Nuclear Level Density within Modified Generalized Superfluid Model with Vibrational Enhancement

O. M.Gorbachenko **[1], *V. A. Plujko*** **[1,2], *B. M. Bondar*** **[1], *E. P. Rovenskykh*** **[1]**

[1] *Taras Shevchenko National University, Kyiv, Ukraine*

[2] *Institute for Nuclear Research, Kyiv, Ukraine*

The nuclear level density (NLD) ρ is crucial parameter to define characteristics of nuclear decay. The collective states have rather strong effect on NLD, specifically at low excitation energies.

Up to now, there exist problems in estimation of this effect. Specifically, there is rather big uncertainties in estimation of the magnitude of the vibrational enhancement of nuclear level densities.

ISINN-21, Alushta, Ukraine, May 20 –25, 2013

COLLECTIVE ENHANCEMENT (VARIATION) FACTOR

$$\boxed{K_{coll} = \rho / \rho_{int}}$$

ρ , ρ_{int} - **NLD with and without allowance for collective states**

$$\rho(U, A) = \Phi\{Z(\alpha^*, \beta^*)\}$$

$$Z(\alpha, \beta) = \mathrm{Tr}\left[\exp(\alpha\hat{A} - \beta H)\right] \text{- partition function}$$

Equation of thermodynamic state:

$$\begin{cases} A = \partial \ln Z / \partial\alpha \,|_{\alpha^*,\beta^*}, \\ E = -\partial \ln Z / \partial\beta \,|_{\alpha^*,\beta^*} = U + E_0 \end{cases} \equiv> \begin{pmatrix} \alpha^* \\ \beta^* \end{pmatrix} = \vec{\Psi}_Z \begin{pmatrix} A \\ U \end{pmatrix}$$

$T = 1/\beta^*$, $\mu = \alpha^*/\beta^*$ **- the temperature and chemical potential**

$$\boxed{Z(\alpha, \beta) = Z_{int}(\alpha, \beta)\Delta Z(\beta)}$$

$$\rho_{int}(U, A) = \Phi\{Z_{int}(\alpha^*_{int}, \beta^*_{int})\}$$

$$\begin{pmatrix} \alpha^*_{int} \\ \beta^*_{int} \end{pmatrix} = \vec{\Psi}_{Z_{int}} \begin{pmatrix} A \\ U \end{pmatrix} \quad => \quad T_{int} = 1/\beta^*_{int}$$

$$T_{int} \neq T$$

NLD WITHIN ENHANCED GENERALIZED SUPERFLUID MODEL (EGSM or EMPIRE GLOBAL SPECIFIC MODEL)

$$\boxed{\rho(U,J) = \bar{\rho}_{EM}(U,J) \cdot K_{EM}(T_{\text{int}})}$$

$\bar{\rho}_{EM}(U,J)$ - **INTRINSIC NLD WITH ROTATIONAL ENHANCEMENT**

K_{EM} - **LDM VIBRATIONAL ENHANCEMENT FACTOR WITH DAMPING** q_{vib}

$$\boxed{K_{EM} = K_{LDM} \cdot (1 - q_{vib}) + q_{vib}}$$

$$K_{LDM} \equiv Z_{boz}(\{n(\omega_L)\}) = \exp\left[C_3 A^{2/3} \cdot T_{\text{int}}^{4/3}\right], \quad \omega_L \text{ => Liquid Drop Model}$$

$$q_{vib} = 1/[1 + \exp\{(T_{1/2} - T_{\text{int}})/DT\}]$$

$$\boxed{\bar{\rho}_{EM}(U,J) = \Xi\{\tilde{a}, \tilde{\delta}_{shift}\}}$$

$\tilde{a}$ -asymptotic value of *a*-parameter of NLD

$\tilde{\delta}_{shift}$ -additional shift of excitation energy to adjust cumulative sum N_{cum} of experimental discrete levels

Herman M., Capote R.,et al. Nucl. Data Sheets 108 (2007) 2655; EMPIRE-3.1, available online at http://www.nndc.bnl.gov/empire/

MODIFIED EGSM

$$\rho(U,J) = \overline{\rho}_{EM}(U,J) \cdot K_{vibr}$$

Vibrational enhancement factors with damping

Boson partition function with damped occupation numbers(DN)

Ignatyuk A.V., Weil J.L., Raman S., Kahane S. PRC 47 (1993) 1504

$$\boxed{K_{vibr} \equiv Z_{boz}(\{\overline{n}_L\}) = \exp\left(\overline{S} - \overline{U}/T_{\text{int}}\right) \equiv K_{DN}(T_{\text{int}})}$$

$$\overline{S} = \sum_L (2L+1)\left[(1+\overline{n}_L)\ln(1+\overline{n}_L) - \overline{n}_L \ln \overline{n}_L\right], \qquad \overline{U} = \sum_L (2L+1)\hbar\omega_L \overline{n}_L,$$

$$\overline{n}_L = \frac{\exp\left[-\Gamma_L/(2\hbar\omega_L)\right]}{\exp(\hbar\omega_L/T_{\text{int}}) - 1}, \qquad \Gamma_L = C \cdot \left[(\hbar\omega_L)^2 + 4\pi^2 T_{\text{int}}^2\right], \quad L = 2,\ 3$$

Boson partition function with average occupation numbers

$$\boxed{\Delta Z(\tau) = \frac{Z_{bos}(\{< n(\omega_L,\tau) >\})}{Z_{bos}(\{< n(\tilde{\omega}_L,\tau) >\})} \equiv \frac{\prod_L (1+ < n(\omega_L,\tau) >)^{2L+1}}{\prod_L (1+ < n(\tilde{\omega}_L,\tau) >)^{2L+1}}}$$

$$< n(\omega_L,\tau) >= \frac{1}{T_p}\int_0^{T_p} n(\omega_L)\exp(-\Gamma_L t)dt = \frac{1-\exp(-2\pi\Gamma_L/\hbar\omega_L)}{\exp(\hbar\omega_L/\tau)-1}\frac{\hbar\omega_L}{2\pi\Gamma_L}$$

$$L^\pi = 2^+,\ 3^-;\ \hbar\tilde{\omega}_{2^+} = \hbar\omega_{shell}/2,\ \hbar\tilde{\omega}_{3^-} = \hbar\omega_{shell},\ \hbar\omega_{shell} = 41A^{-1/3}$$

BAN approximation without allowance for changing temperature

$$\boxed{K_{vib} \equiv K_{BAN}(T_{\text{int}}) = \Delta Z(\tau = T_{\text{int}})}$$

BAN approximation with allowance for changing temperature

$$Z_{\text{int}}(\alpha,\beta) \equiv Z_{BSFG}(\alpha,\beta);\quad Z'(\alpha,\beta) = Z_{BSFG}(\alpha,\beta)\cdot\Delta Z(\beta = 1/\tau)$$

$$\boxed{K_{vib} \equiv K_{BANT}(T = T_{\text{int}} + \delta T_{BSFG}) = \frac{\rho}{\rho_{\text{int}}} = \frac{\Phi\{Z'(\alpha^*,\beta^* = 1/T)\}}{\Phi\{Z_{BSFG}(\alpha^*_{\text{int}},\beta^*_{\text{int}} = 1/T_{\text{int}})\}}}$$

Plujko V.A. et al., Int.J.of Mod.Phys.E 16, 570 (2007); Phys. Atom. Nucl. 70, 1643 (2007)

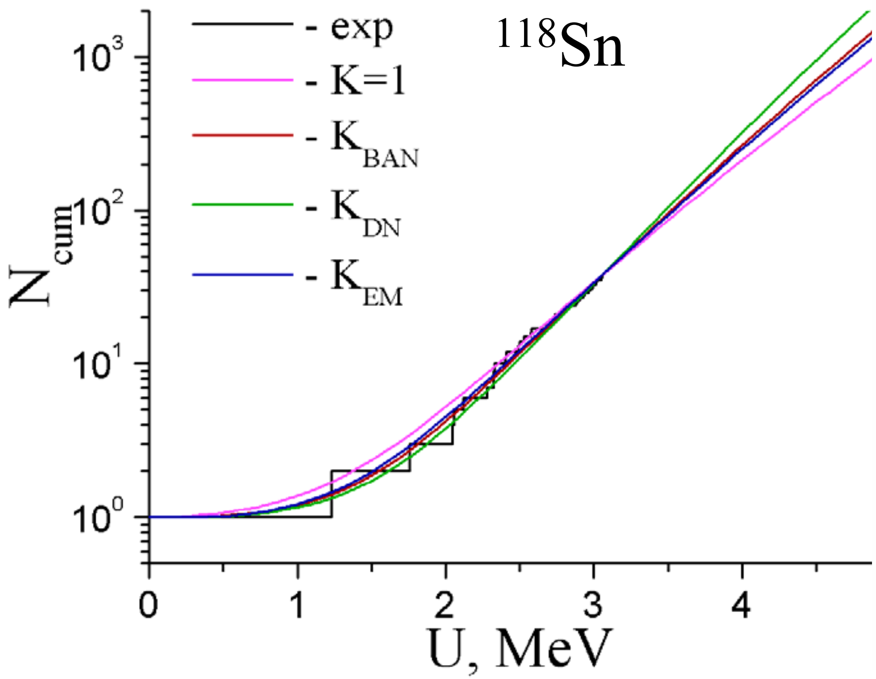
Modified EGSM

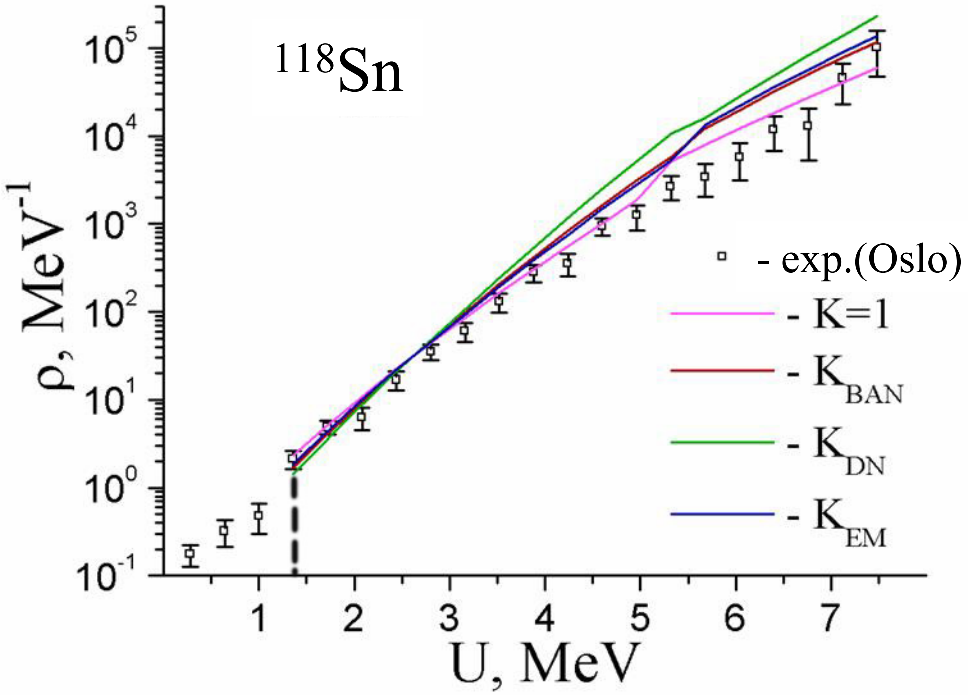
$$\rho(U,J) = \bar{\rho}_{EM}(U,J)\cdot K_{vibr} = \tilde{\Xi}\{\tilde{a},\tilde{\delta}_{shift}\}$$

Determination of $\tilde{a}, \tilde{\delta}_{shift}$

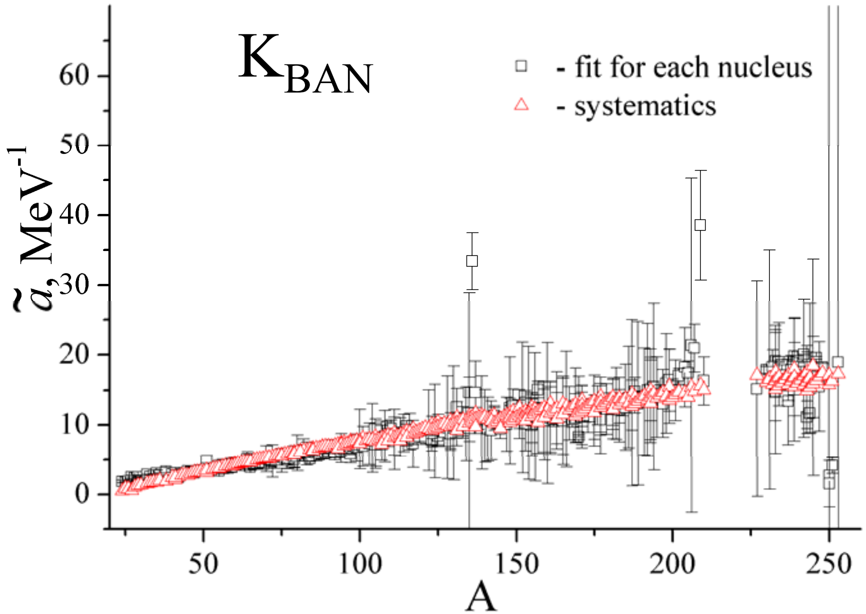
$\tilde{a}$ was found from fitting $D_0^{(theor)} = 1/\rho_{theor}(U = S_n)$ to $D_0^{(\exp)}$ with $\tilde{\delta}_{shift} = 0$

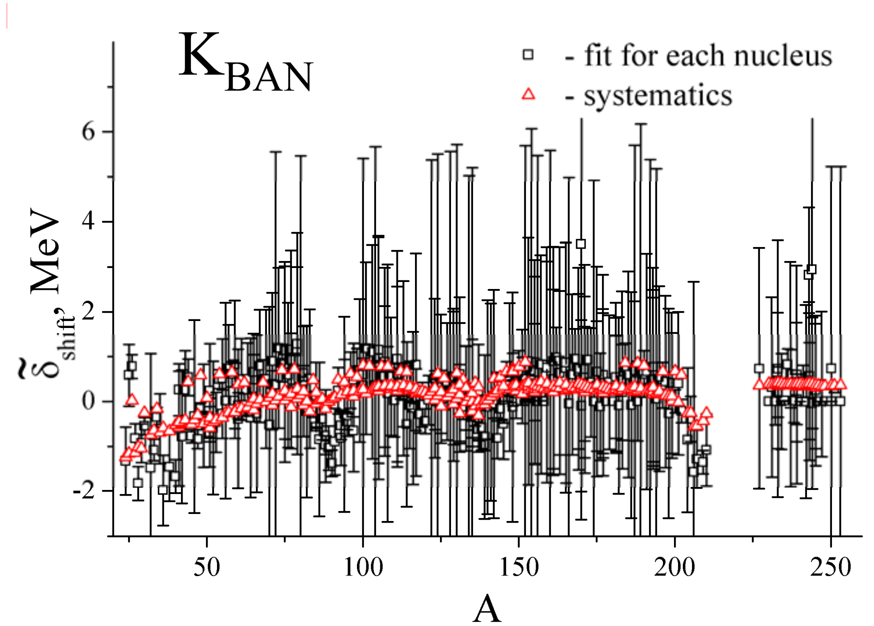
$\tilde{\delta}_{shift}$ => from fit $N_{cum}^{\exp}$ to $N_{cum} = \int_0^{U_{cum}} \sum_{J,\pi} \rho(U,J,\pi)\,\mathrm{d}U$ with previously determined $\tilde{a}$





Systematics of $\tilde{a}$ and $\tilde{\delta}_{shift}$





$$\tilde{a}(A,I) = 0.527(6)\,A(1+4.50(6)\,I^2) \; -1.17(2)\,A^{2/3}(1+13.3(2)I^2)$$
$$-0.0447(6)Z^2/A^{1/3} \;\; (MeV^{-1}), \qquad I=(N-Z)/A$$

$$\tilde{\delta}_{shift}(A,I) = 1.37(2)(1-11.7(5)I^2) + 0.00000003(1)A(1+76503(23974)I^2)$$
$$-0.697(9)E_{2_1^+} \quad (MeV),$$

Contributions of vibrational enhancement in continuum range

Comparisons of ratios of chi-square deviations

$$\Delta\chi^2 \equiv \sum_{i=1}^{N_{nucl}} \chi_i^2(K_{vibr}) / \sum_{i=1}^{N_{nucl}} \chi_i^2(K_{vibr}=1)$$

$$\chi_i^2(K_{vibr}) = \sum_{j=1}^{n_i} (\rho_{theor,i}(U_j) - \rho_{\exp,i}(U_j))^2 / n_i,$$

n_i – number of experimental data for nucleus i, N_{nucl} – number of nuclei

Data	$\Delta\chi^2$			
	K_{EM}	K_{DN}	K_{BAN}	K_{BANT}
Dubna	4.0	1.7	1.5	1.6
Obninsk	1.5	5.5	0.9	0.6
Oslo	1	0.9	1.0	0.9
average	2.2	2.7	1.1	1.0

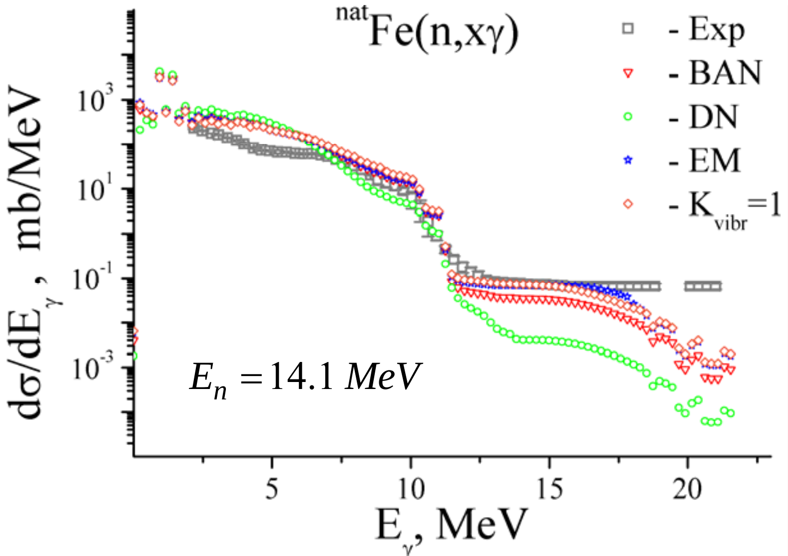
$$K_{BAN}(T_{\text{int}})$$

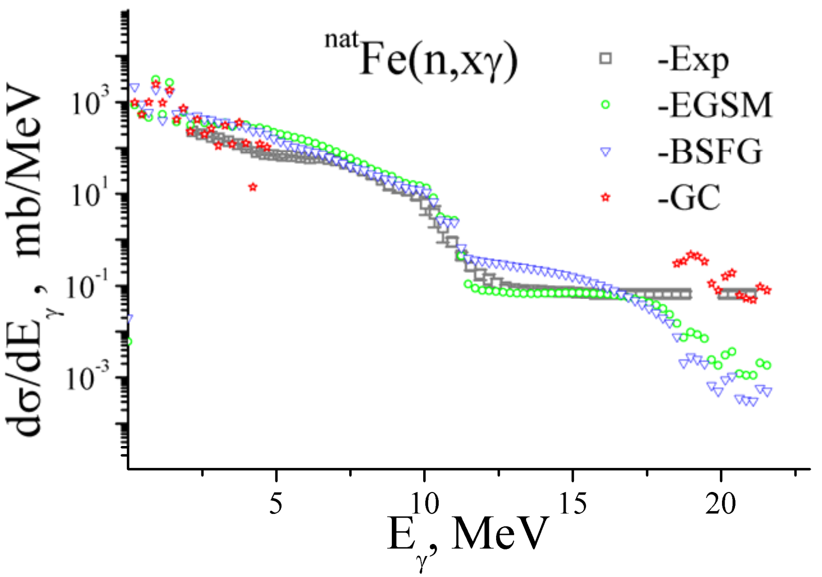
$$K_{BANT}(T = T_{\text{int}} + \delta T_{BSFG})$$

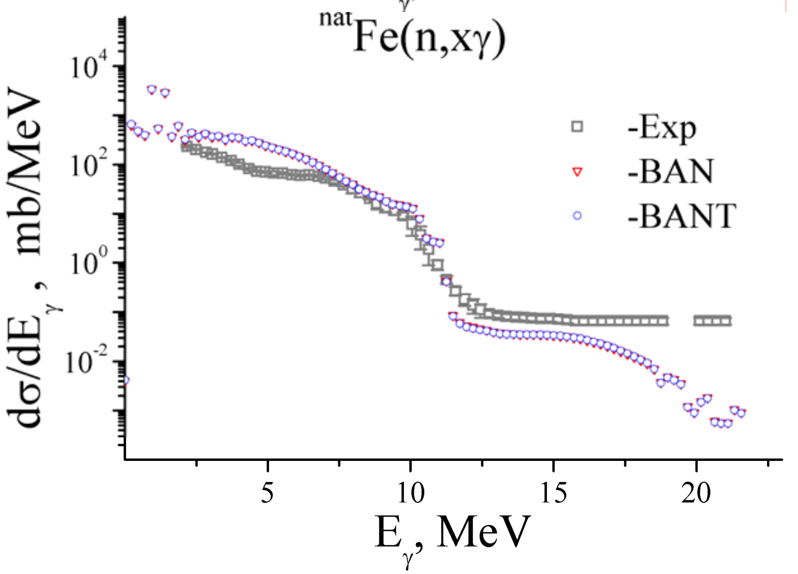
$$K_{vibr}(S_n) <\sim 2 \div 5 \qquad (A \sim 100)$$

Dubna - Sukhovoj A.M.et al., in Proc. of the ISINN-15, Dubna, May 2007, 92 (2007).
Obninsk –Zhuravlev B.V., IAEA, INDC(NDS)-0554, Distr. G+NM, (2009).
Oslo - Agvaanluvsan U. et al., Phys. Rev.C 70, 054611 (2004); http://ocl.uio.no/compilation/.

Effect of vibrational states on gamma-ray spectra







Calculations - EMPIRE 3.1; $E_n = 14.1\ (MeV)$
Experimental data - Bondar V.M., et al. // Proc. of the ISINN-18, Dubna, May 26-29, 2010, 135 (2011)

The scatter of gamma-ray spectra calculated within EGSM for intrinsic and rotational level densities with different K_{vibr} are the same order as scatter of the spectra calculated using different models of NLD

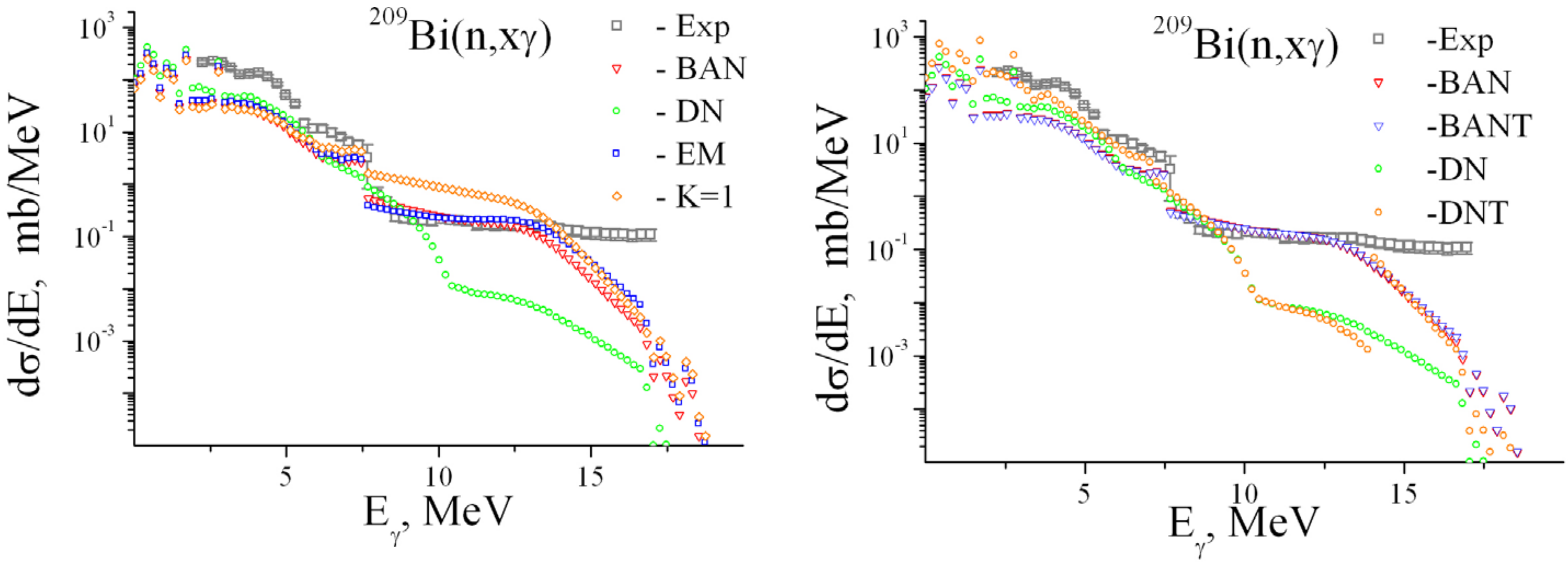
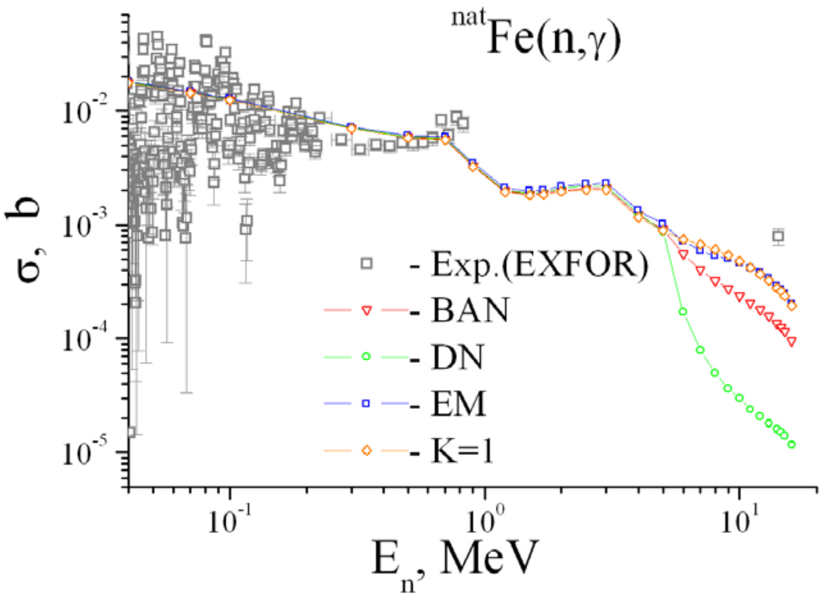
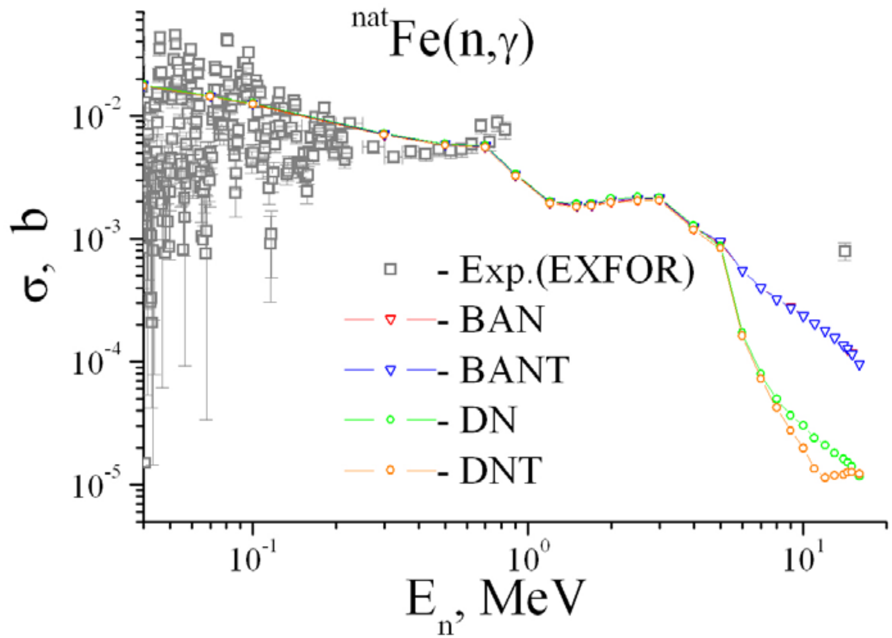


Fig. Dependence of $d\sigma / dE$ on gamma-ray energy for $^{209}\mathrm{Bi}(n, x\gamma)$ reaction.
Calculation was made with the use of the EMPIRE 3.1 code at $E_n = 14.1\ (MeV)$
Experimental data are taken from: [Bondar V.M., Gorbachenko O.M., Kadenko I.M., Leshchenko B.Yu., Onischuk Yu.M., Plujko V.A. // Nuclear Physics and Atomic Energy. – Vol.11 №3. (2010) 246-251.]

The temperature change by the vibrational states can effect on gamma-gay spectra

Effect of vibrational states on excitation function





Calculations- EMPIRE 3.1; experimental data - EXFOR

Results of calculations of excitation functions and gamma-gay spectra are sensitive to vibrational enhancement

SUMMARY

• NLD within modified EGSM (EGSM for shape of intrinsic and rotational level densities with different vibrational enhancement factors) was studied.

• For modified EGSM, approximation of boson partition function with average occupation numbers (BAN) can be considered as the most appropriate approach for calculation of the vibrational enhancement factor and within BAN

$$K_{vibr}(S_n) <\sim 2 \div 5 \qquad (A \sim 100)$$

•Shape and values of excitation function and gamma-ray spectra are sensitive to choice of vibrational enhancement factor.

• The scatter of gamma-ray spectra calculated within EGSM for intrinsic and rotational NLD and different K_{vibr} are the same order as scatter of the spectra calculated using different NLD models.