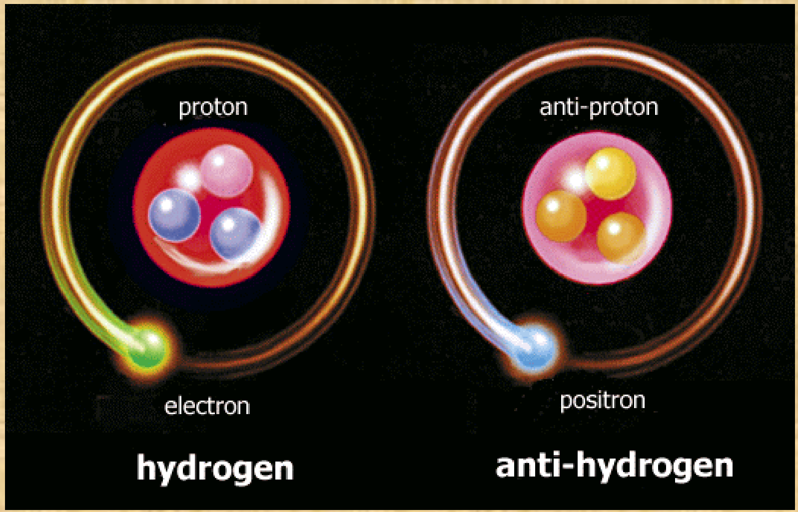
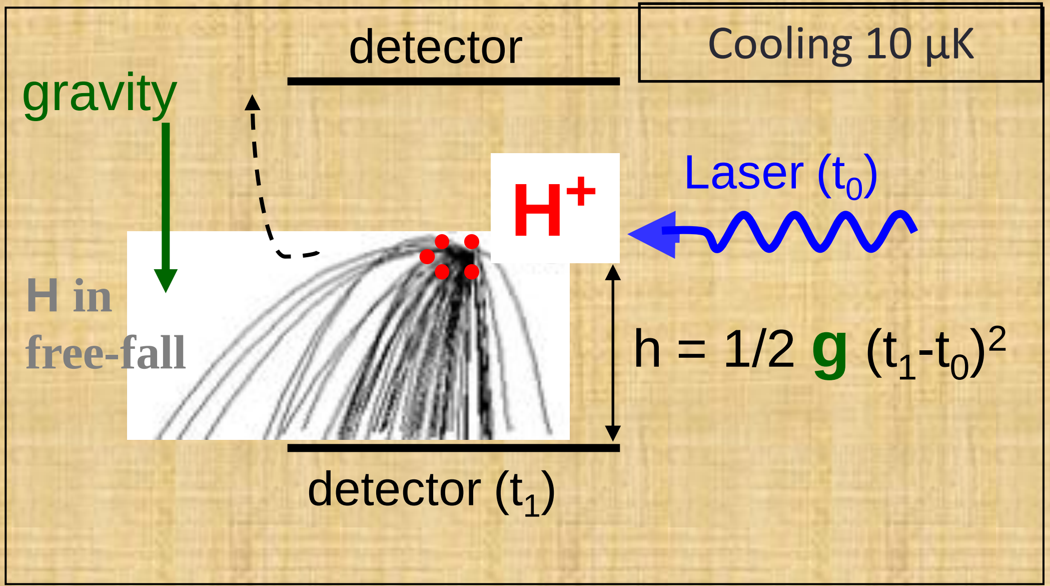
G. Dufour, P. Debu, A. Lambrecht, V.V. Nesvizhevsky, S. Reynaud, A.Yu. Voronin, *Shaping the distribution of vertical velocities of antihydrogen in GBAR*, Europ. Phys. J. C 74 (2014) 2731



GBAR principle: cool H^+ to get ultra-slow H

- *Produce H^+*
- *Capture H^+ in a trap*
- *Cooling H^+ with Be^+*
  $\rightarrow$ 10 μK
- *Photodetachment of one excess e^+*
- *Time of flight*

$H^+ = p\, e^+ e^+$

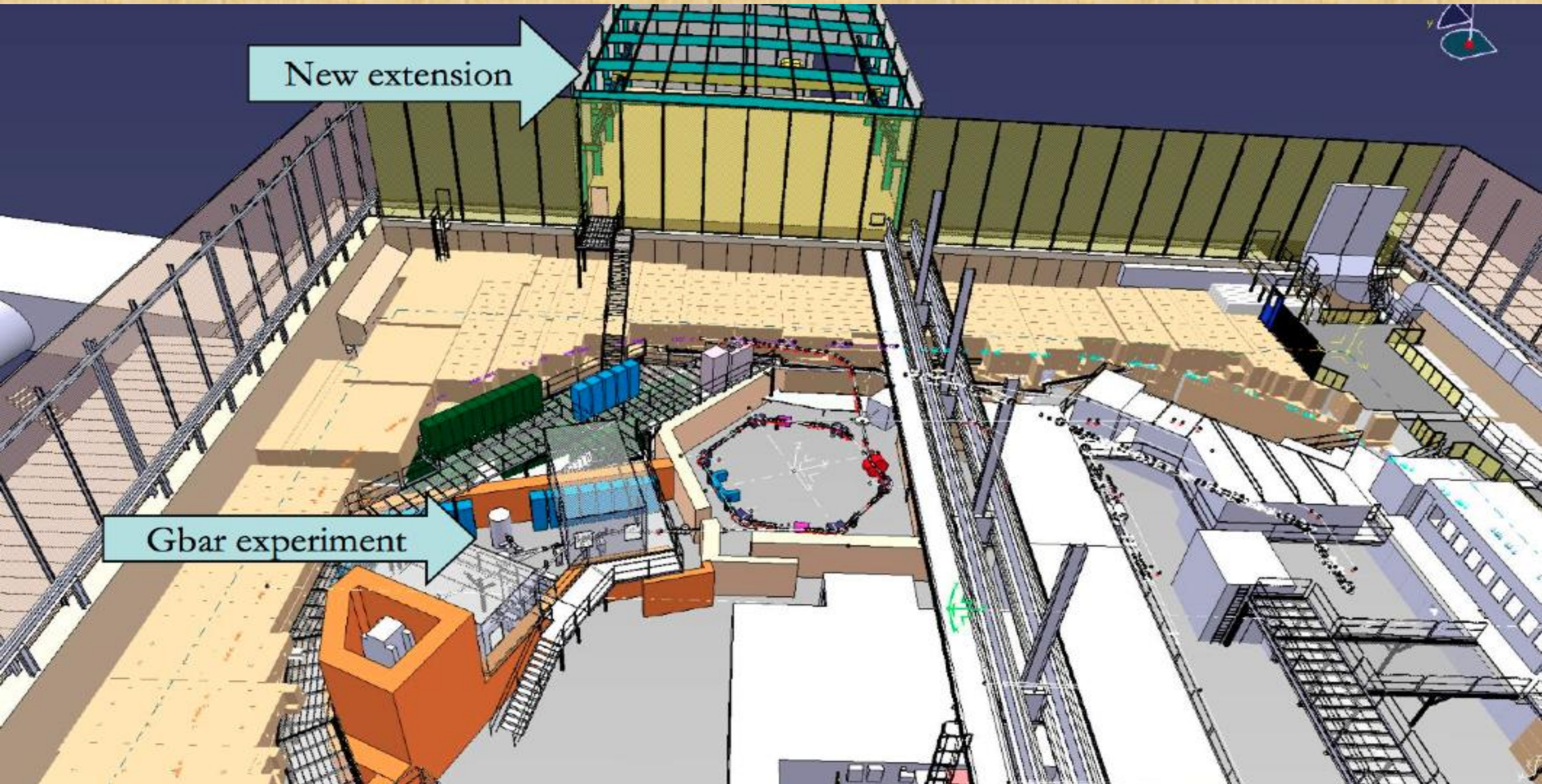


J.Walz & T. Hänsch,

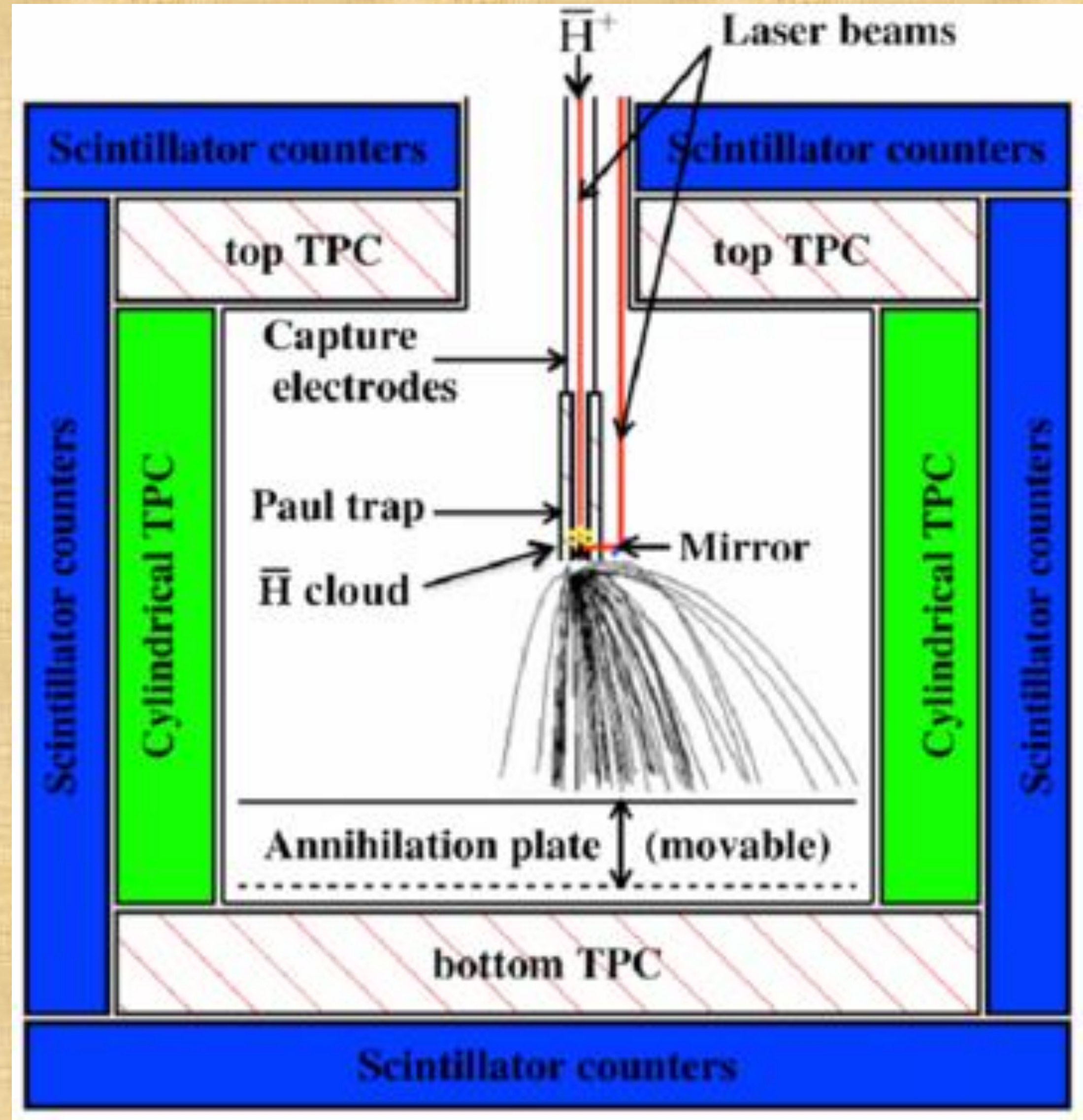
General Relativity and Gravitation, 36 (2004) 561.

Uncertainty dominated by the temperature of H^+

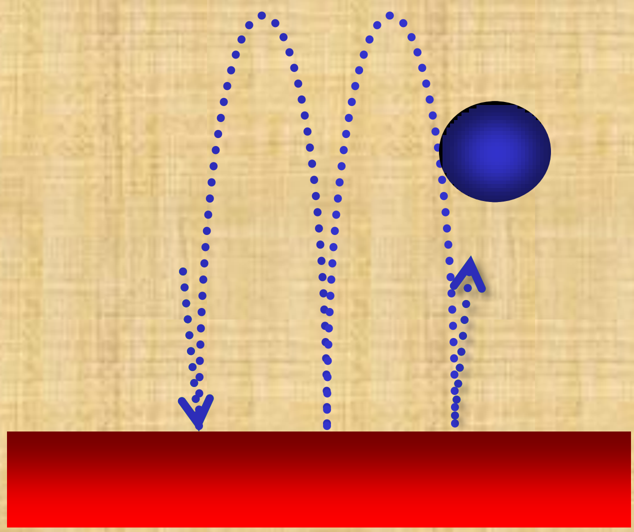
Future installation at CERN



Future installation at CERN



Precision? -> Gravitational Spectroscopy !



1) **Electrical neutrality** (usually the gravitational interaction for an object above a surface is much weaker than other interactions)
2) **Long life-time** $\left(\Delta E \approx \frac{\hbar}{\Delta \tau}\right)$
3) **Small mass** $\left(\Delta \mathrm{v} \cdot \Delta x \approx \frac{\hbar}{m}\right)$
4) **Energy (effective temperature) of UCN, or an atom,** is extremely low; it is not equal to the surface temperature (the effective temperature of a particle in gravitational quantum states is ~10 nK)

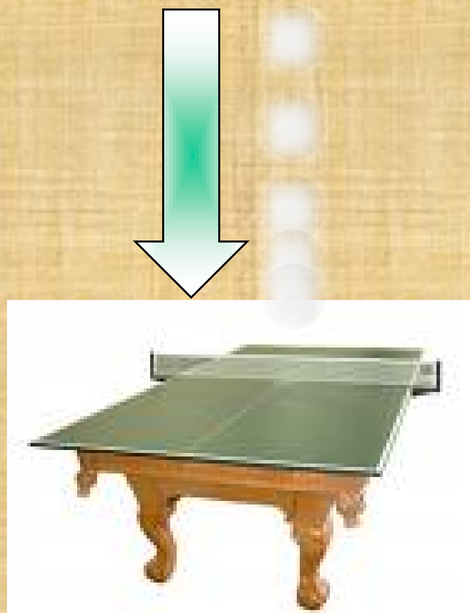
A particle avobe a mirror in the gravity field: An ultracold neutron (V.I. Luschikov, A.I. Frank « Quantum effects occuring when ultracold neutrons are stored on a plane », JETP Lett. 28 (1978) 759**)** **and … an anti-hydrogen atom (A.Yu. Voronin, P. Froelich, V.V. N.** « Gravitational quantum states of antihydrogen », Phys. Rev. A 83 (2011) 032903**)**

Energy of quantum states, in the Bohr-Zommerfeld approximation, equals :

$$E_n \approx \sqrt[3]{\left(\frac{9 \cdot m}{8}\right) \cdot \left(\pi \cdot \hbar \cdot g \cdot \left(n - \frac{1}{4}\right)\right)^2}$$

Matter / Anti-matter

Yesterday's sensation is today's calibration and tomorrow's background [Richard Feynman]



Gravitational properties of matter have never been measured directly!

but aimed at, for instance, in GBAR project at CERN

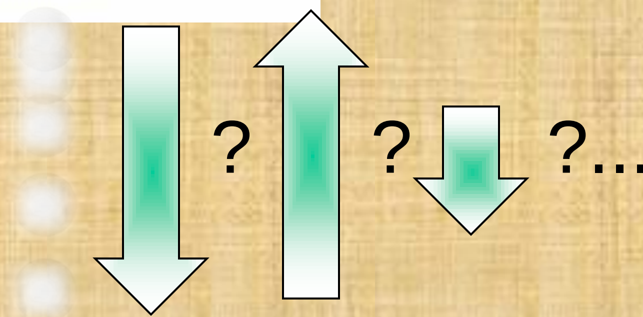
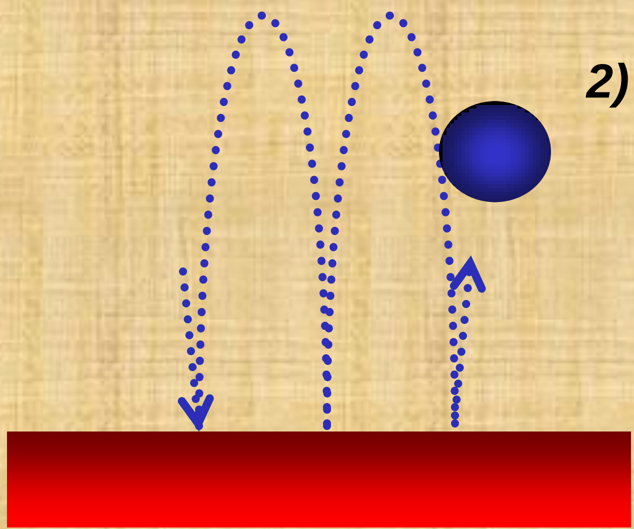


Illustration for quantum motion of a matter object and an anti-matter object above a mirror in a gravitational field.

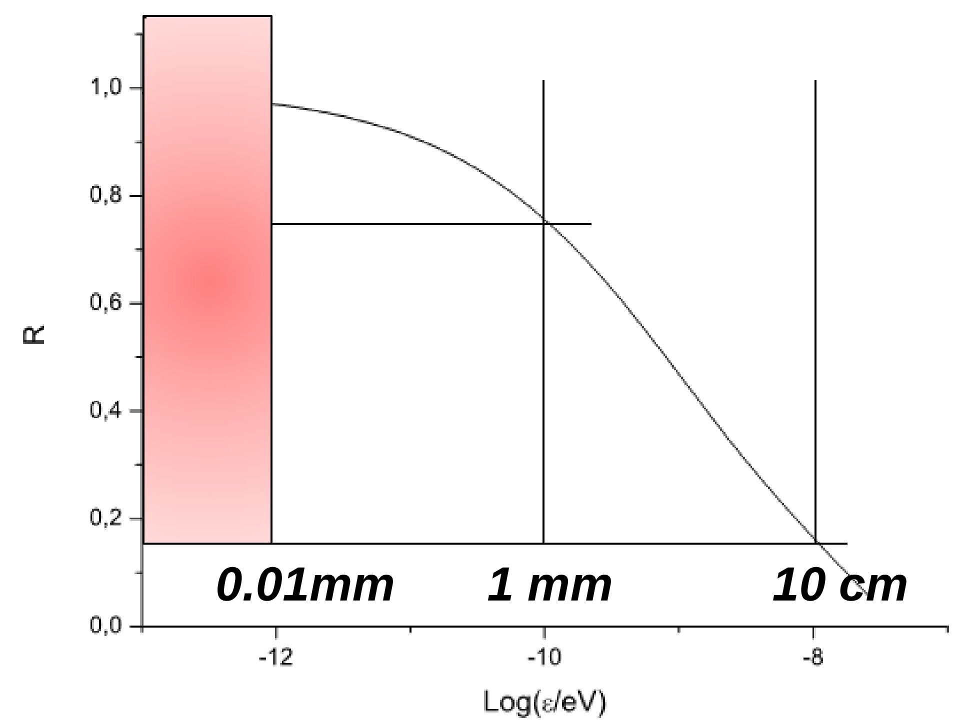
Choosing a quantum system



A(n) (anti)-particle avobe a mirror in the gravity field

1) *A neutron is reflected from the nuclear optical potential of a matter due to averaging of the neutron interaction with a huge number of nuclei*

2) *An anti-matter (matter) atom is reflected from the sharply-changing (negative) van der Waals/ Casimir-Polder (vdW/CP) potential step (originating from vacuum fluctuations)* A.Yu. Voronin, P. Froelich, B. Zygelman, Phys. Rev. A 72 (2005) 062903



Gravitational quantum states of antihydrogen

$$\lambda_n \approx \left(\frac{3\pi}{4}\left(2n - \frac{1}{2}\right) \right)^{2/3}$$

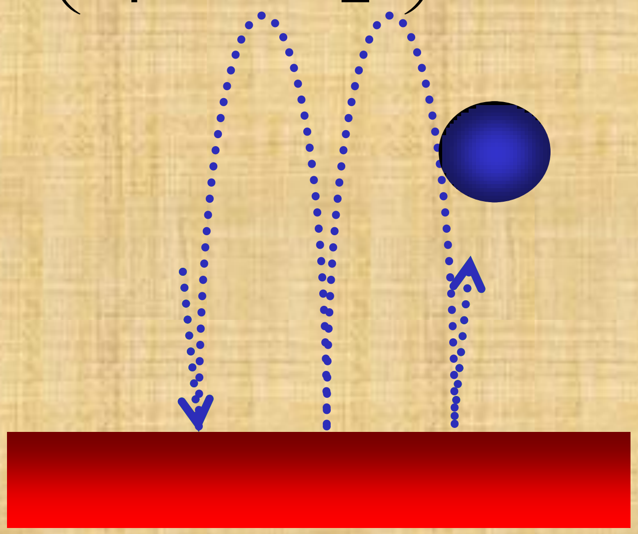


TABLE I. The eigenvalues, gravitational energies, and classical turning points of a quantum bouncer with the mass of (anti)hydrogen in the Earth's gravitational field.

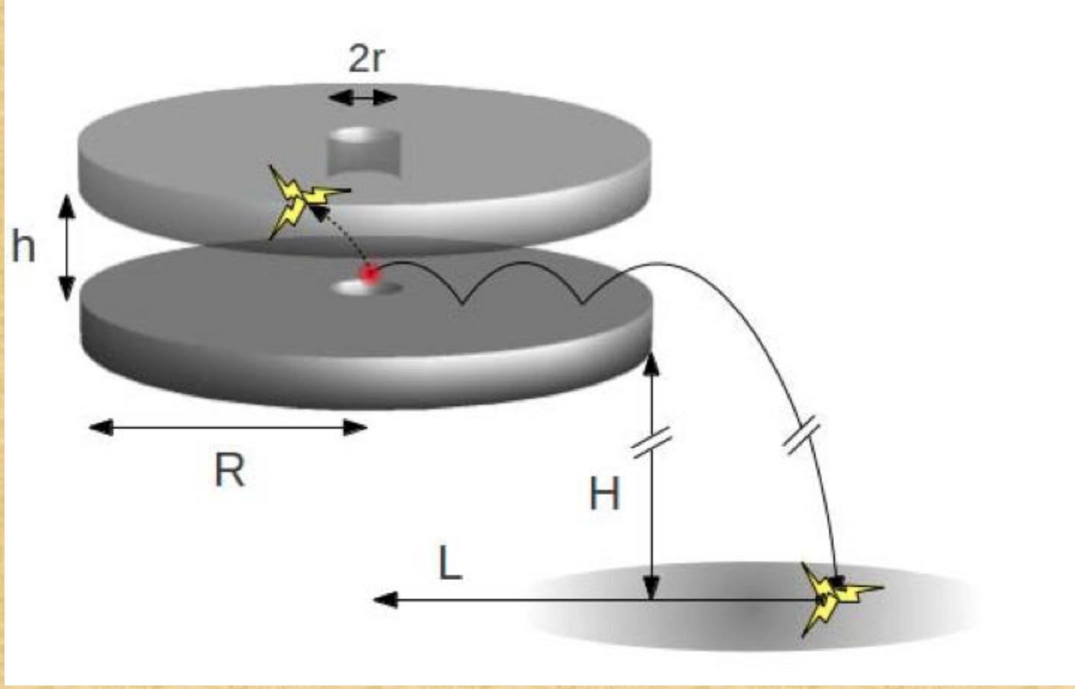
n	λ_n^0	E_n^0 (peV)	z_n^0 (μm)
1	2.338	1.407	13.726
2	4.088	2.461	24.001
3	5.521	3.324	32.414
4	6.787	4.086	39.846
5	7.944	4.782	46.639
6	9.023	5.431	52.974
7	10.040	6.044	58.945

A(n) (anti)-particle avobe a mirror in the gravity field

$$\left. \begin{aligned} &\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + V_{CP}(z) + Mgz - E \right]\Psi(z) = 0 \\ &\Psi(-\infty) = 0 \end{aligned} \right\} \Rightarrow \begin{aligned} &\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + Mgz - E \right]\Psi(z) = 0 \\ &\frac{\Psi(0)}{\Psi'(0)} = -\frac{a_{CP}}{l_g} \approx i0.005 \end{aligned}$$

Methods of their observation

- ***Spectroscopy: induced transitions between gravitational quantum states***
- ***Interference: temporal and spatial oscillations of annihilation signal of a superposition of gravitational states***
- ***Temporal and spatial resolution of free-fall events: mapping of the momentum distribution in gravitational states into time-of-fall or spatial distribution***



Antihydrogen in magnetic field

$$\hat{T}_z + Mgz + \hat{H}_c + \frac{\alpha_{HF}}{2}\left(\boldsymbol{F}^2 - \frac{3}{2}\right) + \boldsymbol{B}(z,t)(\mu_B \hat{\mathbf{s}}_{\bar{e}} + \mu_{\bar{p}} \hat{\mathbf{s}}_{\bar{p}}) + \hat{H}_{st}$$

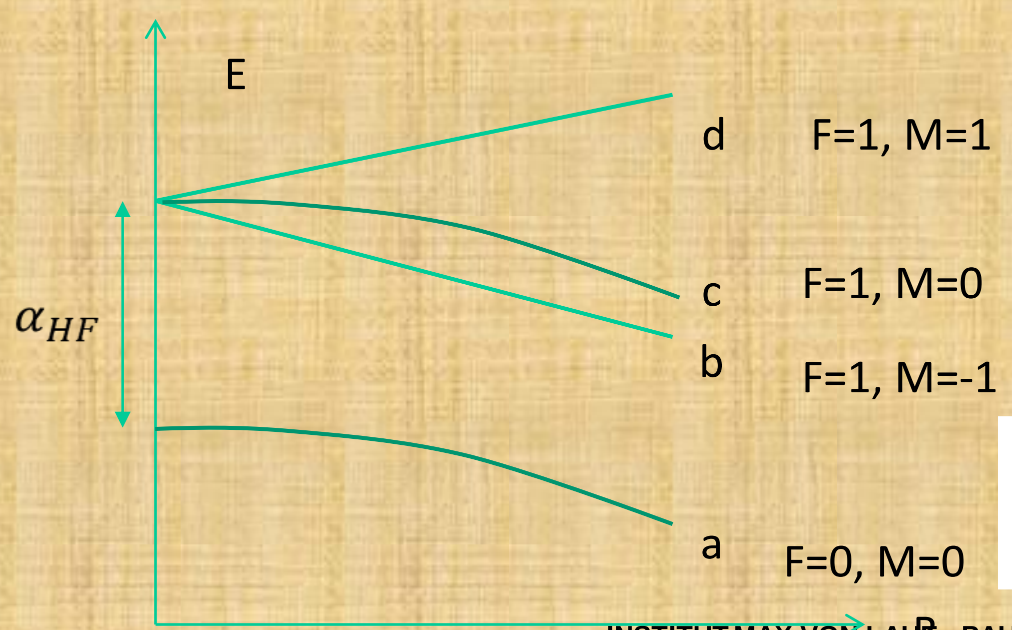
Center-of-mass fall

Coulomb

HF splitting

Field-magnetic moment interaction

Effective Stark-effect



$$\frac{\alpha_{HF}}{\hbar} \cong 1420\ Mhz$$

$$\varepsilon_{grav} \ll \mu B \ll \alpha_{HF}$$

$$E_{a,c} = E_{1s} - \frac{\alpha_{HF}}{4} \mp \frac{1}{2}\sqrt{\alpha_{HF}^2 + |(\mu_B - \mu_{\bar{p}})B(z,t)|^2}$$

$$E_{b,d} = E_{1s} + \frac{\alpha_{HF}}{4} \mp \frac{1}{2}|(\mu_B + \mu_{\bar{p}})B(z,t)|.$$

Interference of gravitational states

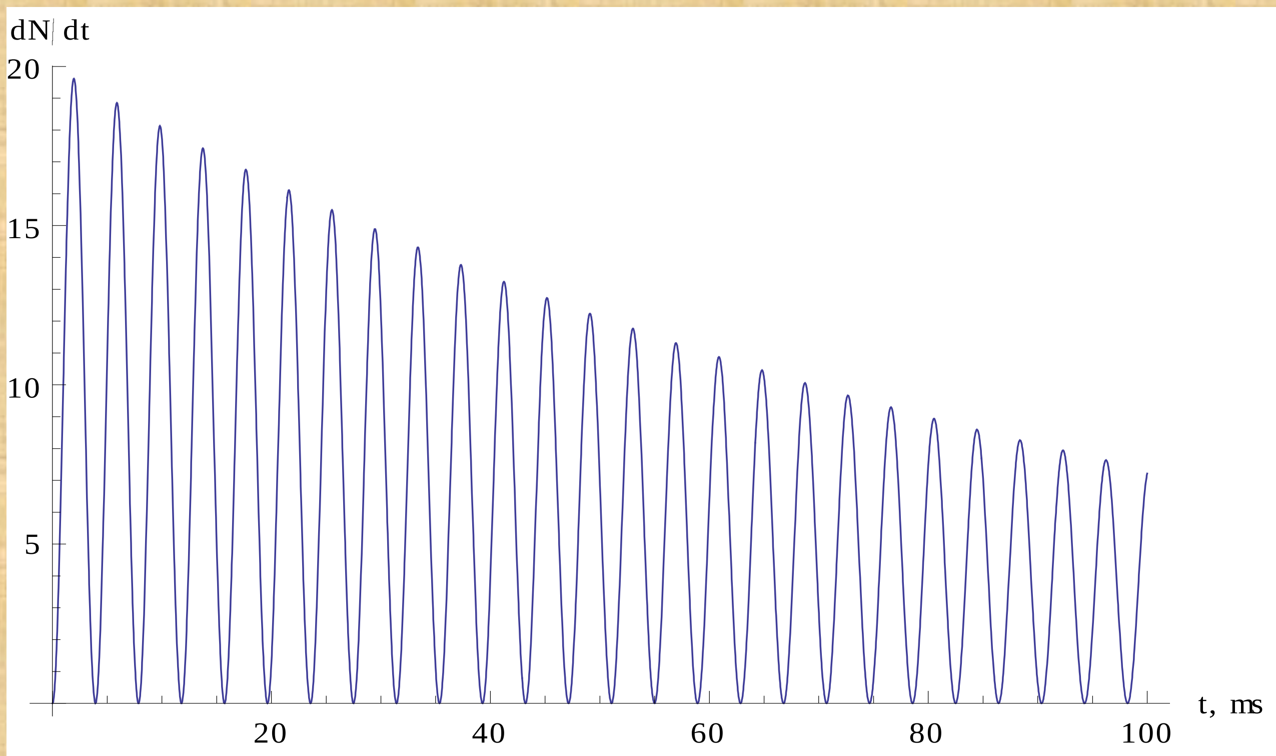
$$\Psi = \sum_{i=1}^{N} C_i \Psi_i$$

$$j(z,t) = \frac{i\hbar}{2m}\left[\Psi(z,t)\frac{d\Psi^*(z,t)}{dz} - \Psi^*(z,t)\frac{d\Psi(z,t)}{dz}\right]$$

$$\frac{dF_{12}}{dz} = -\frac{\Gamma}{\hbar}\exp(-\frac{\Gamma}{\hbar}t)(1-\cos(\omega_{12}t))$$

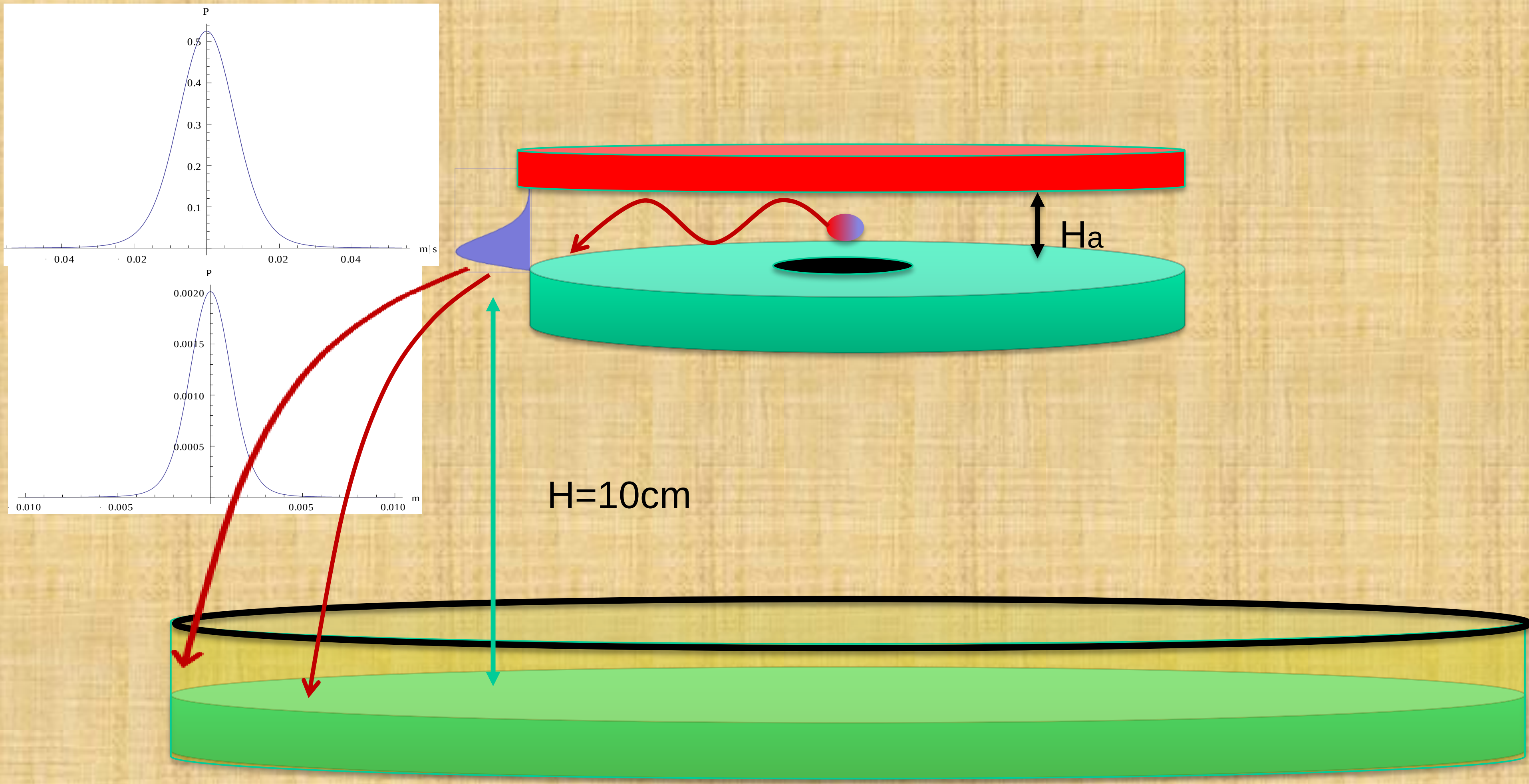
$$\omega_{12} = \frac{E_2 - E_1}{\hbar}$$

$$\hbar\Gamma^{-1} \approx 0.1s$$



Temporal and spatial resolving of gravitational states

Momentum distribution in a gravitational state can be mapped into a measurable time or spatial distribution



Mapping of momentum distribution

$$\Psi(z,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{ipz/\hbar} G(p,t,p') F_0(p') dp dp'$$

$$G(p,t,p') = \exp\left[-\frac{it}{2m\hbar}(p^2 - Mgpt + M^2 g^2 t^2/3)\right]\delta(p - Mgt - p')$$

$$\Psi(z,t) \approx \sqrt{\frac{m}{t}} e^{\frac{imz^2}{2t\hbar} + -\frac{it^3 M^2 g^2}{2m\hbar}} F_0(p_0 - Mgt);\ p_0 = (z + \frac{gt^2}{2})\frac{m}{t}$$

$$|\Psi(z,t)|^2 \approx \frac{m}{t} |F_0(k)|^2$$

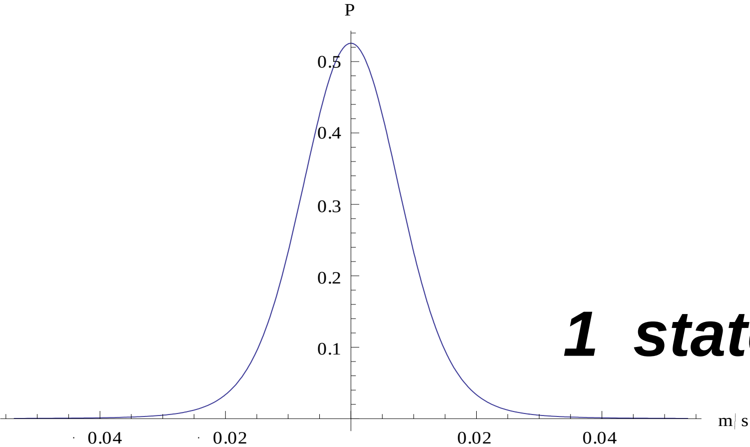
$$1)\, z = z_0 : k = mg(t - t_0),\ t_0 = \sqrt{2g/z_0}$$

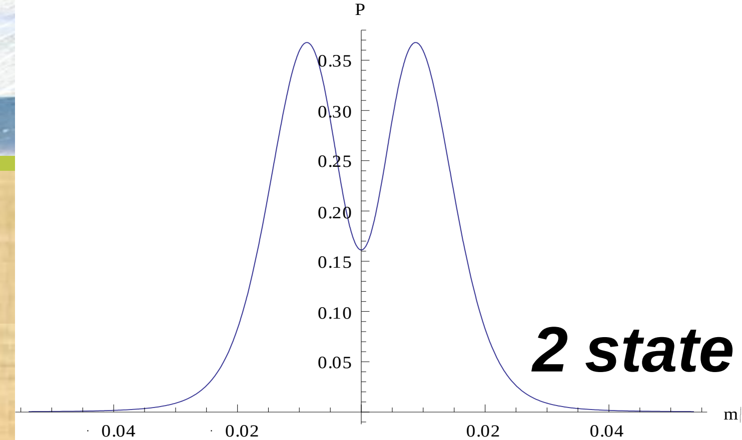
$$2)\, t = t_0 = L/v : k = \frac{m(z - z_0)}{t_0}$$

$$l_g = \frac{\delta(n)\hbar}{\Delta k_n}$$

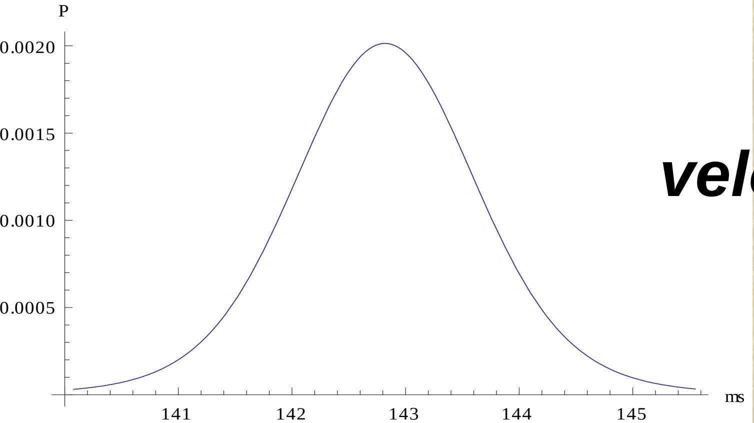
1 state

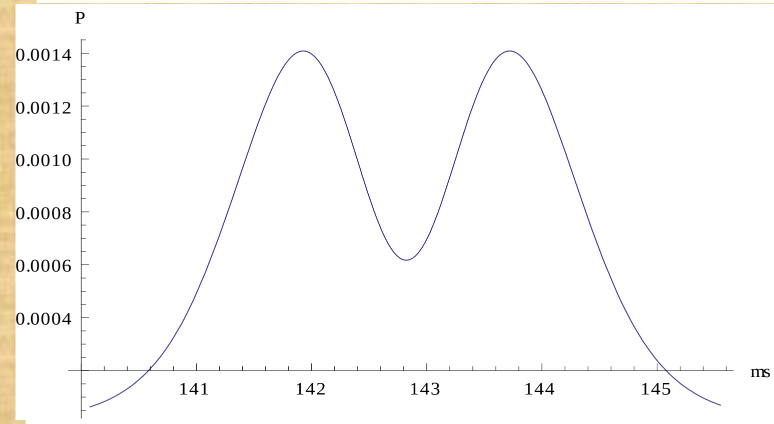
2 state



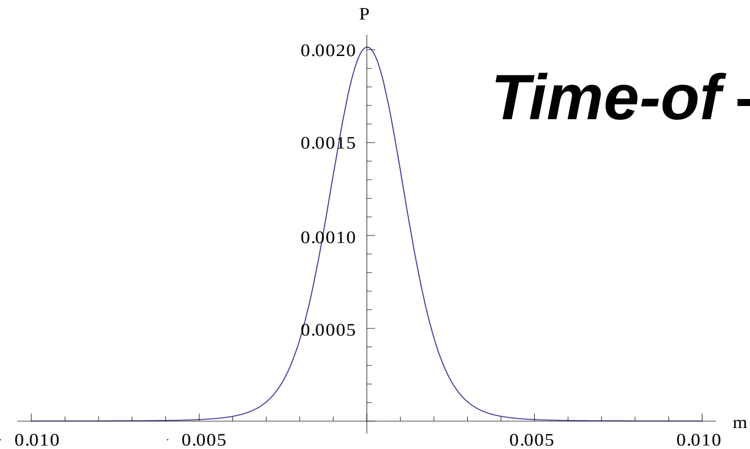


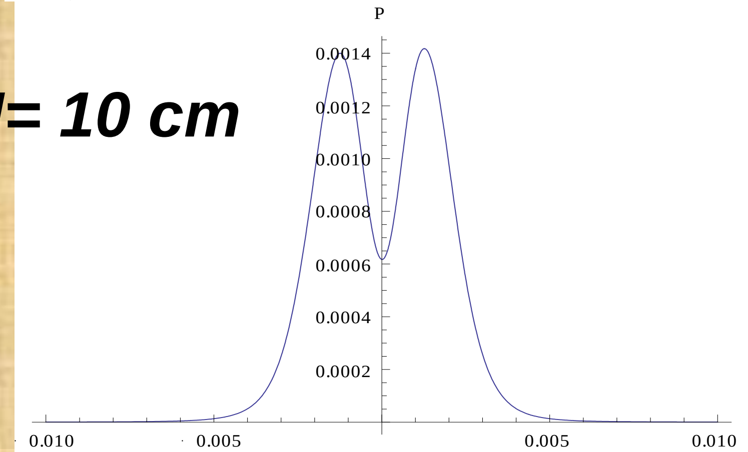
velocity distribution





Time-of –fall distribution H= 10 cm

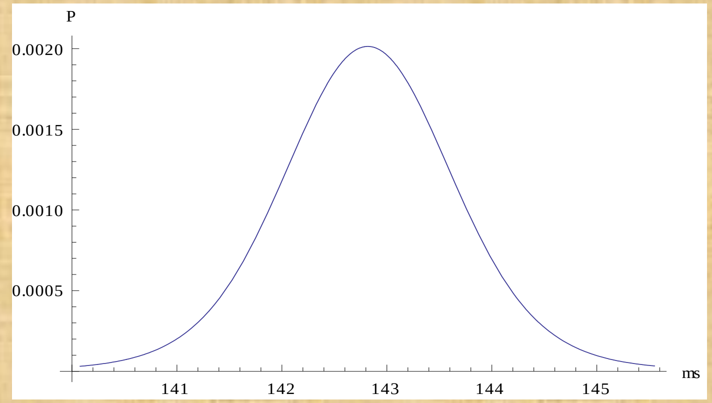




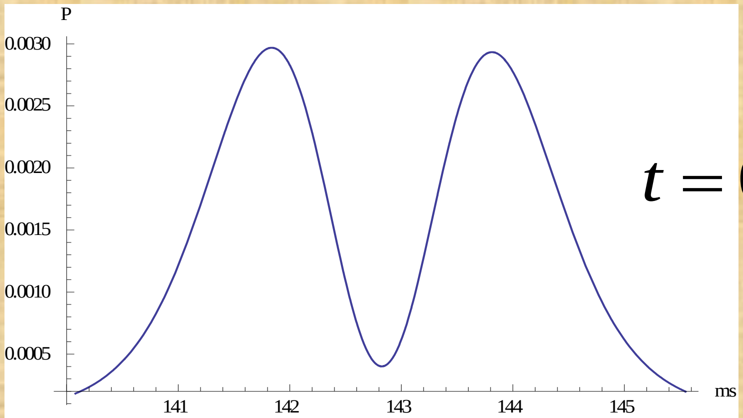
Spatial distribution time-of-flight T=0.1s

Phase monitoring

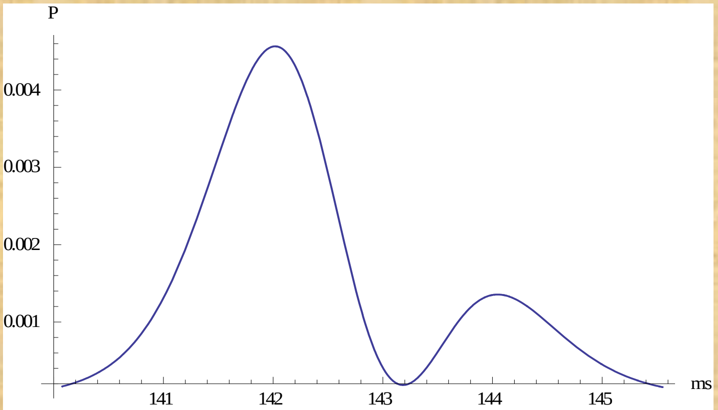
$$\Psi = \Psi_1 + Exp(-i\omega_{21}t)\Psi_2$$



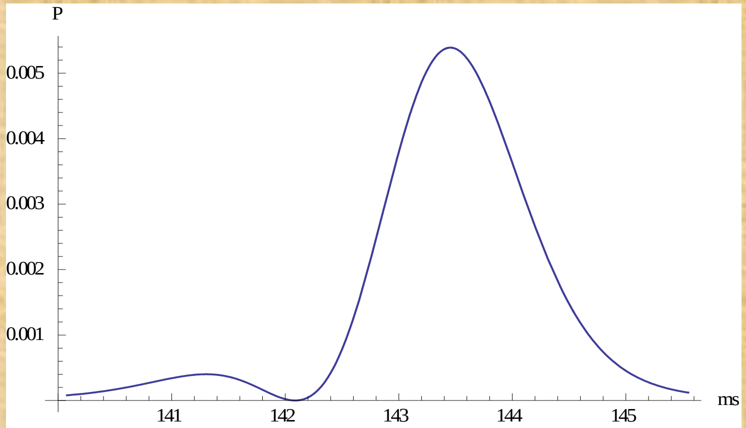
$t = 0$



$t = 0.0019s$

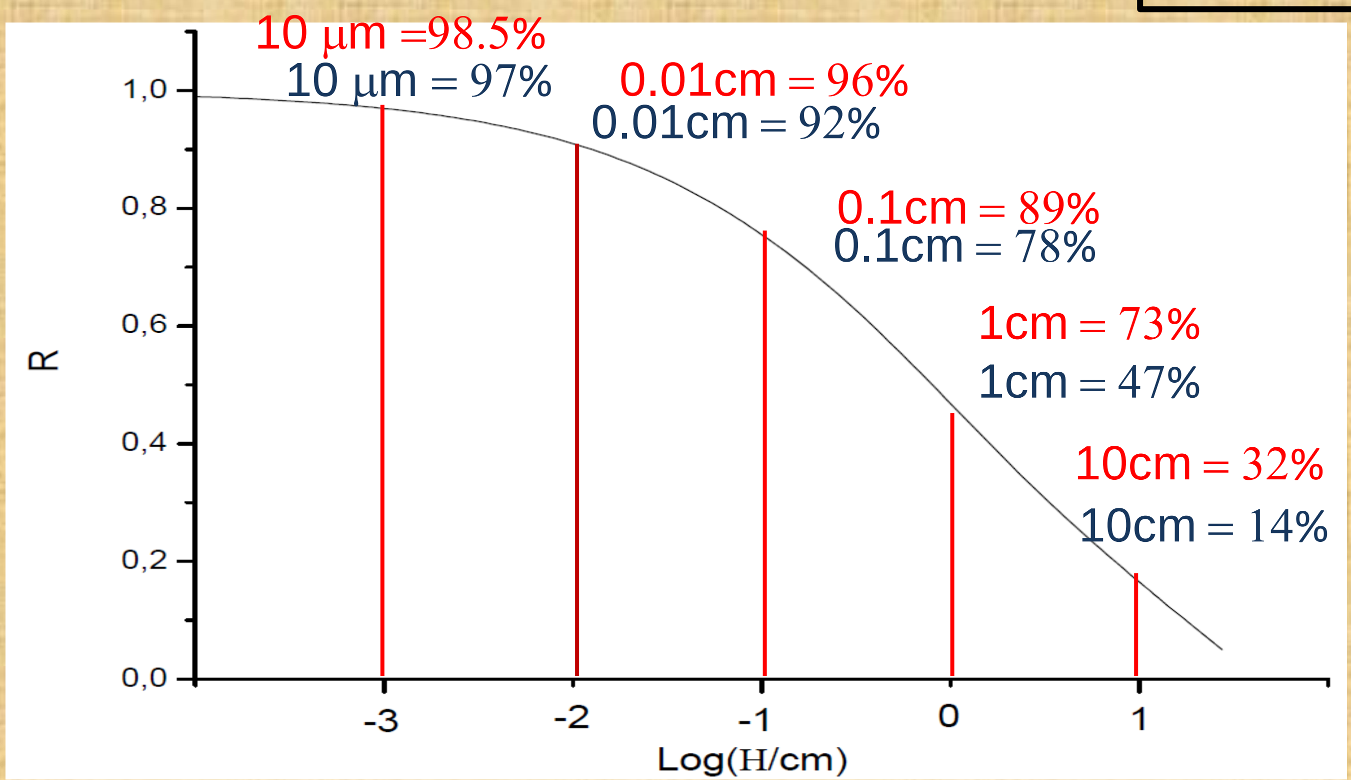


$t = 0.0014s$



$t = 0.0029s$

Red- silica, black- gold



A. Yu. Voronin, P. Froelich, and B. Zygelman, Phys. Rev. A **72**, 062903 (2005).

G. Dufour, A. Gérardin, R. Guérout, A. Lambrecht, V. V. Nesvizhevsky, S. Reynaud, A. Yu. Voronin Phys. Rev. A 87, 012901 (2013)

Motivations for gravitational quantum experiment

1. Precision spectroscopic and interferometric measurements (0.01% and better).
2. Compact experimental design, thus cheap setup.
3. We can profit form major expertise gained in analogous experiments with ultracold neutrons (UCNs).
4. An option of one-to-one prototyping antihydrogen experiments using, for instance, UCNs in GRANIT spectrometer in ILL, Grenoble.
5. Constraints for extra short-range fundamental interactions between matter and antimatter.

NEUTRONS FOR SCIENCE

First step towards fully quantum experiment

1. The goal of GBAR : to measure *precisely* the acceleration of gravity for antimatter
2. Precision depends on the *dispersion* of vertical velocities and on the *control* of their distribution
3. *A new method* for shaping and controlling the distribution of vertical velocities of antihydrogen
4. We estimate statistical and systematical uncertainties as *better than 0.1 %*

Shaping the distribution of vertical velocities of antihydrogen in GBAR

Here, we limit ourselves to simple (*quasi)classical description* of the new scheme for shaping the distribution of vertical velocities of antihydrogen

Rigorous *quantum description* is available in the publication cited on the front page

This scheme is a first step towards *quantum experiment*

By considering classical->quantum limit, we show that quantum experiment will provide better statistical sensitivity, smaller systematical uncertainties, more compact design and cheaper setup *simultaneously*

Analysis of « unrestricted » free fall

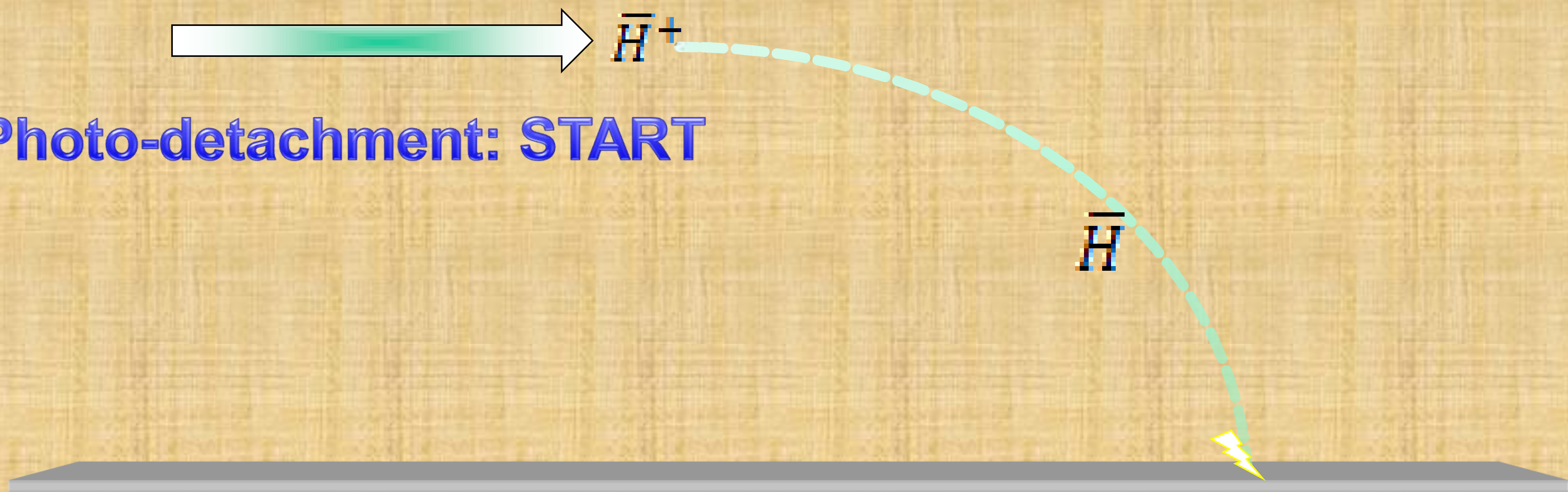


Photo-detachment: START

Annihilation: STOP

$$\bar{g} = Mg/m$$

Analysis of « unrestricted » free fall

$$\bar{H}^{+} \quad mv\zeta = \frac{\hbar}{2}$$

Photo-detachment: START

$$\bar{H}$$

$$(\Delta\bar{g}/\bar{g}) = 2(\Delta t/t)$$

$$v_H = \sqrt{2\bar{g}H} \qquad t_H = \sqrt{2H/\bar{g}}$$

Annihilation: STOP

$$\frac{\Delta t}{t_H} = \sqrt{\left(\frac{\zeta}{2H}\right)^2 + \left(\frac{\upsilon}{v_H}\right)^2} = \sqrt{\left(\frac{\zeta}{2H}\right)^2 + \left(\frac{\hbar}{2mv_H\zeta}\right)^2}$$

$$\zeta_{opt} = \sqrt{\frac{\hbar H}{mv_H}} \qquad \left(\frac{\Delta t}{t_H}\right)_{opt} = \sqrt{\frac{\hbar}{2mv_H H}}$$

Analysis of « unrestricted » free fall

$\zeta_{opt} \approx 88\ \mu m$

Photo-detachment: START

$H = 0.3\ m \quad v_H \approx 2.4\ m/s \quad t_H = \sqrt{2H/\bar{g}}$

Annihilation: STOP

$(\Delta t/t_0)_{opt} \approx 2.0 \cdot 10^{-4}$

$(\Delta \bar{g}/\bar{g})_{opt} \approx 4.0 \cdot 10^{-4}$

$N_{tot} \approx 2.6 \cdot 10^{4}$

$$\frac{\Delta \bar{g}_{opt}}{\bar{g}\sqrt{N_{opt}}} = \sqrt{\frac{2\hbar}{m v_H H N_{tot}}} \approx 2.5 \cdot 10^{-6}$$

Analysis of « unrestricted » free fall

BUT: $0.22\ \mu m > \zeta > 0.07\mu m$

that is 3 orders of magnitude smaller

Photo-detachment: START

$H = 0.3\ m \quad v_H \approx 2.4\ m/s$

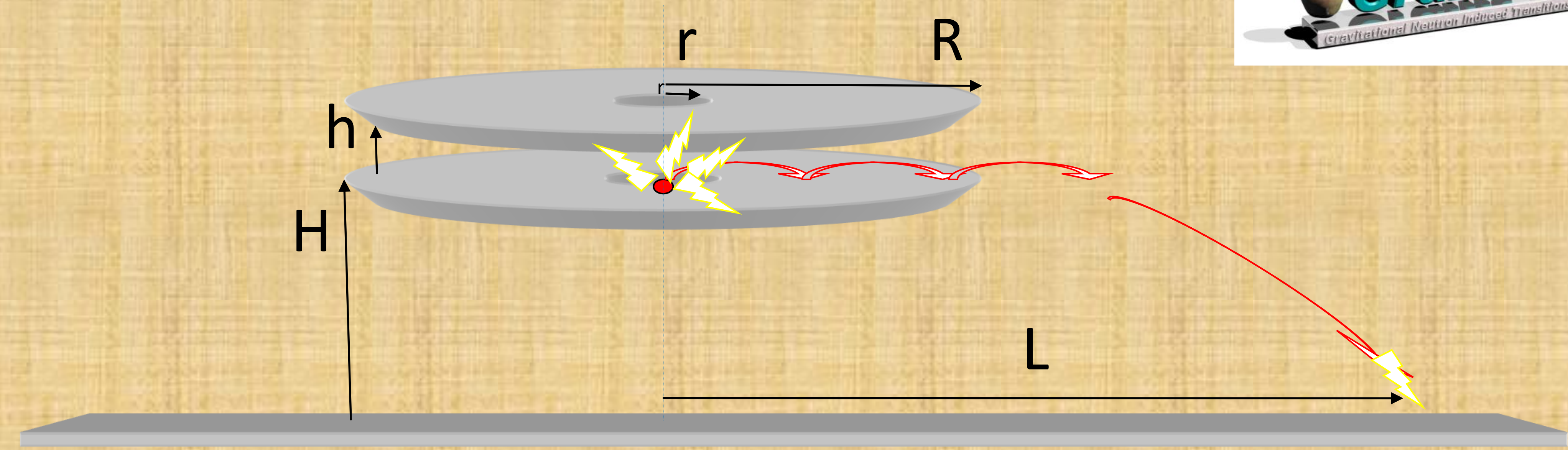
Annihilation: STOP

thus the resolution is limited by the large dispersion of initial velocity

$$\frac{\Delta \bar{g}}{\bar{g}\sqrt{N_{tot}}} = \frac{2v}{v_H\sqrt{N_{tot}}}$$

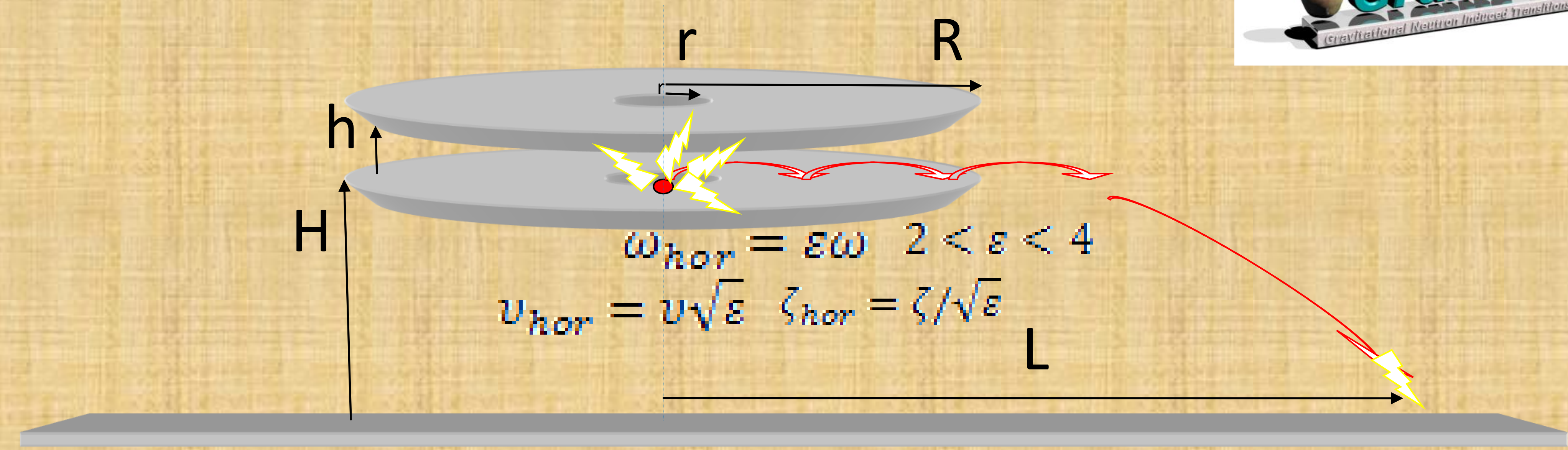
$N_{tot} \approx 2.6 \cdot 10^4$

A new scheme for shaping the distribution of vertical velocities of antihydrogen in GBAR



1) To take into account the possibility of antigravity, the setup should be reversed « upside-down »; 2) The shaping device has to be coupled with the Paul trap; 3) Position-sensitive and time-resolving detectors for counting annihilation events; 4) Cylindrical symmetry.

A new scheme for shaping the distribution of vertical velocities of antihydrogen in GBAR



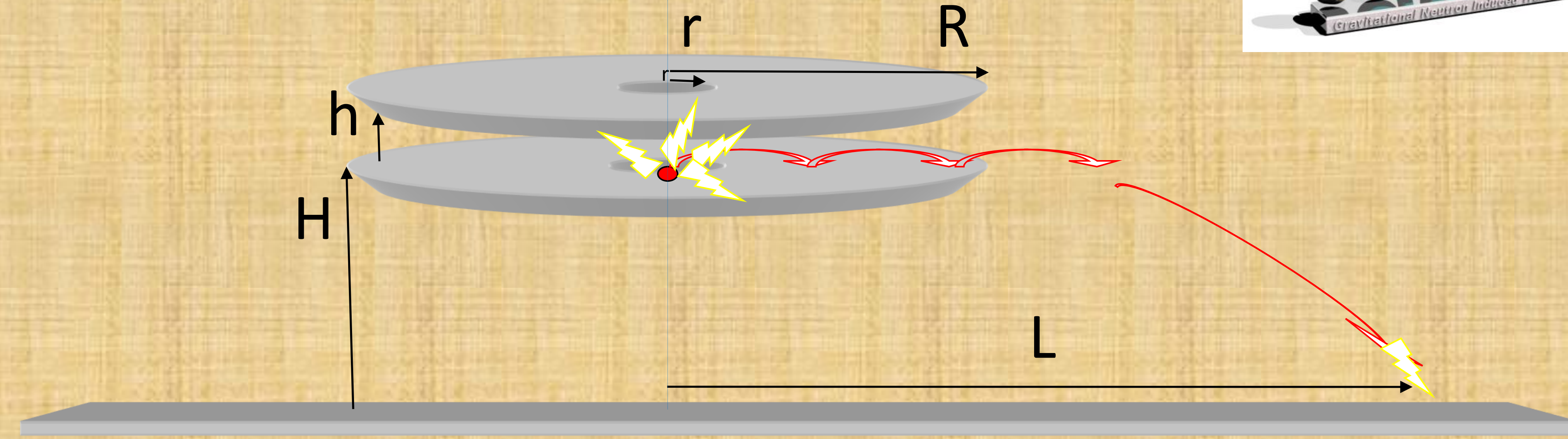
$$\omega_{hor} = \varepsilon\omega \quad 2 < \varepsilon < 4$$

$$v_{hor} = v\sqrt{\varepsilon} \quad \zeta_{hor} = \zeta/\sqrt{\varepsilon}$$

To start: r tends to zero, R tends to infinity, losses are negligible. Then

$$\frac{N}{N_{tot}} = \frac{\Delta v}{v}\sqrt{\frac{1}{2\pi}} \qquad \Delta v = \sqrt{2\bar{g}h}$$

Constraints for r and R values



$$\frac{h}{r} > \frac{\Delta v}{v\sqrt{\varepsilon}}$$

$$T = \frac{R}{v\sqrt{\varepsilon}} > 2t_h = 2\sqrt{\frac{2h}{\bar{g}}}$$

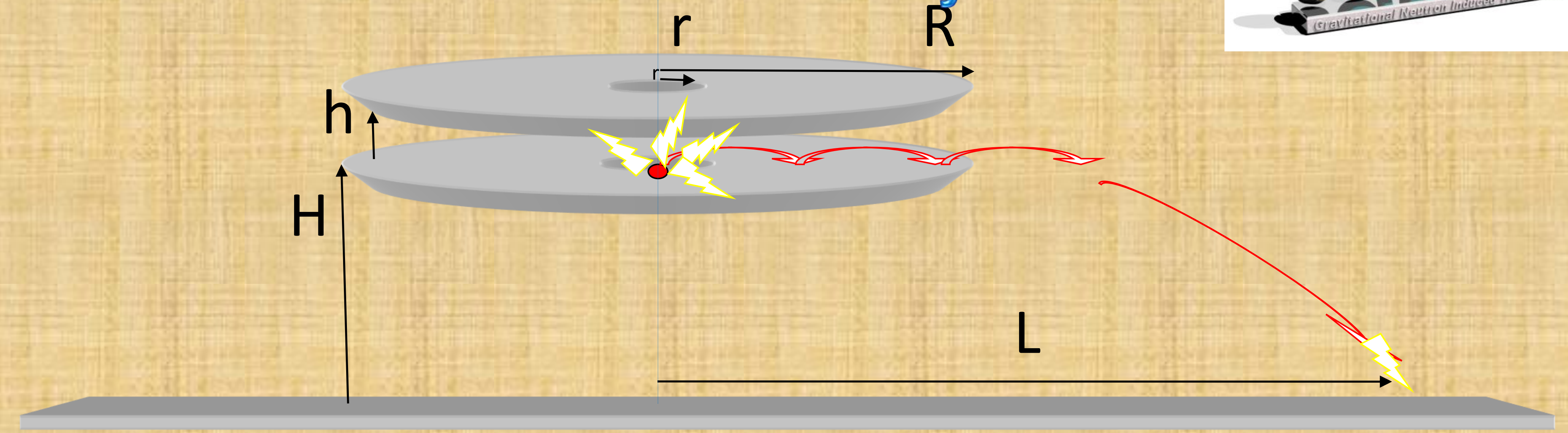
$$r < r_{max} = \frac{v\sqrt{\varepsilon h}}{\sqrt{2\bar{g}}} = \sqrt{\varepsilon h h_{max}}$$

$$R > R_{min} = \frac{4v\sqrt{\varepsilon h}}{\sqrt{2\bar{g}}} = 4r_{max}$$

A new scheme for shaping the distribution of vertical velocities of antihydrogen in GBAR

NEUTRONS FOR SCIENCE

Estimation of statistical uncertainty



$$\frac{\Delta t}{t_H} = \sqrt{\alpha\left(\frac{h}{2H}\right)^2 + \beta\left(\frac{\Delta v}{v_H}\right)^2}$$

$$\Delta v = \sqrt{2\bar{g}h}$$

$$v_H = \sqrt{2\bar{g}H}$$

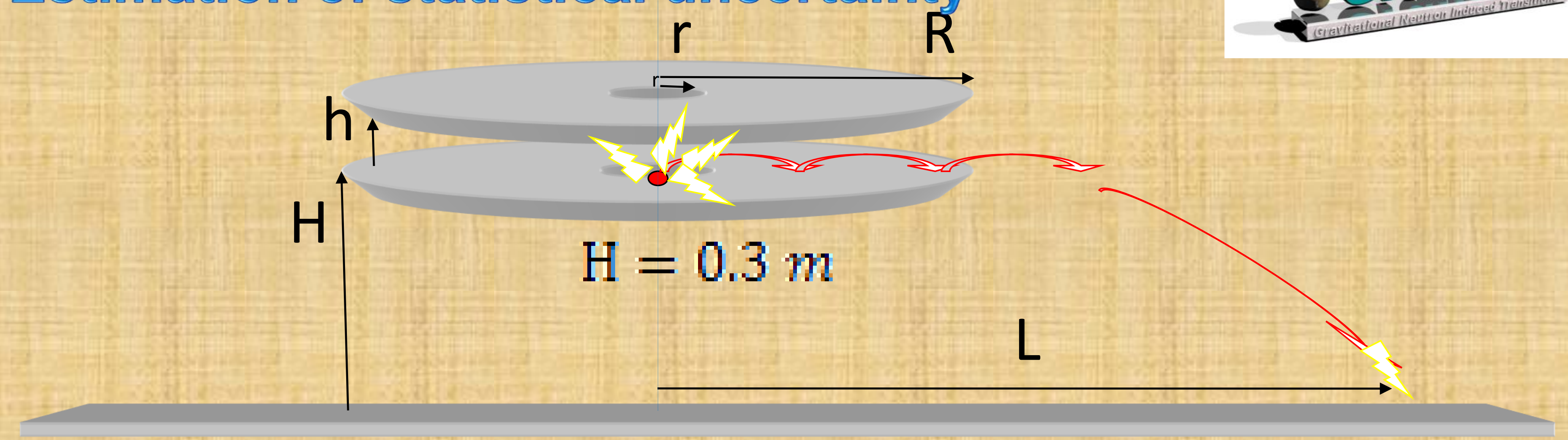
$$h \ll H$$

$$\frac{\Delta t}{t_H} \approx \sqrt{\frac{\beta h}{H}}$$

$$(\Delta\bar{g}/\bar{g}) \approx 2\sqrt{\beta h/H}$$

$$\left(\frac{\Delta\bar{g}}{\bar{g}}\right)\frac{1}{\sqrt{N}} = \frac{2\sqrt{\beta h}}{\sqrt{H}}\sqrt{\frac{v}{N_{tot}\Delta v}\sqrt{2\pi}} = 2\sqrt[4]{\frac{\pi h \beta^2 v^2}{\bar{g}H^2 N_{tot}^2}}$$

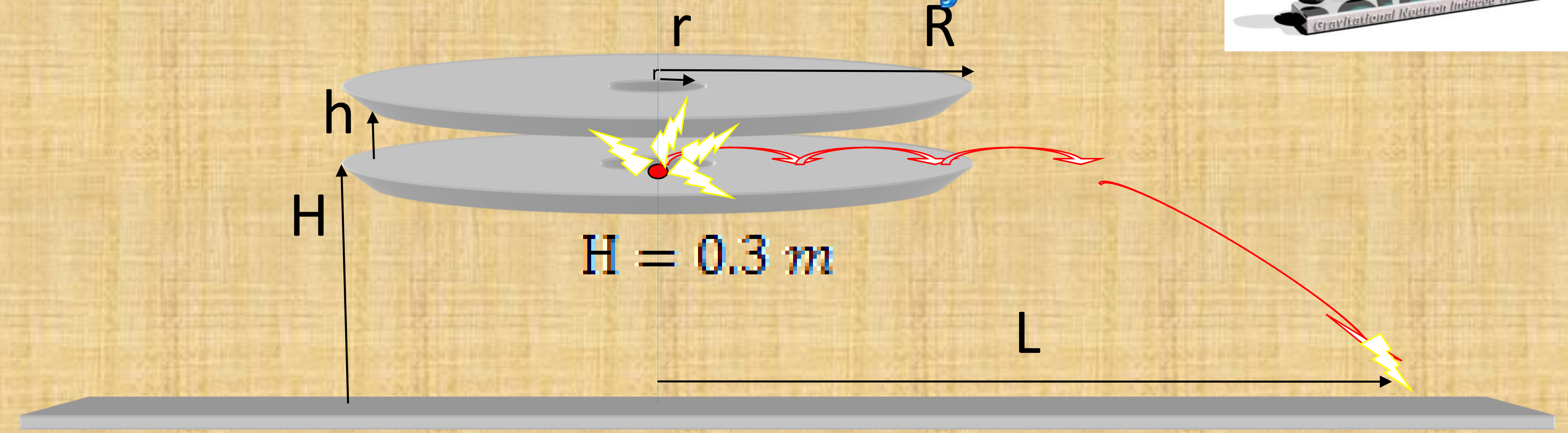
Estimation of statistical uncertainty



The limit of large h: Restricted only by the maximum radius R (because of antihydrogen losses) $0{,}1v = 4.2\ cm$

$h = 3\ mm$ $N \approx 8 \cdot 10^3$ $\Delta g/g \approx 1.5 \cdot 10^{-3}$ $r < 5.2\sqrt{\varepsilon}\ mm$

Estimation of statistical uncertainty



$H = 0.3\ m$

The limit of purely quantum behaviour: $h < 50\ \mu m$

$N \approx 1.1 \cdot 10^3$ $\Delta g/g \approx 0.9 \cdot 10^{-3}$ $1.4\sqrt{\varepsilon}\ mm$

Estimation of systematic effects (all well below 0.1%):

- Uncertainty of shaping/measuring the distribution of vertical velocity components;
- Finite position- and time- resolution of the detectors;
- Correction for the time spent in the shaping device;
- Diffraction of antihydrogen on the mirror edges;
- Residual electromagnetic effects;
- « Patch effects » on mirror surfaces;
- Inclinations of the disks and the reference plate;
- Finite precision of production and adjustment of optical elements;
- Vibrations causing parasitic transitions between gravitational quantum states.

CONCLUSION

1. We propose *a new method* for shaping and controlling the distribution of vertical velocities of antihydrogen
2. We estimate statistical and systematical uncertainties as *better than 0.1 %*
3. Statistical uncertainty decreases for smaller slit sizes, thus for better defined vertical velocities: the range of *vertical velocities « overweights » the loss* in statistics
4. Systematical uncertainties *decrease even more dramatically* for smaller heights of the slit between the two disks
5. The « *vertical temperature* » of antihydrogen in the proposed scheme is as low as about *10 nK*.