Random Walks and the Search for a Neutron edm

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Major Contirbutions by:
S.K. Lamoreaux
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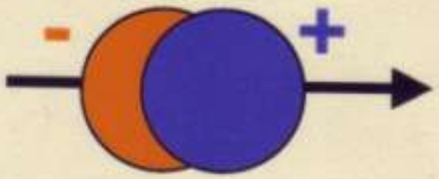
EDM: A sensitive test of symmetry

$\vec{d} \cdot \vec{E}$



s = 1/2

A permanent neutron EDM requires

Parity and Time Reversal Violation

T Violation implies C P Violation, which is linked to the

Baryon Asymmetry of the Universe

Sources of C P Violation

Standard Model	$d_n < 10^{-31}$ e cm
Super Symmetric Extension	$d_n < 10^{-25}$ e cm
Strong C P Problem	$d_n < 10^{-26}$ e cm
Electroweak baryogenesis	$d_n < 10^{-26}$ e cm

If Baryon Asymmetry of the Universe (multiverse?) is due to CP violation

6 x 10^-28 e-cm <dn <2 x 10^-25 e-cm

BUT

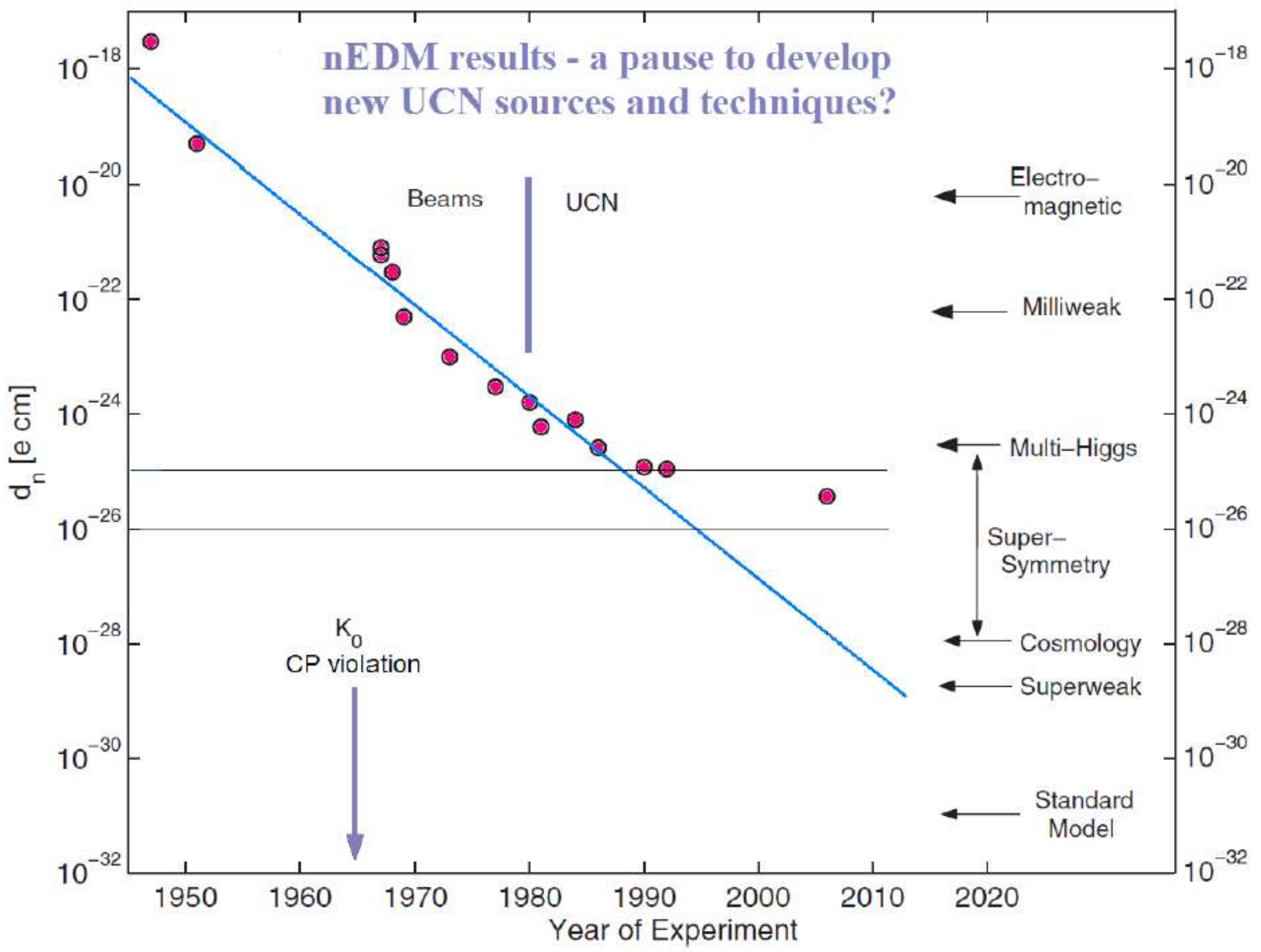
Standard Model (without theta problem)

10^-33 e-cm < dn < 10^-31 e-cm

Theta problem: edm is very large prop to unkown parameter ‘theta’ which must be set to ~10^(-10).

To avoid this Axions were invented => small industry of Axion searches

Edm searches are the best way to look for physics beyound the standard model because the expected standard model background is so small



nEDM results - a pause to develop new UCN sources and techniques?
Beams
UCN
Electro–magnetic
Milliweak
Multi–Higgs
Super–Symmetry
Cosmology
Superweak
Standard Model
K_0
CP violation
d_n [e cm]
Year of Experiment
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-26}
10^{-28}
10^{-30}
10^{-32}
1950
1960
1970
1980
1990
2000
2010
2020

What is Unique About Our Experiment

- Production of ultracold neutrons (UCN) within the apparatus
  – *higher UCN density and longer storage times*

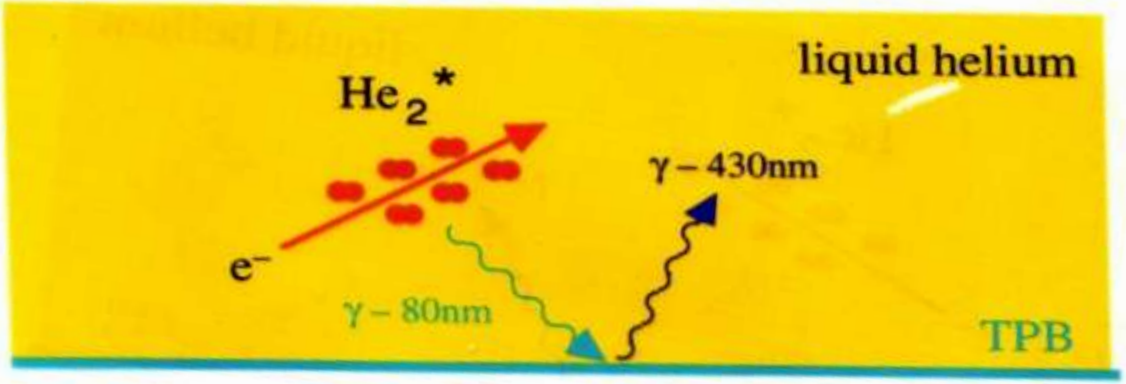
- Use of liquid as a high voltage insulator
  – *higher electric fields*

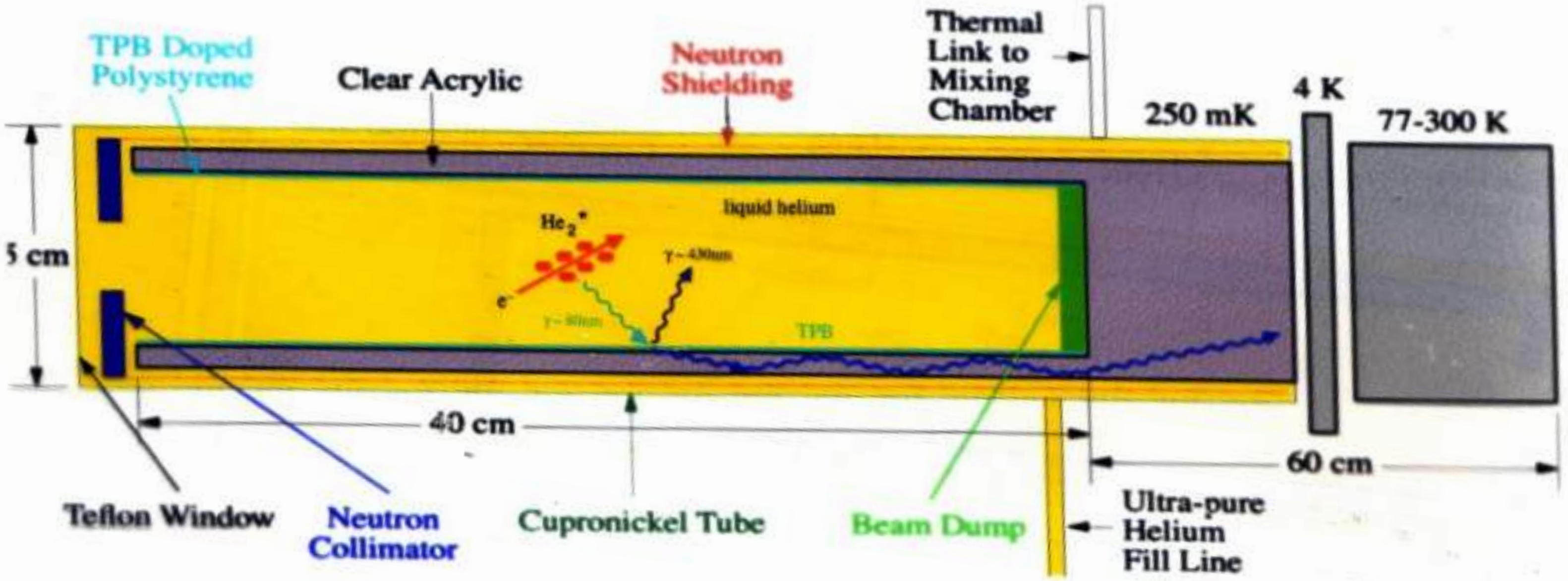
- Use of a ^{3}He co-magnetometer and superconducting shield
  – *better control of magnetic field systematics*

- Employ two different measurement techniques
  – *oscillation of scintillation rate and dressed spin techniques*

Tackling unknown systematic effects requires unique handles in the experiment that can be varied.

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$





Polarized He3 as:

-Polarizer (partial..beam is polarized
-Magnetometer
-Analyzer
-Detector

Important Systematic error

Magnetic field

$$\frac{\vec{v}}{c} \times \vec{E}$$

Perpendiculat to v

Rotates with v

BLOCH-SIEGERT

Frequency shift ~ (Beff)^2

However

If $B_r \sim r$

then $(B_r + v \times E)^2 \propto E$

Important systematic effect $\delta\omega \sim \frac{\partial B_z}{\partial z} v$ E

Discovered by Cummins…molecular beam edm experiment

$$\delta\omega_E = -\frac{abv^2}{\omega_o{}^2 - \omega_r{}^2}$$

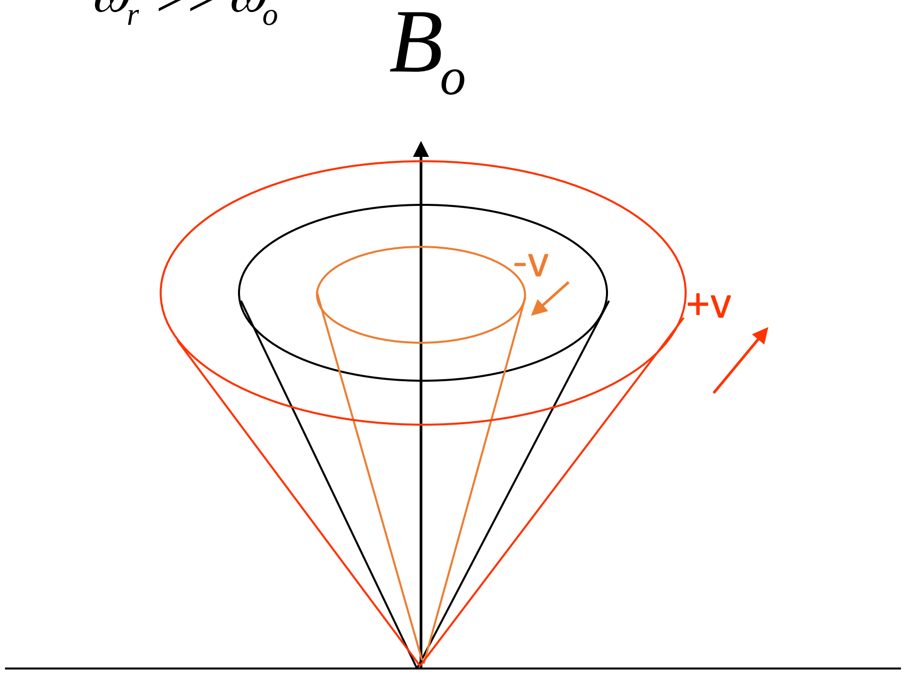
$$\omega_r = \frac{v}{r}$$

$$\omega_{xy} = ar$$

$$b = \frac{E}{c}\gamma$$

$$= -\frac{abv^2}{\omega_{o^2}} \qquad \omega_o >> \omega_r$$

$$= abR^2 \qquad \omega_r >> \omega_o$$



Geometric Phase Picture

Different approaches

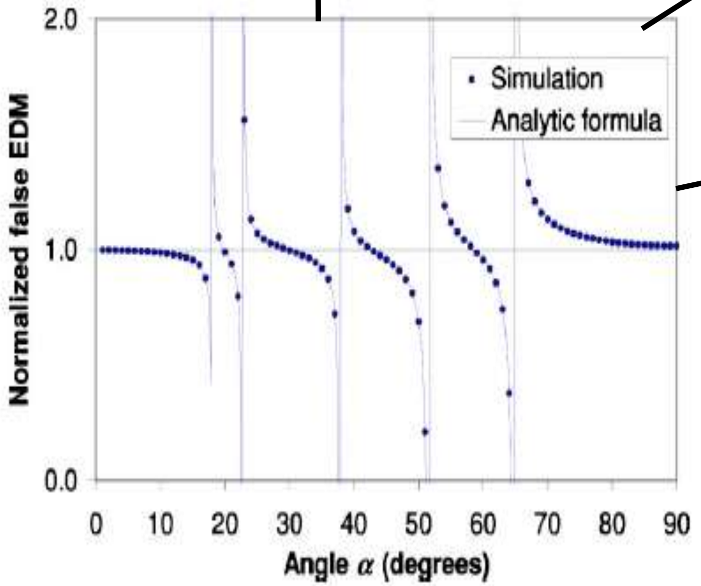
Classical Bloch equations

Quantum Density matrix or Torrey equation

Quantum Schroedinger equation (Steyerl)

Direct solution

Position-velocity correlation function



Single velocity
Cylinder
Specular wall reflections
No gas collisions

Pendlebury et al 2004

Classical Approach

The Bloch equation for the motion of a magnetic moment coupled to an angular momentum can be written

$$\frac{dJ_z}{dt} = \omega_y J_x - \omega_x J_y \quad (1)$$

$$\frac{dJ_x}{dt} = \omega_z J_y - \omega_y J_z \quad (2)$$

$$\frac{dJ_y}{dt} = \omega_x J_z - \omega_z J_x \quad (3)$$

where $\omega_i = \gamma B_i$, with γ the gyromagnetic ratio of the spins, and the $B_{i=x,y,z}$ are arbitrary functions of the time. In terms of the spherical angles ϕ, θ we can write

$$J_z = \cos\theta \quad (4)$$

$$J_x = \sin\theta\cos\phi \quad (5)$$

$$J_y = \sin\theta\sin\phi \quad (6)$$

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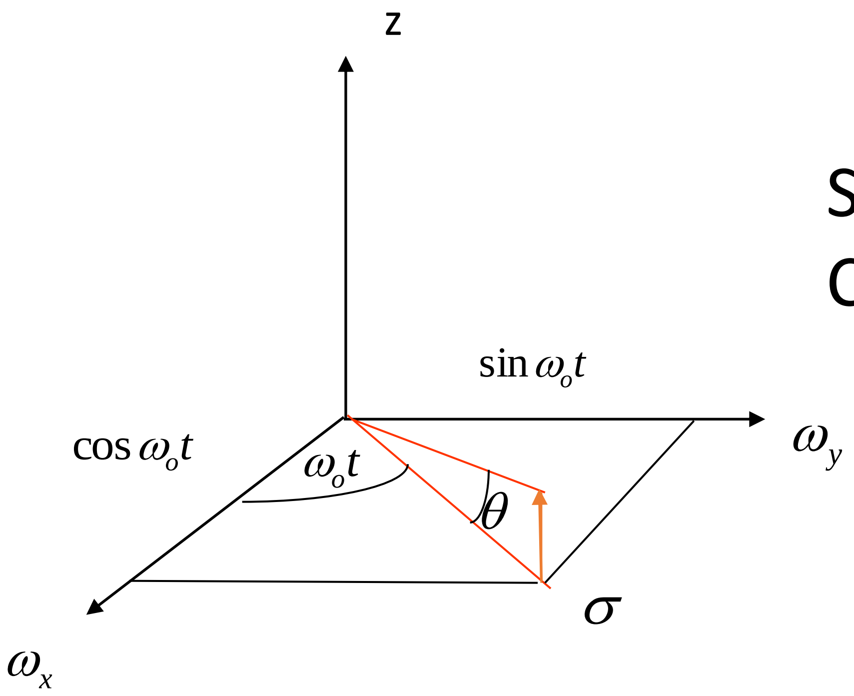


Sobolev
Classical equations

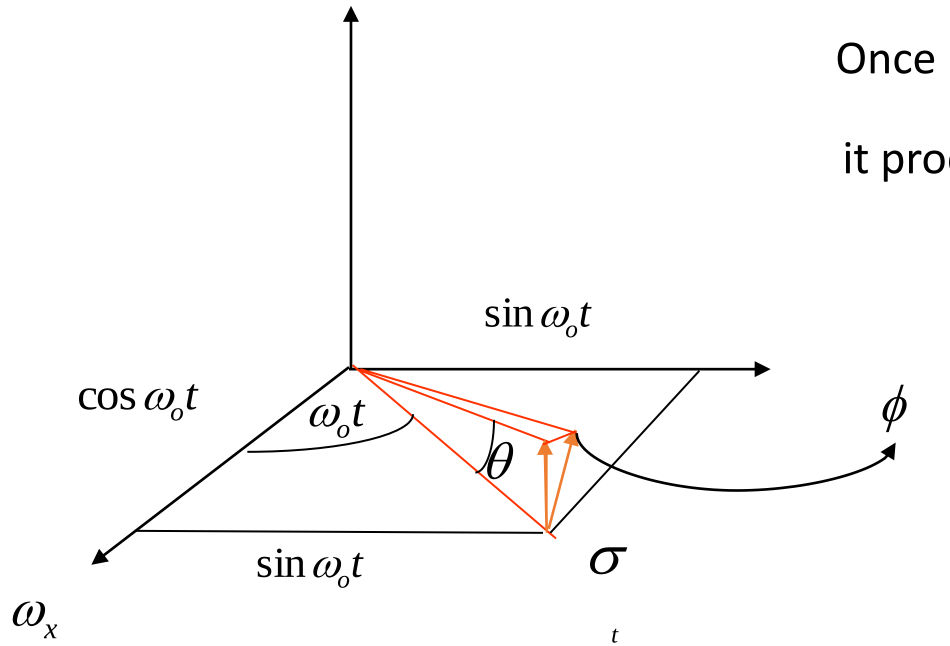
CREATE θ

$$\dot{\theta} = \omega_x(t)\sin\omega_o t - \omega_y(t)\cos\omega_o t$$

$$\theta = \int_0^t dt'\left[\omega_x(t')\sin\omega_o t' - \omega_y(t')\cos\omega_o t'\right]$$

Once θ Builds up

it produces a change in ϕ



$$\theta = \int_0^t dt'\left[\omega_x(t')\sin\omega_o t' - \omega_y(t')\cos\omega_o t'\right]$$

$$\dot{\phi} = \theta(t)\left[\omega_x \cos\omega_o t + \omega_y \sin\omega_o t\right]$$

$$\sim \omega_x(t)\int dt'\omega_y(t')\cos\omega_o t\cos\omega_o t'$$

Now $\omega_x = ax + bv_y$, $\omega_y = ay - bv_x$ where

$$a = \frac{\gamma}{2}\frac{\partial B_z}{\partial z},$$

$$b = \gamma\frac{E}{c},$$

$$\phi(t) \sim \int \omega_x(t)\int dt'\omega_y(t')\cos\omega_o t\cos\omega_o t' dt$$

$$\text{.........} \sim \iint \langle \omega_x(t)\omega_y(t')\rangle \cos\omega_o t\cos\omega_o t' dtdt$$

Inhomogeneous fields -> fluctuations due to different trajectories

Three approaches…all are equivalent

Torrey equation…solve to second order

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H,\rho] + D\nabla^2 \rho .$$

Equation of motion of density matrix to second order

$$\frac{\partial \rho^*}{\partial t} = \frac{1}{i\hbar}[H_1^*(t),\rho^*(0)] + \left(\frac{i}{\hbar}\right)^2 \int_0^t [[\rho^*(0), H_1^*(t')], H_1^*(t)]\, dt',$$

Leads to relaxation matrix in terms of correlation functions…..is same as Torrey equation results when diffusion theory determines correlation functions

Steyerl, Golub Solution of Schroedinger equation

A. Steyerl,* C. Kaufman, G. Müller, S. S. Malik, and A. M. Desai
R. Golub
PHYSICAL REVIEW A **89**, 052129 (2014)

$$H = -\frac{1}{2}\begin{bmatrix} \omega_o' & \omega_x - i\omega_y \\ \omega_x + i\omega_y & -\omega_o' \end{bmatrix} = -\begin{bmatrix} \omega_o & \Omega^* \\ \Omega & -\omega_o \end{bmatrix}$$

The Schroedinger equation is then:

$$i\frac{\partial}{\partial t}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = H\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\ddot{\alpha}_r - \left(\frac{\dot{\Omega}^*}{\Omega^*} - i2\omega_o\right)\dot{\alpha}_r = -\left|\Omega\right|^2 \alpha_r$$

This is exact for any time dependence W(t)

Solve by treating rhs as a perturbation

Geometric phase effect

Considering only non-zero correlations we find

$$\delta\omega = -\frac{1}{2}\int_0^t d\tau\left(\cos\omega_0\tau\right)\left\langle \omega_x(t)\omega_y(t-\tau) - \omega_x(t-\tau)\omega_y(t)\right\rangle$$

With Frequencies

$$\omega_x = ax + bv_y \qquad a = \gamma\frac{G_z}{2}$$

$$\omega_y = ay + bv_x \qquad b = \gamma\frac{E}{c}$$

- Keeping only terms linear in E

$$\delta\omega = \frac{ab}{2}\int_0^\infty R(\tau)\cos(\omega_0\tau)d\tau \qquad R(\tau) = \left\langle \begin{array}{c} y(t)v_y(t-\tau) - y(t-\tau)v_y(t) \\ x(t)v_x(t-\tau) - x(t-\tau)v_x(t) \end{array}\right\rangle$$

R is a position-velocity Correlation Function!

[1] S. K. Lamoreaux and R. Golub. Phys. Rev. A, 71:032104, 2005.

$$\delta\omega = \frac{ab}{2}\int_0^t d\tau \cos\omega_o \tau R(\tau)$$

$$R(\tau) = \left\langle \vec{r}(t)\bullet\vec{v}(t-\tau) - \vec{r}(t-\tau)\bullet\vec{v}(t)\right\rangle$$

$$\vec{r}(t) = \int_0^t dt'\,\vec{v}(t')$$

$$R(\tau) = 2h(\tau)$$

$$h(\tau) = \int_0^\tau dt' \left\langle \vec{v}(t)\bullet\vec{v}(t-\tau)\right\rangle = \int_0^\tau dt'\,\psi(t')$$

Velocity autocorrelation function

$$\delta\omega = -ab\int_{-\infty}^{\infty} d\omega \frac{\psi(\omega)}{\left(\omega_o^{\;2} - \omega^2\right)}$$

BLOCH SIEGERT

(again)

A Family of Correlation functions

$$R_{fg}(\tau) = \int dt \left\langle f(t)\, g(t-\tau) \right\rangle$$

$$R_{\vec{r}\vec{r}}(\tau) = \int dt \left\langle \vec{r}(t) \cdot \vec{r}(t-\tau) \right\rangle$$

$$R_{\vec{r}\vec{v}}(\tau) = -\frac{\partial}{\partial \tau} \int dt \left\langle \vec{r}(t) \cdot \vec{r}(t-\tau) \right\rangle$$

$$R_{\vec{v}\vec{v}}(\tau) = -\frac{\partial}{\partial \tau} R_{\vec{r}\vec{v}}(\tau) = \frac{\partial^2}{\partial \tau^2} \int dt \left\langle \vec{r}(t) \cdot \vec{r}(t-\tau) \right\rangle = \frac{\partial^2}{\partial \tau^2} R_{\vec{r}\vec{r}}(\tau)$$

Correlation of arbitrary space-dependent fields

$$R_{B_iB_i}(\tau) = \iint dx dx_o P(x,\tau\,|\,x_o,0) B_i(x) B_j(x_o)\, p(x_o,0)$$

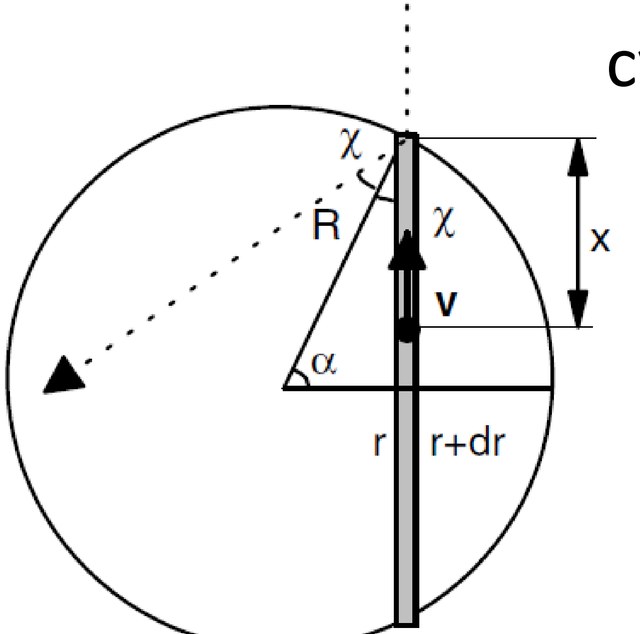
Conditional probability, transition probability

$$S_B(\omega) = \frac{1}{(2\pi)^3}\int_V \mathrm{B}(\mathrm{x_0})d^3\mathrm{x_0}\int_{-\infty}^{\infty}\tilde{\mathrm{B}}(\mathrm{x})d^3\mathrm{x}\int d\omega\int d^3\mathrm{Q}e^{i\mathrm{Q}\cdot(\mathrm{x_0}-\mathrm{x})+i\omega\tau}p(\mathrm{Q},\omega)$$

$$S_B(\omega) = \frac{1}{(2\pi)^3}\int d^3Q\left[\int B(x_o)e^{iQ\cdot x_o}d^3x_o\right]\left[\int \tilde{B}(x)e^{-iQ\cdot x}d^3x_o\right]P(Q,\omega)$$

Introduced by Petukhov and independently by Clayton using P given by diffusion theory

Barabanov found an analytic expression for the velocity auto-correlation function for motion in a cylindrical cell



Trajectory of a particle in a cylindrical cell.

A. L. Barabanov,[1] R. Golub,[2] and S. K. Lamoreaux[3]
PHYSICAL REVIEW A **74**, 052115 (2006)

$$-\Delta\omega(\alpha) = R^2 ab \sin^2\alpha \sum_{m=-\infty}^{\infty} \frac{1}{(\alpha+\pi m)^2} \times \left[\frac{\left(\omega_0'^2 - \frac{(\alpha+\pi m)^2}{\sin^2\alpha}\right)}{\left[\left(\omega_0'^2 - \frac{(\alpha+\pi m)^2}{\sin^2\alpha}\right)^2 + \omega_0'^2 r_0^2\right]} \right]$$

Linear electric field frequency shift (important for next generation electric dipole moment searches) induced in confined gases by a magnetic field gradient.

A.L. Barabanov#, R. Golub+ and S.K. Lamoreaux*

Analytic approach to correlatioin fns

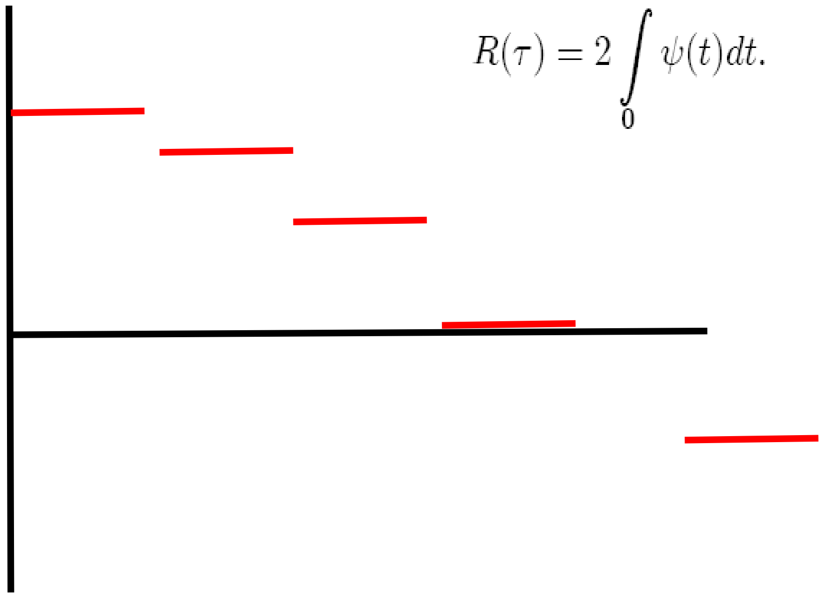
$$\delta\omega = \frac{ab}{2}\lim_{T\to\infty}\int_0^T R(t)\cos(\omega_0 t)dt, \qquad R(\tau) = \langle \vec{r_\perp}(t)\cdot\vec{v_\perp}(t-\tau) - \vec{r_\perp}(t-\tau)\cdot\vec{v_\perp}(t)\rangle .$$

According to [11] the correlation function $R(\tau)$ is determined by the velocity autocorrelation function,

$$\psi(t) \equiv \langle \vec{v}_\perp(t)\cdot\vec{v}_\perp(0)\rangle, \quad (3)$$

namely,

$$R(\tau) = 2\int_0^\tau \psi(t)dt.$$



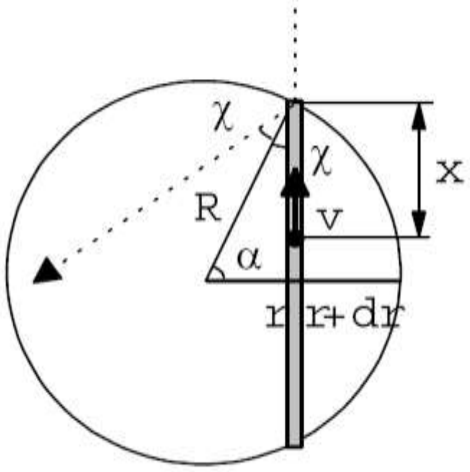


Figure 2: Trajectory of a particle in a cylindrical cell.

$$-\Delta\omega\left(\alpha\right)=R^2ab\sin^2\alpha\sum_{m=-\infty}^{\infty}\frac{1}{\left(\alpha+\pi m\right)^2}\left[\frac{\left(\omega_o'^2-\frac{(\alpha+\pi m)^2}{\sin^2\alpha}\right)}{\left(\left(\omega_o'^2-\frac{(\alpha+\pi m)^2}{\sin^2\alpha}\right)^2+\omega_o'^2 r_o^2\right)}\right] \tag{42}$$

that is we go from the collision free case to the case of gas collisions by replacing

$$f_\alpha\left(\omega'\right)=\frac{1}{\left(\omega_o'^2-\frac{(\alpha+\pi m)^2}{\sin^2\alpha}\right)}$$

in (25) by the square bracket in (42) or by replacing $f_\alpha\left(\omega'\right)$ by $f_\alpha\left(\omega'\sqrt{1+i\frac{r_o}{\omega'}}\right)$ and taking the real part. Since we have evaluated the summation (25) we obtain the frequency shift by making the equivalent transformation to (26)

$$-\Delta\omega\left(\alpha\right)=R^2ab\sin^2\alpha\,\mathrm{Re}\left\{F_P(\alpha,\delta=\delta_o\sqrt{1+\frac{i}{\omega_o\tau_c}})\right\} \tag{43}$$

where

$$F_P(\alpha,\delta)=\left(1+\frac{\sin^2\alpha\sin 2\delta}{2\delta\sin\left(\delta-\alpha\right)\sin\left(\delta+\alpha\right)}\right)\frac{1}{\delta^2}$$

(remember $\delta_o=\omega_o\tau_w/2$). For a fixed velocity we average over α, according to (10):

$$\Delta\omega=\int_0^{\pi/2}d\alpha P\left(\alpha\right)\Delta\omega\left(\alpha\right) \tag{44}$$

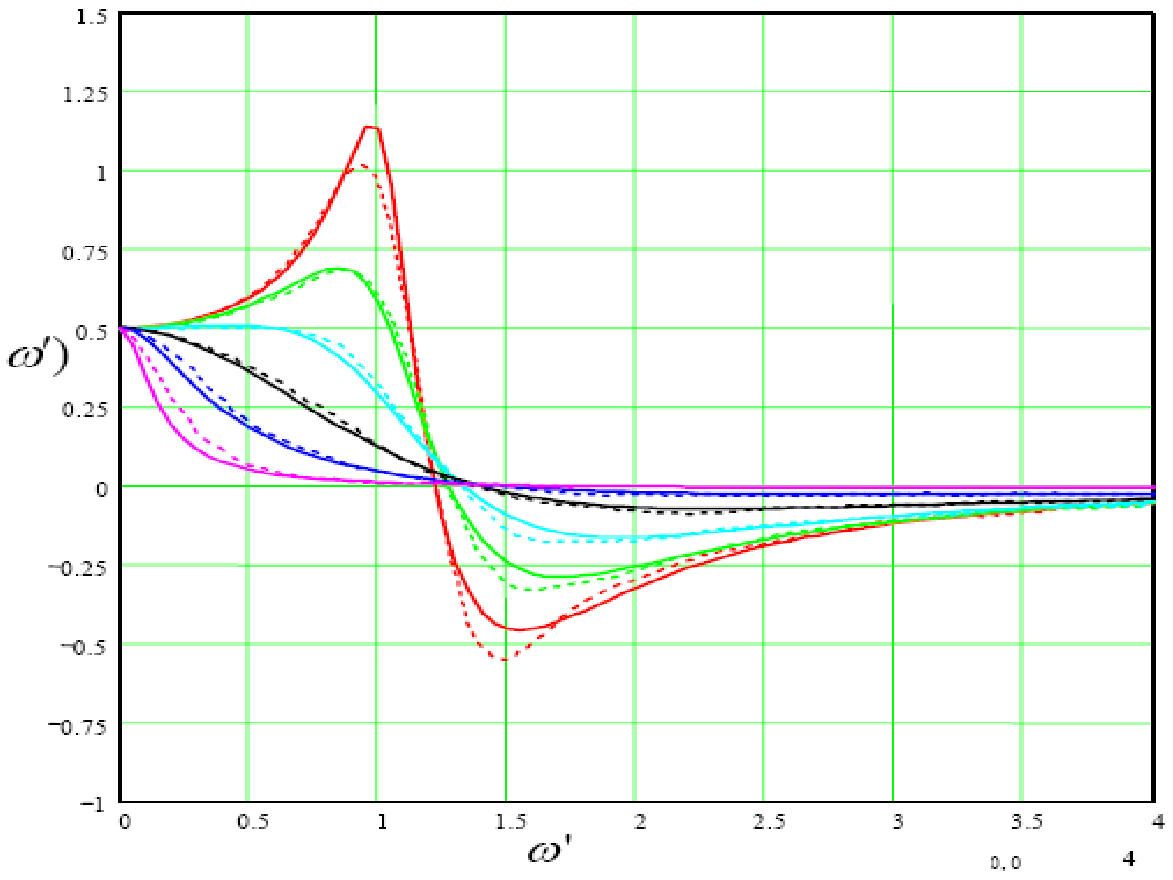
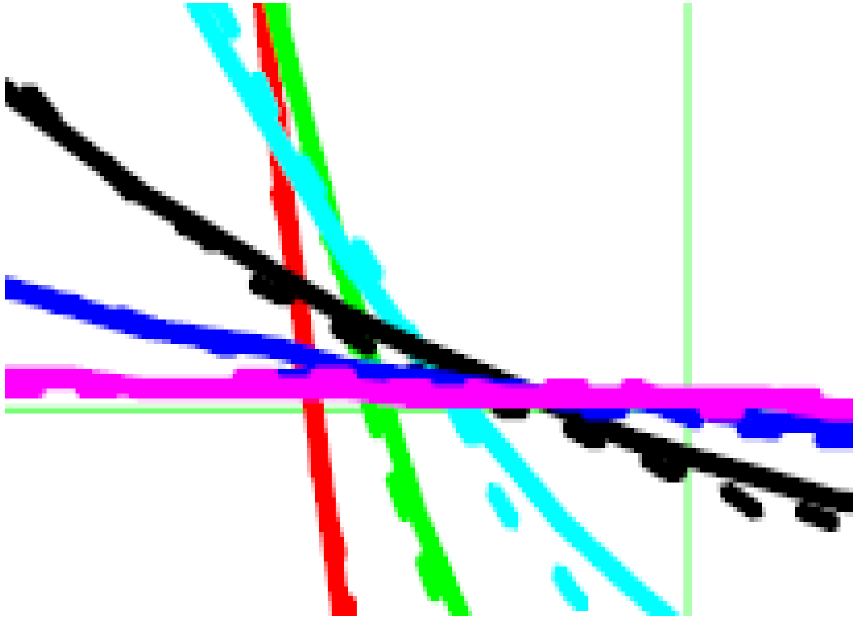


Figure 3: Normalized frequency shift for a constant velocity as a function of normalized applied frequency, $\omega' = \omega_o R/v$, for different values of the damping parameter $r_o = R/\lambda$. Solid curves - results of the analytic function, equations (43) and (44). Dotted lines numerical simulations from ref. [11]. red - $r_o = .2$, green - $r_o = .5$, cyan - $r_o = 1$, black $r_o = 2$, blue - $r_o = 4$, magenta - $r_o = 10$.



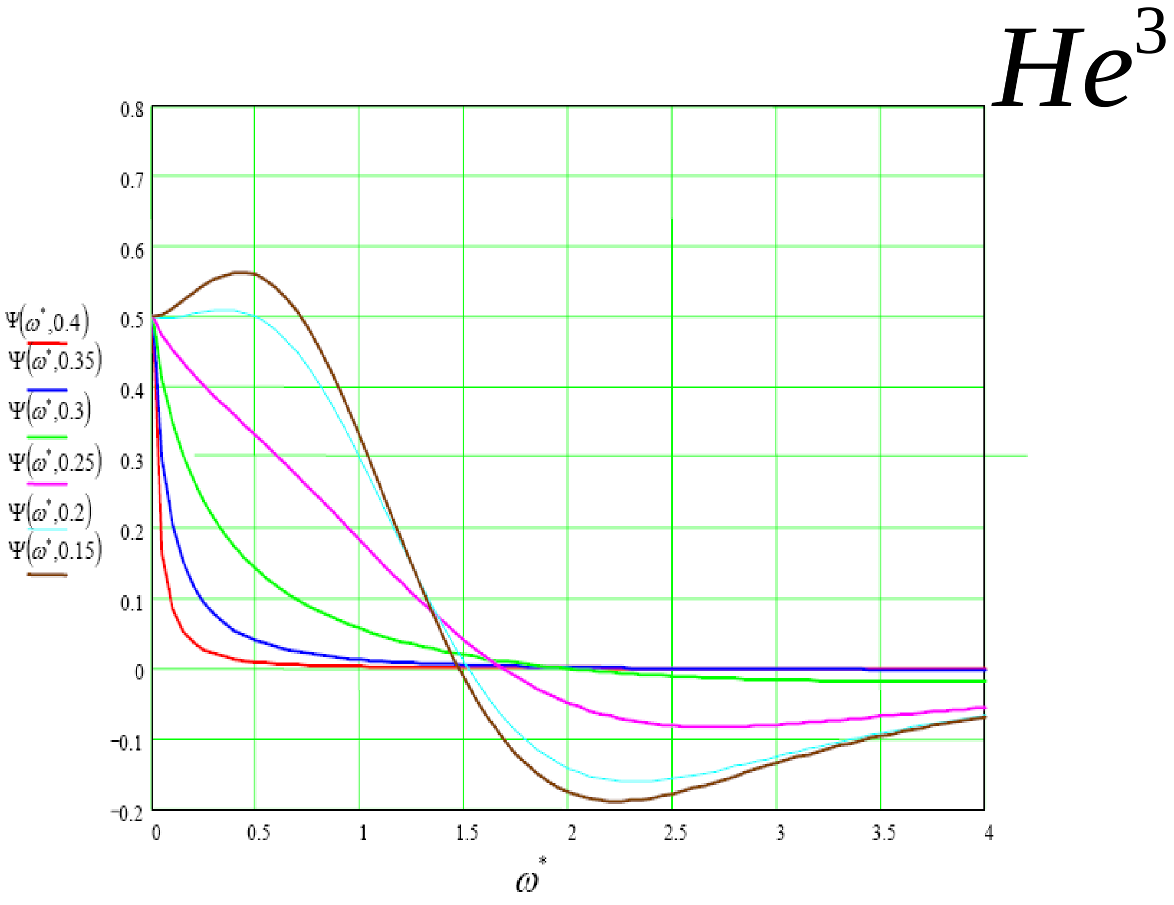


Figure 4: Normalized velocity averaged frequency shift vs. reduced frequency $\omega^{*}=\omega_{o}R/\beta\,(T)$ for various temperatures using the temperature-dependent mean free path for He^{3} in He^{4}.

He^3

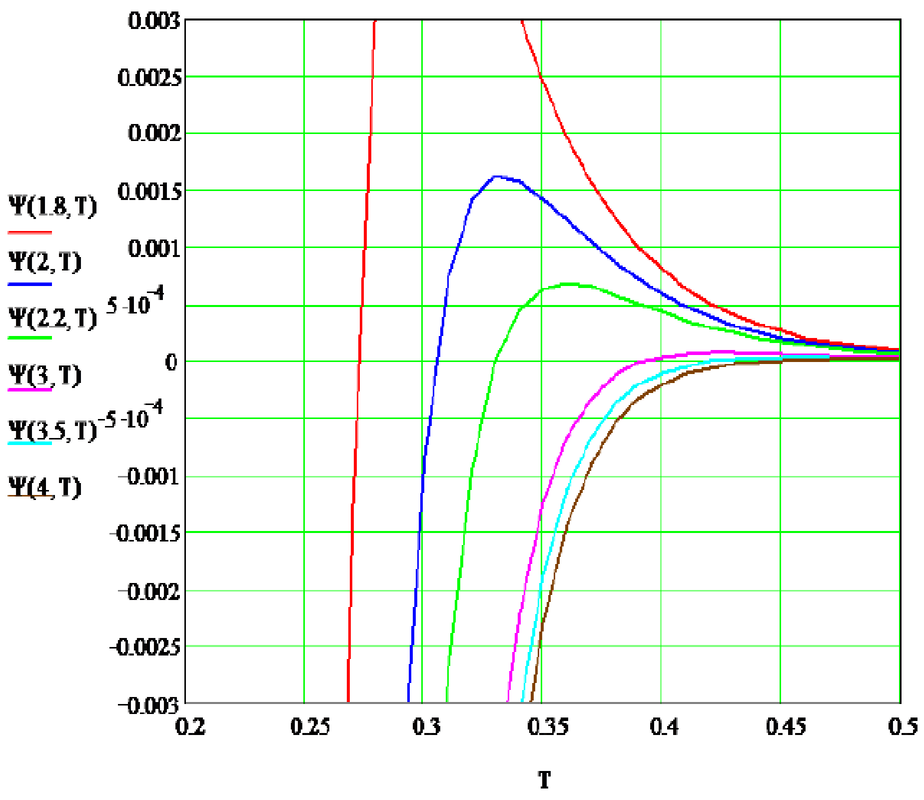


Figure 5: Normalized velocity-averaged frequency shift, $\Psi(\omega^*, T)$ vs. temperature, T, for various reduced frequencies $\omega^* = \omega_o R/\beta(T)$ using the temperature-dependent mean free path for He^3 in He^4

Phase shift in terms of position autocorrelation function

$$\langle x(t)v_x(t+\tau)\rangle = \frac{\partial}{\partial\tau}\langle x(t)x(t+\tau)\rangle$$

$$\langle x(t+\tau)v_x(t)\rangle = \langle x(t)v_x(t-\tau)\rangle = -\frac{\partial}{\partial\tau}\langle x(t)x(t+\tau)\rangle$$

Therefore the position velocity correlation function can be written in terms of the position autocorrelation function.

$$\delta\omega = \frac{\gamma^2 \quad E}{4c}\int_0^\infty d\tau \cos\omega_0\tau \frac{\partial}{\partial\tau}\langle B_x(t)x(t+\tau) + B_y(t)y(t+\tau)\rangle$$

Which can be written in terms of the Fourier transform for an arbitrary B field

$$\delta\omega = \omega_0\frac{\gamma^2 E}{2c}\Im\left\{\int_{-\infty}^{\infty} d\tau \mathrm{e}^{-i\omega_0\tau}\langle B_x(t)x(t+\tau) + B_y(t)y(t+\tau)\rangle\right\}$$

$$-\frac{\gamma^2 E}{c}\langle B_x x + B_y y\rangle$$

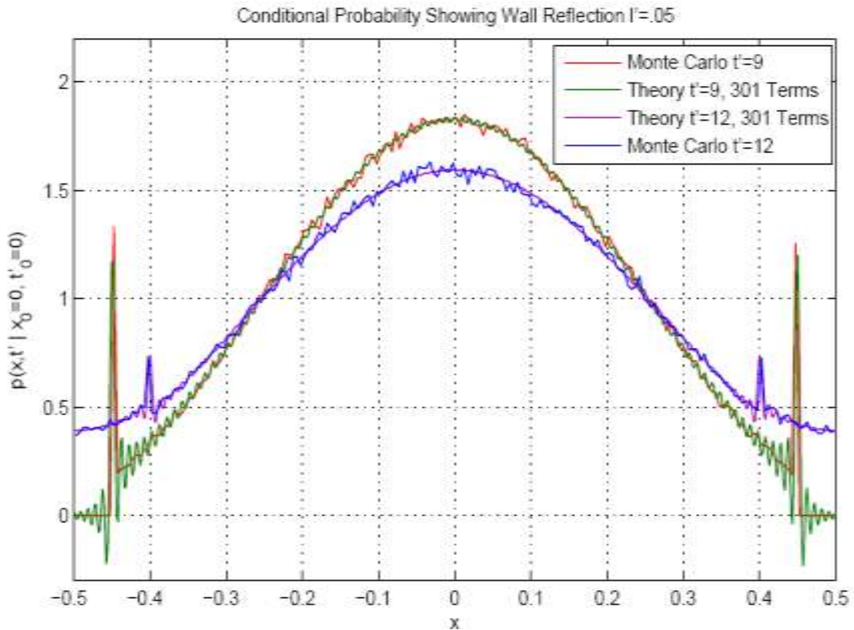
The Correlation Function in 1-D

- $\delta\omega$ is related to position conditional density

$$\int B_x(t)B_x(t-\tau)d\tau = \int B_x(x)B_x(x$$

$$\langle h_x(t)x(t+\tau)\rangle = \int_{-\frac{L}{2}}^{\frac{L}{2}} dx_0 G_x x_0 \; p(x_0,t) \int_{-\frac{L}{2}}^{\frac{L}{2}} x \; p(x,t \mid x_0, t+\tau) \; dx$$

$$H_{xv}(\omega) = \frac{8v^2 G_x}{\pi^2} \sum_{n=odd}^{1\to\infty} \frac{1}{n^2} \frac{(\omega^2 - \omega_n^2)}{(\omega^2-\omega_n^2)^2 + \frac{\omega^2}{\tau_c^2}}$$



[1] C. Swank and R. Golub. arXiv:1012.4006 2010.

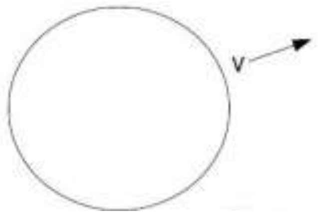
2D & 3D Formulation

First define the scattering probability per unit time.

$$\psi(t) = \frac{1}{\tau_c} e^{-\frac{t}{\tau_c}}$$

Second, define how the probability is transported before or between scattering

$$f(\mathbf{r}, t; \theta) = \delta(x - vt\cos\theta)\delta(y - vt\sin\theta)$$



Now define the distribution after a scatter.

$$\beta(\theta|\theta') = {}^{1}/{}_{2\pi}$$

2D & 3D Conditional densities.

The probability density of scattering at position **r** and time *t* is given as: (from origin)

$$\rho(\mathbf{r},t) = \frac{1}{2\pi}\psi(t)f(\mathbf{r},t) + \frac{1}{2\pi}\int d^2\mathbf{r}' \int_0^t d\tau\rho(\mathbf{r}',\tau)f(\mathbf{r}-\mathbf{r}',t-\tau)\psi(t-\tau)$$

The probability of arriving at position **r** at a time *t*

Probablity of scattering

Probability of not scattering

$$p(\mathbf{r},t) = \frac{1}{2\pi}\Psi(t)f(\mathbf{r},t) + \frac{1}{2\pi}\int d^2\mathbf{r}' \int_0^t d\tau\rho(\mathbf{r}',\tau)f(\mathbf{r}-\mathbf{r}',t-\tau)\Psi(t-\tau)$$

where

$$\Psi(t) = \int_t^\infty \psi(\tau)d\tau = \mathrm{e}^{-\frac{t}{\tau_c}} = \tau_c\psi(t).$$

Masoliver, J., Porrá, J.M. and Weiss, G.H., Phys Rev **E48**, 939 (1993)

2D & 3D Conditional densities.

Substituting

$$g(\mathbf{r},t) = f(\mathbf{r},t)\psi(t)$$

$$G(\mathbf{r},t) = f(\mathbf{r},t)\Psi(t)$$

gives

$$\rho(\mathbf{r},t) = \frac{1}{2\pi}\left(g(\mathbf{r},t) + \int d^2\mathbf{r}' \int_0^t \rho(\mathbf{r}',\tau)g(\mathbf{r}-\mathbf{r}',t-\tau)d\tau\right)$$

$$p(\mathbf{r},t) = \frac{1}{2\pi}\left(G(\mathbf{r},t) + \int d^2\mathbf{r}' \int_0^t \rho(\mathbf{r}',\tau)G(\mathbf{r}-\mathbf{r}',t-\tau)d\tau\right)$$

Taking the Laplace transform the equations can be linearized. Defined as,

$$f(\mathbf{Q},s) = \mathcal{L}\left(f(r,t)\right) = \int d^2\mathbf{r} \int_0^\infty f(r,t)\mathbf{e}^{i\mathbf{Q}\cdot\mathbf{r}-st}dt, \quad s = \sigma + i\omega.$$

2D & 3D Conditional densities.

The solved equations in Laplace space

$$\rho(\mathbf{Q}, s) = \frac{g(\mathbf{Q}, s)}{2\pi - g(\mathbf{Q}, s)}$$

$$p(\mathbf{Q}, s) = \frac{G(\mathbf{Q}, s)}{2\pi - g(\mathbf{Q}, s)}$$

$$G(\mathbf{r}, t) = \tau_c g(\mathbf{r}, t)$$

Taking the transform of g

$$g(\mathbf{Q}, s) = \int d^2\mathbf{r} \int_0^\infty f(r,t)\psi(r,t)\mathbf{e}^{i\mathbf{Q}\cdot\mathbf{r} - st} dt = \frac{2\pi}{\sqrt{(1 + \frac{s}{\tau_c})^2 + \frac{v^2 Q^2}{\tau_c^2}}}$$

Therefore

$$p(Q, s) = \frac{1}{\sqrt{(\frac{1}{\tau_c} + s)^2 + v^2 Q^2} - \frac{1}{\tau_c}}$$

2D & 3D Conditional densities.

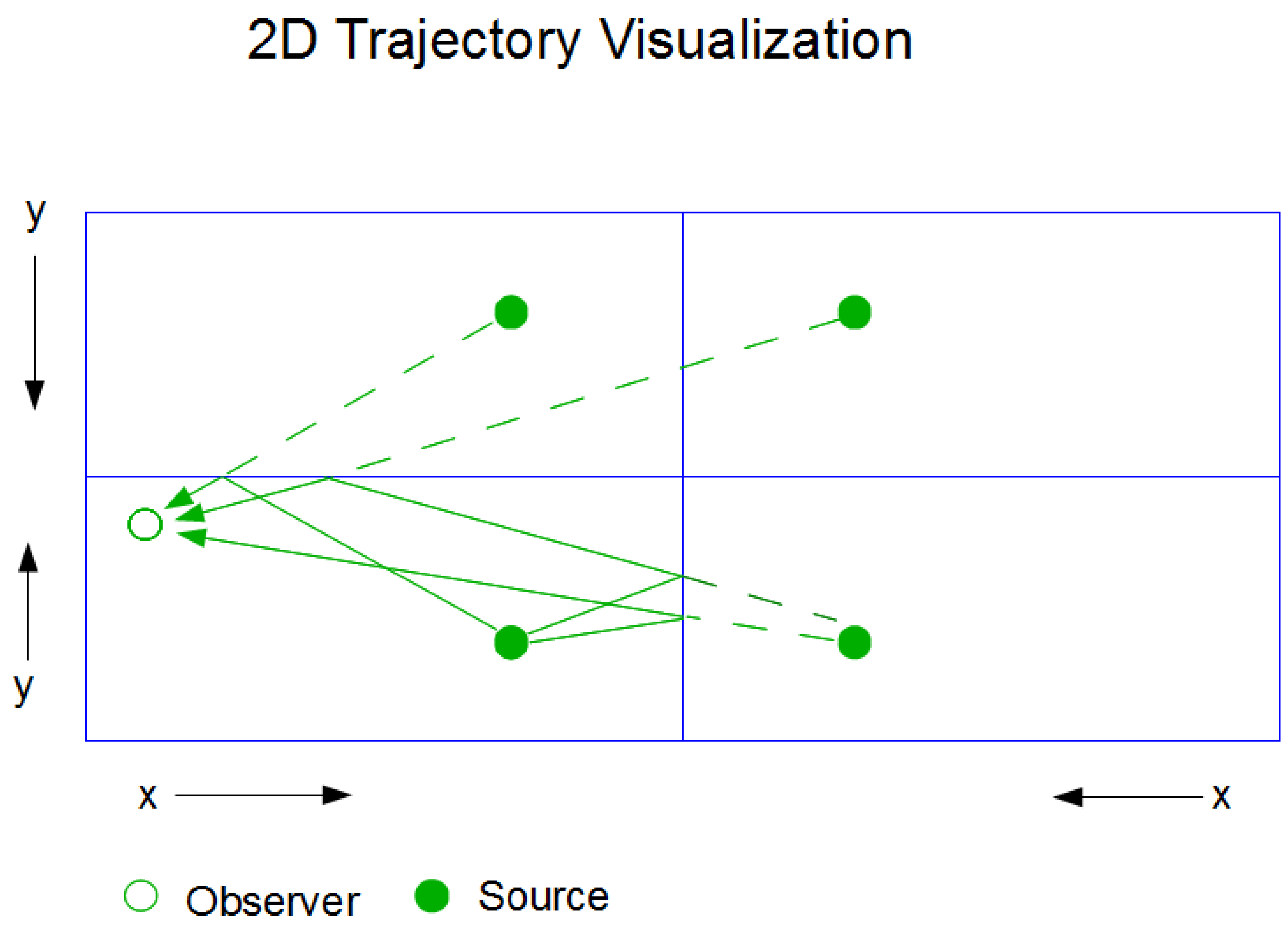
This 2D spectrum can be converted back to space and time by an inverse Fourier transform

$$p(r,t) = \mathrm{e}^{-\frac{t}{\tau_c}} \left[\frac{\delta(r - vt)}{2\pi r} + \frac{1}{2\pi v \tau_c \sqrt{v^2 t^2 - r^2}} \exp\left(\frac{1}{\tau_c} \sqrt{t^2 - \frac{r^2}{v^2}} \right) \right]$$

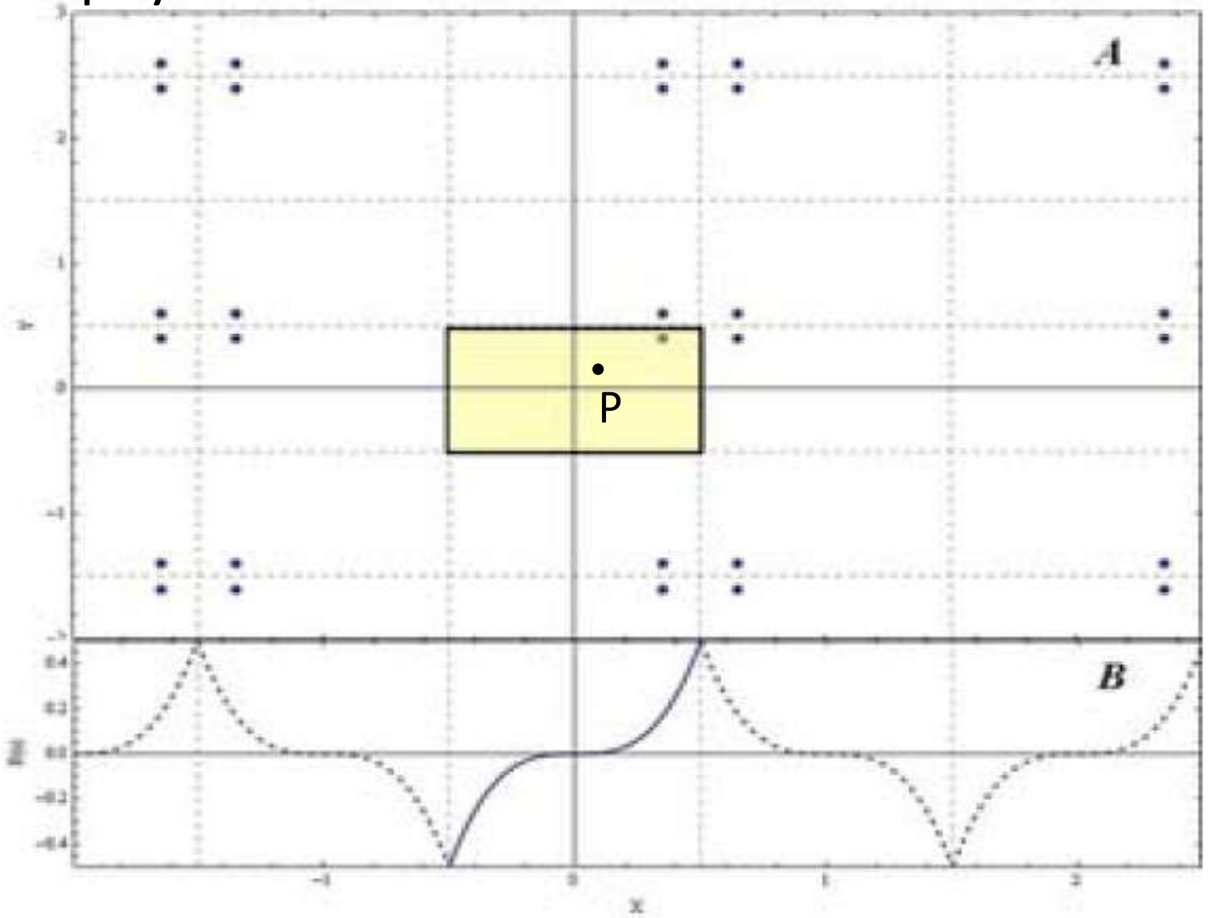
The 3D spectrum can be found in the same way as the 2D, however it does not have an inverse Fourier transform

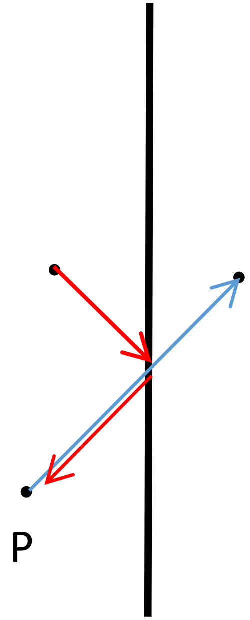
3D $$p(Q,s) = \frac{\arctan\left(\frac{vQ}{\frac{1}{\tau_c} + s} \right)}{vQ - \frac{1}{\tau_c} \arctan\left(\frac{vQ}{\frac{1}{\tau_c} + s} \right)}$$

Restricted random walk



Solve with infinite domain P(r,t) and continued periodic function in unphysical cells





The geometric frequency shift derived from the restricted random walk

Define the spectrum.

$$S_B(\omega) = \int_{-\infty}^{\infty} \langle B(t)B(t+\tau)\rangle\, \mathrm{e}^{-i\omega\tau}\, d\tau$$

$p(x,\tau\,|\,x_o,0)$

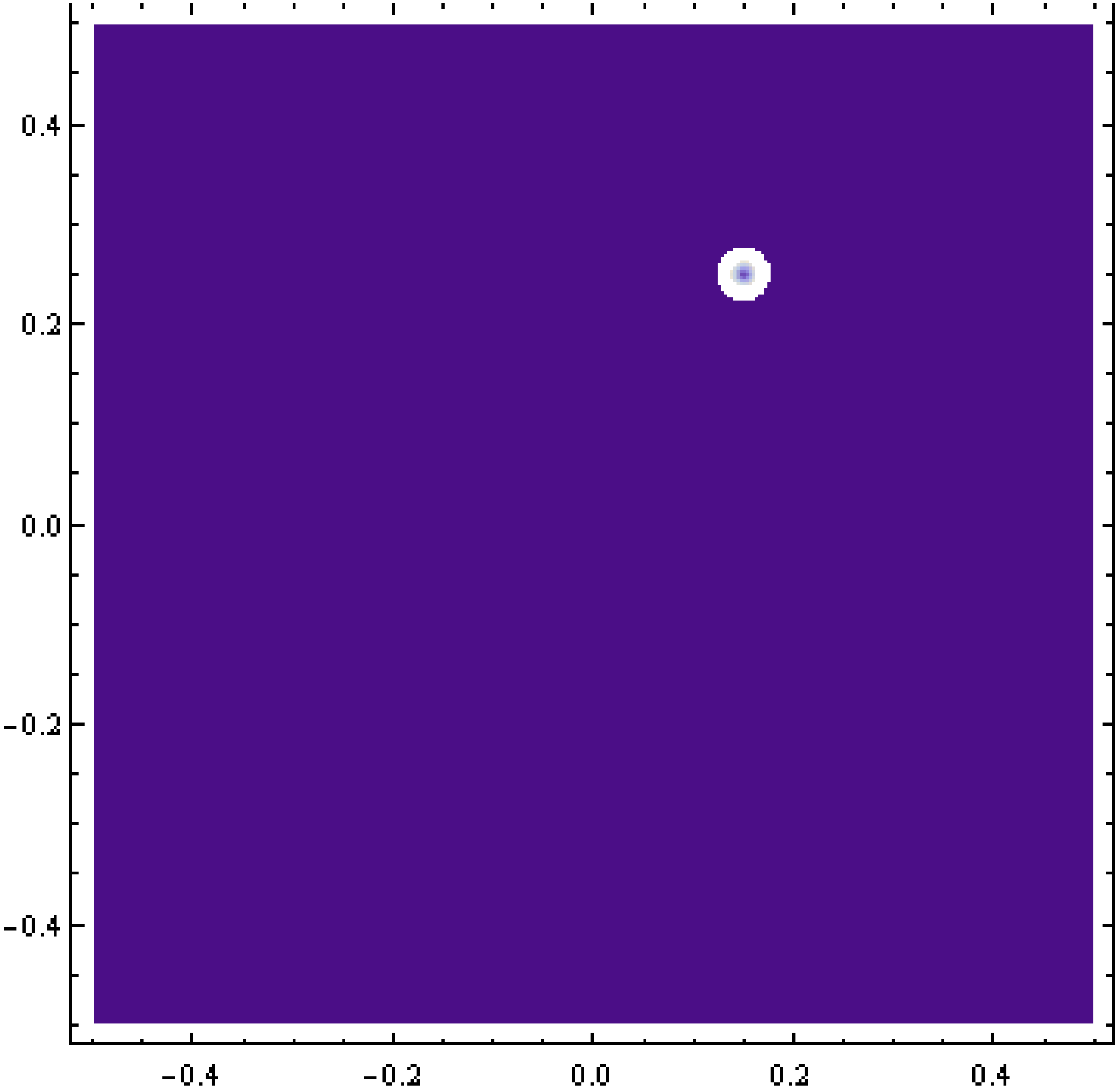
Written in terms of the 3D conditional probability.

$$S_B(\omega) = \frac{1}{(2\pi)^3}\int_V \mathbf{B}(\mathbf{x_0})d^3\mathbf{x_0}\int_{-\infty}^{\infty}\widetilde{\mathbf{B}}(\mathbf{x})d^3\mathbf{x}\int d\omega\int d^3\mathbf{Q}\mathrm{e}^{i\mathbf{Q}\cdot(\mathbf{x_0}-\mathbf{x})+i\omega\tau}p(\mathbf{Q},\omega)$$

One can take advantage of the periodicity of the variables via Fourier series,
Simplifying the integration.

$$\int_{-\infty}^{\infty}\widetilde{\mathbf{B}}(\mathbf{x})d^3\mathbf{x}\mathrm{e}^{-i\mathbf{Q}\cdot\mathbf{x}} = (2\pi)^3\sum_{l_x,l_y,l_z=-\infty}^{\infty}\int_{-\frac{L_x}{2}}^{\frac{3L_x}{2}}\int_{-\frac{L_y}{2}}^{\frac{3L_y}{2}}\int_{-\frac{L_z}{2}}^{\frac{3L_z}{2}} d^3\mathbf{x}\mathbf{B}(\mathbf{x})\ldots$$

$$\times\mathrm{e}^{-i\mathbf{Q}\cdot\mathbf{x}}\delta(Q_x-\frac{l_x\pi}{L_x})\delta(Q_y-\frac{l_y\pi}{L_y})\delta(Q_z-\frac{l_z\pi}{L_z})$$



0.4
0.2
0.0
−0.2
−0.4
−0.4
−0.2
0.0
0.2
0.4

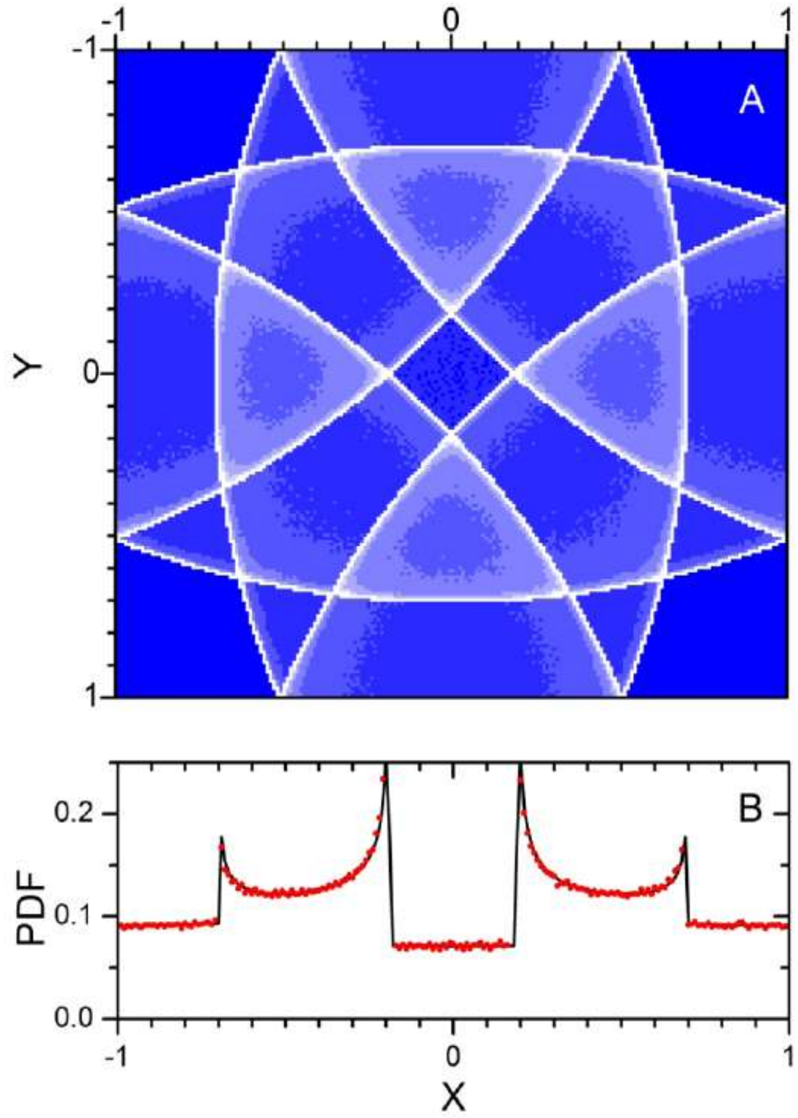
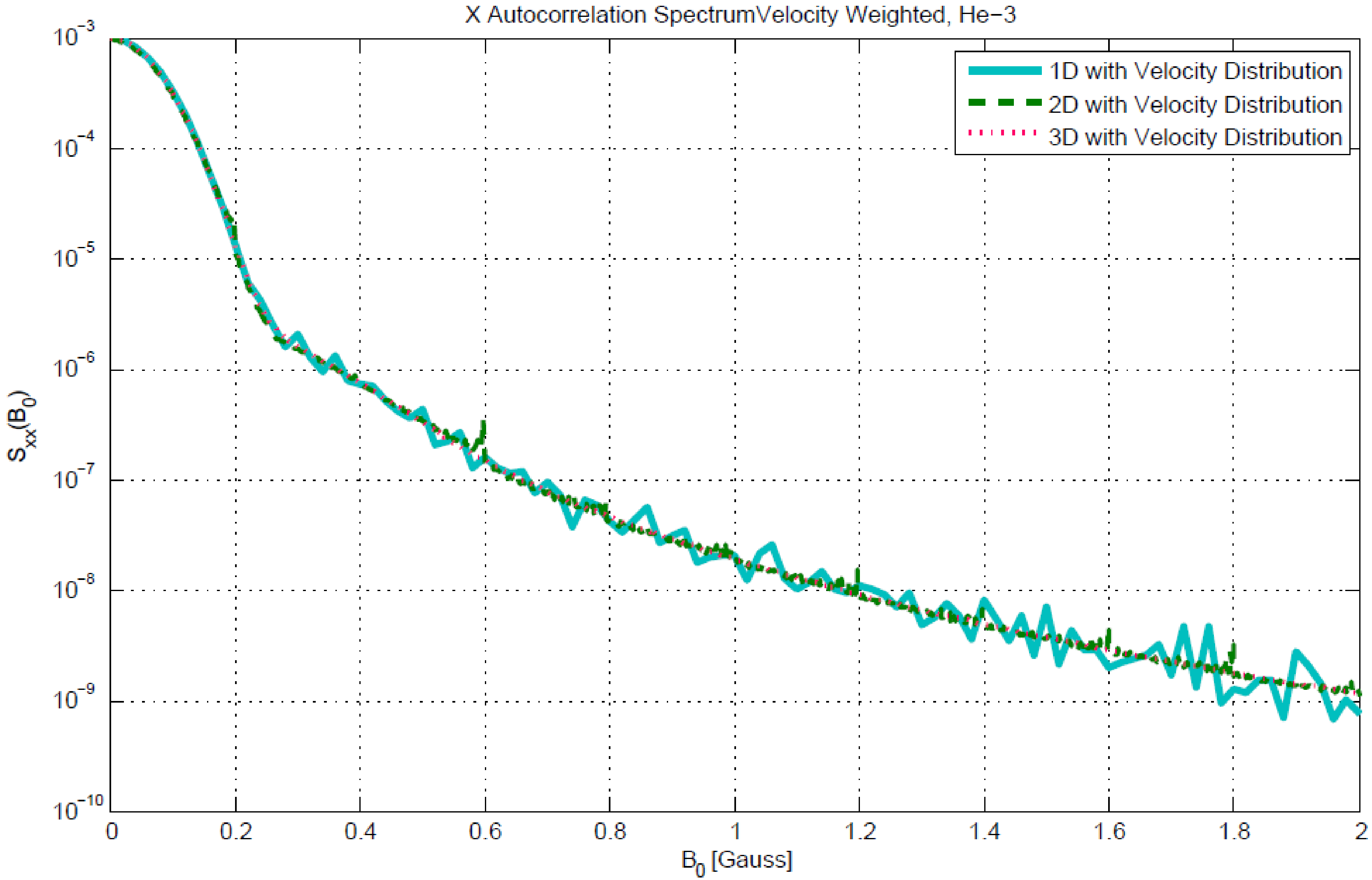
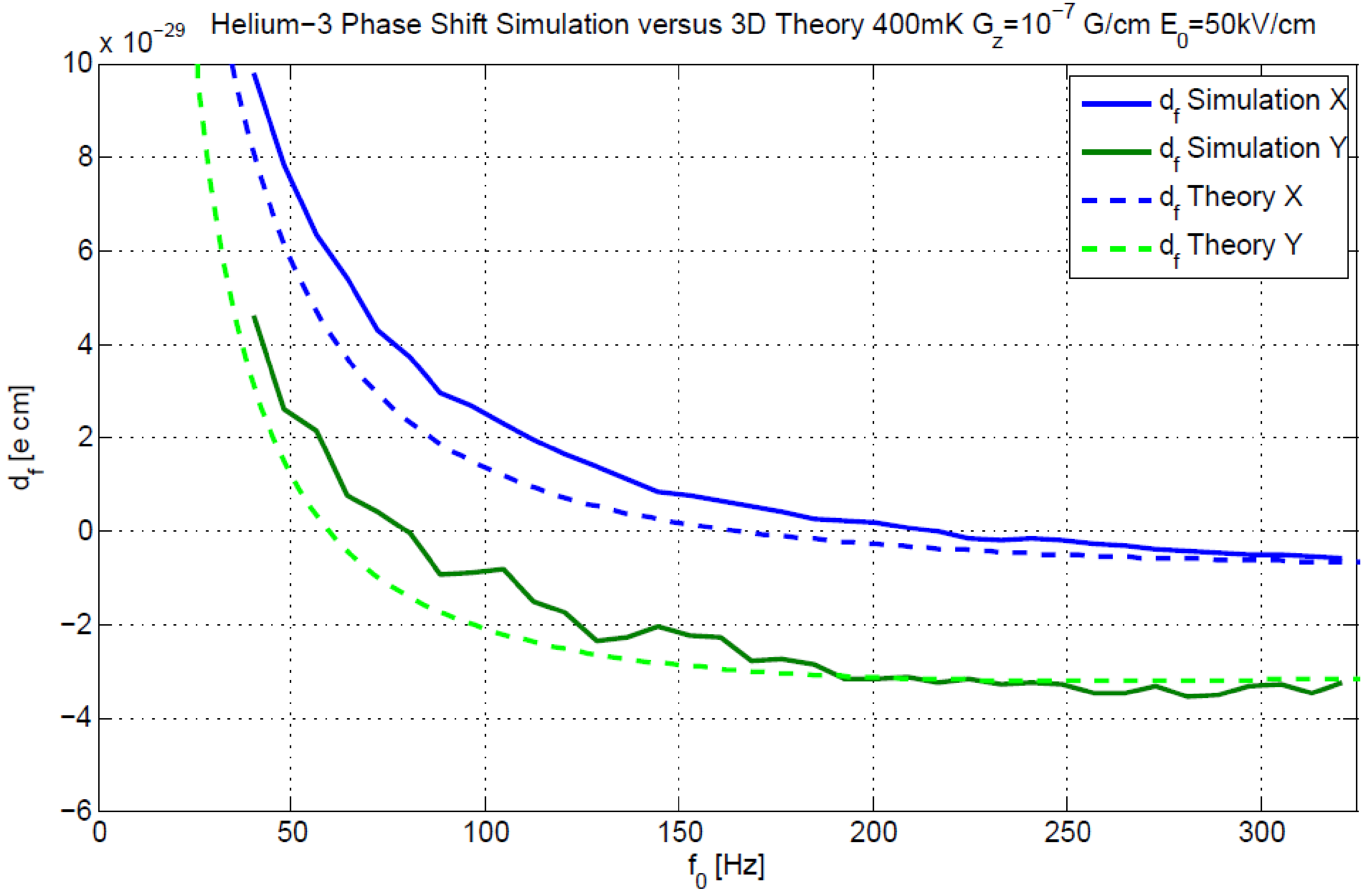


FIG. 2. A) Showing a snapshot of $P(\vec{r}, \tau)$ for parameters given in the text. The white lines are the peak of unscattered particles while the brightness of the contrast indicates the build up of the wake of scattered particles. B) A cross section along the x axis of the function $P_{02}(\vec{r}, \tau = 2.7)$ shown in A). The smaller peaks at ± 0.7 are the particles that made a collison with the walls at $x = \pm 1$. The larger peaks represent the crossing of the two "waves" emanating from the image points at $(\pm 2, \pm 2)$ and are larger because they are the sum of the two "waves". Solid lines are the result of equation (??) while the dots are the results of a Monte-Carlo simulation.

Velocity Weighted Distributions



False EDM due to frequency shift



$$\Delta\omega = \frac{2ed_n\Delta E}{\hbar}$$

$$d_f = 4.56 \times 10^{-29} S(\omega)$$

Figure 2.9: Comparison of the simulated false EDM in helium-3 versus the 3D theory 400mK. 200,000 particles where simulated.

Measuring the correlation function

The relaxation can be derived from the same density matrix as the phase shift. This time ignoring the **E** field. Keeping only non-zero correlations.

Longitudinal Relaxation

$$\frac{1}{T_1} = \frac{\gamma^2}{2}\left(\int_{-\infty}^{\infty} B_x(t)B_x(t+\tau)e^{-i\omega_0\tau}\,d\tau + \int_{-\infty}^{\infty} B_y(t)B_y(t+\tau)e^{-i\omega_0\tau}\,d\tau\right)$$

Transverse Relaxation

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{\gamma^2}{2}\int_{-\infty}^{\infty} B_z(t)B_z(t+\tau)d\tau$$

In a linear gradient the position autocorrelation function spectrum is proportional to the relaxation rate, therefore we can measure the spectrums used to predict the geometric phase.

Measuring the correlation function

Spectrums proposed to model Relaxation/Phase shift.

$$S_{1D}(\omega) = \sqrt{\frac{2}{\pi}\left(\frac{m}{kT}\right)} \sum_{n=\text{integers}} \int_0^\infty dv\ e^{-\frac{mv^2}{2kT}} \ldots$$

$$\times \frac{2v\tau_c}{\left(\frac{1}{2}\pi + n\pi\right)\left(\left(\omega^2 - \omega_n^2\right)^2 \tau_c^4 + \tau_c\omega^2\right)}$$

$$S_{2D}(\omega) = \frac{m}{kT} \sum_{lx=odd} \int_0^\infty dv\ ve^{-\frac{mv^2}{2kT}} \frac{8L_x^2}{\pi^4 l_x^4} \frac{1}{\sqrt{\left(\frac{1}{\tau_c} + i\omega\right)^2 + \frac{v^2\pi^2 l_x^2}{L_x^2}} - \frac{1}{\tau_c}}$$

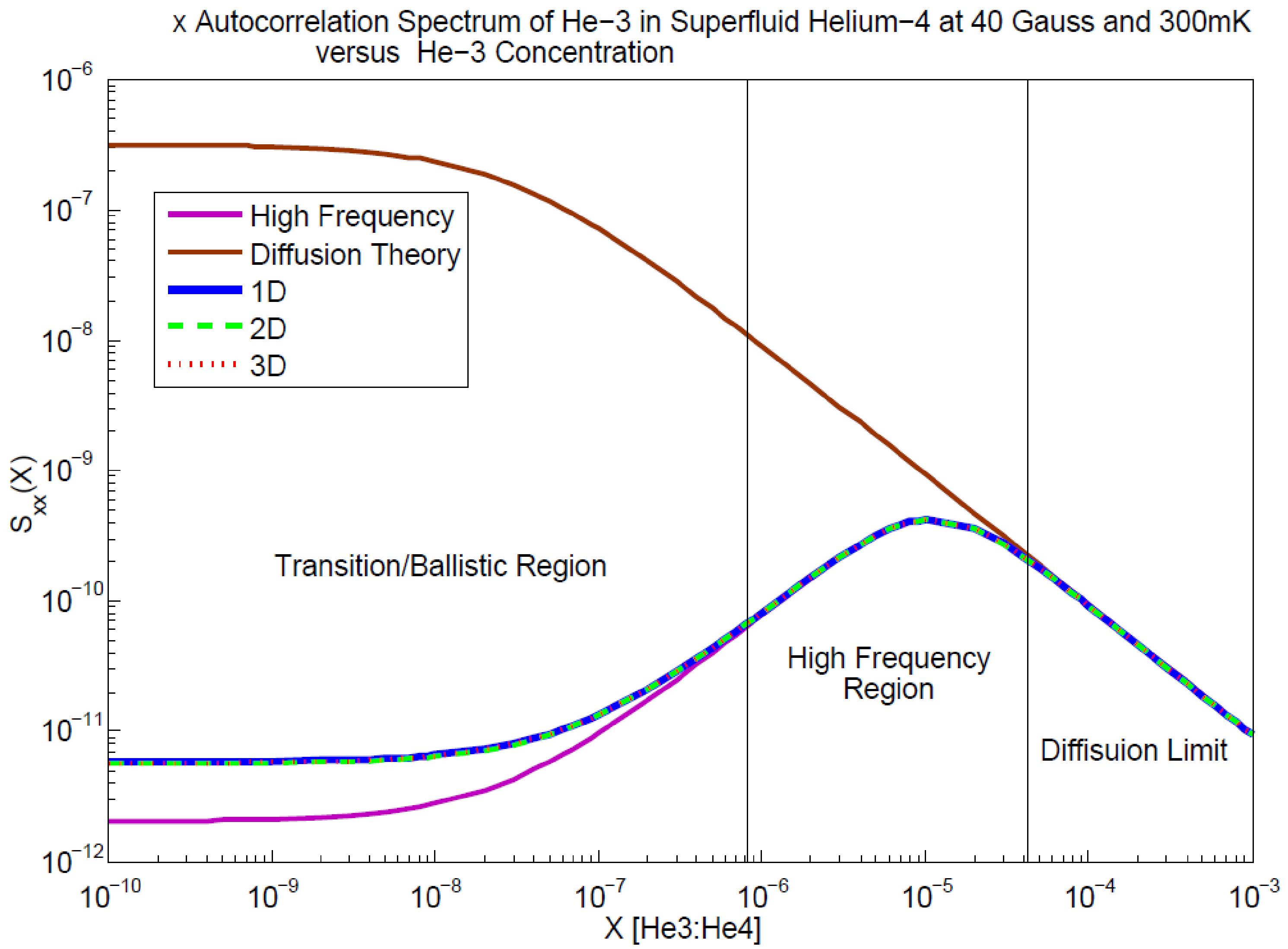
$$S_{3D}(\omega) = \sqrt{\frac{2}{\pi}\left(\frac{m}{kT}\right)^3} \sum_{lx=odd} \int_0^\infty dv\ v^2 e^{-\frac{mv^2}{2kT}} \frac{8L_x^2\tau_c}{\pi^4 l_x^4} \frac{1}{\frac{q_x}{\arctan\left(\frac{q_x}{1+i\omega\tau_c}\right)} - 1}$$

Spectrums used to measure Relaxation/Phase in the High Frequency and Diffusive Models.

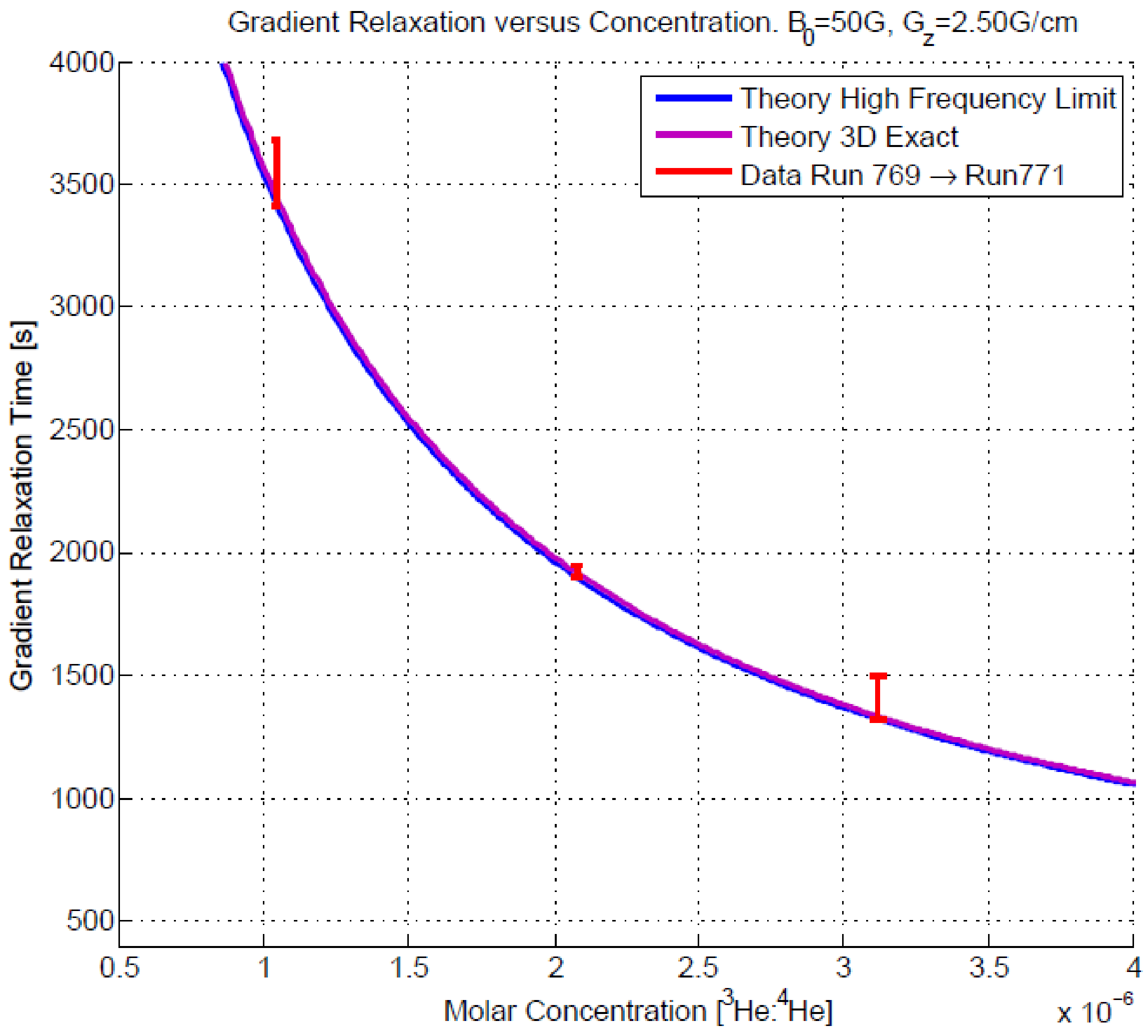
$$S_{\text{HF}} = \frac{2\left\langle v_{1D}^2\right\rangle \tau_c}{\omega^2(1+\omega^2\tau_c^2)}$$

$$S_{\text{Diff}} = \sum_n \frac{16D}{\pi^2 n^2} \frac{1}{\left(\frac{n^2\pi^2 D}{L^2}\right)^2 + \omega^2}, \quad n = \text{positive odd}$$

Measuring the correlation function



Results



High Frequency

$$\chi^2_\nu = 2.46$$

3D Theory

$$\chi^2_\nu = 0.88,$$