Goos – Hänchen effect in neutron optics

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Goos – Hänchen effect–
Longitudinal shift of the wave beam at total reflection

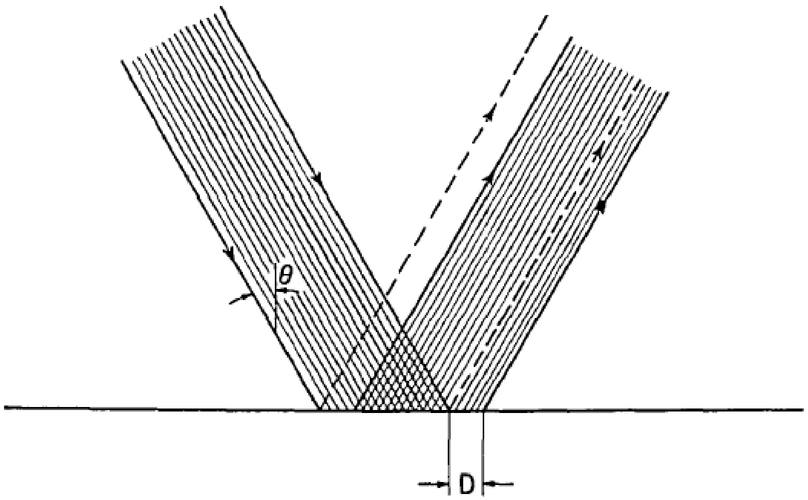
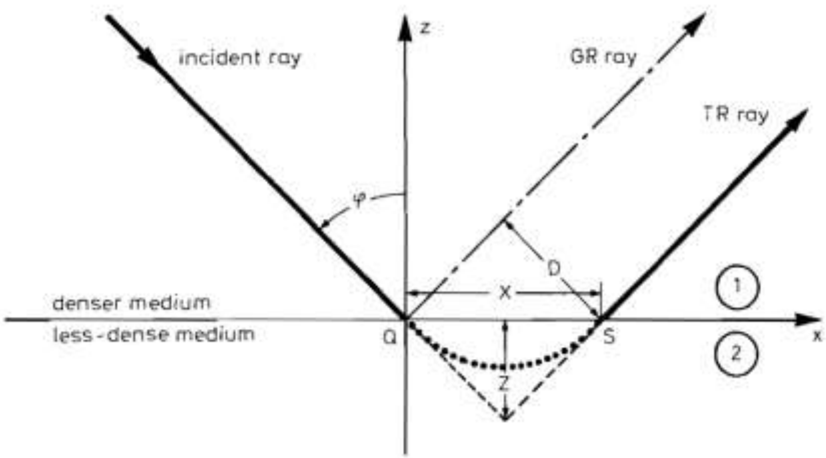


FIG. 1. Lateral displacement of actual reflected beam (solid lines) relative to geometrically reflected beam (dashed lines). The upper medium is the denser medium.



Total inner reflection

First observation of the G.-Ch. effect at total reflection 1947-49 yrs.

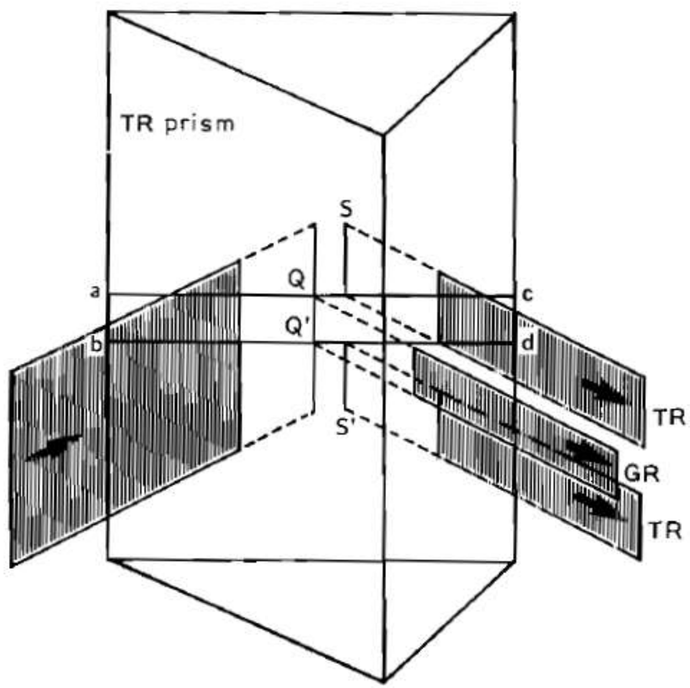


Fig. 2. Schematical illustration of the historical experiment devised by *Goos* and *Hänchen* [1a]. The strip ab-cd on the back side of the totally reflecting (TR) prism is silvered. The beam incident from the left-hand side, therefore, produces totally reflected (TR) beams as well as a geometrically reflected (GR) beam due to the metallic reflection.

F. Goos und O. Hänchen, Ann. der Phys. 1, 333 (1947).
F. Goos und H. Lindberg-Hanchen, Ann. der Phys. 5, 251 (1949)

Goos-Hänchen effect in acoustic

A. Schoch, Acustica 2, 1 (1952)

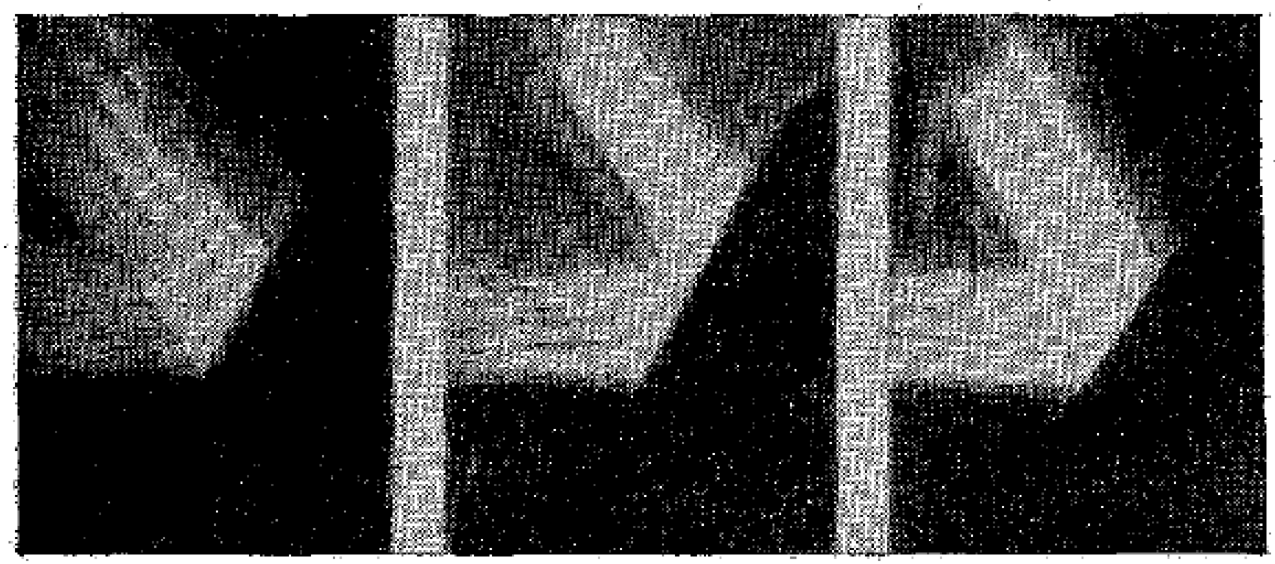
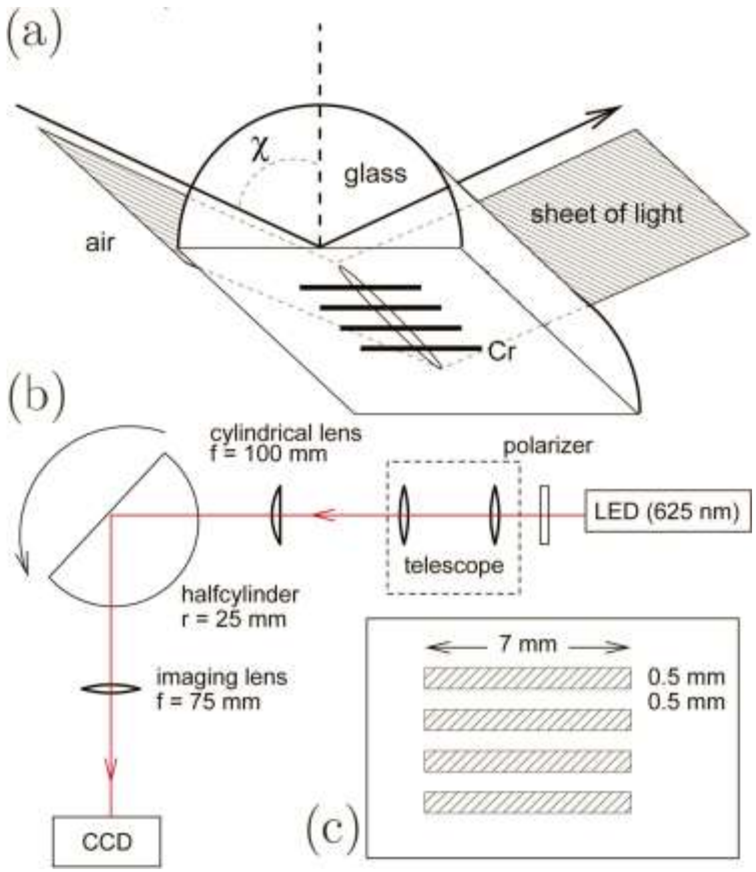


Рис. 4. Смещение ультразвукового пучка при отражении его от границы ксилол-алюминий для частоты $5{,}6\cdot10^6$ *гц* и при трёх различных углах падения.

From: L.M. Brechovskikh, Usp. Fiz. Nauk [Sov.Phys. Uspechy] 50, 539 (1953)

Modern optical experiment for the observation G.-H. effect



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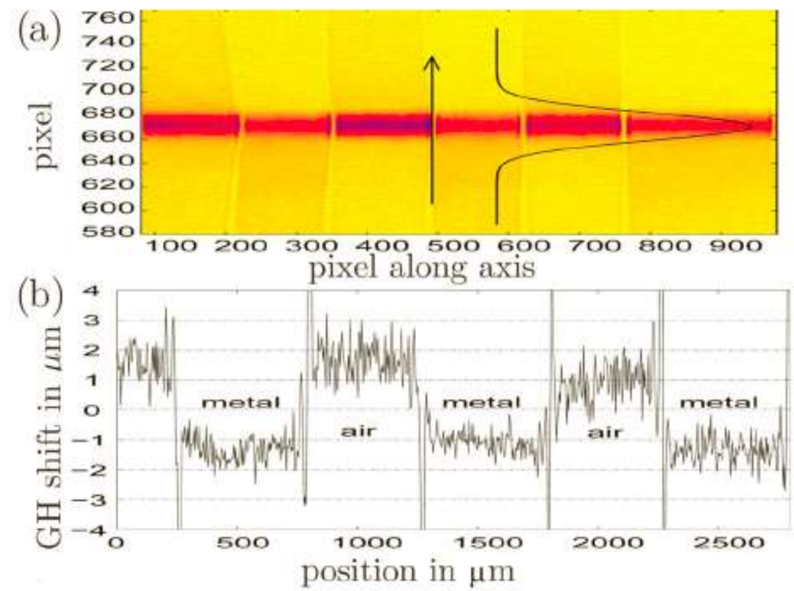


Fig. 2. (Color online) (a) CCD image for a TE polarized light incident at 39°. The stripes correspond to different reflection amplitudes at the glass–air and glass–metal interface. The arrow shows the direction in which the data are fitted by a Gaussian distribution. (b) Peak position of the fitted Gaussian with respect to the height on the cylinder (metal–air indicates the reflecting interface).

G.-H. shift and neutron optics

1. A.A. Seregin. *Surface shift of neutron at reflection*. Yadernaya Physica [Sov. Journ. Nuclear Physics] 33, 1173 (1981). **First proposal**

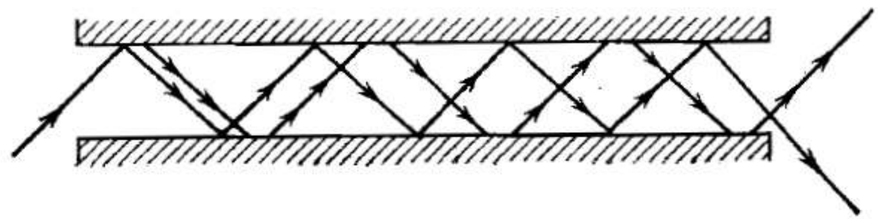


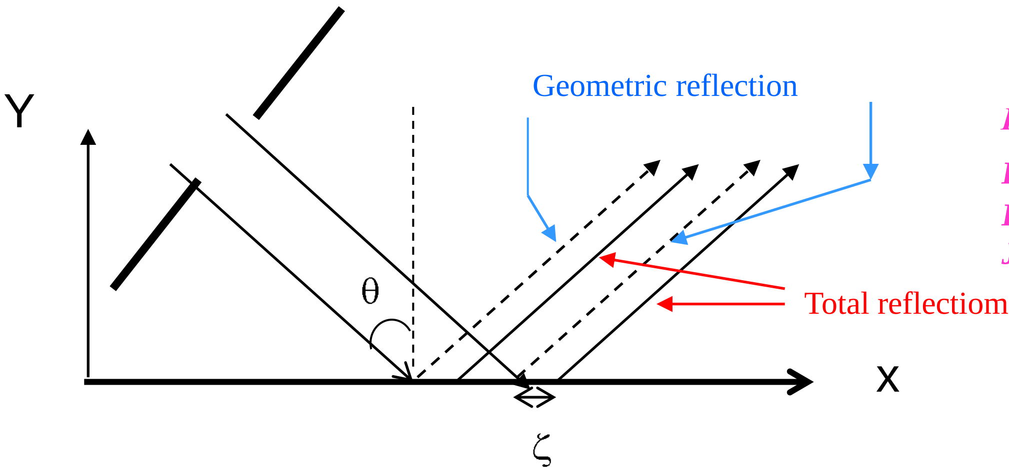
Рис. 2. Схематическое изображение суммирования поверхностных смещений в результате многократных отражений

2. M. Maaza, B.Pardo. *On the possibility to observe the longitudinal Goos-Hänchen shift with cold neutrons*. Opt.Comm. 142, 84 (1997). **(Proposal and calculations based on the Renard's theory)**

3. V.K. Ignatovich. *Neutron reflection from condensed matter, the Goos–Hänchen effect and coherence.* Phys. Lett. A, 36, 322 (2004). **(Theory)**

4. V. -O. de Haan, J.Plomp, Th. M. Rekveldt, W. H. Kraan, and Ad A. van Well. *Observation of the Goos-Hänchen Shift with Neutrons.* Phys.Rev.Lett. 010401 (2010). **(Observation of the pseudo Larmor precession)**

G.-Ch shift and the stationary phase principle



K. Artmann, Ann. der Phys. 2, 87 (1948)
L.M. Brechovskikh, Usp. Fiz. Nauk 50, 539 (1953)
H. Hora, Optik 17, 409 (1960)
J. L. Carter and H. Hora. J. Opt. Soc. Am, 61, 1640, (1971)

Two waves are falling at angles θ and θ+Δθ at the border of matter Z=0

X – component of the reflected wave function in XZ plane is a superposition

$$\exp\left[i(k_x x + \varphi)\right]\left\{1 + \exp\left[i\left(\Delta k_x x + \Delta\varphi\right)\right]\right\}$$

Maximum of intensity

$$\Delta k_x x + \Delta\varphi = 2\pi$$

Geometric reflection

$$\varphi = 0, \quad \Delta\varphi = 0 \qquad \Delta k_x x_0 = 2\pi$$

$$\varsigma = x - x_0 = -\frac{\Delta\varphi}{\Delta k_x} \qquad \Delta\theta \to 0$$

$$\varsigma = -\frac{d\varphi}{dk_x}$$

Artmann's formula

G.-Ch shift the matter wave and its relation with the group delay time at reflection of particle

$$\varsigma = -\frac{d\varphi}{dk_x}$$

$$\varsigma = -\frac{d\varphi}{dk_x} = -\frac{d\varphi}{dE_\perp}\frac{dE_\perp}{dk_x}$$

$$E_\perp = \frac{\hbar^2}{2m}k_y^2 = \frac{\hbar^2}{2m}\left(k^2 - k_x^2\right)$$

$$\hbar\frac{d\varphi}{dE_\perp} = \tau$$

$$\frac{dE_\perp}{dk_x} = -\frac{\hbar^2}{m}k_x$$

$$\varsigma = \frac{\hbar}{m}k_x\tau = v_x\tau$$

$$\tau = \hbar\frac{d\varphi}{dE_\perp}$$

Group delay time

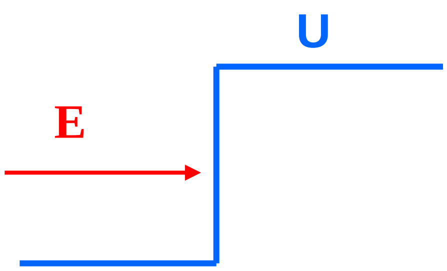
(Bohm, Wigner, 1952-55)

Delay time and the G.-Ch shift at reflection from the potential barrier

$$\varsigma = \frac{\hbar}{m} k_x \tau = v_x \tau$$

$$\tau = \hbar \frac{d\varphi}{dE_\perp}$$

$$\tau = \frac{\hbar}{\sqrt{E_\perp (U - E_\perp)}}$$



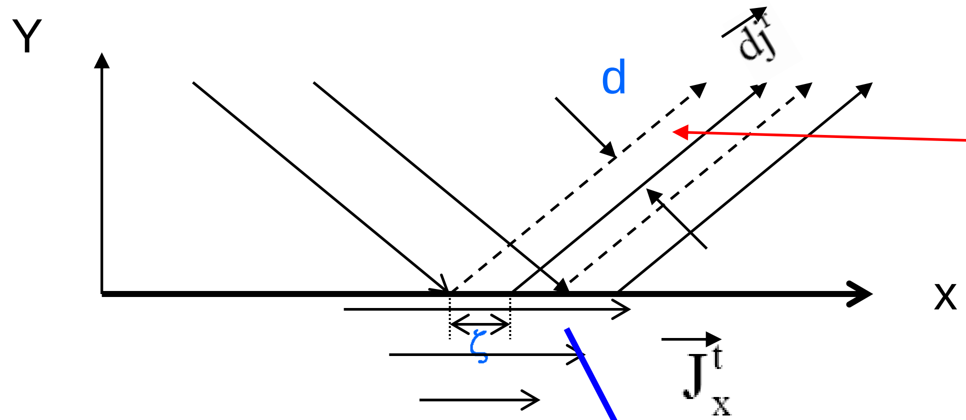
$$k_c^2 = \frac{2m}{\hbar^2} U$$

$$\varsigma = \frac{k_x}{m} \frac{\hbar^2}{\sqrt{E_\perp (U - E_\perp)}}$$

$$\varsigma = \frac{2k_x}{k_y \sqrt{k_c^2 - k_y^2}}$$

Renard's theory

R. H. Renard. Total Reflection: *A New Evaluation of the Goos-Hanchen Shift*. J. Opt. Soc. Am. 54, 1190 (1964)



$$J_x^t = d \cdot j^r$$

$$\psi^t(x,y) = t\exp\left[i\left(k_x x + k_y^t y\right)\right] \qquad k_y^t = i\sqrt{k_c^2 - k_y^2}$$

$$t = \frac{2k_y}{k_y + k_t} \qquad |t|^2 = \frac{4k_y^2}{k_c^2} \qquad J_y^t = 0$$

$$J_x^t = v_x \int_0^\infty \left|\psi^t(y)\right|^2 dy \qquad J_x^t = \frac{\hbar}{m}\frac{k_x}{\sqrt{k_c^2 - k_y^2}}\frac{2k_y^2}{k_c^2} \qquad d = \frac{k_x}{k_c^2 k_0}\frac{2k_y^2}{\sqrt{k_c^2 - k_y^2}}$$

$$\varsigma = \frac{k_x}{k_c^2}\frac{2k_y}{\sqrt{k_c^2 - k_y^2}}$$

Comparing of the result of Artmann-Hora-Brechovskikh with Renards'result

Artmann-Hora-Brechovskikh

$$\varsigma = \frac{2k_x}{k_y\sqrt{k_c^2 - k_y^2}}$$

$$k_c^2 = \frac{2m}{\hbar^2}U \qquad k_x = k_{\parallel} \qquad k_y = k_{\perp}$$

$$\varsigma = \hbar\frac{k_x}{m}\frac{\hbar}{\sqrt{E(U-E)}}$$

$$\varsigma = v_x\frac{\hbar}{\sqrt{E(U-E)}} = v_x\hbar\frac{d\varphi}{dE_{\perp}} = v_x\tau$$

$$\tau = \frac{\hbar}{\sqrt{E(U-E)}}$$

Renard

$$\varsigma_{Ren} = \frac{k_x}{k_c^2}\frac{2k_y}{\sqrt{k_c^2 - k_y^2}}$$

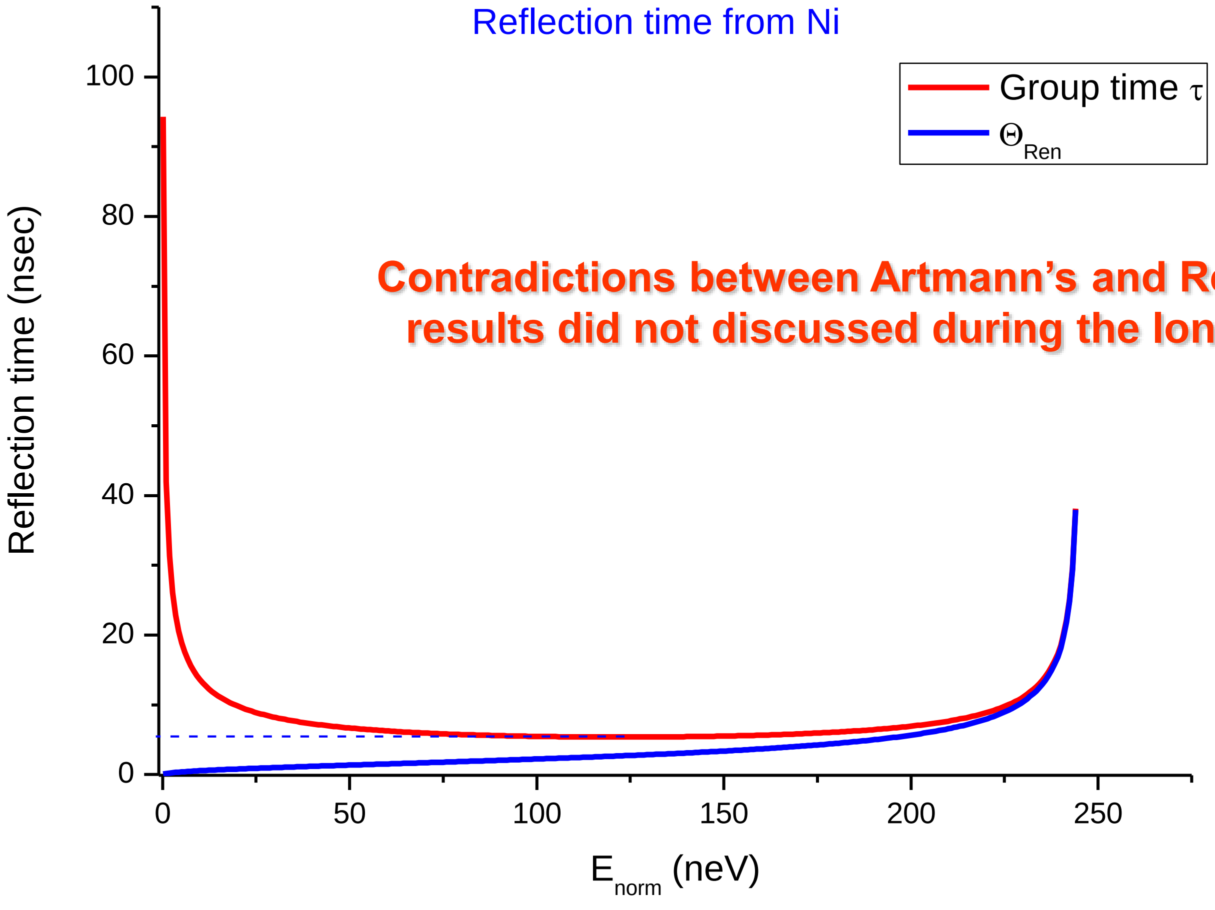
$$\varsigma_{Ren} = \hbar\frac{k_x}{m}\frac{\hbar\sqrt{E}}{U\sqrt{(U-E)}}$$

$$\varsigma_{Ren} = v_x\hbar\frac{\sqrt{E}}{U\sqrt{(U-E)}} = v_x\theta$$

$$\theta = \hbar\frac{\sqrt{E}}{U\sqrt{(U-E)}} \neq \hbar\frac{d\varphi}{dE_{\perp}}$$

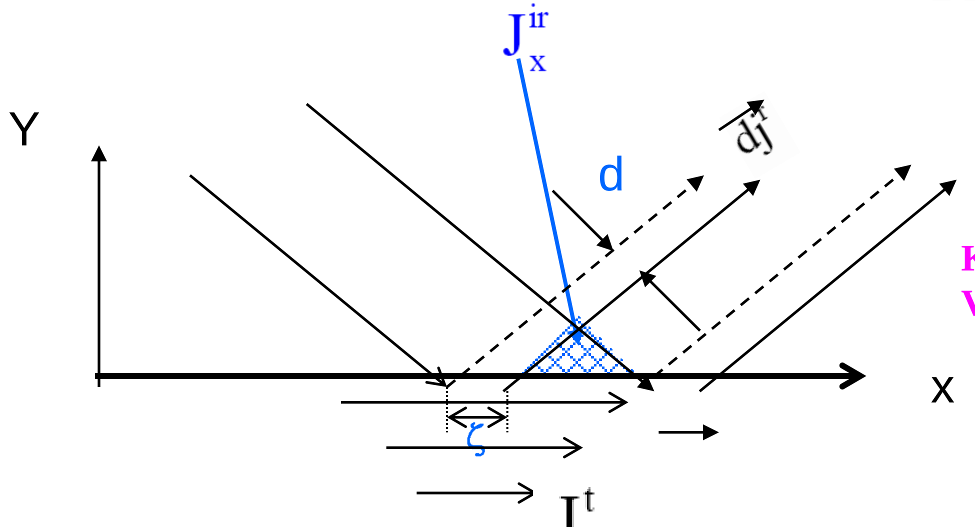
Group delay time at reflection from barrier

Group delay time at reflection τ and effective time θ, following from the Renard's formlula



Contradictions between Artmann's and Renard"s results did not discussed during the long time

The reason of contradiction and correction to the Renard's approach



$$J_x^t = d \cdot j^r$$

K. Yasimoto, Y.Oishi. J.Apll. Phys. 54, 2170 (1983)
V. G. Fedoseyev J. Opt. Soc. Am. A 3, 826, (1986)

$$J_x^t + J_x^{ir} = d \cdot j^r$$

$$\psi_{ir}(x,y) = \exp\left[i\left(k_x x + k_y y\right)\right] + r\exp\left[i\left(k_x x - k_y y\right)\right] \qquad r = e^{i\varphi}$$

$$j_x^r = 2v_x\left[1 + \cos\left(2k_y y - \varphi\right)\right] \qquad J_x = J_x^t + J_x^{ir} = \frac{\hbar}{m}\frac{2k_x}{\sqrt{k_c^2 - k_y^2}} \qquad d = \frac{2k_x}{k_0\sqrt{k_c^2 - k_y^2}}$$

$$\varsigma = \frac{2k_x}{k_y\sqrt{k_c^2 - k_y^2}}$$

Artmann-Hora formula

Since Atrmann's and Renard's approaches (with correction of Yasimoto-Oishi-Fedoseev) lead to the identical result

$$\varsigma = -\frac{d\varphi}{dk_x} \rightleftarrows \varsigma = \frac{2k_x}{k_y\sqrt{k_c^2 - k_y^2}}$$

We can believe that our conclusion concerning the relation of Goos-Hänchen shift with group delay time is correct

$$\varsigma = -\frac{d\varphi}{dk_x} \Rightarrow \varsigma = \frac{\hbar}{m}k_x\tau = v_x\tau$$

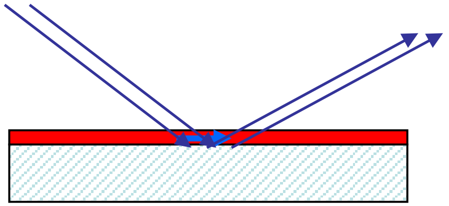
Giant G.-Ch. shift at reflection from multilayred structures

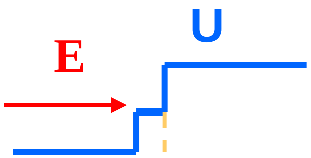
T. Tamir, H.L. Bertoni, J.Opt.Soc.Am. 61, 1397 (1971)

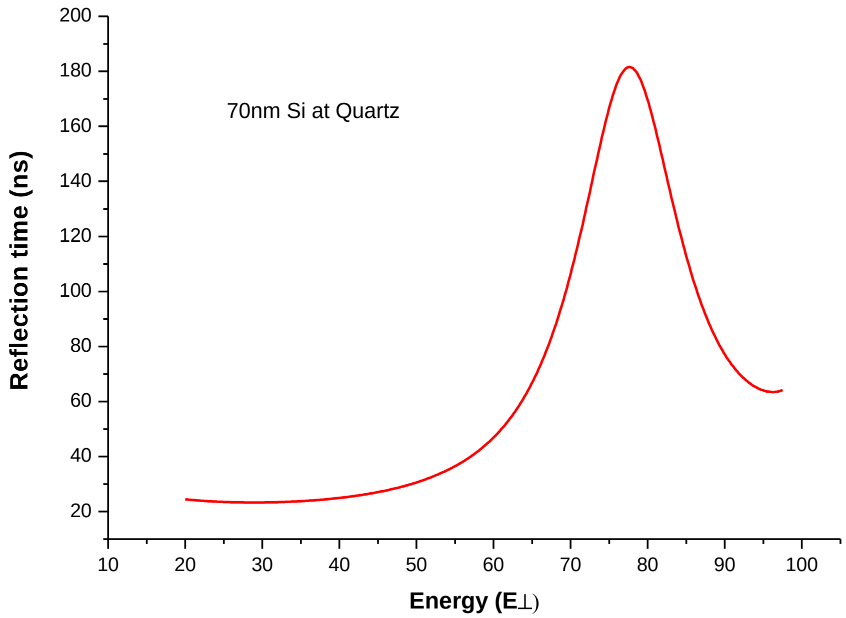
Giant G-Ch. shift of the neutron beam

A films at the wafer (total reflection)

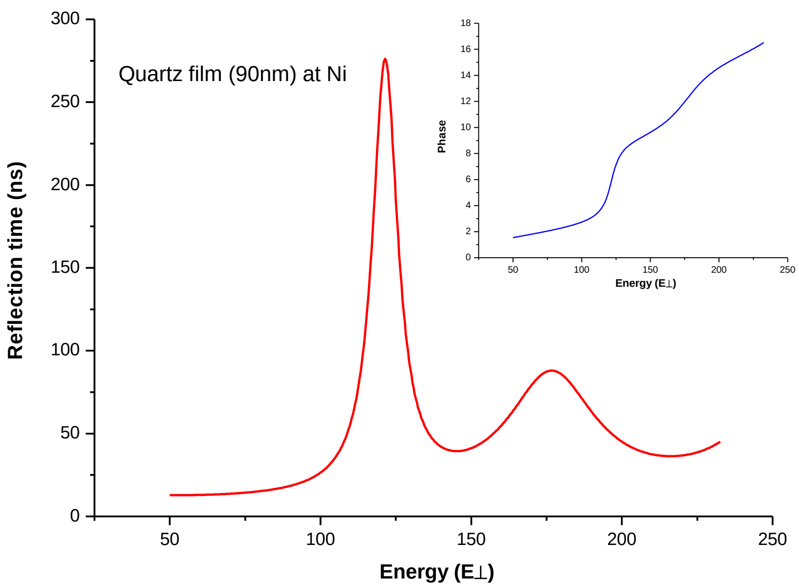
T. Tamir, H.L. Bertoni, J.Opt.Soc.Am. 61, 1397 (1971)
V.Ignatovich, 2004







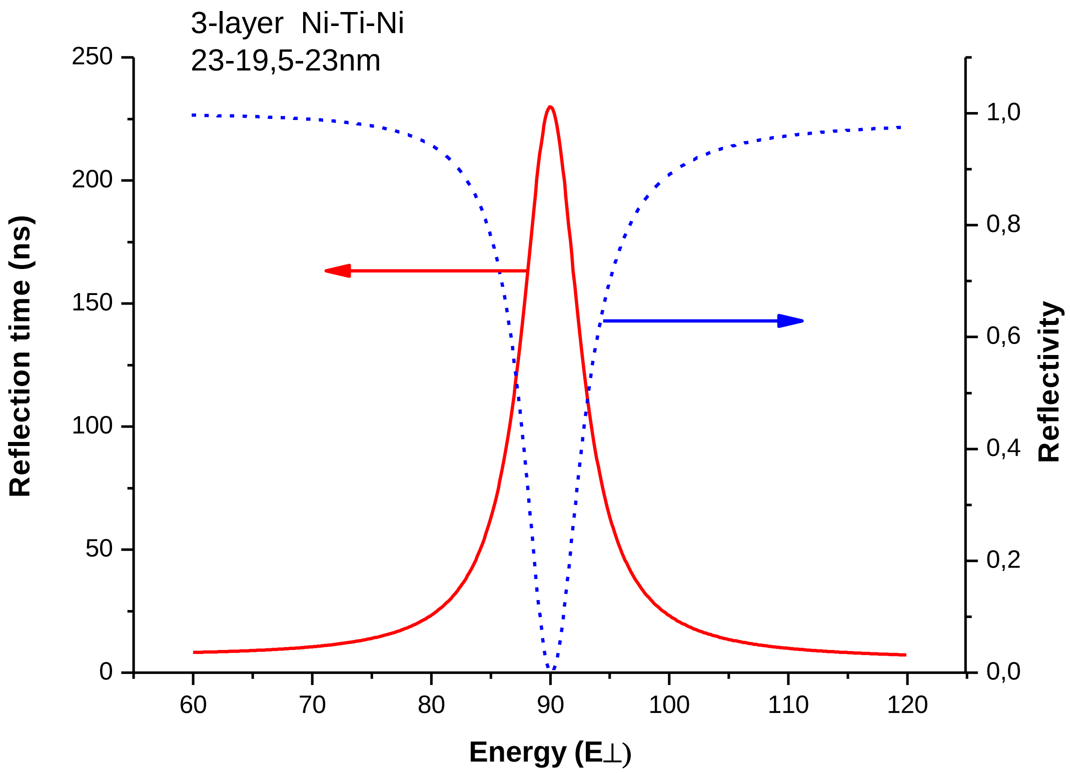
Reflection time at neutron reflection from Si film (60nm) deposed at the quartz substrate (ns)

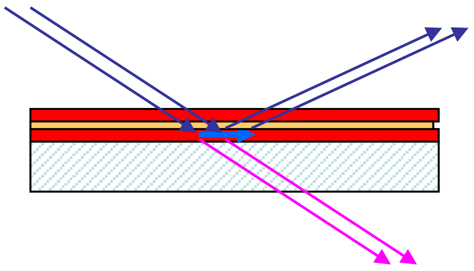


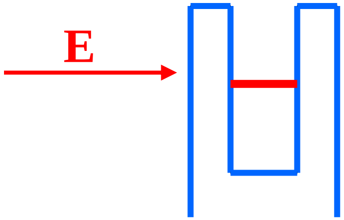
Reflection time at neutron reflection from quatz film (90nm) deposed at Ni substrate
The inset shows the phase of the reflected wave

Giant longitudinal shift

Fabry-Perrot interferometer (Neutron Interference filter)







Reflection time and reflection coefficient for neutron reflection from the three layered resonant structure NiMo-Ti-NiMo

Negative longitudinal shift

Negative longitudinal shift in optics and acoustics

1. *T. Tamir, H.L. Bertoni, J. Opt. Soc. Am. 61 1397(1971)*
2. *M. A. Breazeale and M. A. Torbett. Appl. Phys. Lett. 29 456 (1976)*

A. Aniĉin, R. Fazlić and M. Koprić . J. Phys. A: Math. Gen. 11, 1657 (1978).

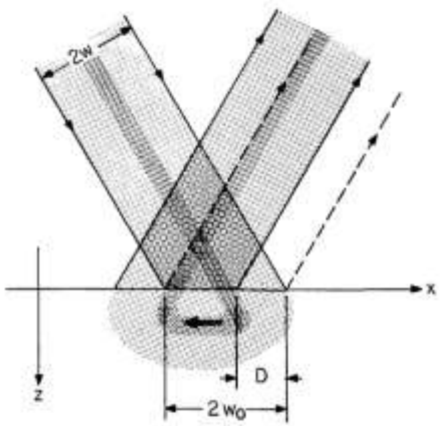


FIG. 8. Lateral beam shift due to reflection by a backward leaky-wave structure. The thick arrow indicates the direction of energy flow within the leaky-wave structure in the region z>0; the dashed lines show the reflected beam predicted by geometrical optics.

T. Tamir, H.L. Bertoni, 1971

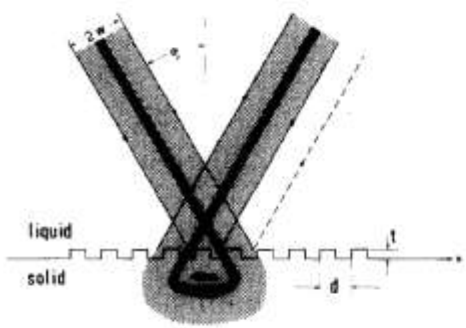


FIG. 1. Diagram of incident beam coupling to a backward-directed leaky wave to produce backward displacement of reflected beam.

FIG. 2. Backwared displacement of 6-MHz ultrasonic beam at a water-brass grating interface.

M. A. Breazeale and M. A. Torbett, 1976

An example of the negative longitudinal shift detection in optics

1432 OPTICS LETTERS / Vol. 32, No. 11 / June 1, 2007

Observation of large positive and negative lateral shifts of a reflected beam from symmetrical metal-cladding waveguides

Lin Chen,* Zhuangqi Cao, Fang Ou, Honggen Li, Qishun Shen, and Huicong Qiao

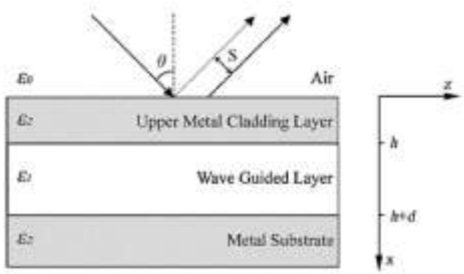
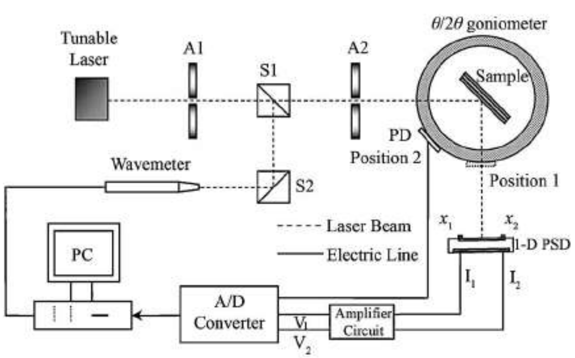


Fig. 1. Structure of the SMCW.



(a)

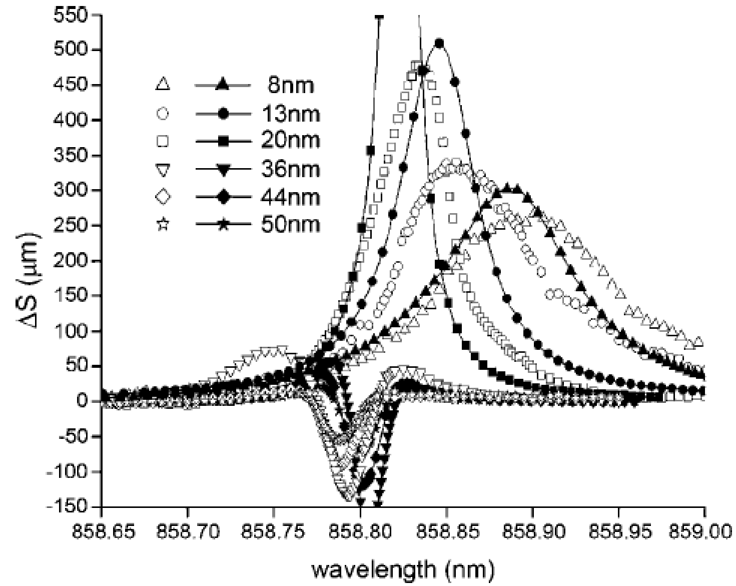
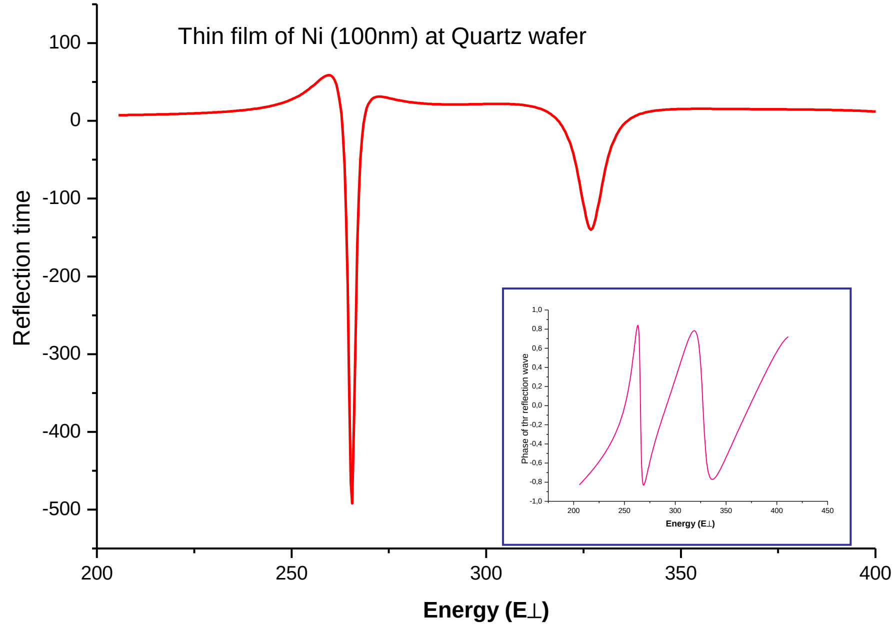
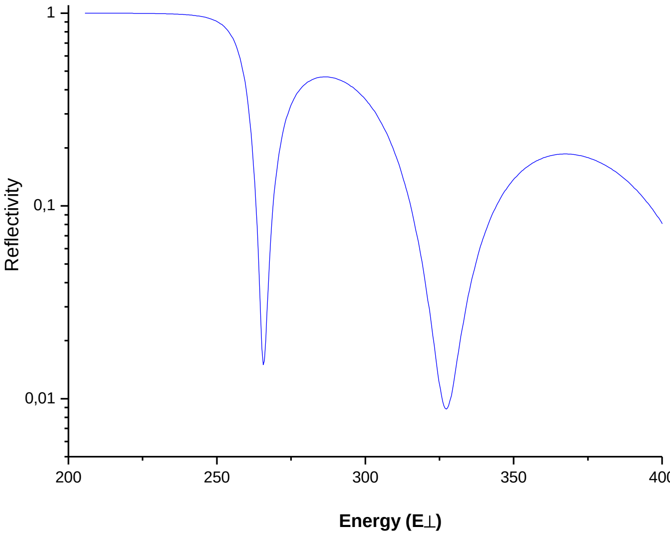


Fig. 5. Contrastive graph of relative theoretical (line +scatter) and experimental (circles) displacement ΔS versus wavelength with various h. The parameters are as follows: $\theta=8.11°$, glass slab ($\epsilon_1=2.278$ $d=0.38$ mm), gold film ($\epsilon_2=-28+1.8i$), waist radius 800 μm.

Negative group time and negative longitudinal shift

Film at the substrate (above barrier reflection)



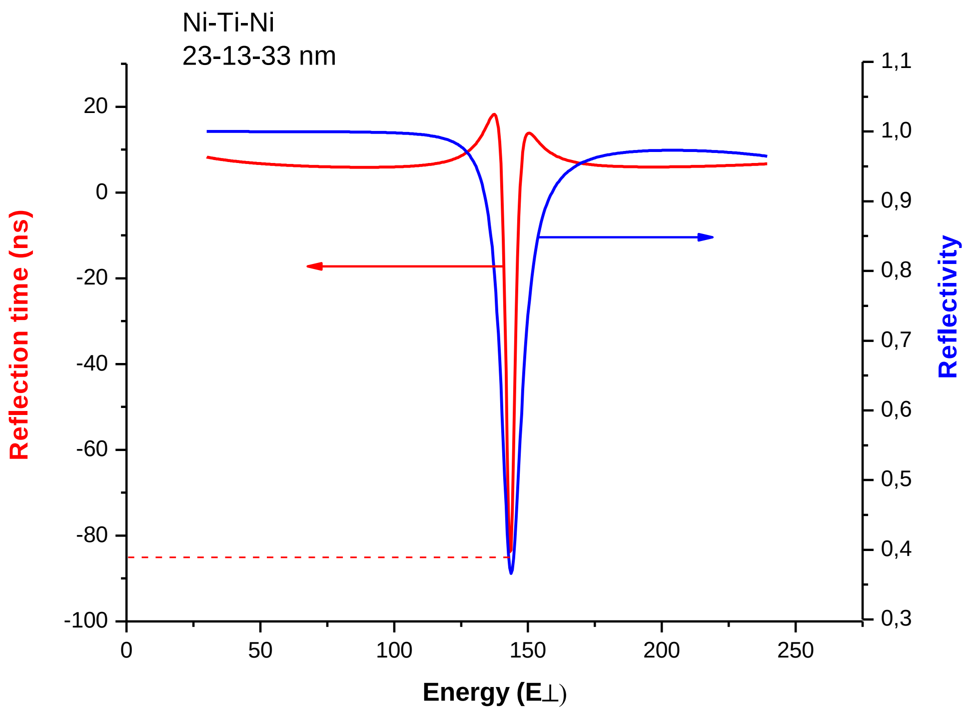


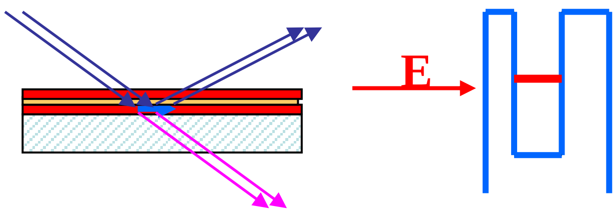
Reflection time for the Ni film (100nm) deposed at the quartz wafer. The inset shows the phase of the reflected wave.

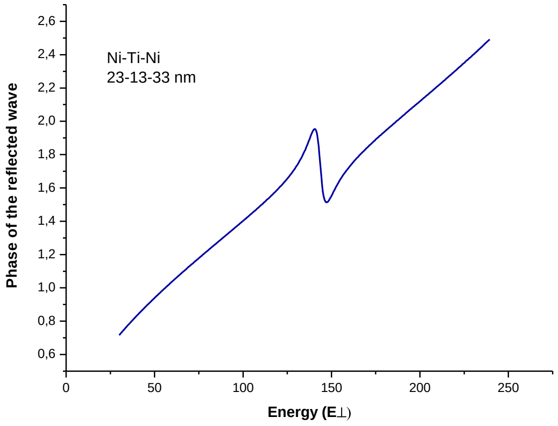
Coefficient of reflection for the Ni film (100nm) deposed at the quartz wafer.

Negative group time and negative lateral shift

Asymmetric interference filter







Phase of the reflection wave

Group reflection time and reflection coefficient for the asymmetric three-layered resonant structures NiMo-Ti-NiMo

On the measurement of the group reflection time: Larmor clock

Reflection of sate with precessing spin (in magnetic field B).

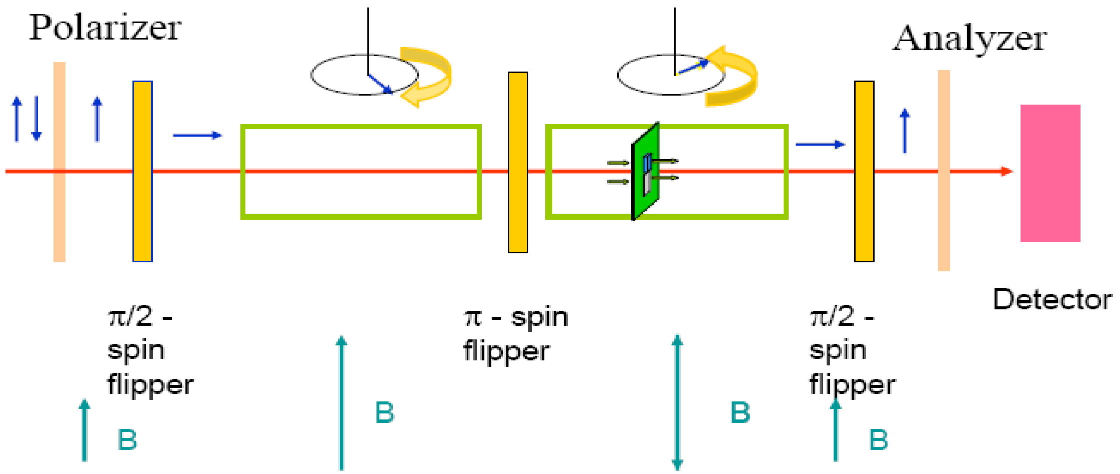
$$\phi = \omega_L \tau \qquad \tau = \frac{\Delta\phi}{\omega_L} = \hbar\frac{\Delta\phi}{2\mu B} \qquad 2\mu B = \frac{\hbar^2}{2m}\left(k_+^2 - k_-^2\right) = \Delta E$$

$$\tau = \hbar\frac{\Delta\phi}{2\mu B} = \hbar\frac{\Delta\phi}{\Delta E} \longrightarrow \hbar\frac{d\phi}{dE}$$

In the limit $B, \ \Delta E \to 0$

$$\tau = \frac{\Delta\phi}{\omega_L} = \hbar\frac{d\varphi}{dE}$$

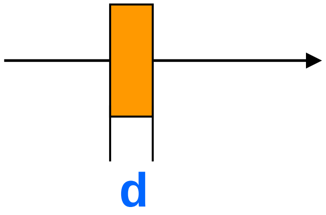
Group reflection time



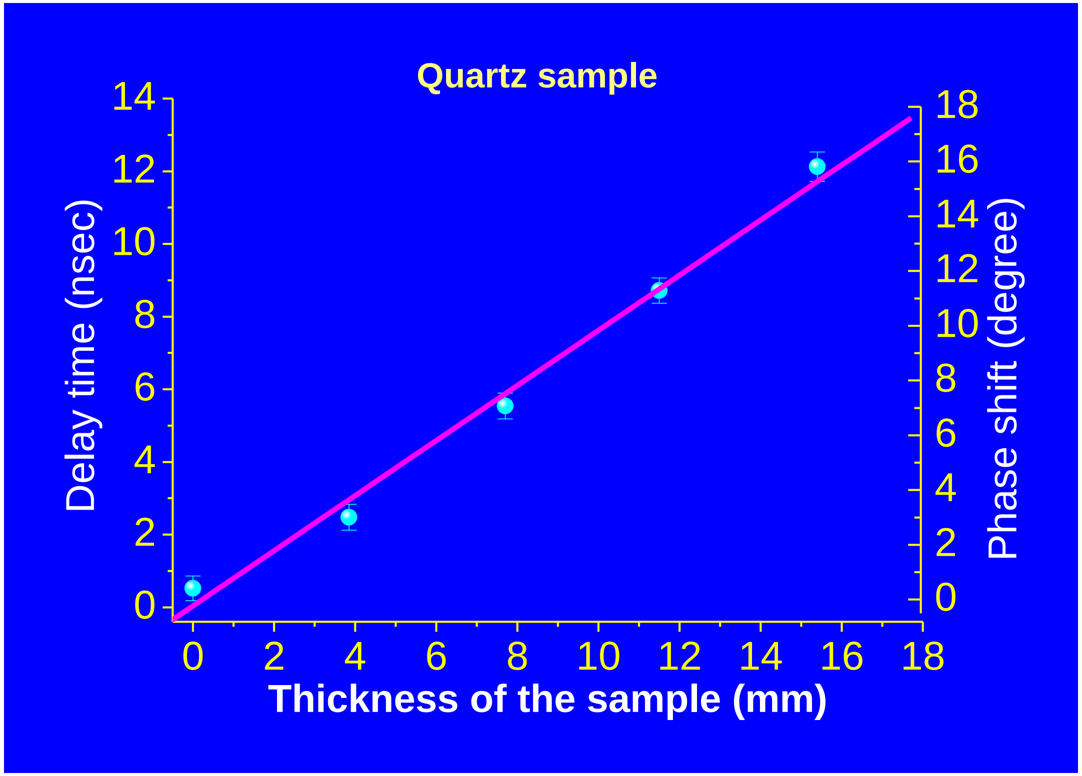
Baz' 1967
A.I. Frank et al.. 2001

Delay time or precession angle was measured

a) for the case of refraction



$$\Delta t = \left(\frac{d}{nv} - \frac{d}{v} \right) = \frac{d}{v} \left(\frac{1-n}{n} \right)$$

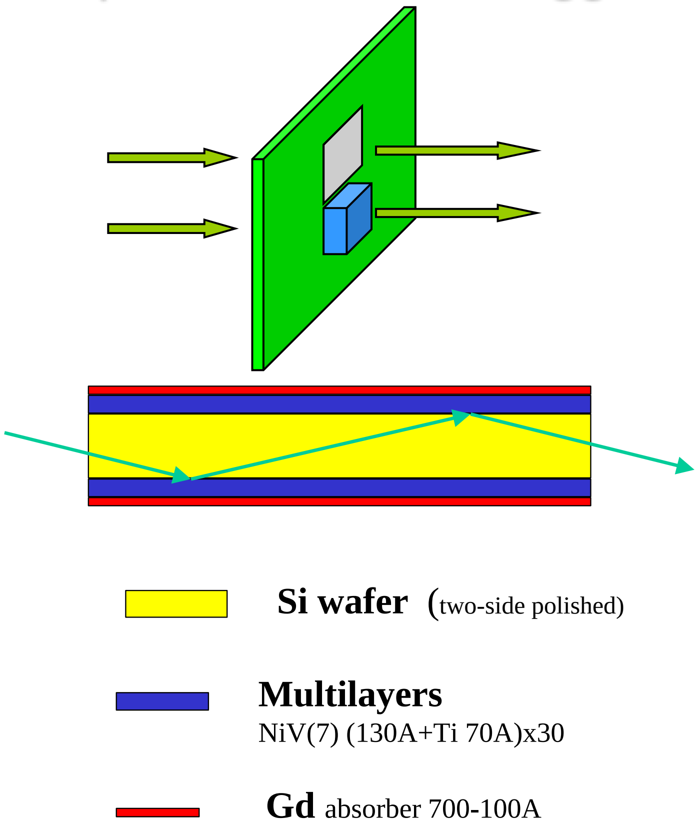


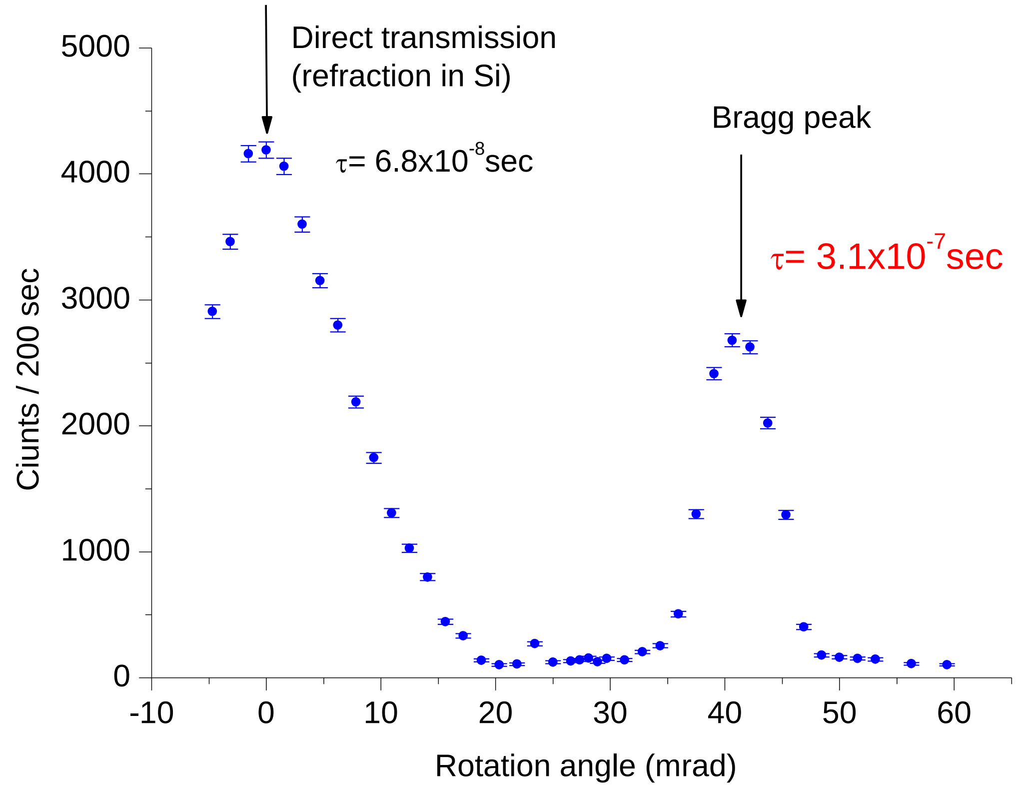
$$\frac{\Delta t}{t} \approx 10^{-8}$$

A.I. Frank, I.V. Bondarenko, A.V. Kozlov, et al.. Physica B 297, (2001) 307

Delay time or precession angle was measured

b) for the case of Bragg reflection

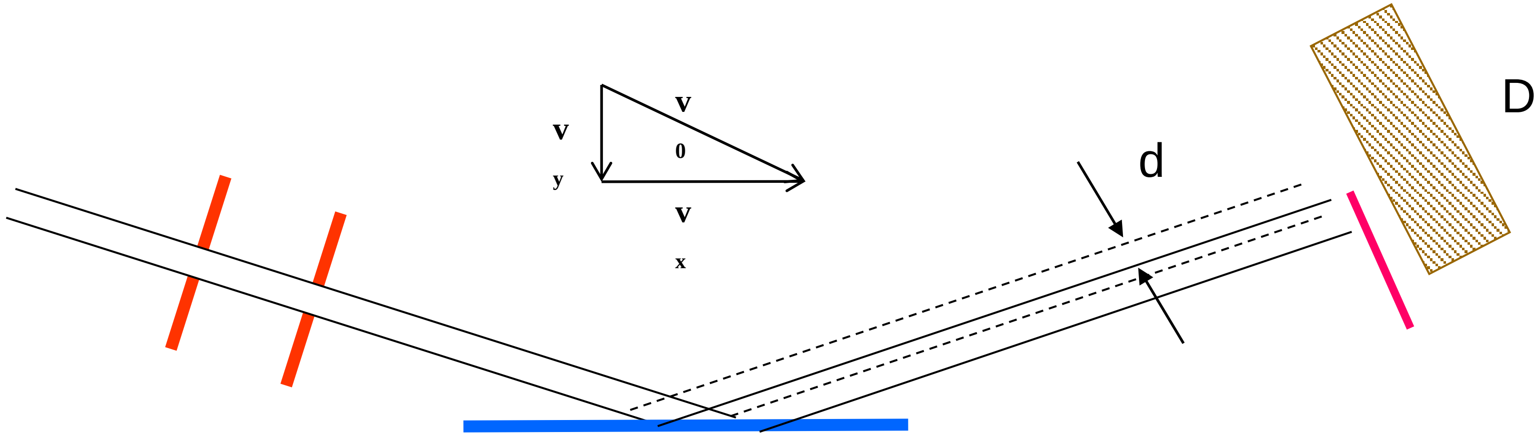




A.I. Frank, I.Anderson, I.V. Bondarenko, et al. *Phys.of At. Nuclei, 65, 2009 (2002)A.I.Frank, I.V.Bondarenko, A.V.Kozlov, G.Ehlers and P. Høghøj. In: Neutron Spin Echo Spectroscopy, Eds. F.Mezei, C.Pappas and T.Gutbertlet. Springer, pp164-175.*

To the measurement of the giant G.-Ch. shift.
Time of flight reflectometry

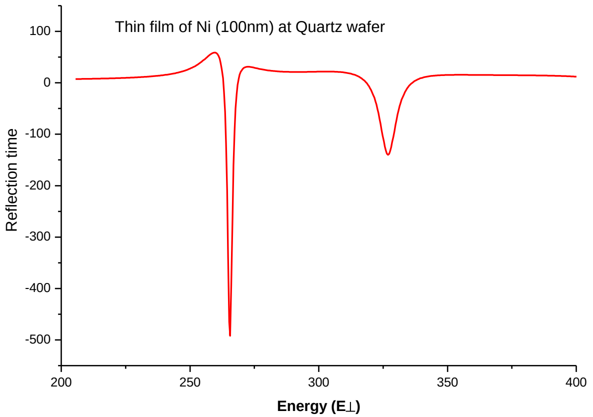
Neutron time of flight reflectometer



$$\varsigma = V_x \tau \qquad d = \varsigma \frac{V_y}{V_0}$$

$$d = \frac{V_x V_y}{V_0}\tau = \frac{V_y}{\sqrt{1+V_y^2/V_x^2}}\tau$$

$$V_x >> V_y \qquad d \approx V_y \tau$$



$V_y \approx 7$ m/s $\qquad \tau \approx 400$ns $\qquad d \approx 2.8$ mkm

Resonant character of the effect makes possible to extract signal in reflectometric time of flight experiment

Conclusion

1. Longitudinal Goose – Chänchen shift always proportional to the reflection group delay time
2. Group delay time of the neutron wave from the multilayered structures may exceed the total reflection time by two order of magnitude and (or) may be negative
3. The method of the measurement of group delay time, Larmor clock is exist.
4. Giant and negative G. – Ch. shift are essentially resonant phenomena and probably may be detected in time of flight reflectometric experiment

Thank you for your attention