Collective states in nuclear level density within closed-form methods

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A level density of atomic nuclei is one of the most crucial ingredients for a reliable theoretical analysis and prediction of the nuclear reaction observables (cross sections, spectra, angular distributions, abundance of elements in the Universe, and other) within statistical models. In this contribution the level densities in spherical and deformed atomic nuclei are investigated using different semi-phenomenological approaches with allowance for quasiparticle and collective excitations.

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Effect of the collective states

The collective states can strongly effect on the level density, specifically, at low excitation energies. The simplest method to estimate effect of the vibrational states on level densities is calculation of collective enhancement factor

$$K = \rho / \rho_0$$

ρ , ρ_0 - level densities with and without allowing for collective states

Collective enhancement factor

$$K = K_{vibr} \cdot K_{rot}$$

Problems with estimation of K

1. Uncertainties in value of K_{vibr}

Closed-form approach

Ignatyuk A.V. Statistical properties of excited nuclei. 1983; Yad. Fiz. 21(1975) 20 ; Izv.AN SSSR 38 (1974) 2612 (RPA approach);
Ignatyuk A.V., Weil J.L., Raman S., Kahane S. PRC. 47 (1993) 1504

$$K_{vibr}(S_n) \sim 15 \div 30$$

$$(A \sim 100); \quad RIPL-2,3$$

Microscopic calculations within quasiparticle-quasiphonon model

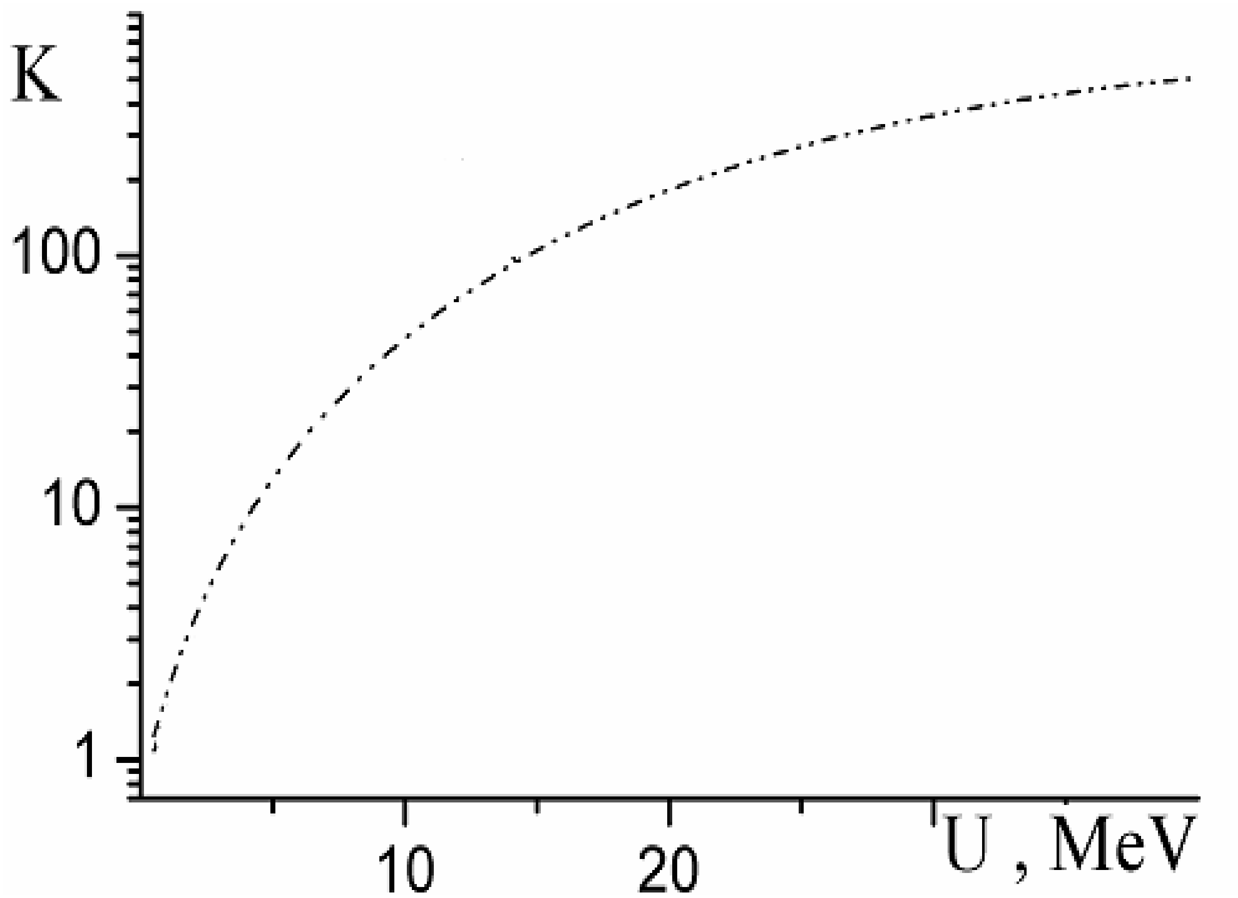
Soloviev V.G., Stoyanov Ch., Vdovin A.I. NPA224 (1974) 411; Voronov V.V., Malov L.A., Soloviev V.G. Yad.Fiz. 21 (1975) 40; Malov L.A., Soloviev V.G., Voronov V.V. Phys.Lett. B55 (1975) 17; Vdovin A.I., Voronov V.V., Malov L.A. Soloviev V.G., Stoyanov Ch. Fiz.El.Chas.At.Yad. 7 (1976) 952

$$K_{vibr}(S_n) \sim 2 \div 5 \qquad (A \sim 100)$$

$$K_{rot}(S_n) \approx J_{\perp} T \sim 100 \qquad (A \sim 100)$$

2. Energy dependence

Unrealistic dependence of vibrational enhancement factor
on excitation energy without collective state damping



Enhancement calculations

Response function method

[Plujko V.A., Gorbachenko O.M. // INDC(NDS)-462,Distr:G+NM,IAEA, (2004)65]

$K = \rho / \rho_0$ with ρ within saddle-point approximation

$$\rho(U, A) = \left(4\pi^2 D\right)^{-1/2} \exp S(\alpha_0, \beta_0),$$

$$S(\alpha_0, \beta_0) = -\alpha_0 A + \beta_0 E + \ln Z(\alpha_0, \beta_0)$$

- nuclear entropy;

$$Z(\alpha, \beta) = \mathrm{Tr}\left[\exp(-\beta \tilde{H})\right]$$

- partition function;

$\Omega(\alpha,\beta) = -\ln Z(\alpha,\beta)/\beta$ - thermodynamic potential;

$$\tilde{H} \equiv \hat{H} - \mu \hat{A}; \quad \mu = \alpha / \beta;$$

Equations of thermodynamic state:

$$\begin{cases} A = \partial \ln Z / \partial \alpha \,|_{\alpha_0, \beta_0}, \\ E = -\partial \ln Z / \partial \beta \,|_{\alpha_0, \beta_0} \end{cases}$$

$1/\beta_0 = T$ - the temperature; $\mu = \alpha_0 / \beta_0$ - chemical potential

Coherent separable interactions

$$V_{k_L,res}(i,j) = k_L \sum_{L,\mu} q^*_{L\mu}(\vec{r}_i) q_{L\mu}(\vec{r}_j),$$

$$q_{L\mu}(\vec{r}) = r^L Y_{L\mu}(\hat{r})$$

The total Hamiltonian: $\hat{H}_k = \hat{H}_0 + \hat{V}_{k,res},$

Vibrational state addition to partition function

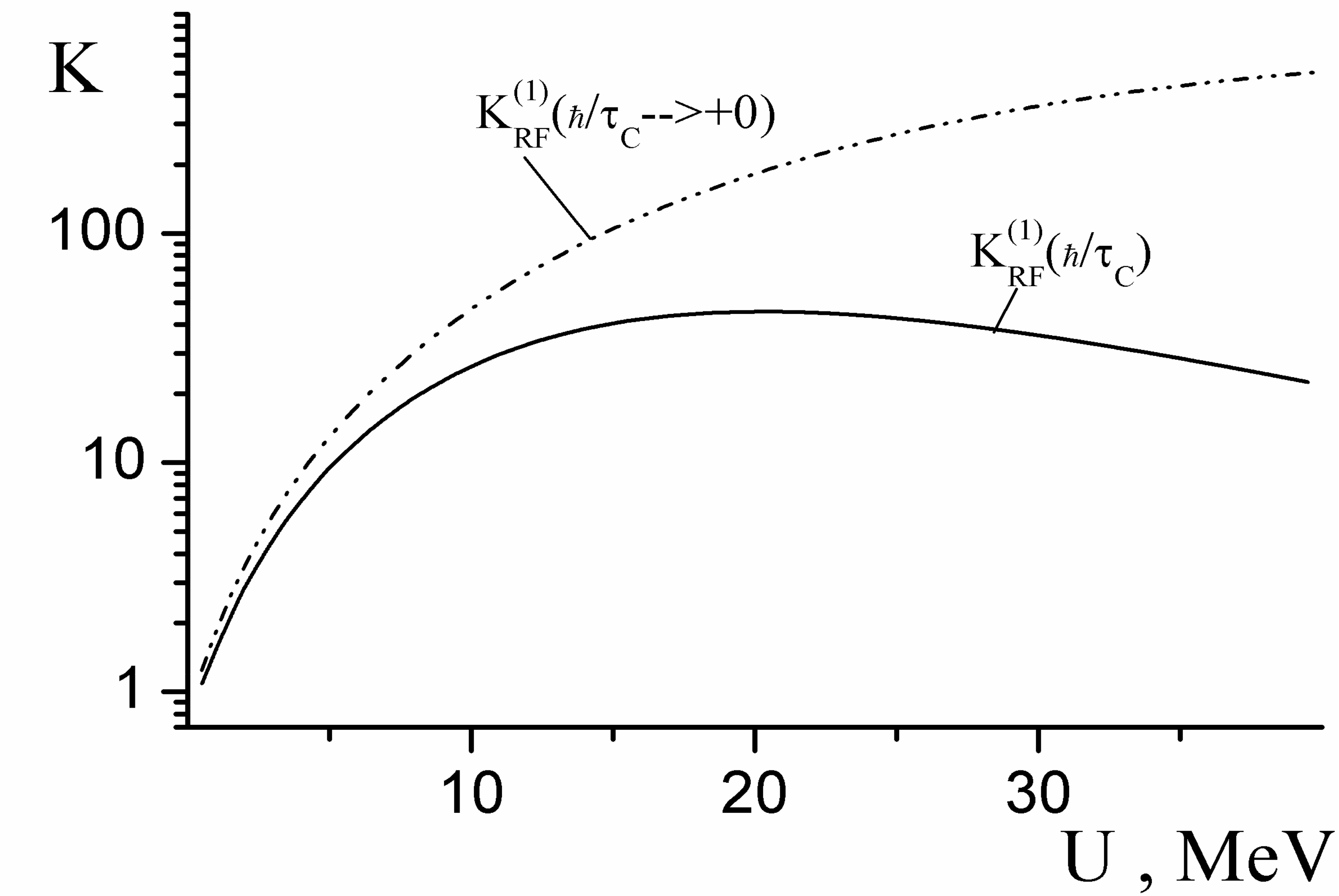
$$\Delta Z = Z / Z_0 = \exp(-\beta \Delta \Omega), \quad Z_0 => \hat{H}_0$$

Green's function method result for thermodynamic potential

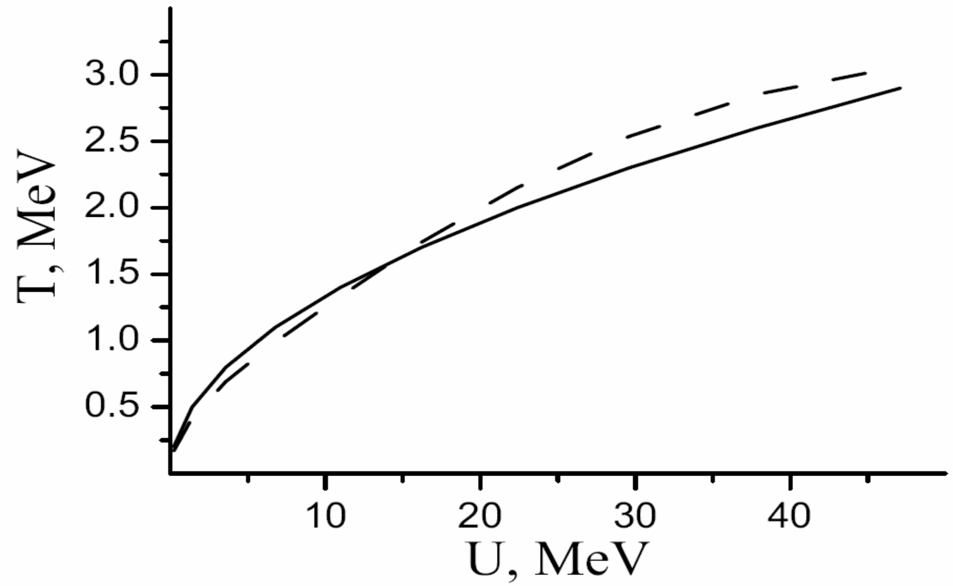
$$\Delta\Omega = \sum_L \Delta\Omega_L \;;$$

$$\Delta\Omega_L = -\frac{2L+1}{2\pi} \int_0^{k_L} dk' \int_{-\infty}^{+\infty} \frac{\hbar}{1-e^{-\beta\hbar\omega}} \mathrm{Im}(\chi_L^{k'}(\omega) - \chi_L^0(\omega)) d\omega \;;$$

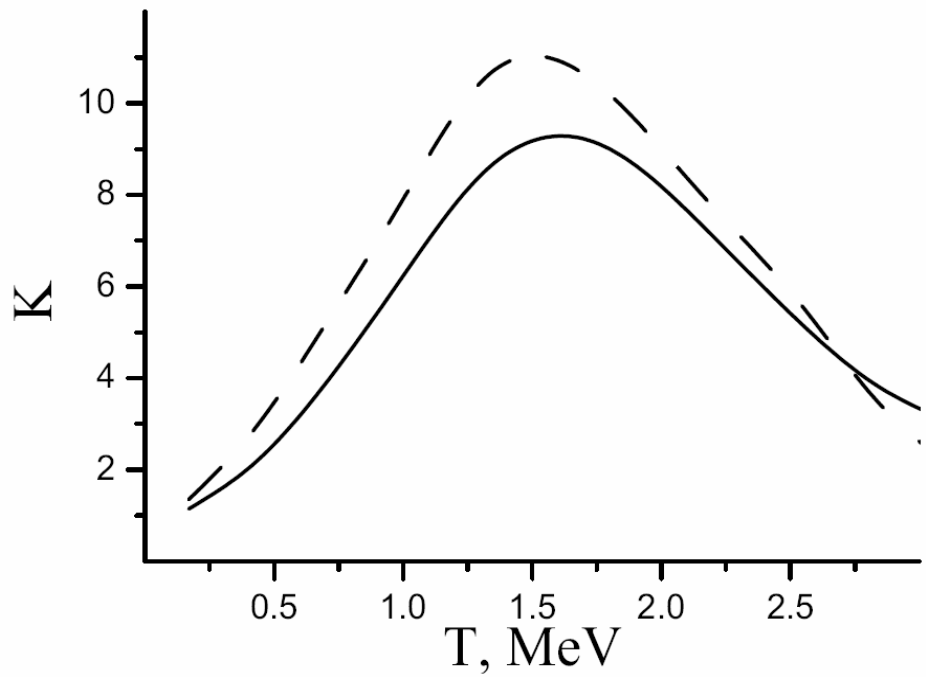
$\chi_L^{\kappa'}(\omega) = \mathrm{Tr}_{\{1\}}\left(q_{L0}(\vec{r}) \cdot \delta\rho(\vec{r};\omega)\right)_{k'}$ - RF within Vlasov-Landau kinetic equation



Effect of relaxation on K in ^{56}Fe:
$K^{(1)}_{RF}$ - RF method in one-resonance approach



The temperature in ^{56}Fe:
(– – –) with allowance for (2^+) vibrational state and without(–––)



Enhancement factor in ^{56}Fe:
——–- saddle-point method,
––– - adiabatic approach

$$K = \Delta Z(T_{FG})$$

$$T_{FG} = \sqrt{U/a}$$

a – level density parameter

RF method calculations for ^{56}Fe

RF within Vlasov-Landau kinetic equation

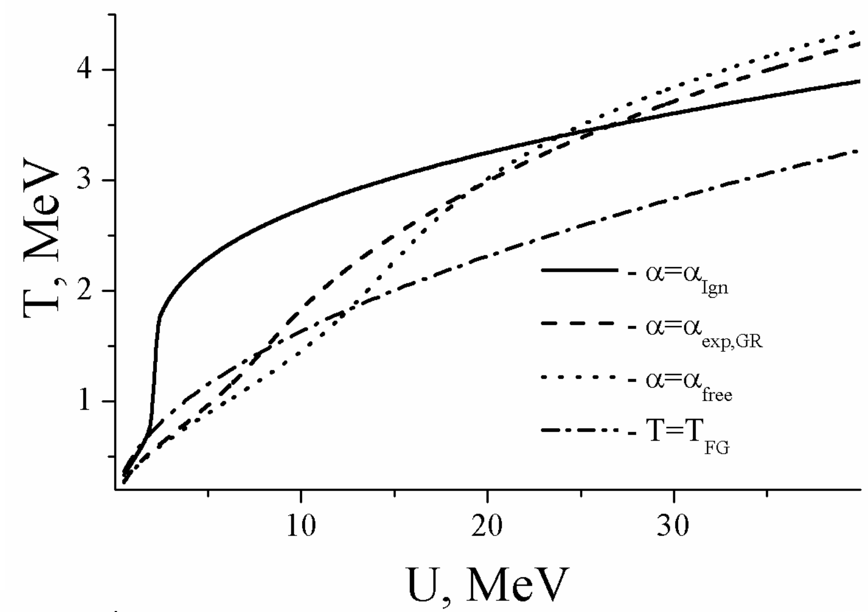
$$\frac{\hbar}{\tau_c\left(\hbar\omega, T_f\right)} = \frac{(\hbar\omega)^2 + 4\pi^2 T^2}{4\pi\alpha}$$

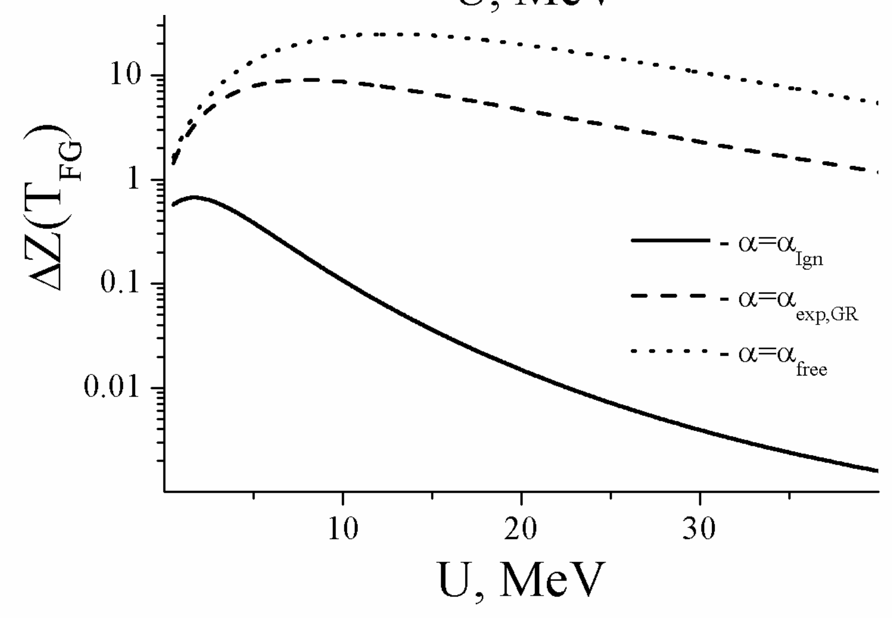
$$\alpha_{Ign} = 2/(4\pi^2 0.0075 \mathrm{A}^{1/3})$$

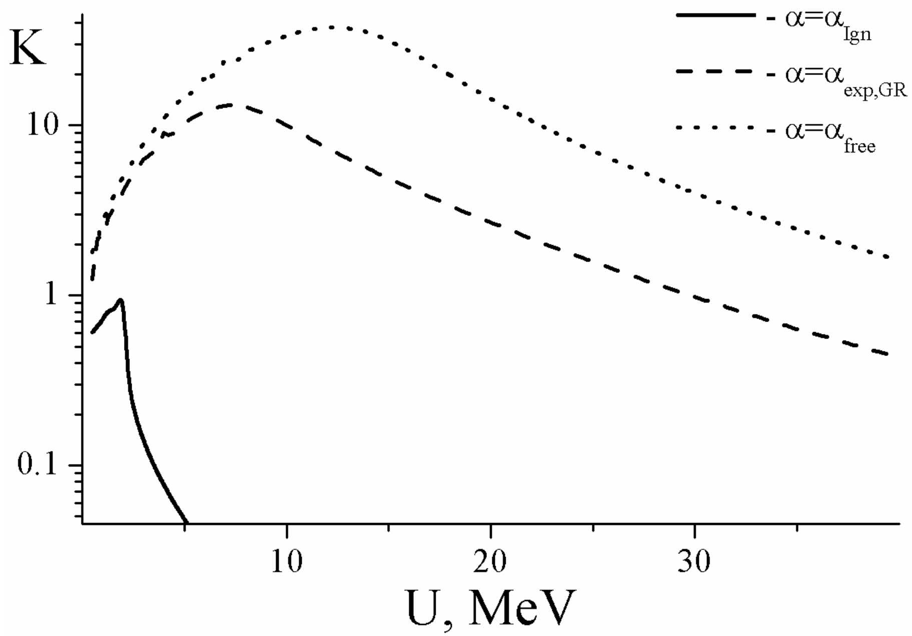
$$\alpha_{\exp,GR} = 2/(4\pi^2 b_{GR}), \quad b_{GR,2^+} = \Gamma_{GR,2^+}/E^2_{GR,2^+}$$

$$b_{GR,3^-} = \Gamma_{GR,3^-}/E^2_{GR,3^-}$$

$$\alpha_{free} = \frac{9\hbar^2/16m}{\sigma^{free}(np)},$$





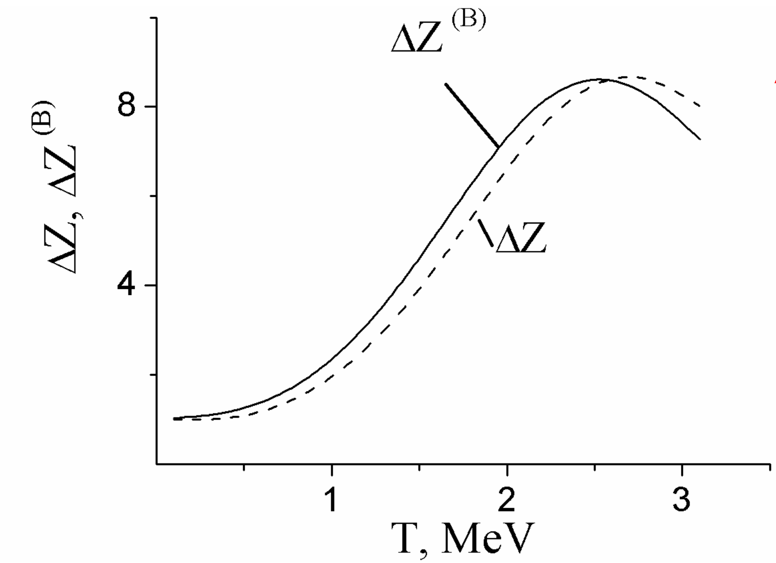


One resonance RF

$$\chi_{k,L}(\omega) = \frac{\chi_{0,L}(\omega)}{1 - k\,\chi_{0,L}(\omega)}$$

$$\chi_{0,L}(\omega) = B\left(\frac{1}{\omega - \tilde{\omega}_0} - \frac{1}{\omega + \tilde{\omega}_0^*}\right), \quad \tilde{\omega}_0 = \omega_0 - i\eta_0$$

$$\Delta\Omega_L = -\frac{2L+1}{\beta}\ln\left(\Delta Z^{(B)}\Delta Z^{(M)}\Delta Z^{(0)}\right)$$



$$\Delta Z^{(B)} = \frac{\exp(\beta\hbar\omega_0/2)\,|1-\exp(-\beta\hbar(\omega_0 + i\eta_0))|}{\exp(\beta\hbar\omega_k/2)\,|1-\exp(-\beta\hbar(\omega_k + i\eta_k))|}$$

$$\Delta Z^{(M)} = \prod_{n=1}^{\infty}\left|\frac{\exp(k\chi_{0,L}(-\omega_n))(1-k\chi_{0,L}(-\omega_n))}{\exp(k\chi_{0,L}(\omega_n))(1-k\chi_{0,L}(\omega_n))}\right|^{1/2}$$

$$\Delta Z^{(0)} = \exp\left(\beta\hbar\frac{\omega_k^2 - \omega_0^2}{4\omega_0}\operatorname{Re}\left(cth\left(\beta\hbar(\omega_0 + i\eta_0)/2\right)\right)\right)$$

$K = \Delta Z(T_{FG})$, in first change of $\Delta T = T - T_{FG}$

Phenomenological expressions for vibrational enhancement factor

Boson partition function with damped occupation numbers(DN)

[Ignatyuk A.V., Weil J.L., Raman S., Kahane S. PRC 47 (1993) 1504]

$$K = \exp\left(\bar{S} - \bar{U}/T\right) \equiv K_{DN},$$

$$\bar{S} = \sum_L (2L+1)\left[(1+\bar{n}_L)\ln(1+\bar{n}_L) - \bar{n}_L \ln \bar{n}_L\right],$$

$$\bar{U} = \sum_L (2L+1)\hbar\omega_L \bar{n}_L,$$

$$\bar{n}_L = \frac{\exp\left[-\Gamma_L/(2\hbar\omega_L)\right]}{\exp(\hbar\omega_L/T) - 1},$$

$$\Gamma = C \cdot \left[(\hbar\omega)^2 + 4\pi^2 T^2\right],$$

$$C = 0.0075 A^{1/3}\ MeV^{-1}$$

Boson partition function with complex energies (CE)

[Blokhin A.I, Ignatyuk A.V., Shubin Yu.N. Yad. Fiz. 48 (1988) 371]

$$K = \prod_L \left| \frac{\delta Z_{B,L}(\hbar\omega_L + i\gamma_L; T)}{\delta Z_{B,L}(\hbar\tilde{\omega}_L + i\tilde{\gamma}_L; T)} \right| =$$

$$= \prod_L \left| \frac{1-\exp\left[-(\hbar\omega_L + i\gamma_L)/T\right]}{1-\exp\left[-(\hbar\tilde{\omega}_L + i\tilde{\gamma}_L)/T\right]} \right|^{-(2L+1)} \equiv K_{CE}$$

$$[\hbar\omega_L]^2 = [\hbar\tilde{\omega}_L]^2 - \xi(T)\{[\hbar\tilde{\omega}_L]^2 - [\hbar\omega_{L,\exp}]^2\},$$

$$\xi(T) = \exp\{-C_1 T^2 / [\hbar\omega_{L,\exp}]\}$$

K using boson partition function with average occupation numbers (BAN)

$$K = K_{BAN} \equiv K(\Delta Z = \Delta Z_{BAN})$$

$$\Delta Z = \prod_L (1 + \overline{n}_L(\omega_L))^{2L+1} \equiv \Delta Z_{BAN},$$

$$\overline{n}_L = \frac{1}{T_p} \int_0^{T_p} n_L \exp(-\Gamma_L t) dt = \frac{\left(1 - \exp(-2\pi\Gamma_L/\hbar\omega_L)\right)}{\left(\exp(\hbar\omega_L/T) - 1\right)} \frac{\hbar\omega_L}{2\pi\Gamma_L}$$

Liquid drop partition function with reduction (EM)

[Empire-II code by Herman M., Capote-Noy R., Oblozinsky P. et al. Journ. Nucl. Sc. Techn. Suppl.2 (2002) 116]

$$K = K_{LDM}(1 - Q_{damp}) + Q_{damp} \equiv K_{EM},$$

$$Q_{damp} = 1/[1 + \exp\{(T_{1/2} - T)/DT\}],$$

$$DT = 0.1 \; MeV, \quad T_{1/2} = 1 \; MeV$$

$$K_{LDM} = \exp\left[C_3 A^{2/3} \cdot T^{4/3}\right]$$

Rotational contribution

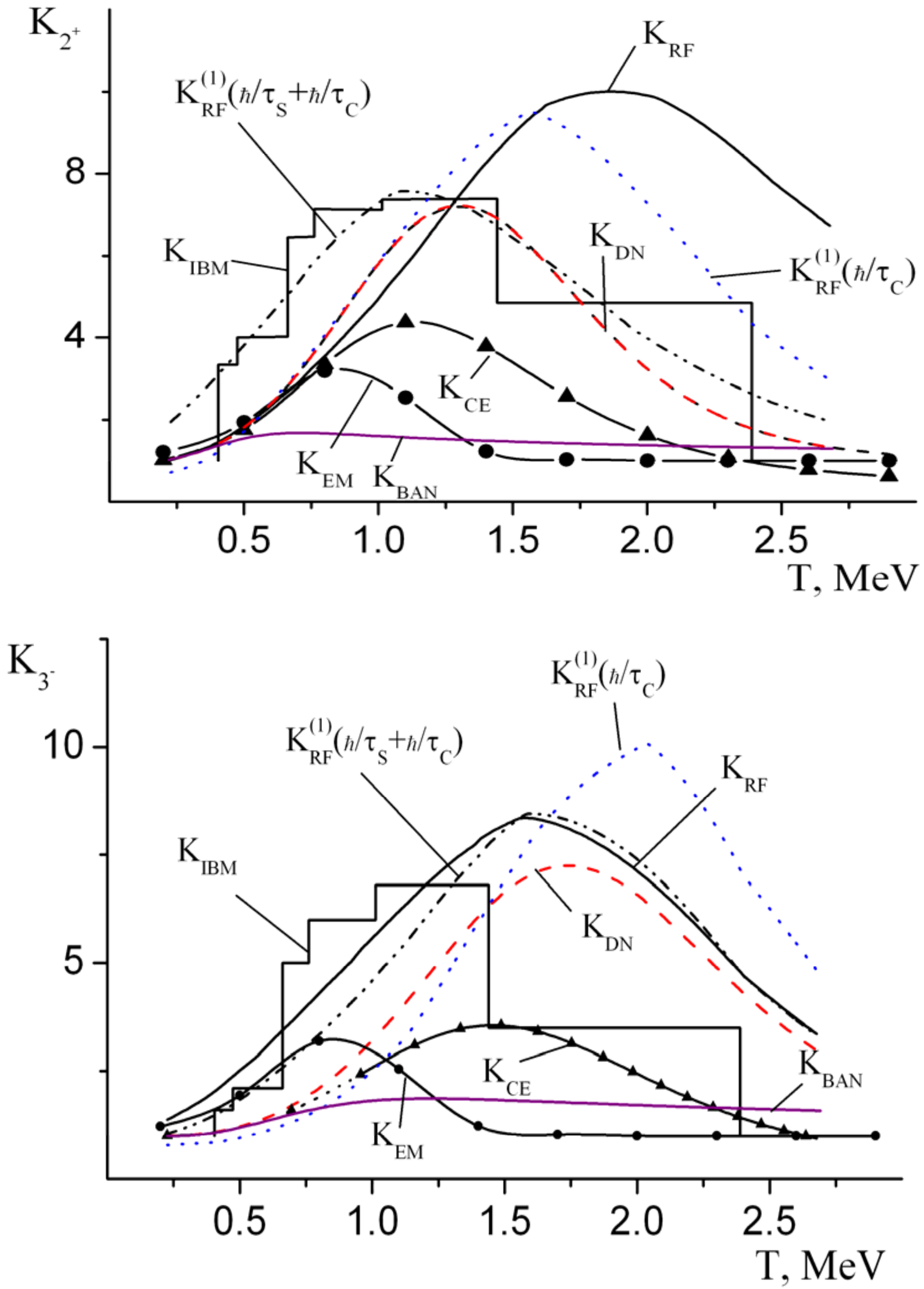
[Ignatyuk A.V.,et al, PRC 47 (1993) 1504; RIPL2]

$$K_{rot}(U) = \frac{(J_{\perp}T(1+\beta/3)-1)}{(1+\exp(U-U_{cr})/d_{cr})} + 1,$$

$$U_{cr} = 120 A^{1/3}\beta^2 \ MeV,$$

$$d_{cr} = 1400 A^{-2/3}\beta^2 \ MeV$$

$J_{\perp}$ - moment of inertia, β - quadrupole deformation



Comparison of enhancement factor in ^{146}Sm : K_{IBM} - finite temperature IBM [Mengoni A., et al. Journ. Nucl. Sc. Techn. Suppl.2 (2002) 766]; $K^{(1)}_{RF}$ - RF method in one-pole approach

NUCLEAR LEVEL DENSITY

$$\rho(U) = \rho_0(U', T) \cdot K(T)$$

Generalized Superfluid Model (GSM) for ρ_0
is used by default in codes TALYS and EMPIRE

Different expressions for GSM model is used in:

1) [Ignatyuk A.V.,et al, PRC 47 (1993) 1504; RIPL2]

$$U' = U + n\Delta_0 + \delta_{shift}, \quad \Delta_0 = 12/\sqrt{A} \quad (MeV)$$

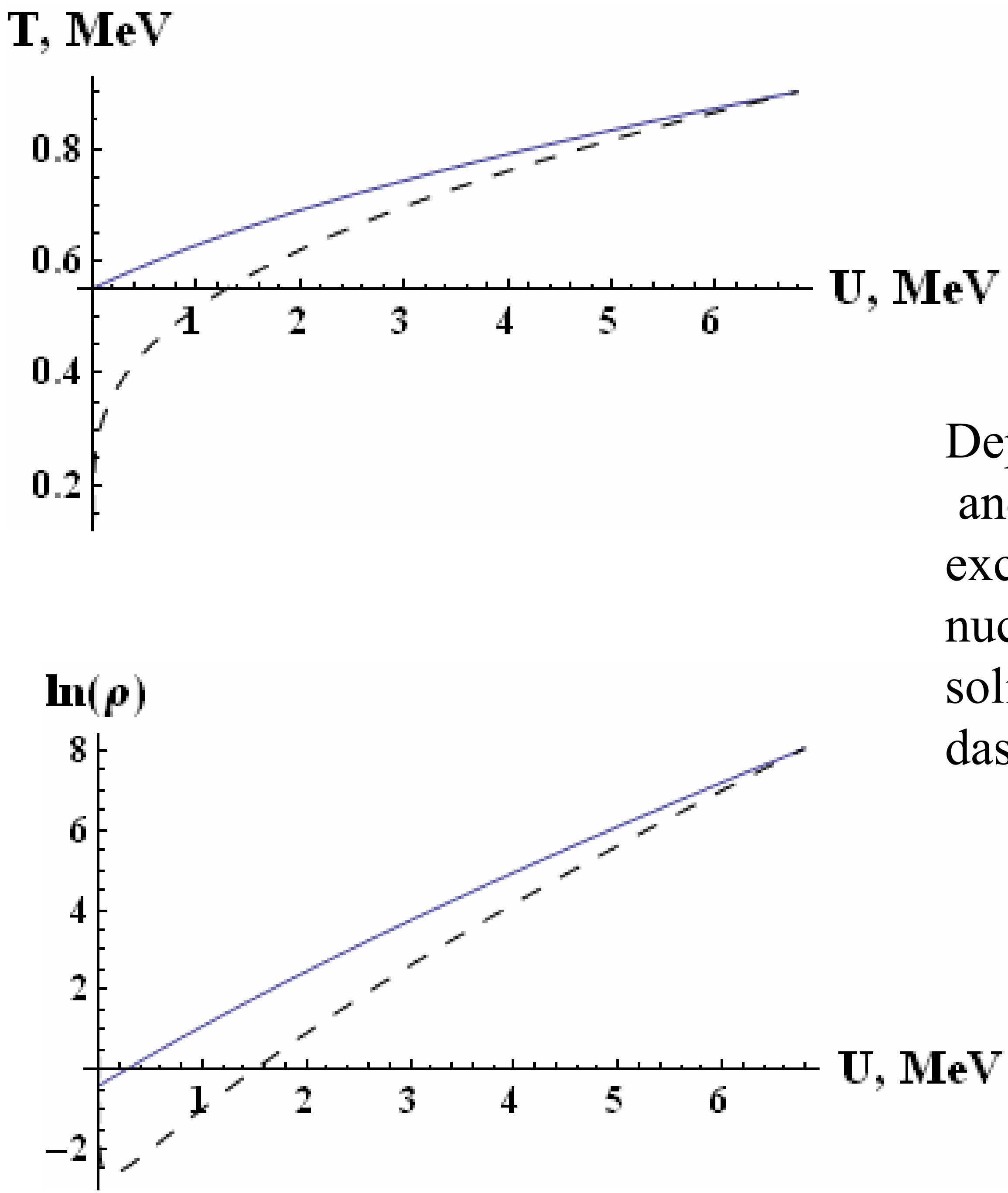
$$E_{cond} = \frac{3}{2\pi^2} a_{cr} \Delta_0^2$$ -condensation energy

δ_{shift} -additional shift to excitation energy

2) [Svirin M.I., Fiz.El.Chas.At.Yad (Particles and Nucleus), 37(2006)901.]

$$U' = U + \delta_{shift}, \quad E_{cond} = \frac{3}{2\pi^2} a_{cr} \Delta_0^2 - n\Delta_0$$

$$n = (2 - even - even, 1 - odd, 0 - odd - odd)$$



Dependence of temperature (T) and level density (ρ) on excitation energy (U) for odd nucleus ^{57}Fe ($\delta_{shift}=0$) :
solid line – GSM of Ignatyuk,
dash line – GSM of Svirin

Comparison with experimental data

Observable values:

•Neutron resonance spacing

$$D = 1/\overline{\rho}$$

$$\overline{\rho} = \begin{cases} \rho(S_n, I = 1/2), & if \quad I_0 = 0, \\ \dfrac{\rho(S_n, I = I_0 - 1/2) + \rho(S_n, I = I_0 + 1/2)}{2}, & if \quad I_0 \neq 0 \end{cases}$$

•Cumulative number of nuclear levels

$$N_c = \int_0^{U_0} \rho(U)dU$$

Fitting of level density within the EMPIRE code

Enhanced GSM model recommended with the parameters $\tilde{a},\ \delta_{shift}$

For prolate nuclei

$$\text{if}\ \ U \le U_{cr} \Rightarrow \text{GSM (BCS model with } K_{coll} = K_{vibr} K_{rot})$$

$$\text{if}\ \ U \ge U_{cr} \Rightarrow$$

$$\rho(E,J,\pi) = \frac{1}{16\sqrt{6\pi}} \left(\frac{\hbar}{J_{||}}\right)^{\frac{1}{2}} a^{\frac{1}{4}} \sum_{K=-J}^{J} \left(U - \frac{\hbar^2 K^2}{2J_{eff}}\right)^{-\frac{5}{4}} \exp\left\{2\left[a\left(U - \frac{\hbar^2 K^2}{2J_{eff}}\right)\right]^{\frac{1}{2}}\right\}$$

$$U = E - E_{cond} + n\Delta + \delta_{shift},$$

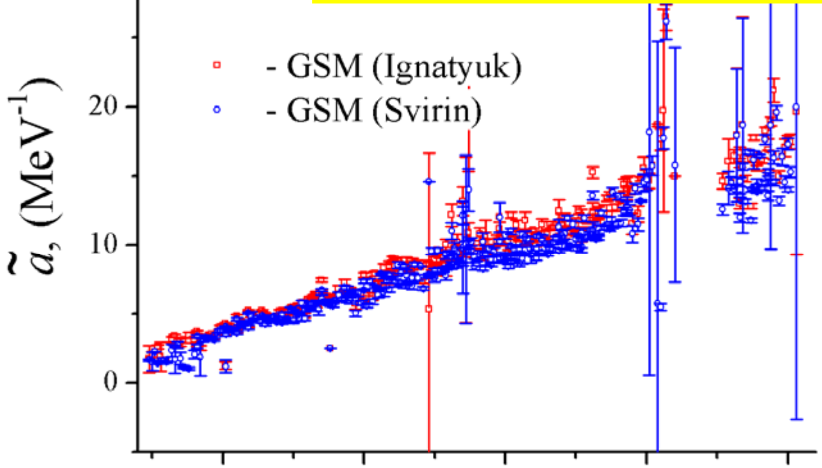
$$a(U) = \tilde{a}\left(1 + f(U)\frac{\delta W}{U}\right),\ \ f(U) = 1 - \exp(-\gamma U), \qquad \frac{1}{J_{eff}} = \frac{1}{J_{||f}} - \frac{1}{J_{\perp}}$$

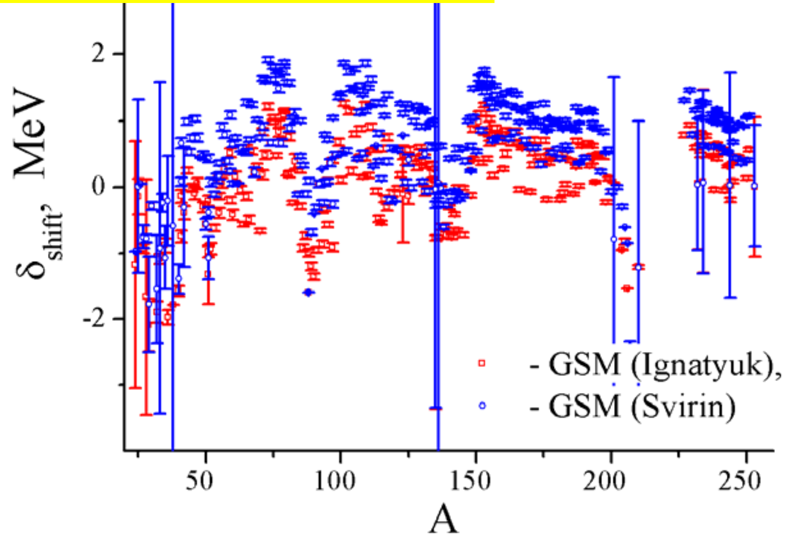
Fitting of the parameters $\tilde{a},\ \delta_{shift}$ **by the minimum of**

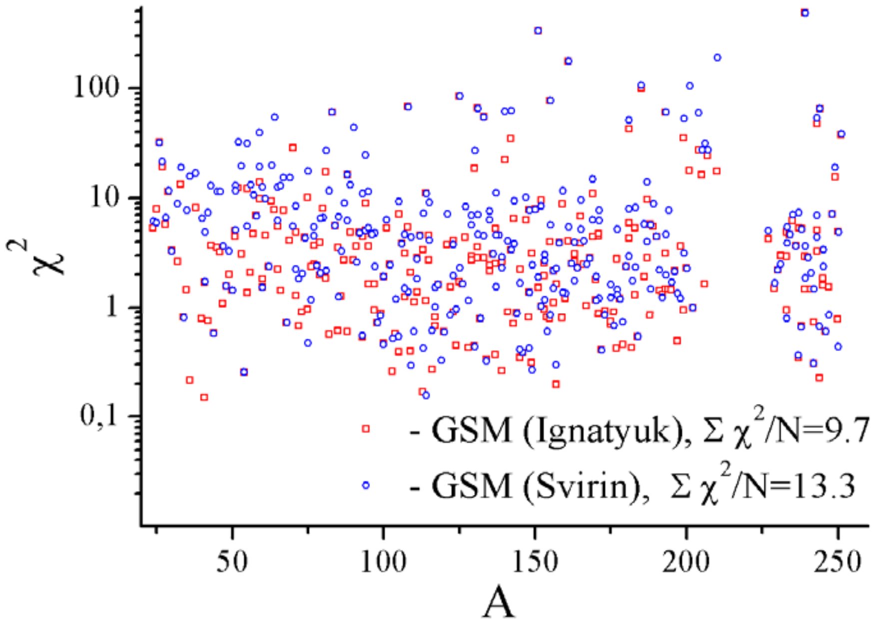
$$\chi^2 = \sum_{i=N_{cum,\min}}^{N_{cum,\max}} (i - N_{cum,theor})^2 / i + \left(\frac{\rho_{\exp} - \rho_{theor}}{\Delta\rho_{\exp}}\right)^2$$

$$N_{cum,theor} = \int_0^U \rho(E)dE \qquad \rho_{\exp} = 1/D_0, \ \ \rho_{theor} = \sum_{J=|I-1/2|}^{I+1/2} \rho(S_n, J, \pi)$$

Comparision of GSM models of Svirin and Ignatyuk
($K = K_{EM} K_{rot}$)





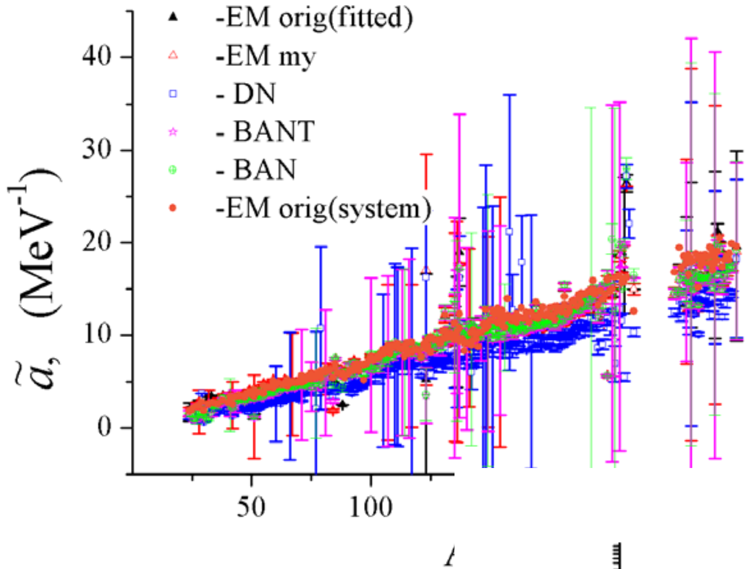


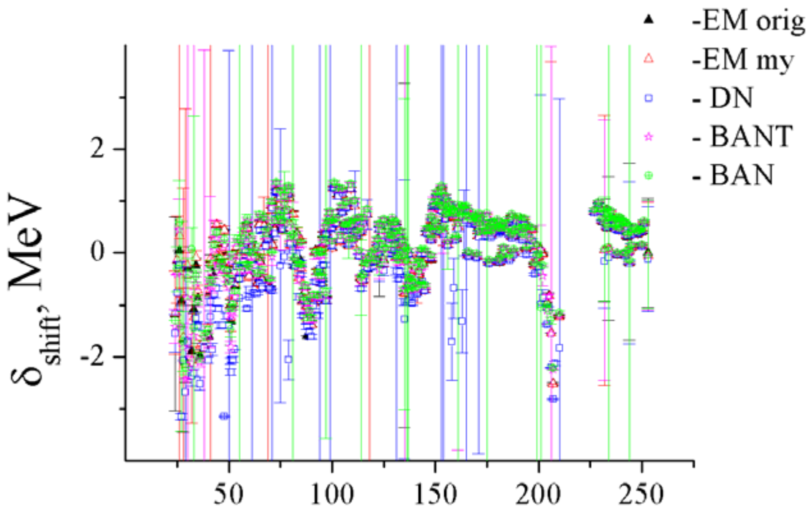
GSM (Ignatyuk) have the best fitt of experimental data, because of

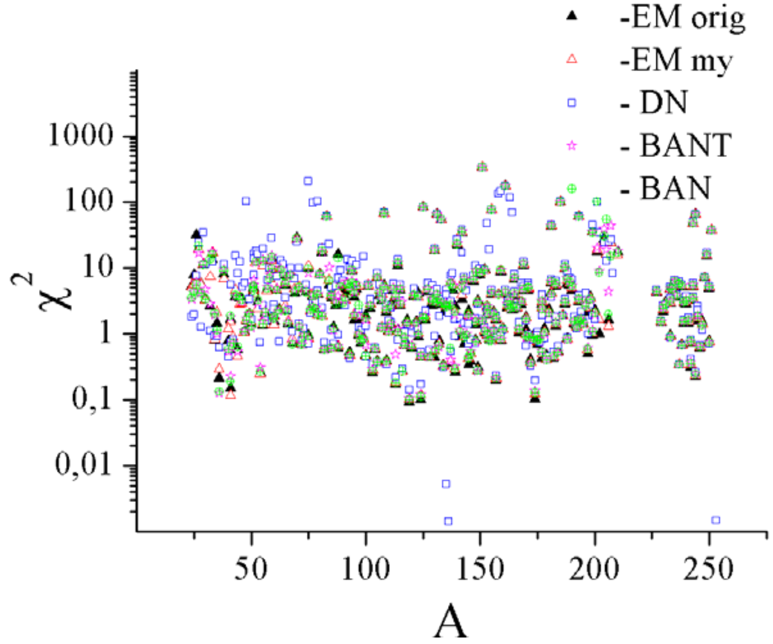
χ^2(Ignatyuk) < χ^2(Svirin)

δ_{shift}(Ignatyuk) < δ_{shift}(Svirin)

Fitting of the parameters $\tilde{a},\ \delta_{shift}$:





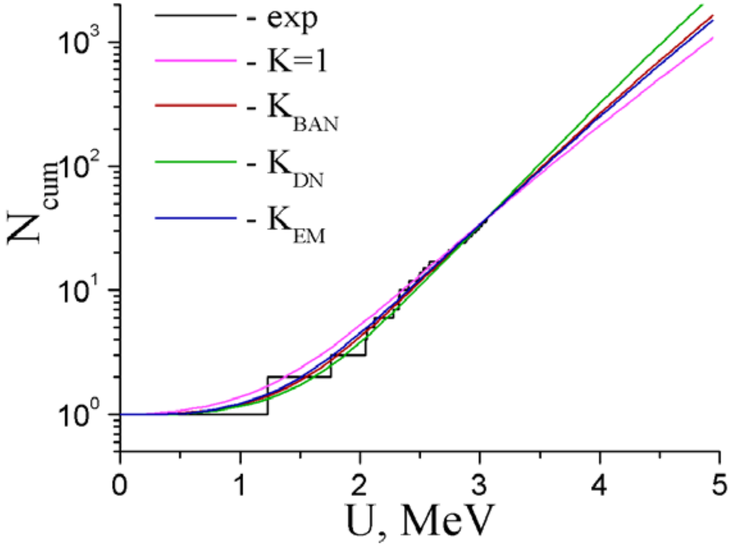


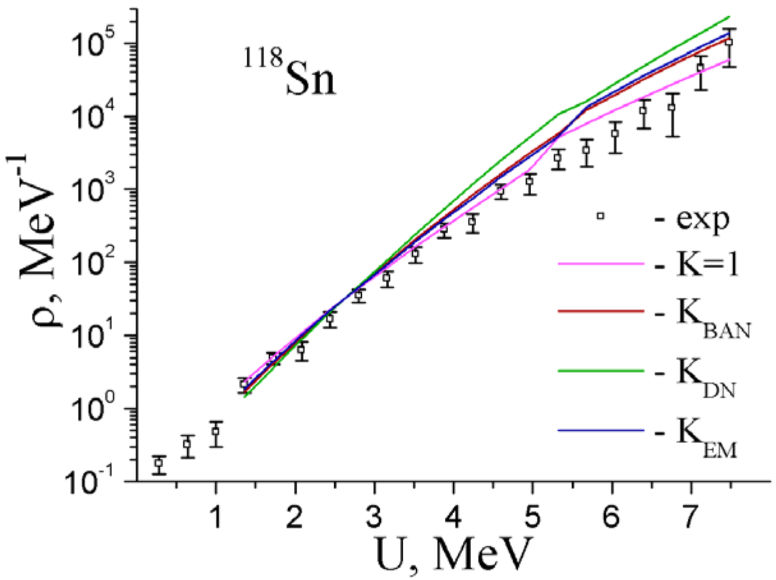
EMPIRE fit

$\tilde{a} = \tilde{a}_{sys} \cdot norm_{\tilde{a}}$

$\delta_{shift,\max}$ fitted by $N_{cum,\max}$

$(\delta_{shift} = f(U))$

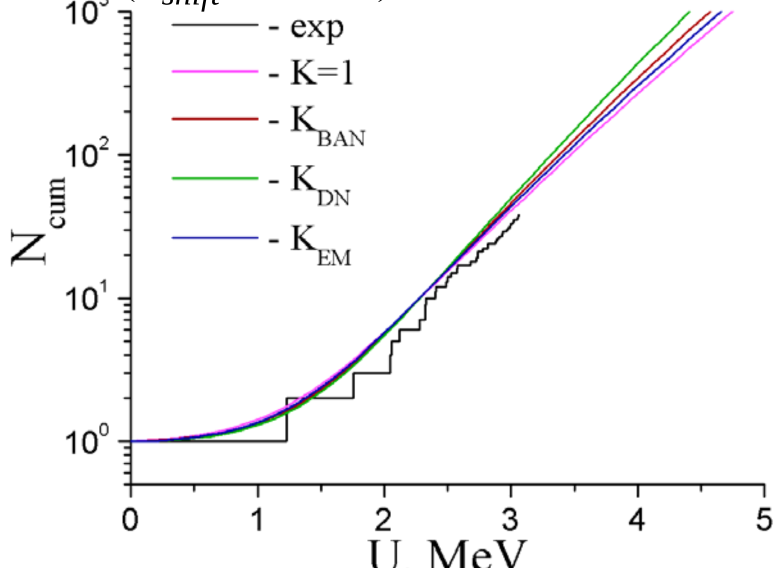


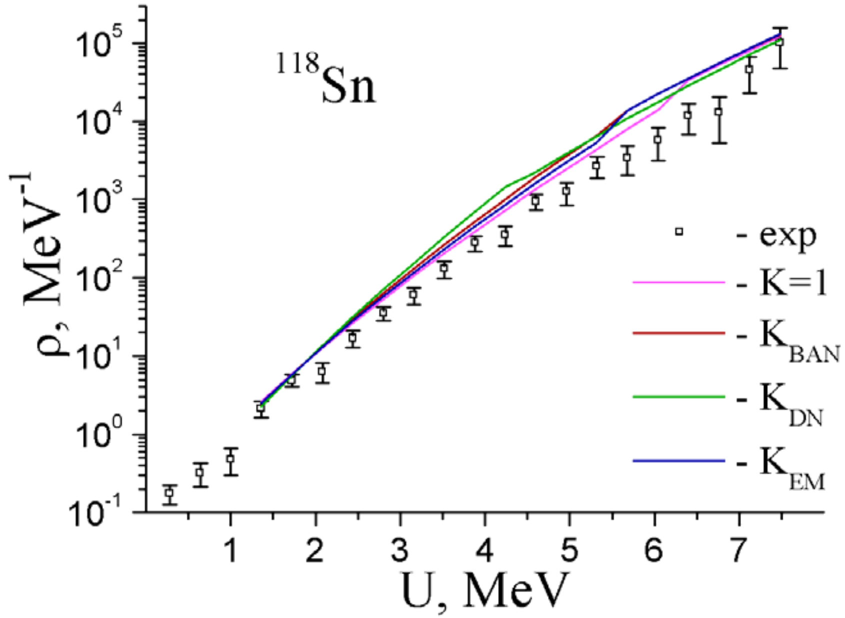


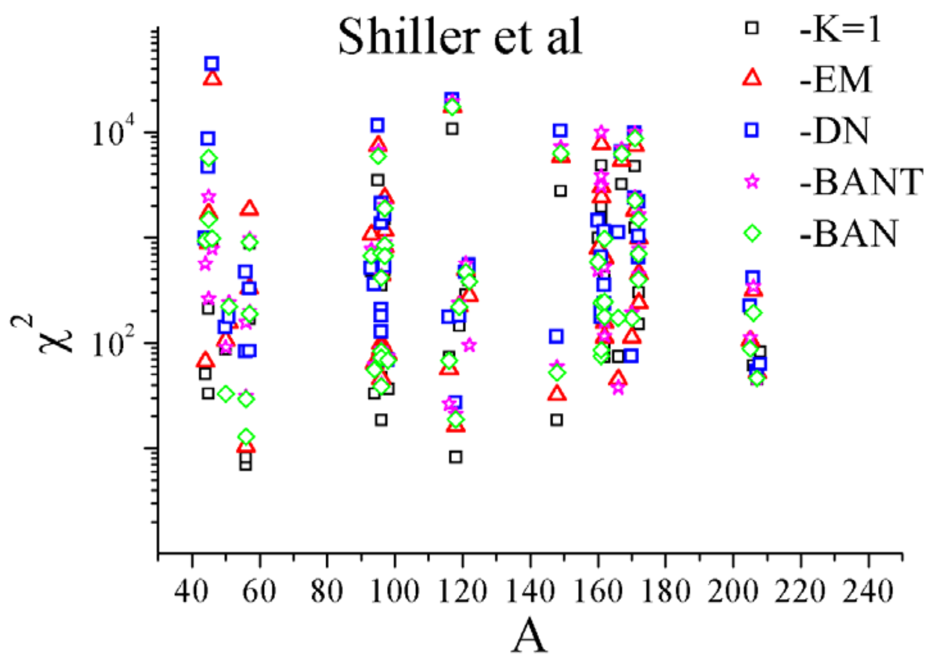
OUR fit

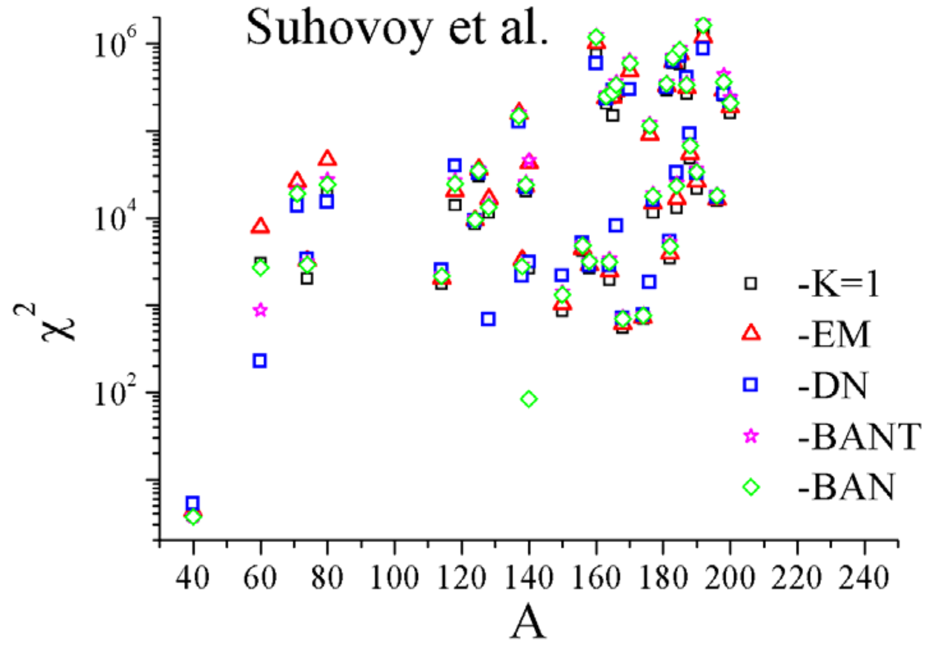
$\tilde{a}$, δ_{shift} **are fitted by** χ^2

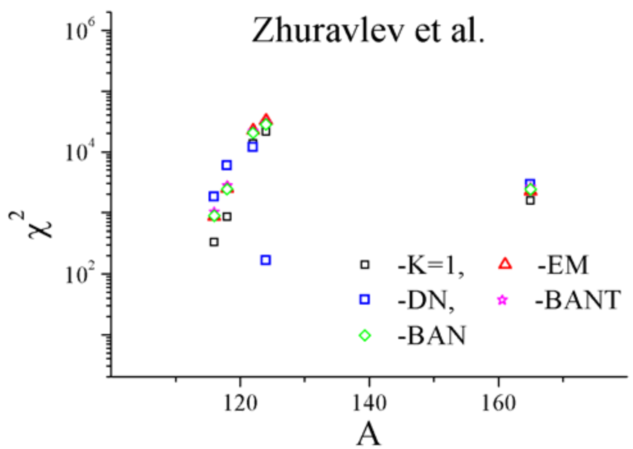
$(\delta_{shift} = const)$



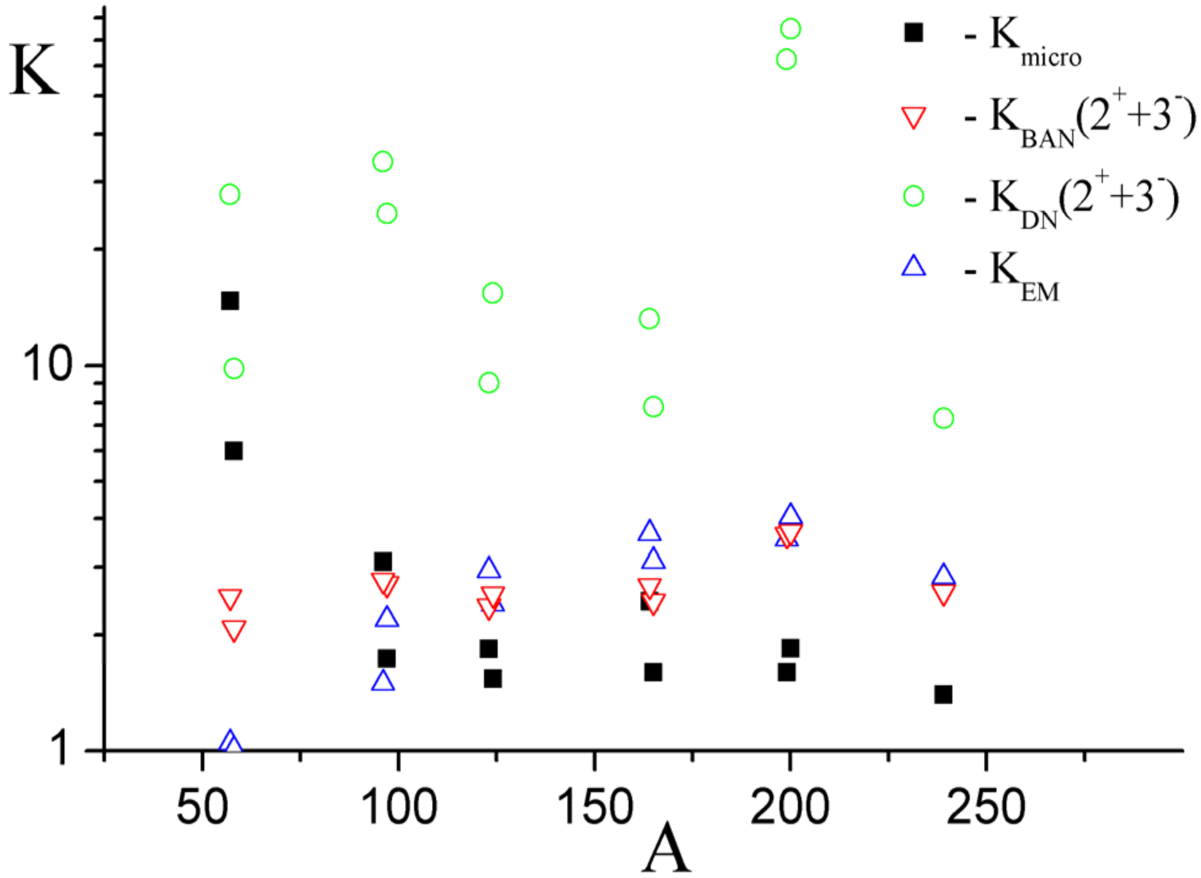






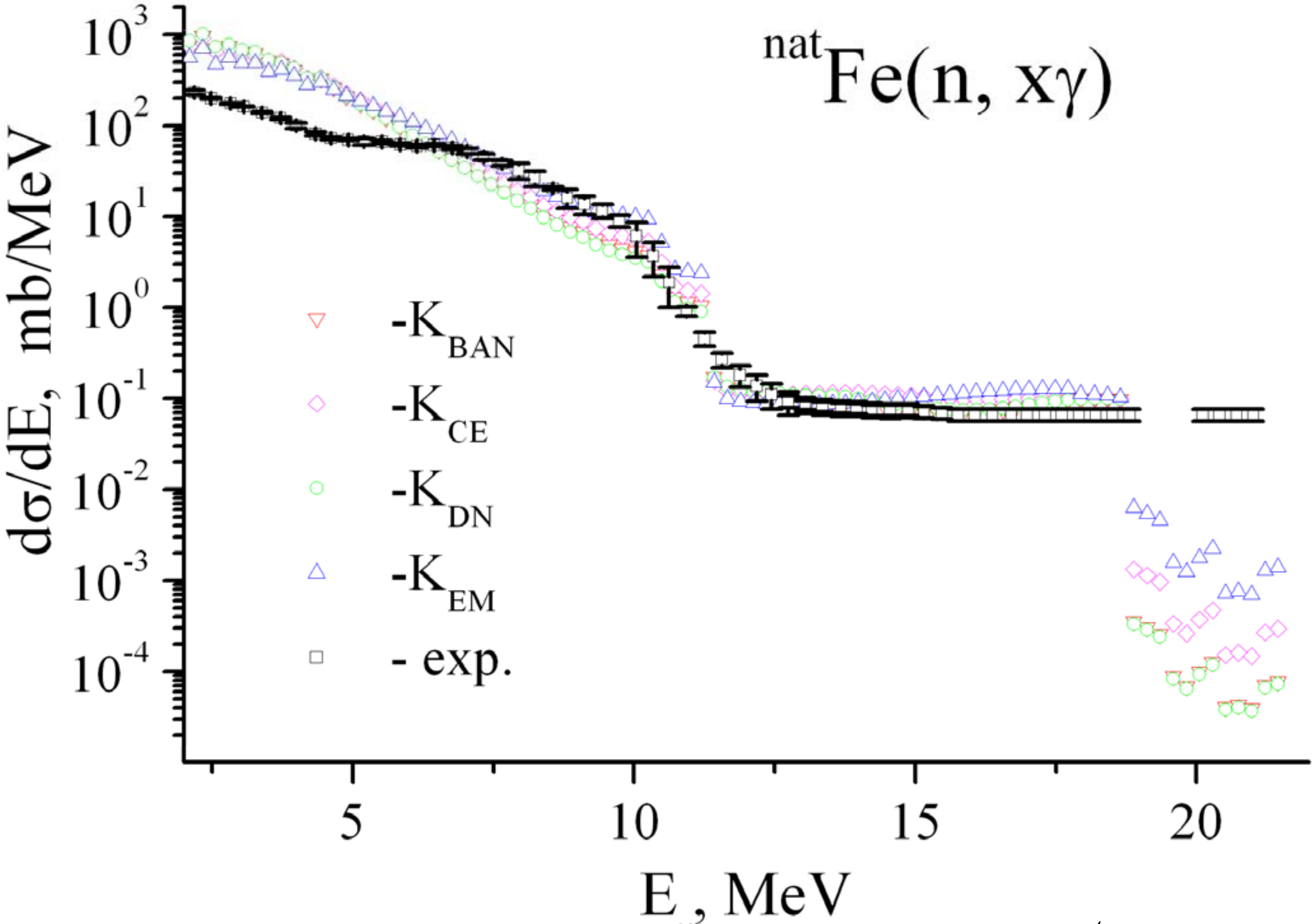


Data	$\sum_{i=1}^{N} \chi_i^2 / N$				
	K=1	K_{EM}	K_{DN}	K_{BANT}	K_{BAN}
Shiller et al.	927	2223	2839	1821	1471
Zhuravlev et al.	6,3E7	6,3E7	5,7E7	8,1E7	7,5E7
Suhovoy et al.	151102	170518	138788	216113	200046

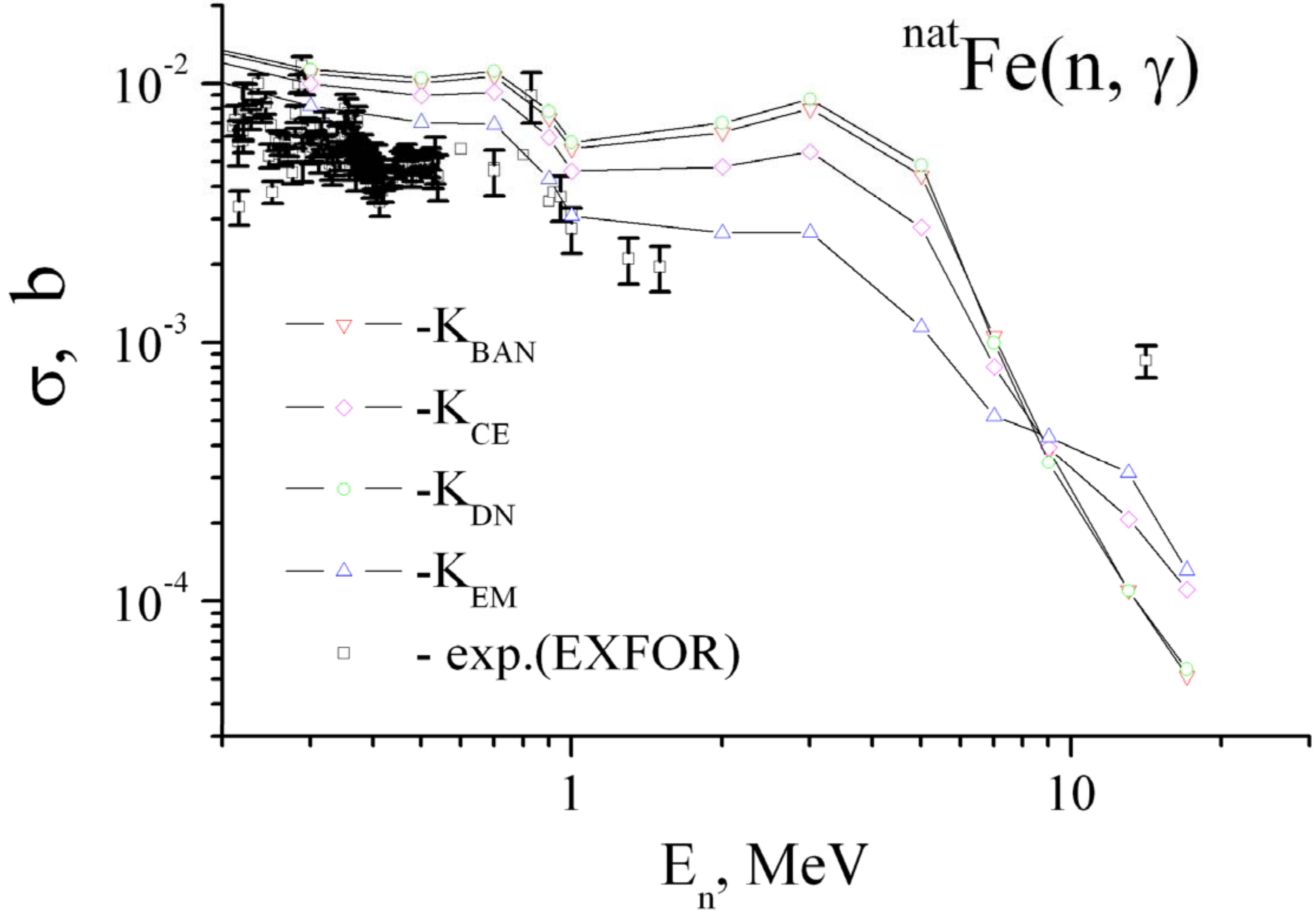


Comparision of different parametrizations for enhancement factor with microscopical calculation K_{micro}.

[Vdovin A.I., Voronov V.V., Malov L.A. Soloviev V.G., Stoyanov Ch. Fiz.El.Chas.At.Yad. 7 (1976) 952]



Dependence of $d\sigma / dE$ on gamma-ray energy for $^{nat}Fe(n, x\gamma)$ reaction.
Calculation was made with the use of the EMPIRE code 3.0
Experimental data are taken from Bondar V.M., Gorbachenko O.M., Kadenko I.M., Leshchenko B. Yu., Onishchuk Yu.M., Plujko V.A. //Proceedings of the 18th International Seminar on Interaction of Neutrons with Nuclei "Neutron Spectroscopy, Nuclear structure, Related topics", Dubna, May 26-29, 2010. (2011)135



Dependence of cross section σ on neutron energy for $^{nat}Fe(n,\gamma)$ reaction. Calculation was made with the use of the EMPIRE code 3.0. Experimental data are taken from EXFOR data library.

SUMMARY

• The calculation of different GSM models with phenomenological approaches for vibrational enhancement factors with allowance for damping were compared. An effect of collective state enhancement on gamma-emission in neutron-induced nuclear reactions is investigated. The calculations demonstrate rather strong dependence of the level density on collective states damping.

• *K* using boson partition function with average occupation numbers (BAN) is the best method of the vibrational enhancement factor calculation when *N/Z* dependence of asymptotic level density parameter is taken into account in the form

$$\tilde{a}(A,I) = 0.71786A(1+24.906I^2) - 0.54324A^{2/3}(1+127.27I^2) - 0.13220Z^2/A^{1/3}\ (MeV^{-1})$$

$$\delta_{shift}(A,I) = 0.2309(1+94.36I^2) - 0.0000449A(1+24586I^2) - 0.4402E_{2_1^+}$$

$$K_{vibr}(S_n) \sim 2 \div 5 \quad (A \sim 100)$$

• for method of boson partition function with damping occupation numbers (DN)

$$\tilde{a}(A,I) = 0.0090735A(1-558.61I^2) - 0.017619A^{2/3}(1-1527.6I^2) + 0.020852Z^2/A^{1/3}\ (MeV^{-1})$$

$$\delta_{shift}(A,I) = 0.14986(1-356.03I^2) - 0.016985A(1-31.413I^2) - 0.51393E_{2_1^+}$$

$$K_{vibr}(S_n) \sim 15 \div 30 \quad (A \sim 100)$$

$$I = (N-Z)/A$$

Thank you for the attention

GSM model proposed by the Ignatyuk
[Ignatyuk A.V.,et al, PRC 47 (1993) 1504; RIPL2]
(used in codes TALYS and EMPIRE)

$$\rho_{\text{int}}(U,I) = \frac{(2I+1)\omega(U)}{2\sqrt{2\pi}\,\sigma_{eff}^3}\exp\left\{-\frac{I(I+1)}{2\sigma_{eff}^2}\right\}$$

if $\beta = 0$ then $\sigma_{eff}^2 = \sigma_{\|}^2 \equiv \mathcal{F}_{\|}T$

if $\beta \neq 0$ then $\sigma_{eff}^2 = \sigma_{\perp}^{4/3}\sigma_{\|}^{2/3} \equiv \mathcal{F}_{\perp}^{2/3} \cdot \mathcal{F}_{\|}^{1/3}T$

$$\rho_{\text{int}}(U) = \int_0^\infty \rho_{\text{int}}(U,I)dI \cong \omega(U)/\sigma_{eff}$$

$$\rho(U) = \rho_{\text{int}}(U)K_{coll}(U;a_{GSM})$$

$$K_{coll}(U;a_{GSM}) = K_{rot}(U,T=\sqrt{U/a_{GSM}})K_{vibr}(U,a_{GSM})$$

$$\omega(U) = \exp S(U) / Det(U)^{1/2}$$

At first the definition of the critical values $T_{cr} = 0.567\Delta_0, \quad \Delta_0 = 12/\sqrt{A} \quad (MeV)$

a_{cr} solution of the equation $a_{cr} = \tilde{a}(1 + (1 - \exp(-\gamma a_{cr} T_{cr}^2))\delta\varepsilon(Z,A)/a_{cr}T_{cr}^2)$

$$E_{cond} = \frac{1}{4} g_{cr}\Delta_0^2 = \frac{3}{2\pi^2} a_{cr}\Delta_0^2 \equiv 0.152\, a_{cr}\Delta_0^2, \quad U_{cr} = a_{cr}T_{cr}^2 + E_{cond} = 0.473\, a_{cr}\Delta_0^2$$

$$S_{cr} = 2a_{cr}T_{cr} \quad Det_{cr} = 45.84 a_{cr}^3 T_{cr}^5$$

$$\mathcal{F}_{\|,cr} = 0.607927 a_{cr} < m^2 > (1 - 2\varepsilon/3) \cong 0.607927 a < m^2 > (1 - \alpha - 3\alpha^2/20)$$

$$\mathcal{F}_{\perp,cr} = 0.607927 a_{cr} < m^2 > (1 + \varepsilon/3) \cong 0.607927 a < m^2 > (1 + \alpha/2 + 5\alpha^2/8)$$

$$< m^2 >= 0.24 A^{2/3} \quad \alpha \equiv \alpha_2 = \sqrt{5/4\pi}\,\beta \quad \varepsilon = 0.946175\beta$$

$$U' = U + n\Delta_0 + \delta_{shift} \quad n = \begin{cases} 0, & even - even \\ 1, & odd \\ 2, & odd - odd \end{cases}$$

If $U' \geq U_{cr}$ *the Back-shifted Fermi model used*

$$U^* = U' - E_{cond}, \quad a = \tilde{a}(1 + (1 - \exp(-\gamma U^*))\delta\varepsilon(Z, A)/U^*), \quad T_{GSM} = \sqrt{U^*/a}$$

$$S = 2aT_{GSM}, \quad Det = 45.84a^3T^5$$

$$\mathcal{F}_{||} = 0.607927a < m^2 > (1 - 2\varepsilon/3) \cong 0.607927a < m^2 > (1 - \alpha - 3\alpha^2/20)$$

$$\mathcal{F}_{\perp} = 0.607927a < m^2 > (1 + \varepsilon/3) \cong 0.607927a < m^2 > (1 + \alpha/2 + 5\alpha^2/8)$$

If $U' < U_{cr}$ *the Superfluid model used*

$$\boxed{\varphi^2 = 1 - U'/U_{cr},} \qquad T_{GSM} = 2T_{cr}\,\varphi/\ln\big((1+\varphi)/(1-\varphi)\big)$$

$$S = S_{cr}(1 - \varphi^2)T_{cr}/T_{GSM}, \qquad Det = Det_{cr}(1 - \varphi^2)(1 + \varphi^2)^2$$

$$\mathcal{F}_{||} = \mathcal{F}_{||,cr}(1 - \varphi^2)T_{cr}/T_{GSM}, \quad \mathcal{F}_{\perp} = \mathcal{F}_{\perp,cr}(1 + 2(1 - \varphi^2)T_{cr}/T_{GSM})/3$$

GSM model proposed by the Svirin

[Svirin M.I., Fiz.El.Chas.At.Yad (Particles and Nucleus), 37(2006)901.]

$$\rho_{\text{int}}(U,I) = \frac{(2I+1)\omega(U)}{2\sqrt{2\pi}\,\sigma_1^3}\exp\left\{-\frac{I(I+1)}{2\sigma_2^2}\right\}$$

$$\text{if} \quad \beta = 0 \quad \text{then} \quad \sigma_1^2 = \sigma_{\|}^2 \equiv \mathcal{F}_{\|}T, \qquad \sigma_2^2 = \sigma_{\|}^2 \equiv \mathcal{F}_{\|}T$$

$$\text{if} \quad \beta \neq 0 \quad \text{then} \quad \sigma_1^2 = \sigma_{\perp}^{4/3}\sigma_{\|}^{2/3} \equiv \mathcal{F}_{\perp}^{2/3}\cdot\mathcal{F}_{\|}^{1/3}T, \quad \sigma_2^2 = \sigma_{\perp}^2 \equiv \mathcal{F}_{\perp}T$$

$$\rho_{\text{int}}(U) = \int_0^{\infty}\rho_{\text{int}}(U,I)dI \cong \omega(U)/\sigma_{\|}$$

$$\rho(U) = \rho_{\text{int}}(U)K_{coll}(U;a_{GSM}) \qquad K_{coll}(U;a_{GSM}) = K_{rot}(U,T=\sqrt{U/a_{GSM}})K_{vibr}(U,a_{GSM})$$

$$\omega(U) = \exp S(U)/Det(U)^{1/2}$$

with such differences:

$$E_{cond} = \frac{1}{4} g_{cr} \Delta_0^2 - n\Delta_0 = \frac{3}{2\pi^2} a_{cr}\Delta_0^2 - n\Delta_0 \equiv 0.152\ a_{cr}\Delta_0^2 - n\Delta_0$$

$$U_{cr} = a_{cr}T_{cr}^2 + E_{cond} = 0.473\ a_{cr}\Delta_0^2 - n\Delta_0$$

$$\boxed{U' = U + \delta_{shift}} \qquad \alpha = \sqrt{5/4\pi}\,\beta$$

$$\mathcal{F}_{\|,cr} = 0.607927 a_{cr} < m^2 > (1 - \alpha - 3\alpha^2/20)$$

$$\mathcal{F}_{\perp,cr} = 0.014 A^{5/3}(1 + \alpha/2 + 5\alpha^2/8)$$

If $U' \geq U_{cr}$ the Back-shifted Fermi model used

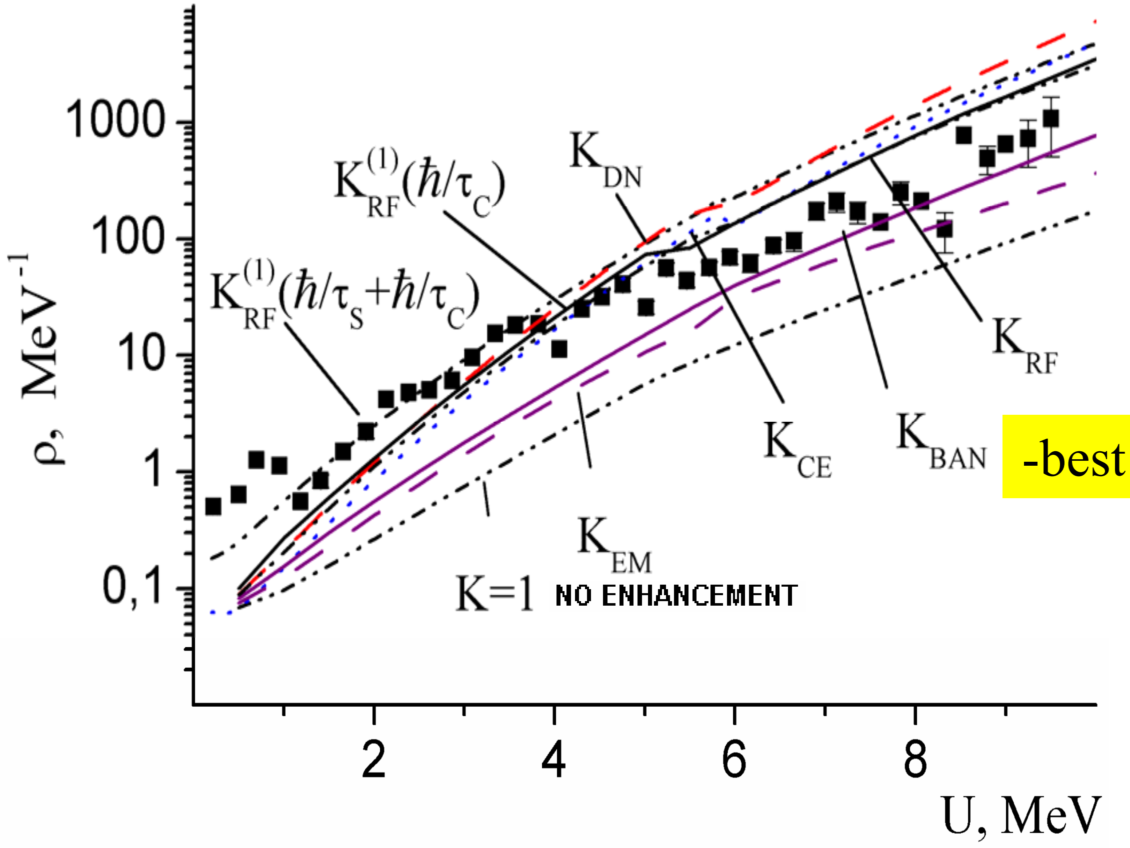
$$\mathcal{F}_{\|} = 0.607927 a < m^2 > (1 - \alpha - 3\alpha^2/20)$$

$$\mathcal{F}_{\perp} = 0.014 A^{5/3}(1 + \alpha/2 + 5\alpha^2/8)$$

If $U' < U_{cr}$ the Superfluid model used

$$Det = Det_{cr}(1 - \varphi^2)(1 + \varphi^2)^3$$

$$\boxed{\varphi^2 = 1 - U'/U_{cr}, = 1 - (U'^{(Ignatyuk)} - n\Delta_0)/(U_{cr}^{(Ignatyuk)} - n\Delta_0)}$$



Dependence of total level density on excitation energy in ^{56}Fe : ✎ -experimental data.

$$\tilde{a}(A,I) = 0.71786A(1+24.906I^2) - 0.54324A^{2/3}(1+127.27I^2) - 0.13220Z^2/A^{1/3} \ (MeV^{-1})$$

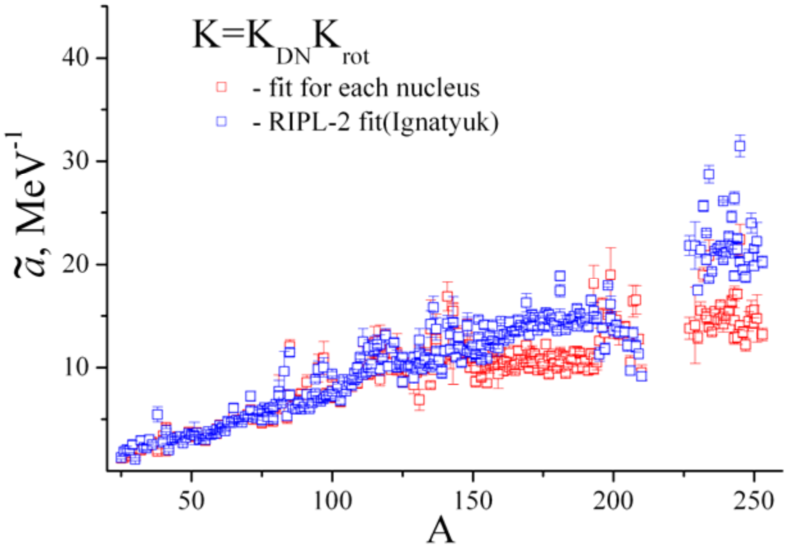
$$\delta_{shift}(A,I) = 0.2309(1+94.36I^2) - 0.0000449A(1+24586I^2) - 0.4402E_{2_1^+}$$

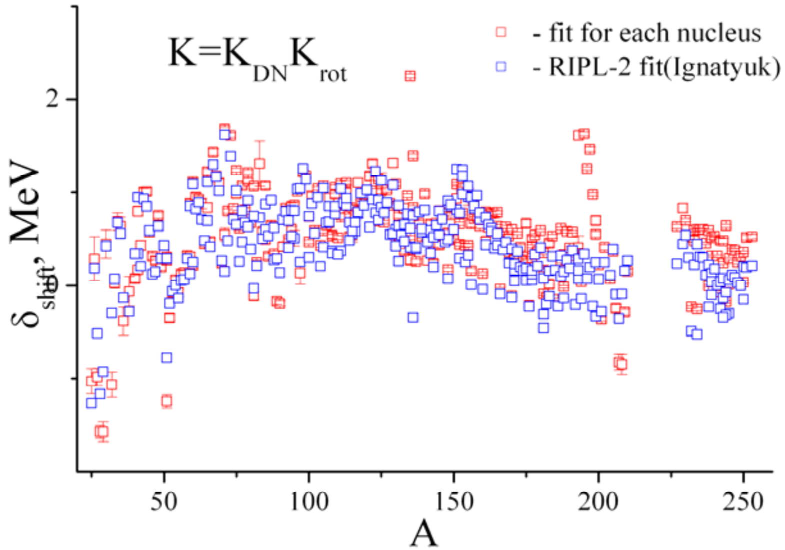
Test of approach acceptance → χ^2 minimum

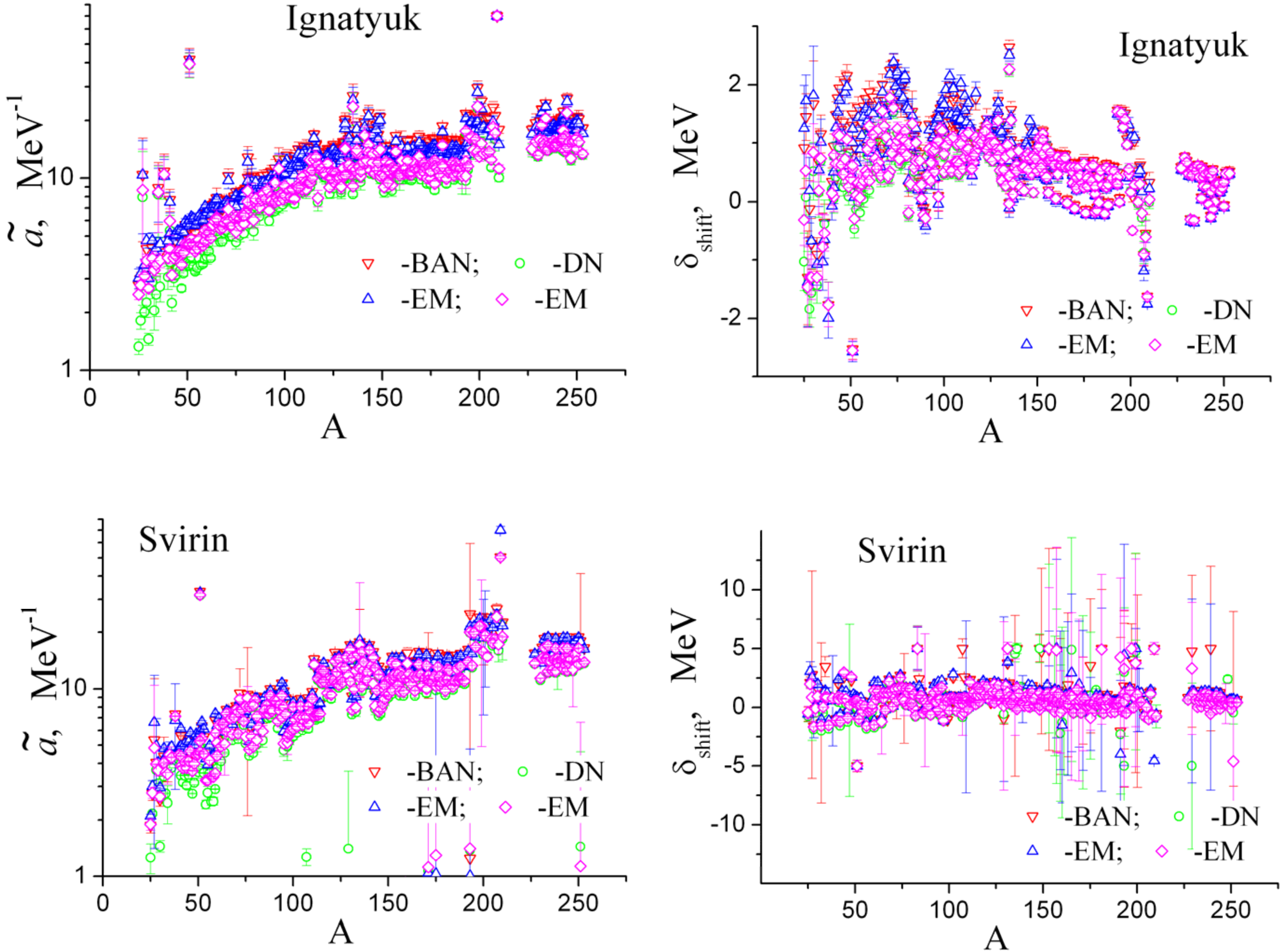
$$\chi_i^2 = \left(\frac{\bar{\rho}(A_i) - \bar{\rho}_{\exp}(A_i)}{\Delta\bar{\rho}_{\exp}(A_i)} \right)^2 + \sum_{N_{c,\exp}^{\min}}^{N_{c,\exp}^{\max}} \left(\frac{N_c(A_i) - N_{c,\exp}(A_i)}{\sqrt{N_{c,\exp}(A_i)}} \right)^2,$$

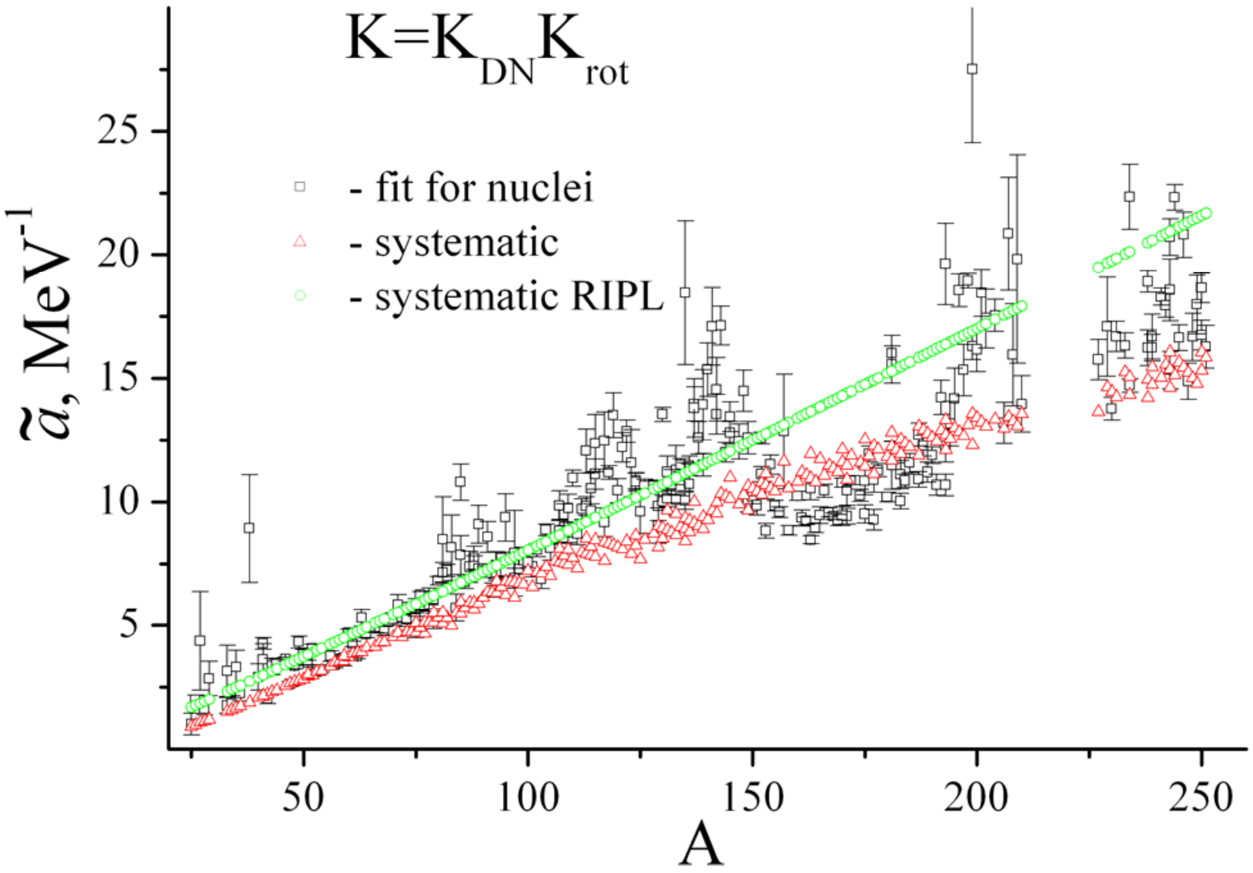
$\bar{\rho}_{\exp}(A_i)$ – from RIPL-2, $N_{c,\exp}^{\min}$ – from RIPL-3("level-densities-gsmcol.dat")

$N_{c,\exp}^{\max}$ – from RIPL-3(up to maximum well known level)





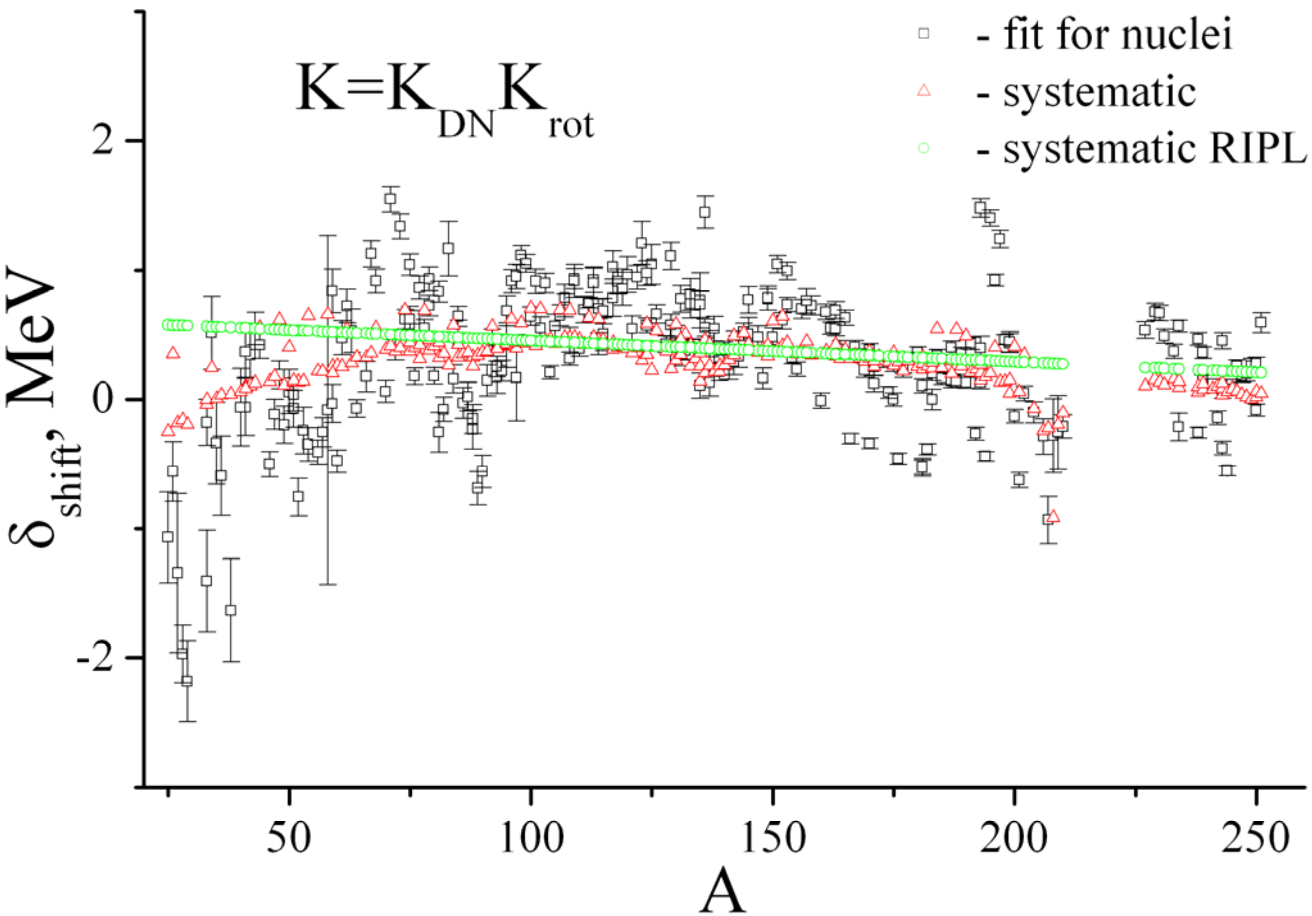

Ignatyuk
ã, MeV^-1
A
-BAN;
-DN
-EM;
-EM
δshift, MeV
Svirin



RIPL systematic: $\tilde{a} = 0.103A - 0.105A^{2/3}\ (MeV^{-1})$

Best systematic for $\tilde{a}$:

$$\tilde{a}(A,I) = 0.0090735A(1-558.61I^2) - 0.017619\,A^{2/3}(1-1527.6\,I^2) + \\ +0.020852Z^2/A^{1/3}\ (MeV^{-1})$$



RIPL systematic: $\delta_{shift} = 0.617 - 0.00164A \ (MeV)$

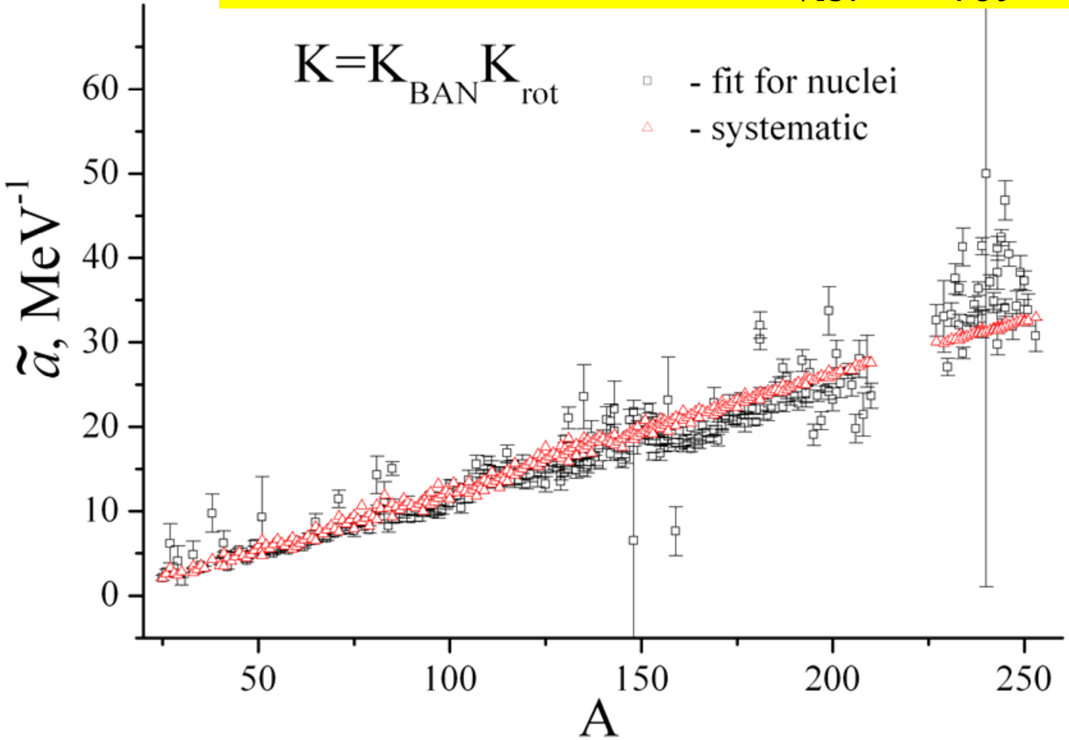
Best systematic for δ_{shift} :

$$\delta_{shift}(A,I) = 0.14986(1 - 356.03I^2) - 0.016985A(1 - 31.413I^2) - 0.51393E_{2_1^+}$$

Fitting of the parameters $\tilde{a}$, δ_{shift} for each nuclei by the minimum of

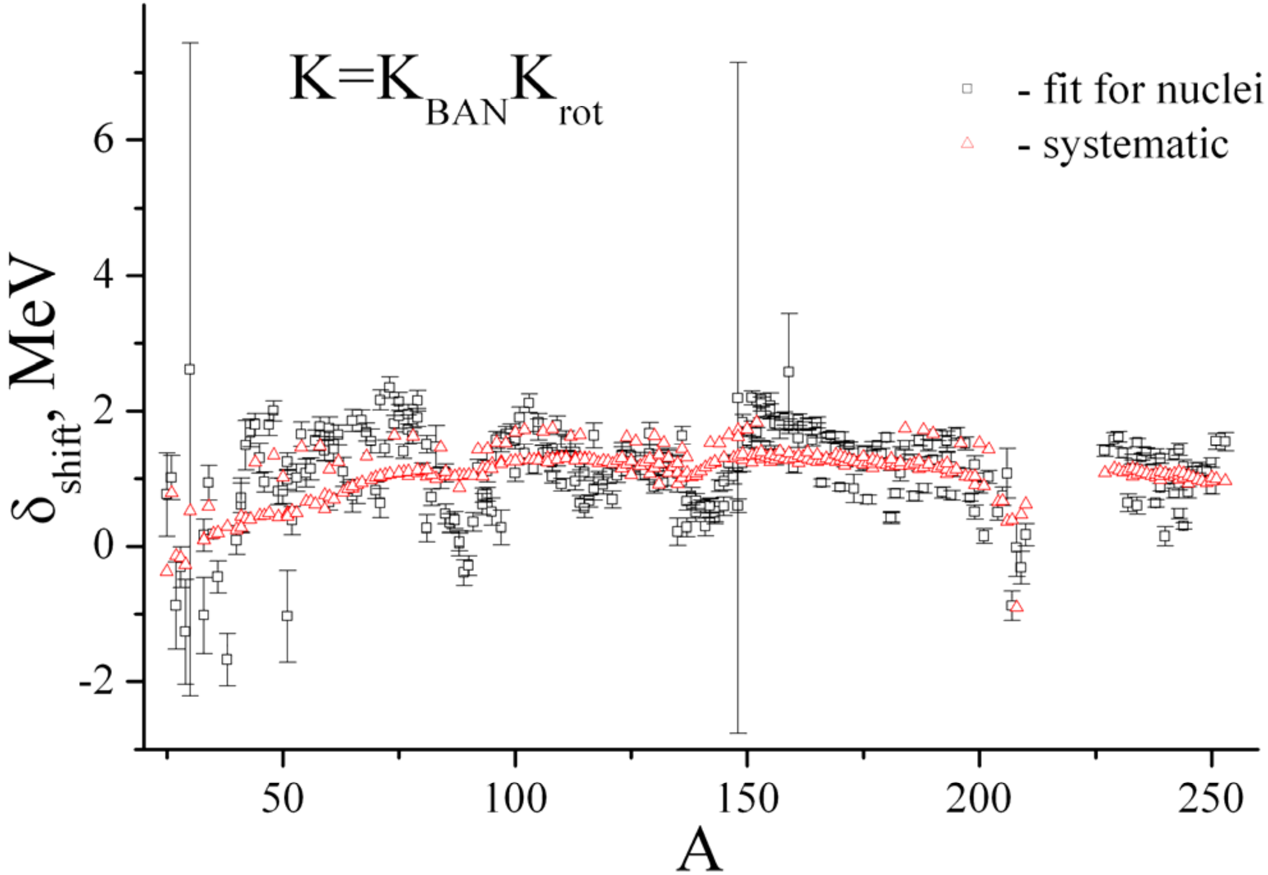
$$\chi_i^2 = \left(\frac{\rho_i - \rho_{\exp,i}}{\Delta\rho_{\exp,i}} \right)^2 + \frac{\left(\mathbb{N}_i - \mathbb{N}_{\exp,i} \right)^2}{\mathbb{N}_{\exp,i}}$$

Best model for $K = K_{vibr} \cdot K_{rot}$ is minimum of $\sum_i \chi_i^2$



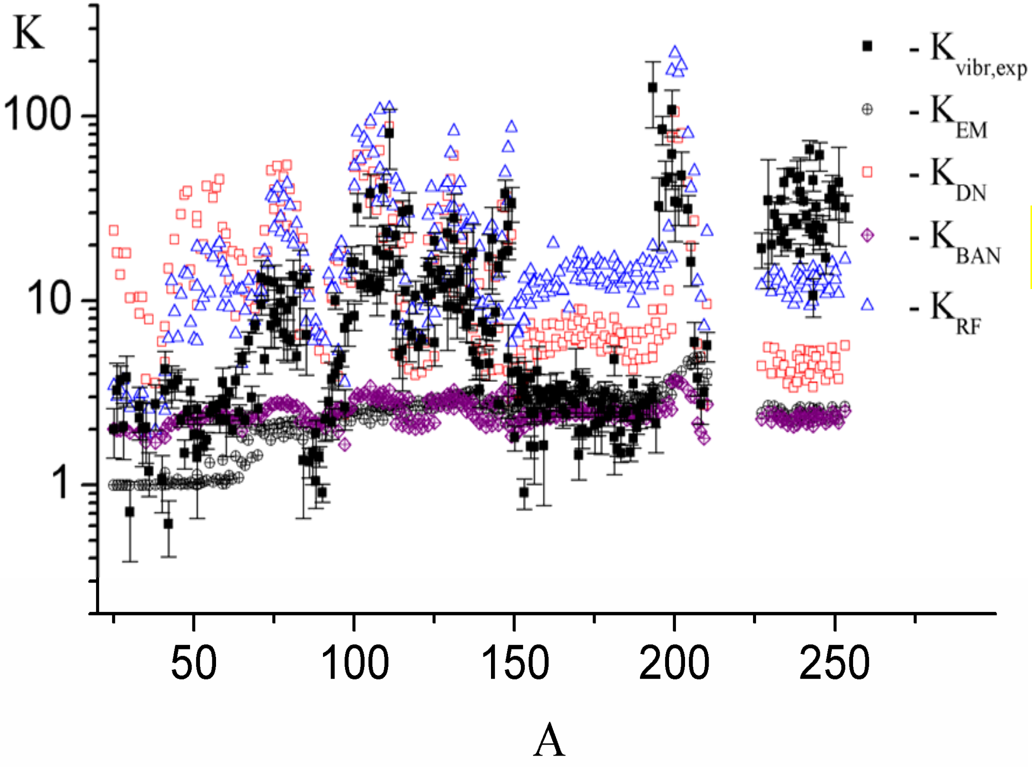
Best systematic for $\tilde{a}$:

$$\tilde{a}(A,I) = 0.71786A(1 + 24.906I^2) - 0.54324A^{2/3}(1 + 127.27I^2) - 0.13220Z^2/A^{1/3} \quad (MeV^{-1})$$



Best systematic for δ_{shift} :

$$\delta_{shift}(A,I) = 0.2309(1 + 94.36I^2) - 0.0000449A(1 + 24586I^2) - 0.4402E_{2_1^+}$$



-best method

The comparison of the different vibrational $(2^+ + 3^-)$ enhancement factors with experimental data: $U = S_n$

$$\tilde{a}(A,I) = 0.71786A(1+24.906I^2) - 0.54324A^{2/3}(1+127.27I^2) - 0.13220Z^2/A^{1/3} \quad (MeV^{-1})$$

$$\delta_{shift}(A,I) = 0.2309(1+94.36I^2) - 0.0000449A(1+24586I^2) - 0.4402E_{2_1^+}$$

Smallest χ^2 value corresponds to method of boson partition function with average occupation numbers (BAN approach).