Abnormal neutron dispersion in crystal close to Bragg reflex

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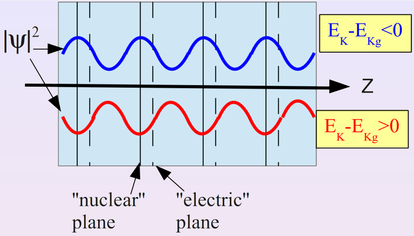
Neutron close to Bragg reflex



Value of electric field

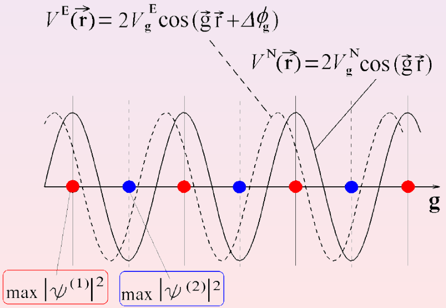
$$E =< \psi|E(\mathbf{r})|\psi >= E_g \frac{2|V_g^N|}{E_k - E_{k_g}}$$

$E_g = g\, v_g^E\, sin(\Delta\Phi_g)$ – g-harmonics.

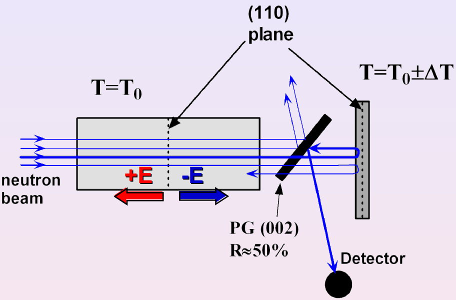
Neutron wave function

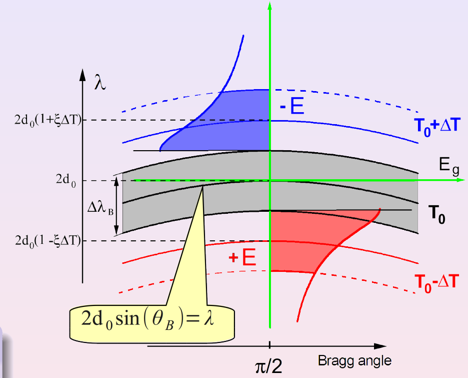
$$\psi(\mathbf{r}) = e^{i(\mathbf{kr})} + \frac{|V_g^N|}{E_k - E_{k_g}} e^{i(\mathbf{k_g r})}$$

$$|\psi(\mathbf{r})|^2 = 1 - \frac{2|V_g^N|}{E_k - E_{k_g}} \cos(\mathbf{gr})$$



Neutron EDM search by crystal-diffraction method



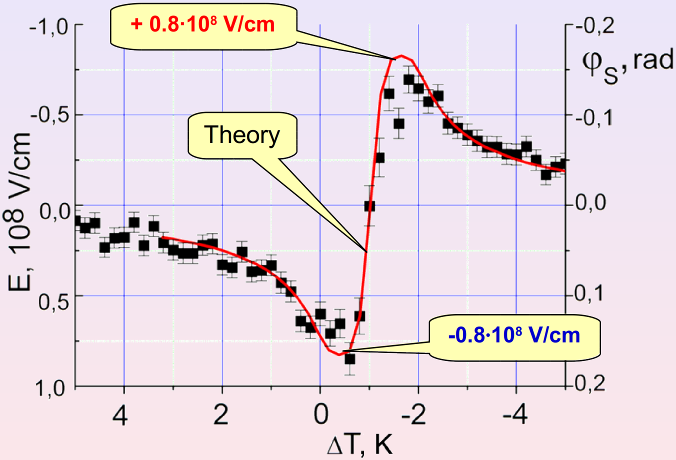


Bragg width in K

$\Delta\lambda_B/\lambda \simeq 10^{-5}$ и $\chi \simeq 10^{-5}$

Т.е. $\boxed{\Delta T = 1} \Rightarrow \boxed{\Delta\lambda = \Delta\lambda_B}$

Electric field in quartz, (110) plane



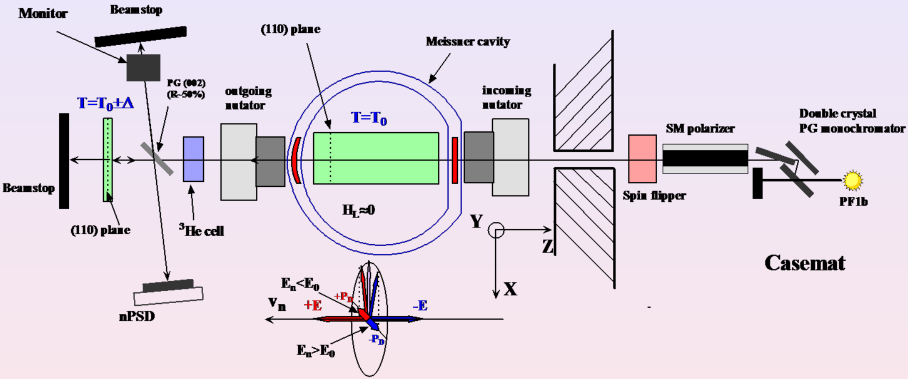
Varying the crystal temperature at $\Delta T = 1$ K

. $\Updownarrow$

$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\lambda_B}{\lambda} \simeq 10^{-5}$

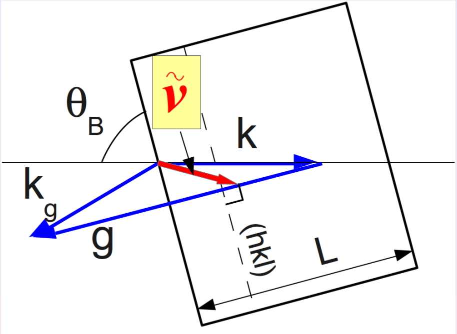
we switch the sign of field with value $\sim 10^8$ V/cm.

Scheme of the nEDM search experiment



The aim is to reach accuracy $\sigma(d_n) \simeq (2-3) \cdot 10^{-26} e \cdot cm$ using quartz crystal in three years.

Neutron propagation in crystal



Neutron velocity in crystal

$$\psi(\mathbf{r}) = e^{i(\mathbf{kr})} + a_g e^{i(\mathbf{k_g r})}$$

где $a_g = \frac{V_g^N}{E_k - E_{k_g}}$

$$\boxed{\tilde{\mathbf{v}} = \frac{\hbar}{m}(\mathbf{k} + |a_g|^2 \mathbf{g})}$$

For the Bragg angle close to 90^0

$$k \simeq -g/2 \Longrightarrow \boxed{\tilde{v} = v_0(1 - 2|a_g|^2)}$$

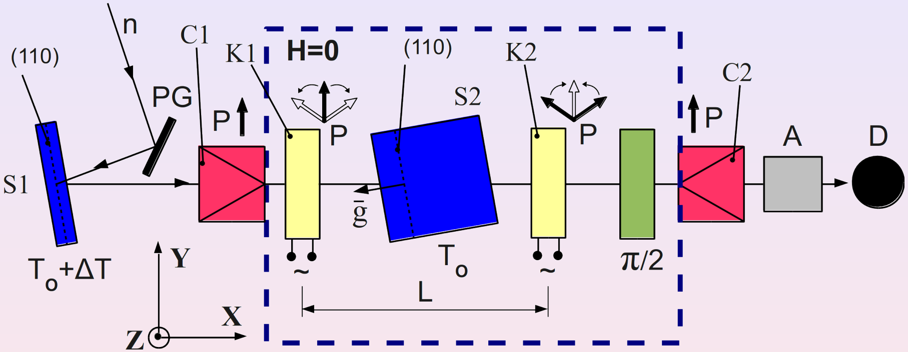
Dispersion close to Bragg condition

$$\frac{d\tilde{v}}{dE} = \frac{v_B}{2E_B}\left(1 - \boxed{\frac{|V_g|^2 E_B}{2\Delta E^3}}\right)$$

here $\Delta E = E_k - E_B$ — deviation from Bragg condition.
$v_B = \sqrt{2E_B/m}$ — Bragg neutron velocity
First term is a normal dispersion, second is abnormal term caused by reflected wave. This term has resonance shape and reverses the sign on Bragg condition.
For the case ($\Delta E \simeq |V_g|$), second term is equal to

$$\boxed{\frac{E_B}{2|V_g|} \sim \frac{1}{n-1} \sim 10^5}$$

Experiment layout



Method of separated oscillated field was used. $\nu \simeq 4\text{kHz}$.

Quartz, (110) plane ($\lambda \simeq 4.9\text{Å}$), $\theta_B = 87^0$, crystal length L=10 cm.

Spin rotation angle

After travelling through the two coils K1—K2.

$$\varphi^s(t) = \frac{2\mu B_0 \tau_B}{\hbar} \cdot 2cos(\omega(t+\tau/2))cos(\omega\tau/2),$$

here t – time of neutron entry into the first coil, τ_B – time of neutron stay in the coil, τ – time of neutron flight between coils.
If $\omega\tau = (2n+1)\pi$, then

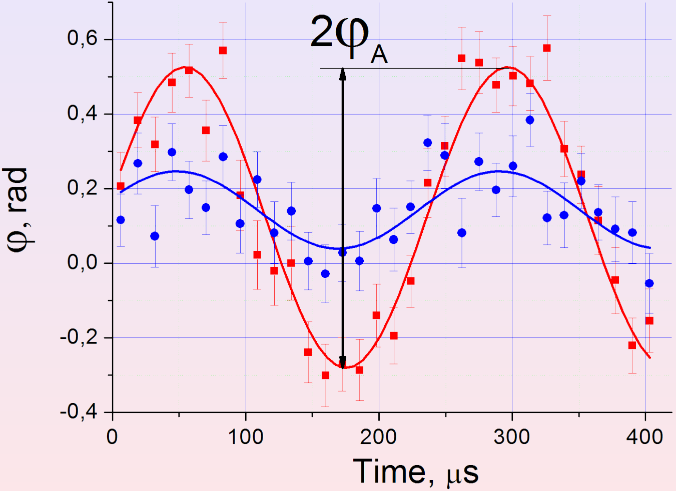
$$\boxed{\varphi^s(t) \equiv 0}$$

for the case of absence of desired effect.
Additional delay in crystal on the time τ_0 gives a time dependence of φ^s

$$\boxed{\varphi^s(t) \simeq \frac{2\mu B_0 \tau_B}{\hbar}\omega\tau_0 \cdot \cos(\omega(t+\tau/2))},$$

i.e. a time depending polarisation along X axis arises.

Time dependence of neutron spin rotation angle



Deviation from Bragg condition

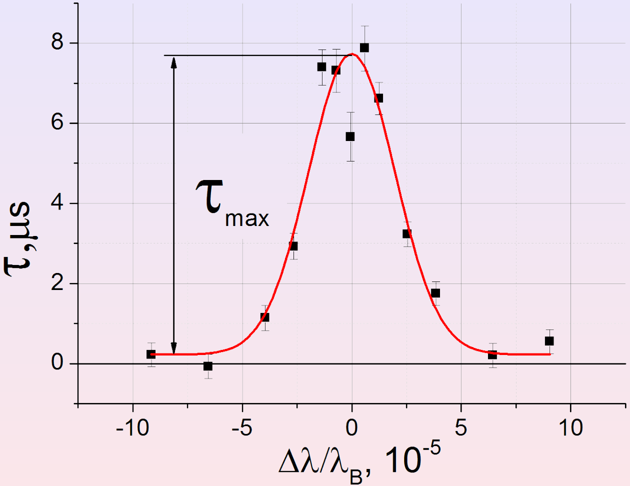
$\Delta E/(2|V_g|) \simeq \mathbf{1.2}$

$\Delta E/(2|V_g|) \simeq \mathbf{9}$

Oscillation amplitude

$$\varphi_A = \frac{2\mu B_0 \tau_B}{\hbar}\omega\tau_0$$

Time of neutron delay in crystal

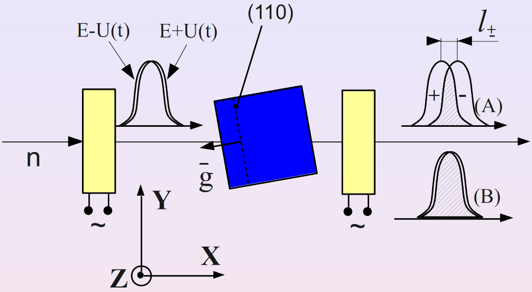


Dispersion value

The $\Delta\lambda/\lambda_B \simeq \mathbf{5 \cdot 10^{-5}}$ gives $\tau_0 \simeq 7.5\ \mu$s, for the $\tau_L = 125\ \mu$s, i.e. $\tau_0/\tau_L \simeq \mathbf{(6 \cdot 10^{-2})}$,

Abnormal part has alternating sign and is about 1000 times more than standard.

Neutron acceleration in magnetic field



Let in a region

$x = [-l/2, +l/2]$

$B(t) = B_0 \sin(\omega t)$

and

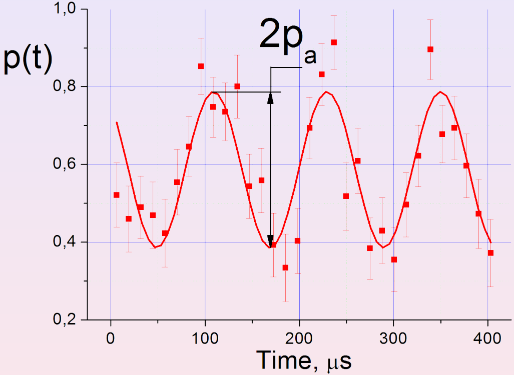
$B(t) = 0, \ -l/2 > x > l/2$

Energy difference of two state

$$\boxed{\Delta E_{\pm}(t) = 4\mu B_0 \cdot \sin \omega t \sin \frac{\omega t_B}{2} = 2\sqrt{2}\mu B_0 \sin \omega t},$$

the last term written for our experimental setup (time of neutron stay in coil t_B equal to 1/4 oscillation period, i.e. $\omega t_B = \pi/2, \ \nu = 4$kHz .

Value of depolarization

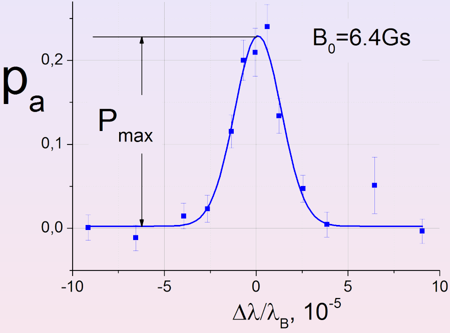


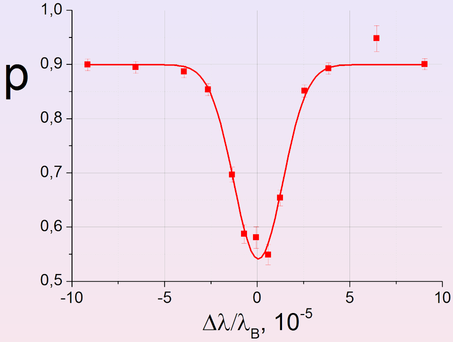
Depolarization value is determined by absolute value of spatial splitting of wave packets $\sim |\Delta E_{\pm}(t)|$, i.e. polarization of the transmitted beam will be

$$p(t) \simeq 1 - p_a \cdot |\sin \omega t|,$$

Therefore, we should see the depolarisation of whole beam after time averaging and time oscillation with the doubled frequency 2ω.

Oscillation amplitude and depolarization





Value of beam polarization after time averaging and amplitude of the time oscillation with the doubled frequency 2ω have to increase close to the Bragg condition.

Wave packet size

Beam polarization is proportional to the packet overlapping area

$$p \simeq \frac{l_p - |l_\pm|}{l_p},$$

Value of spatial splitting

$$|l_\pm|_a = |\Delta\tau_\pm|_a v_n \simeq \frac{|\Delta E_\pm|_a}{E} \cdot K_d\, v_n,$$

The value of $K_d = \Delta\tau/(\Delta E/E) \approx 0.1$ second.

$$l_p = \frac{|\Delta E_\pm|_a}{E} \cdot K_d\, v_n \frac{1}{1-p} = 2\,10^{-5} \frac{B_0}{1-p} \simeq 4,3(4) \cdot 10^{-4}\text{см}.$$

The wave size from the uncertainty principle

$$l_{p0} > \frac{1}{2\Delta k} \simeq 4 \cdot 10^{-4}\text{см},$$

in our experiment $\Delta k = 1.2\,10^3$ cm^{-1} is the Bragg width for (110) quartz plane.

Summary

- In general case, the crystal is a media with the abnormal dispersion.
- Abnormal part is **alternating-sign** and value of dv/dE can exceed corresponding value for a free neutron **in a few orders** close to Bragg reflex.
- A method to measure small turning of neutron energy was demonstrated. Small difference of neutron energy for two spin state in a magnetic field gives essential spatial splitting and neutron beam depolarization.
- Large value of abnormal dispersion allows to observe change of neutron energy on a level $(10^{-11} - 10^{-10})$ eV.