The Relation between The Isoscalar And Isovector Interaction Potential

El-Nohy N.A.[a], Motaweh H.A.[b], Attia A.[a] and
El-Hammamy M.N. [b]

a) Physics Department, Faculty of Science, Alexandria University, Egypt.
b) Physics Department, Faculty of Science, Damanhur University, Egypt.

The interaction potentials of (n,n), (p,p) and (p,n) scattering by different nuclei have been calculated in the framework of the semi-macroscopic approach, using isoscalar and isovector parameters of different types of potentials.

The interaction potentials are used to calculate the differential scattering cross section of (n,n), (p,p) and (p,n) reactions by some nuclei at low energy region. The relation between the isoscalar and isovector potential is studied.

The idea of this report is summarized in the following: To any level are the calculated values of U_{nn} , U_{pp} and U_{pn} are consistence with the relation between them.

For the nuclear part of the nucleon-nucleus, optical potential expressed in terms of the isospin operators in the following simple term of Lane form [5]

$$U = U_o + \frac{4tT}{A} U_1$$

where t, T are the isospin of the particle and target nucleus respectively and A is the mass number of the target.

The matrix elements resulting from previous equation give the following relationships [6].

$$U_{pp} = U_O - \varepsilon U_1$$

$$U_{nn} = U_O + \varepsilon U_1$$

Similarly, the transition matrix element or (p,n) form factor for the charge exchange reaction is

$$U_{pn} = 2(\frac{\varepsilon}{A})^{\frac{1}{2}} U_1$$

$$U_{nn} - U_{pp} = 2\varepsilon U_1 = 2(\frac{\varepsilon}{A})^{\frac{1}{2}} U_{pn} = (N - Z)^{1/2} U_{pn}$$

Where $\varepsilon = \frac{(N-Z)}{A}$

Introducing the main effects of the Coulomb field so that

$$U_{pp} = U_O - \varepsilon U_1 + \Delta U_C$$

$$U_{nn} = U_O + \varepsilon U_1$$

Then the Coulomb correction term must satisfy the form

$$\Delta U_C = U_{pp} - U_{nn} + 2\epsilon U_1$$

Then the final relation must be in the form

$$U_{nn} - U_{pp} + U_c = 2\epsilon U_1 = 2\frac{\epsilon^{\frac{1}{2}}}{A} U_{pn}$$

$$= (N - Z)^{\frac{1}{2}} U_{pn}$$

Method of Calculations

In the semi-macroscopic approach the folded potential has the form

$$\boldsymbol{U(r) = \left[U_O(r) + I_{O,O}(r)\right] + T_z\left[U_1(r) + I_{1,O}(r)\right]}$$

The first bracket refers to the isoscalar potential and the second bracket refers to the isovector potential, in which U_0(r) and U_1(r) are defined as

$$U_o\,(r) = \int \rho_o(r)\; V_{oD}\,(r)\; dr$$

$$U_1\,(r) = \int (\,\rho_p\,(r) - \rho_{\,n}(r))\; V_{1D}\,(r)\, dr$$

The exchange term is defined by

$$I_{k,o}(r) = V_{KE} \int F(s)\rho_{Ko}(r,s) \; J_o(K_o(r)s)s^2 ds,$$

$$K_0^2(r) = \frac{2m}{h^2} \, [E - U_o(r) + T_z U_1(r)].$$

where V_{KE} is the exchange parameter, F(s) its radial dependence and has a Wood Saxon form, $J_o(K_o(r)$ is the spherical Bessel function. T_z= -1 for proton and +1 for neutron.

The calculated form factor for each of the (n,n) and the (p,p), showed a small difference between them, which is due to the difference between the proton and the neutron isospin.

In our calculations we have used four different types of effective two-body interactions:

The first is derived from the DGN-potential

$$V(r) = {}^{ts}X\{V_{C1}\, e^{-r^2/a_1^2} - V_{C2} e^{-r^2/a_2^2}\},$$

where V_{C1} = -56 MeV, α_1= 1.635 fm, V_{C2}=91 MeV, and α_2= 0.55 fm. The operator ${}^{ts}X$ depends on the spin and the isospin quantum numbers of the two nucleons as well as the exchange-force constants

$$^{ts}X = C_w + (-1)^{s+t+1} C_M + (-1)^{S+1} C_B + (-1)^{t+1} C_H,$$

$C_W = C_M = -0.41$ and $C_B = - C_H = -0.09$

S.B. Doma, K.K. Gharib and N.A. El-Nohy. Egypt. J. of Phys., 29 (3): 323 (1998).

The direct parameters of the otentials are derived from the nucleon-nucleon interaction as follows [11].

$$V_{OD} = \frac{1}{16}(9V^{TO} + 3V^{SE} + 3V^{TE} + V^{SO}),$$

$$V_{1D} = \frac{1}{16}(3V^{TO} + 3V^{SE} - 3V^{TE} - V^{SO}),$$

And the exchange parameters are given by [12]

$$V_{OE} = \frac{1}{16}(-9V^{TO} + 3V^{SE} + 3V^{TE} - V^{SO}),$$

$$V_{1E} = \frac{1}{16}(-3V^{TO} + V^{SE} - 3V^{TE} - V^{SO}),$$

Accordingly, our parameters corresponding to the DGN-potential are given by:

$$V_{OD} = V_{OE} = -17.22\, e^{-r^2/(1.635)^2} + 27.98\, e^{-r^2/(0.55)^2}$$

$$V_{1D} = V_{1E} = -7.9\, e^{-r^2/(1.635)^2} + 13.42\, e^{r^2/(0.55)^2}$$

The second potential is derived from the DNG-potential.

$$V(r) =^{ts} X V_C \, e^{-r2} / a^2 ,$$

where Vc = -38 MeV and $\alpha = 1.910 \; fm. \quad Hence,$

$$V_{OD} = V_{OE} = -11.68 \, e^{-r^2 / a^2} ,$$

$$V_{ID} = V_{1E} = -5.6 \, e^{-r^2 / a^2}$$

S.B. Doma, N.A. El-Nohy and K.K. Gharib, Helvetica, Phys. Acta, 69 : 90 (1996).

The third potential is derived from the HJ-potential

$$V_{OD} = V_{OE} = 3604\ e^{-4r}/(4r) – 1158\ e^{2.5r} / (2.5r) – 3.92\ e^{-.707r}/(.707r),$$

$$V_{1D} = V_{1E} = -1442\ e^{-4r}/(4r) + 476\ e^{-2.5r} /(2.5r) +1.3\ e^{-.707r} /(.707r).$$

The fourth potential is taken from the Reid-potential

V_{OD}=5773e^{4r}/(4r) – 1461 $e^{2.5r}$/(2.5r) –5.885 $e^{.707r}$/(.707r).

V_{OE} = 2405e^{-4r}/(4r)–1113$e^{-2..5r}$/(2.5r)–5.885$e^{-.707r}$/(.707r),

V_{1D} = V_{1E} = -3202 e^{-4r}/(4r) + 1002 $e^{-2..5r}$/(2.5r) + 1.308 $e^{-.707r}$/(.707r).

The density distribution function for the nuclei ^{56}Fe, ^{64}Zn and ^{116}Sn are calculated according to the fermi distribution

$$\rho(r) = \rho_o (1 + \omega r^2/c^2) / (1 + \exp(r - c)/z),$$

where ρ_o, c, z and ω are constants defined as the density parameters of the Fermi-model.

Results and Conclusion

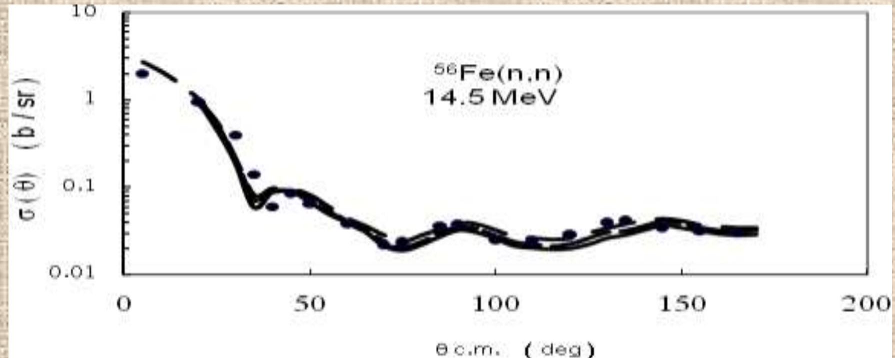
The analysis of the protons and neutrons elastic scattering by using the semi-macroscopic approach shows a **good description of the elastic scattering cross-sections** for protons and neutrons as well as for the quasi-elastic scattering, for the different types of the used potentials.

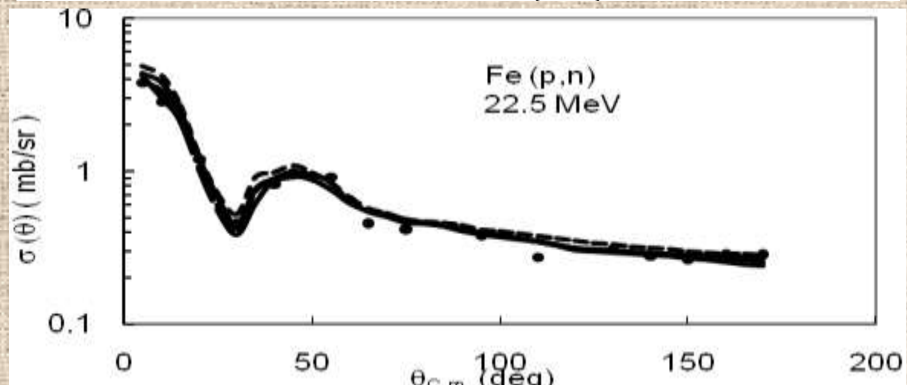
Table1 : $U_{nn,}U_{pp}$ and U_{pn} characteristics of ^{56}Fe

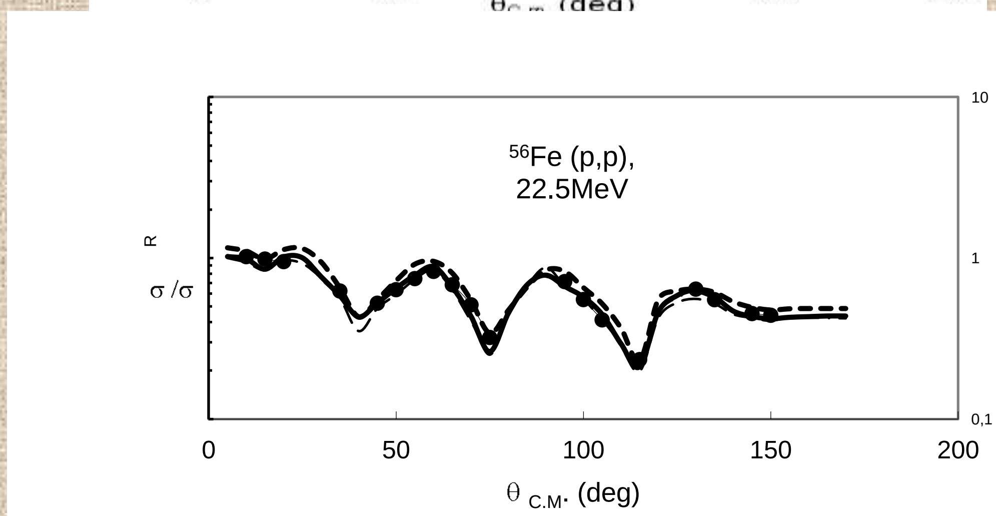
Form-factor	Potential	Depth (MeV)	Vol. Integral	R (fm)
U_{nn}	DGN	-65.69	22999	4.255
U_{nn}	DNG	-73.686	26684	4.394
U_{nn}	HJ	-65.998	24648	4.452
U_{nn}	Reid	-53.316	22898	5.091
U_{pp}	DGN	-64.74	22700	4.258
U_{pp}	DNG	-72.743	26342	4.344
U_{pp}	HJ	-65.181	24343	4.452
U_{pp}	Reid	-52.631	22604	5.091
U_{pn}	DGN	-1.109	389.2	4.230
U_{pn}	DNG	-1.011	367.5	4.388
U_{pn}	HJ	-1.910	702.9	4.375
U_{pn}	Reid	-2.975	1068.5	4.310

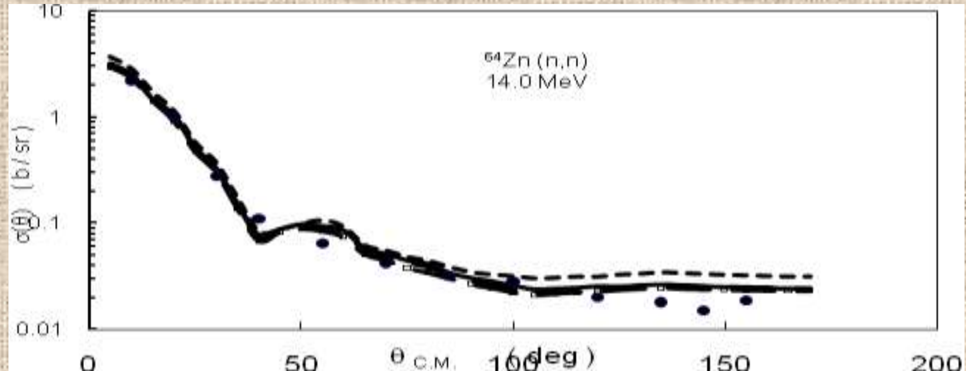
Table2 U_{nn}, U_{pp} and U_{pn} characteristics of nucleus ^{64}Zn

Form-factor	Potential	Depth (MeV)	Vol. Integral	R (fm)
U_{nn}	DGN	-64.728	26211	4.468
U_{nn}	DNG	-73.210	31304	4.578
U_{nn}	HJ	-66.866	27411	4.624
U_{nn}	Reid	-53.965	25838	5.229
U_{pp}	DGN	-63.868	25881	4.468
U_{pp}	DNG	-72.271	30903	4.578
U_{pp}	HJ	-66.009	27034	4.624
U_{pp}	Reid	-53.273	25507	5.229
U_{pn}	DGN	-0.946	430.53	4.466
U_{pn}	DNG	-0.884	377.63	4.572
U_{pn}	HJ	1.682	729.69	4.551
U_{pn}	Reid	2.646	1119.8	4.490

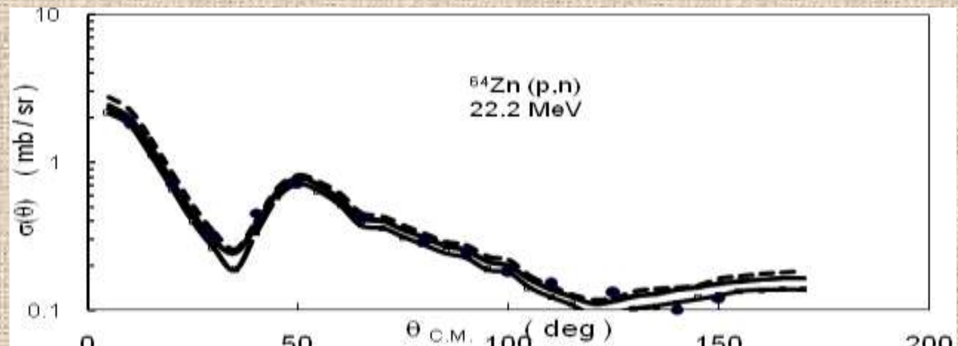

56Fe(n,n)
14.5 MeV
σ(θ) (b/sr)
10
1
0.1
0.01
0
50
100
150
200
θ c.m. (deg)


Fe (p,n)
22.5 MeV
σ(θ) (mb/sr)
10
1
0.1
0
50
100
150
200
(deg)

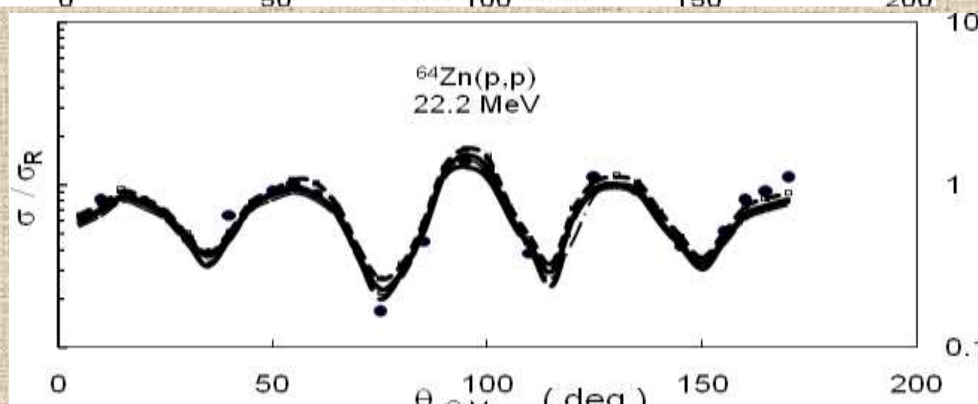

56Fe (p,p),
22.5MeV
σ /σ R
10
1
0,1
0
50
100
150
200
θ C.M. (deg)



⁶⁴Zn (n,n)
14.0 MeV
σ(θ) (b/sr)
10
1
0.1
0.01
0
50
100
150
200
θ C.M. (deg)



⁶⁴Zn (p,n)
22.2 MeV
σ(θ) (mb/sr)
10
1
0.1
0
50
100
150
200
θ C.M. (deg)



⁶⁴Zn(p,p)
22.2 MeV
σ / σR
10
1
0.1
0
50
100
150
200
θ C.M. (deg)

It is easy to notice from these figures that, all the used potentials give a good results for the scattering cross sections of each of the reactions (n,n), (p,p), and (p,n) although these potentials have different characteristic values. This is due to the fact that the calculations of the interaction cross sections depend also up on the imaginary potential

The calculations were performed for **U_{nn}** at **14 MeV** and for **U_{pp}** and U_{pn} at energy **22.2 MeV.** The Coulomb force reduces the kinetic energy of protons by about **9 MeV,** i.e proton and neutron interact with the atomic nucleus with the same energy. There are a great similarity in the behavior of the differential elastic scattering of **U_{nn} at 14 MeV** and **U_{pn} at 22.2 MeV** we can insure that the difference between the proton and the neutron for the same colliding energy is very small.

The Coulomb force plays an essential rule in studies of U_{nn} and U_{pp} and consequently U_{pn}.

The relation between U_{nn} ,U_{pp} and U_{pn}

Potential	Element	$(U_{nn} - U_{pp})$	$U_{nn} - (U_{pp} + U_C)$	$(N-Z)^{1/2}U_{pn}$
DNG	^{56}Fe	0.95	3.668	2.218
DGN		.0943	3.761	2.022
HJ		0.817	3.525	3.820
Reid		0.685	3.403	5.950
DNG	^{64}Zn	0.860	3.859	1.892
DGN		0.939	3.939	1.768
HJ		0.857	3.857	3.364
Reid		0.692	3.692	5.292

The difference between U_{nn} and U_{pp} is a small value for all the used potentials and fare from $(N–Z)^{1/2}U_{pn}$ value.

Introducing the Coulomb correction to the proton incident energy we find that the difference become more close to the value $(N–Z)^{1/2}\, U_{pn}$

The Coulomb force has an essential rule in satisfying the validity of equation (2.6).

We notice also that the values of U_{nn}, U_{pp} and U_{pn} obtained by HJ potential gave results closer to satisfy equation (2.6) for the two elements in compared with the other three potentials.

This indicates that the

HJ potential give values of

U_{nn}, U_{pp} and U_{pn} better than

the other three potentials, for

elements ^{56}Fe and ^{64}Zn.

The question now is the imaginary potentials obtained in the cases of (n,n), (p,p) and (p,n) reactions have the same relations as that of the real potentials.

To answer on this question, we must study the imaginary potentials obtained in these reactions.

Then it is more convenient to list the data of imaginary potentials obtained by HJ potential for the three reactions , which gave best fitted values for the calculated cross-section with the experimental data

As indicated in the following table

The imaginary potential for the (n,n),(p,p) and (p,n) reactions

Reaction	W_v ,MeV	$r_{v\,,}$ fm	a_v, fm
^{56}Fe (n,n)	10	1.2	0.98
^{64}Zn (n,n)	8	1.2	0.98
^{56}Fe (p,p)	3.25	1.2	0.98
^{64}Zn (p.p)	2.32	1.2	0.98
^{56}Fe (p,n)	2.686	1.26	0.58
^{64}Zn (p,n)	2.472	1.26	0.58

Table 6: The relation between W_{nn} ,W_{pp} and W_{pn}

Element	W_{nn} - W_{pp}	W_{pn}	$(N-Z)^{1/2}W_{pn}$
^{56}Fe	6.75	2.686	5.372
^{64}Zn	5.68	2.472	4.944

Conclusion

1-For non-symmetric nuclei the validity of equation (2.6) can be easily verified in our approach.

2-The HJ potential is the more convenient potential in that analysis.

3-The relation between U_{nn}, U_{pp} and U_{pn} for the real potential can be applied for the corresponding imaginary potentials.

Thank you