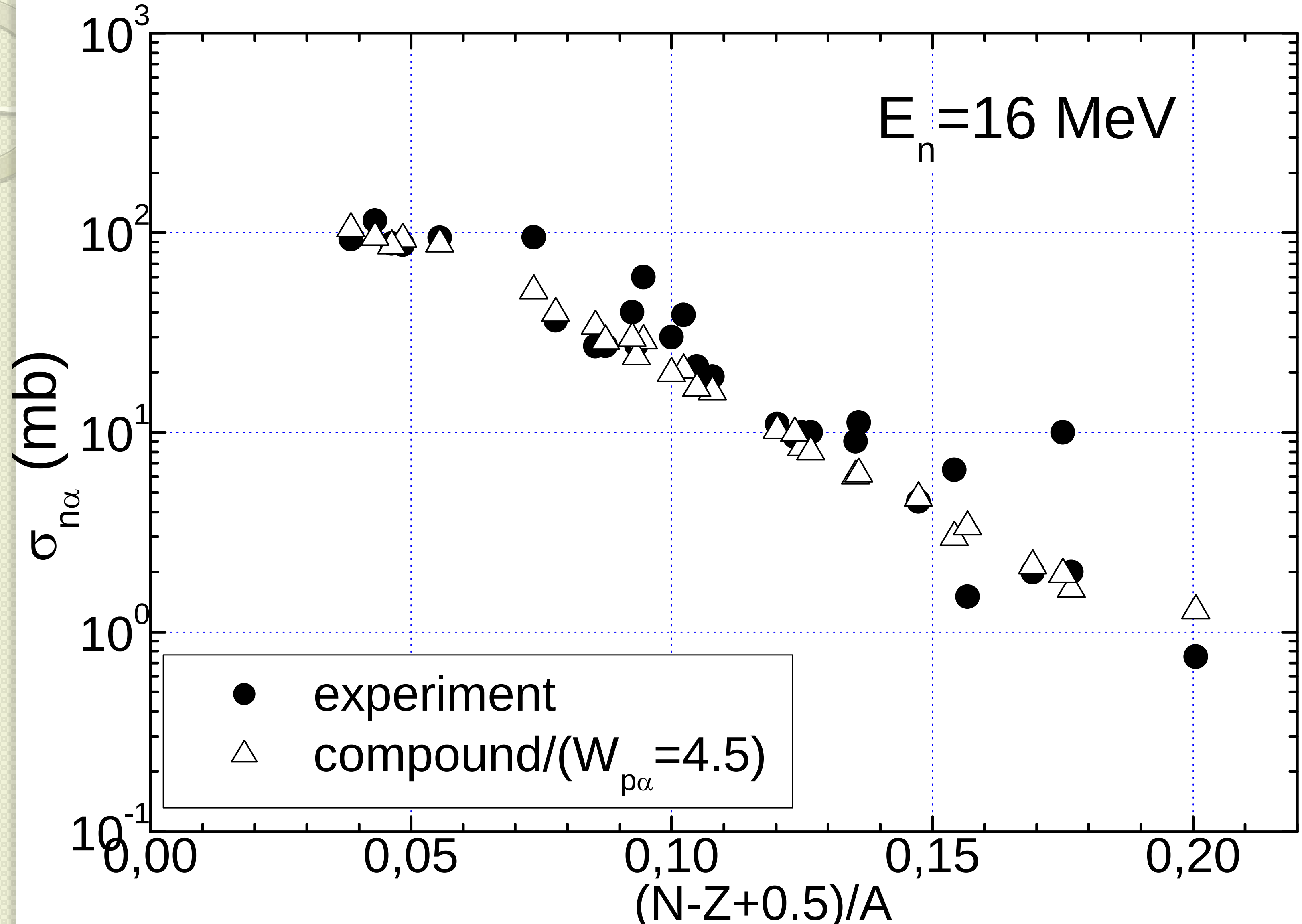


The (n,α) cross sections calculated by statistical model and experimental data



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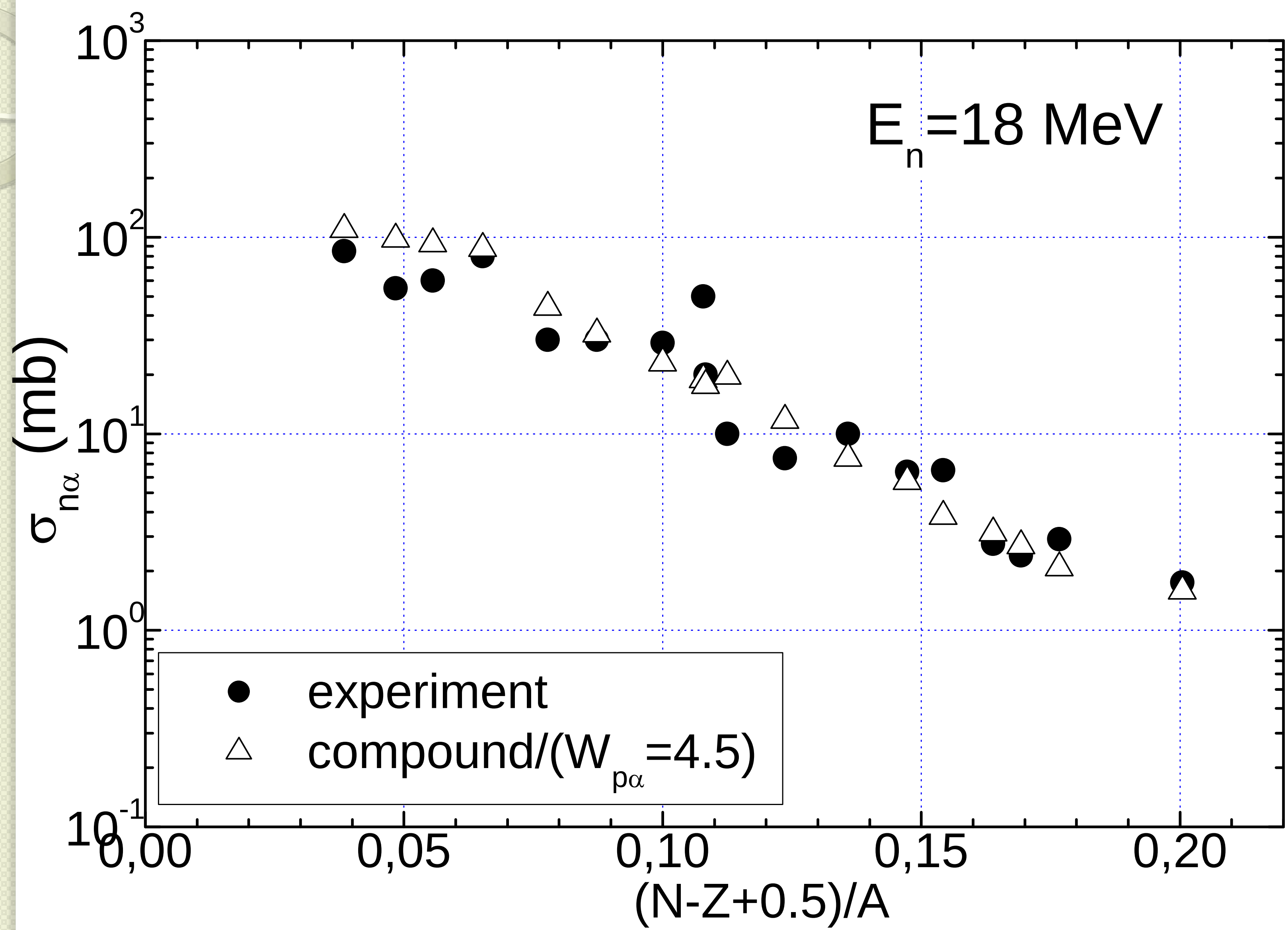
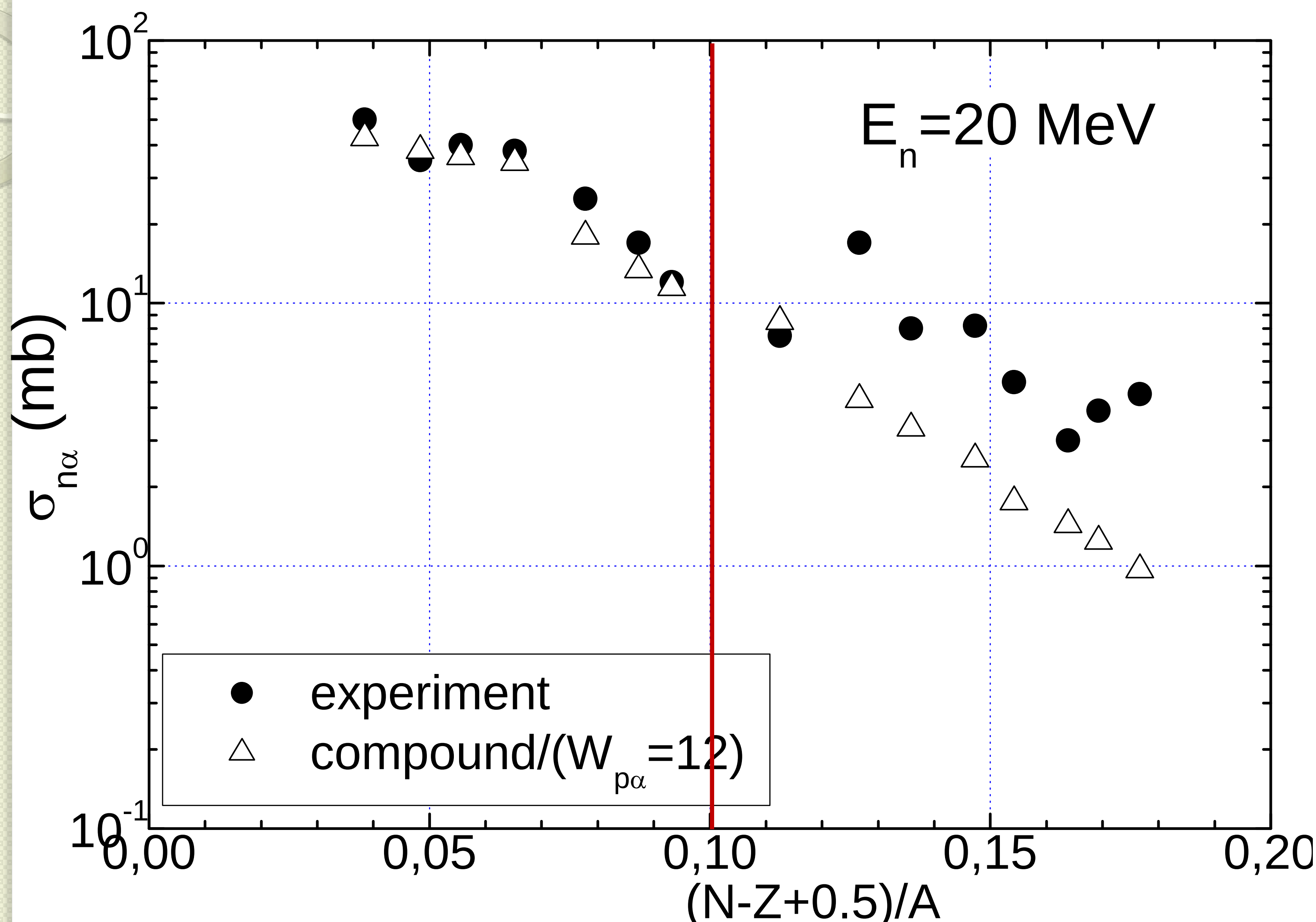
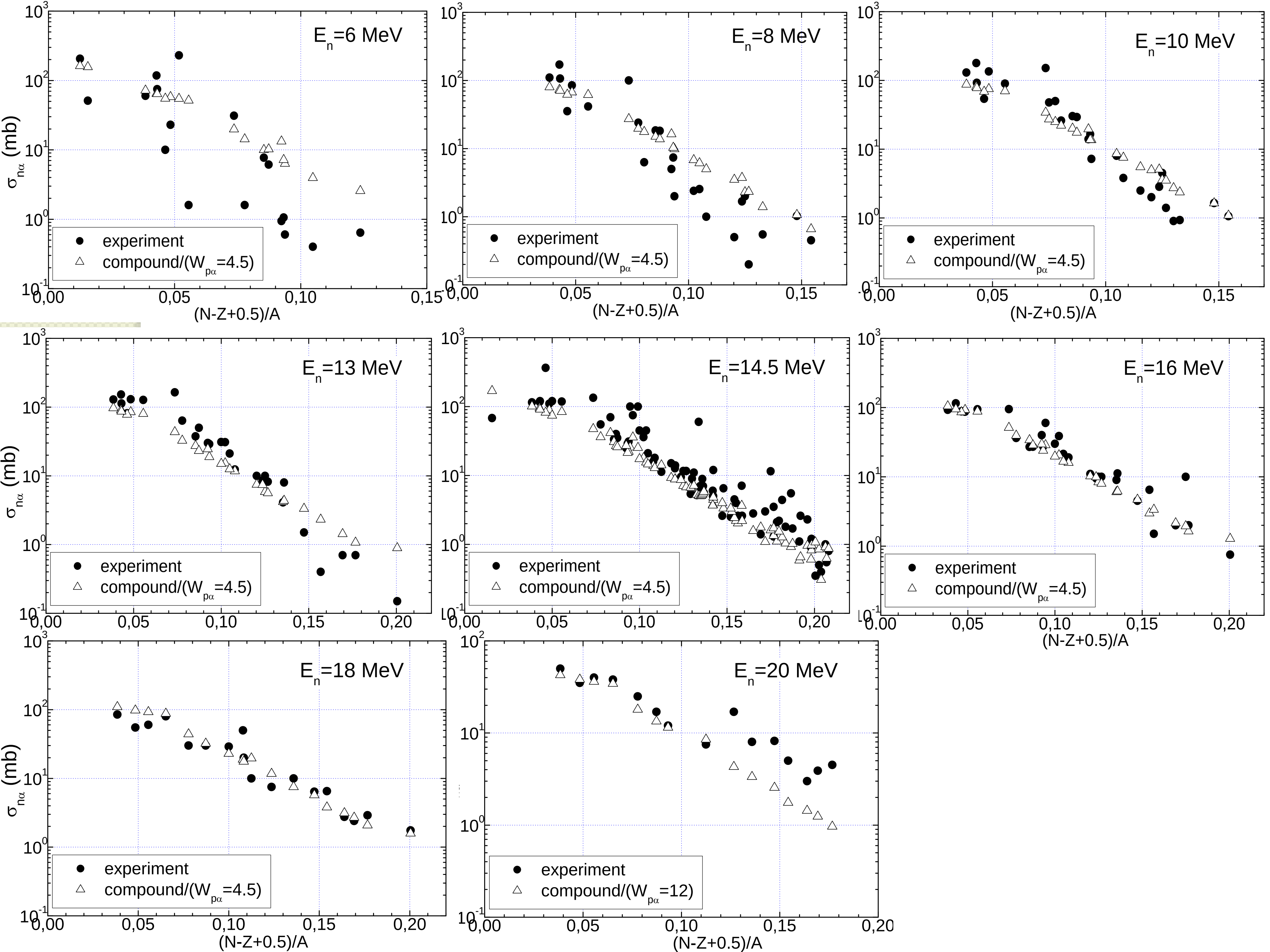


The (n,α) cross sections calculated by statistical model and experimental data





E_n=6 MeV
E_n=8 MeV
E_n=10 MeV
E_n=13 MeV
E_n=14.5 MeV
E_n=16 MeV
E_n=18 MeV
E_n=20 MeV
σ_nα (mb)
(N-Z+0.5)/A
experiment
compound/(W_pα=4.5)
compound/(W_pα=12)
10^-1
10^0
10^1
10^2
10^3
0,00
0,05
0,10
0,15
0,20

Conclusion

- The reduced (n,α) cross sections depend on the asymmetry parameter of neutron and proton numbers for the target nuclei $(N-Z+0.5)/A$ at neutron energy of 6 to 20 MeV.
- The parameter K in the statistical model formula for (n,α) cross section is linear in the neutron energy.
- The comparison of the theoretical and experimental (n,α) cross sections shows that statistical model formula gives overestimated values at all energy points of neutrons.

The discrepancy between the theoretical and experimental (n,α) cross sections was explained by the α-clusterization effect on the surface of nuclei. It was shown that the α-clusterization factor for (n,α) reaction depends on the