Proton-neutron electromagnetic interaction.

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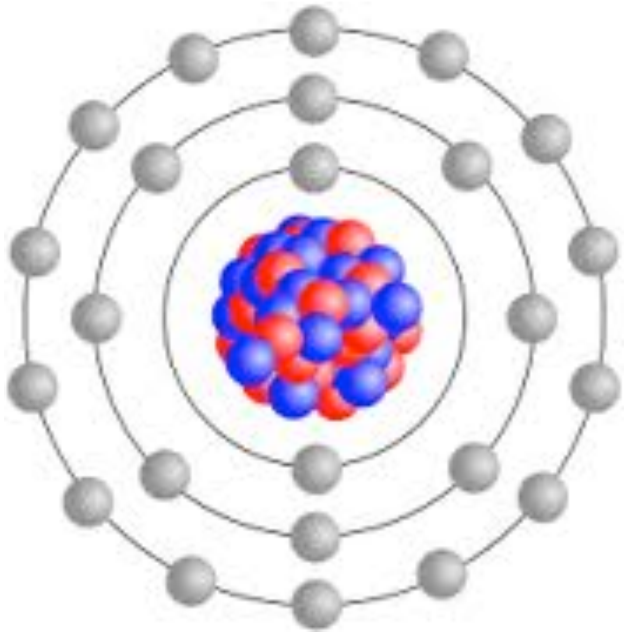
Electromagnetic interactions in a nucleus

- A proton attracts **a not so neutral neutron** as amber (ἤλεκτρον, elektron) attracts small **neutral** pieces of paper.
- Nucleons repulse themselves as **collinear and opposite magnets** (μαγνήτης, magnetis).
- Only the Coulomb repulsion between protons is taken into account in mainstream nuclear physics.

The nuclear shell model

The binding energy of even the simplest bound nucleus, the deuteron ^{2}H cannot be calculated with the nuclear shell model, the fundamental laws of the strong force being unknown.



The force center of the nucleus being undefined, we may **assume** that, in contrast with electrons,

nucleons do not move,

as suggested by the drawings of the atom and its nucleus.

Estimation of ^2H binding energy

Applying electric Coulomb's law with the measured radius of the proton, $\boldsymbol{r_p = 0.88\ fm}$, one obtains,

$$\frac{e^2}{4\pi\epsilon_0 r_p} = 1.6\ MeV$$

not far from **2.2 *MeV***, the measured value of the deuteron binding energy, indicating a possible electric origin of the nuclear interaction.

Proton electric charge

- It is *well known* that the proton contains one positive elementary electric charge, assumed here to be punctual:
  $+e = +1.6 \times 10^{-19}$ Coulomb

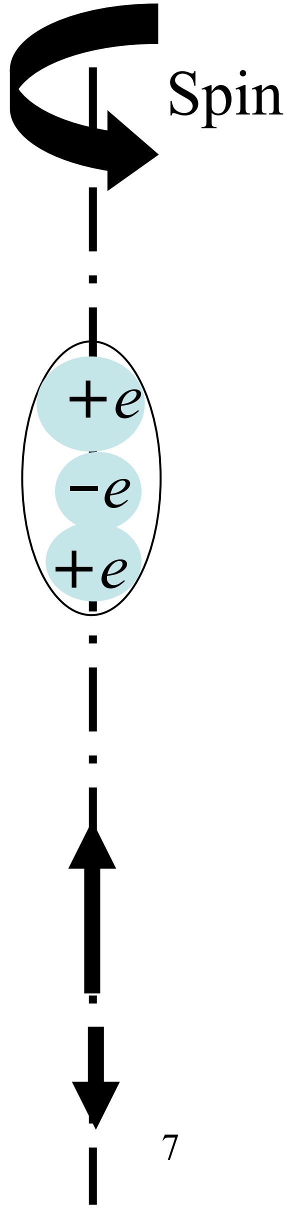
- The proton-proton repulsion, inexistent in the deuteron, is not the only Coulomb force in a nucleus.

Neutron electric charges

It is *less known* that the **not so neutral neutron** contains electric charges with no net charge, assumed *for the sake of simplicity* to be punctual charges +*e* and –*e* (+2e/3, -2e/3 would give a 30% error).

A **proton** attracts the **negative** charge of a nearby **neutron** and repulses its **positive** charge farther away, resulting in a **net attraction**, according to the 1/r Coulomb's law.

Electromagnetic contents of ^2H



3 electric charges (1/r law):

The proton repulses the **+e** charge of the neutron and attracts its **-e** charge, resulting in a **net attraction**.

2 magnetic moments (1/r³ law):

Proton: $\mu_p > 0$

Neutron: $\mu_n < 0$

$$|\mu_p| > |\mu_n|$$

Thus, for collinear and opposite magnetic moments, the resulting magnetic moment is positive, thus **repulsive**.

Exact electric dipole formula

The **$2a/r^2$** approximation of the dipole is **invalid** in the deuteron ^2H where the proton touches the neutron (**a ≈ r**).

Therefore, the **exact electric dipole formula** [1] has to be used instead of the usual simplified formula:

$$\frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r_{np}+a}-\frac{1}{r_{np}-a}\right)\approx-\frac{e^2}{4\pi\epsilon_0}\frac{2a}{r^2}$$

- ***r_{np}*** : separation distance between the proton and the neutron.
- **2*a*** : separation distance between the neutron positive and negative charges of the deuteron.

[1] B. Schaeffer, Advanced Electromagnetics, Vol. 2, No. 1, September 2013.

Proton-neutron electric potential energy.

Electrostatic energy of 3 aligned punctual electric charges e_p, e_{n-}, e_{n+}:

$$U_e^{^2H} = \frac{1}{4\pi\epsilon_0}\left(\frac{|e_p e_{n_+}|}{r_{np}+a} - \frac{|e_p e_{n_-}|}{r_{np}-a} - \frac{|e_{n_+} e_{n_-}|}{2a}\right)$$

The third term is *unphysically infinite* for an isolated neutron where ***a*** **≃ 0**. The *intuitive* solution is to replace 1/2*a* by the **exact** electric dipole potential energy 1/(r+a) - 1/(r-a). The potential is thus **doubbled** and zero for a = 0:

$$U_e^{^2H} = \frac{e^2}{4\pi\epsilon_0}\left(\frac{2}{r_{np}+a} - \frac{2}{r_{np}-a}\right)$$

The potential, being negative, is attractive.

No empirical polarizability, only fundamental electric Coulomb’s law.

Proton-neutron magnetic potential energy

Magnetic dipole-dipole interaction potential energy (interaction between two magnetic moments after Maxwell [2]):

$$U_m = \frac{\mu_0}{4\pi r_{np}^3}\left[\vec{\mu}_n \bullet \vec{\mu}_p - \frac{3\left(\vec{\mu}_n \bullet \vec{r}_{np}\right)\left(\vec{\mu}_p \bullet \vec{r}_{np}\right)}{r_{np}^2}\right] = \frac{\mu_0|\mu_n\mu_p|}{4\pi r_{np}^3}\left[-1-3(-1)(1)\right] = 2\frac{\mu_0|\mu_n\mu_p|}{4\pi r_{np}^3}$$

The potential energy of the ^2H interacting magnetic moments, collinear and opposite, is positive, thus **repulsive**:

$$U_m^{^2H} = 2\left(\frac{\mu_0|\mu_n\mu_p|}{4\pi r_{np}^3}\right)$$

μ_n and μ_p are the neutron and proton magnetic moments.

[2] J. C. Maxwell, A treatise on electricity and magnetism . Vol. II. 1995, art. 384-387

Proton-neutron Coulomb-Maxwell electromagnetic potential energy

With electric attraction and magnetic repulsion combined, the **total** binding energy potential of ^2H is:

$$U_{em}^{^2H} = 2\left[\frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r_{np}+a} - \frac{1}{r_{np}-a}\right) + \left(\frac{\mu_0|\mu_n\mu_p|}{4\pi r_{np}^3}\right)\right]$$

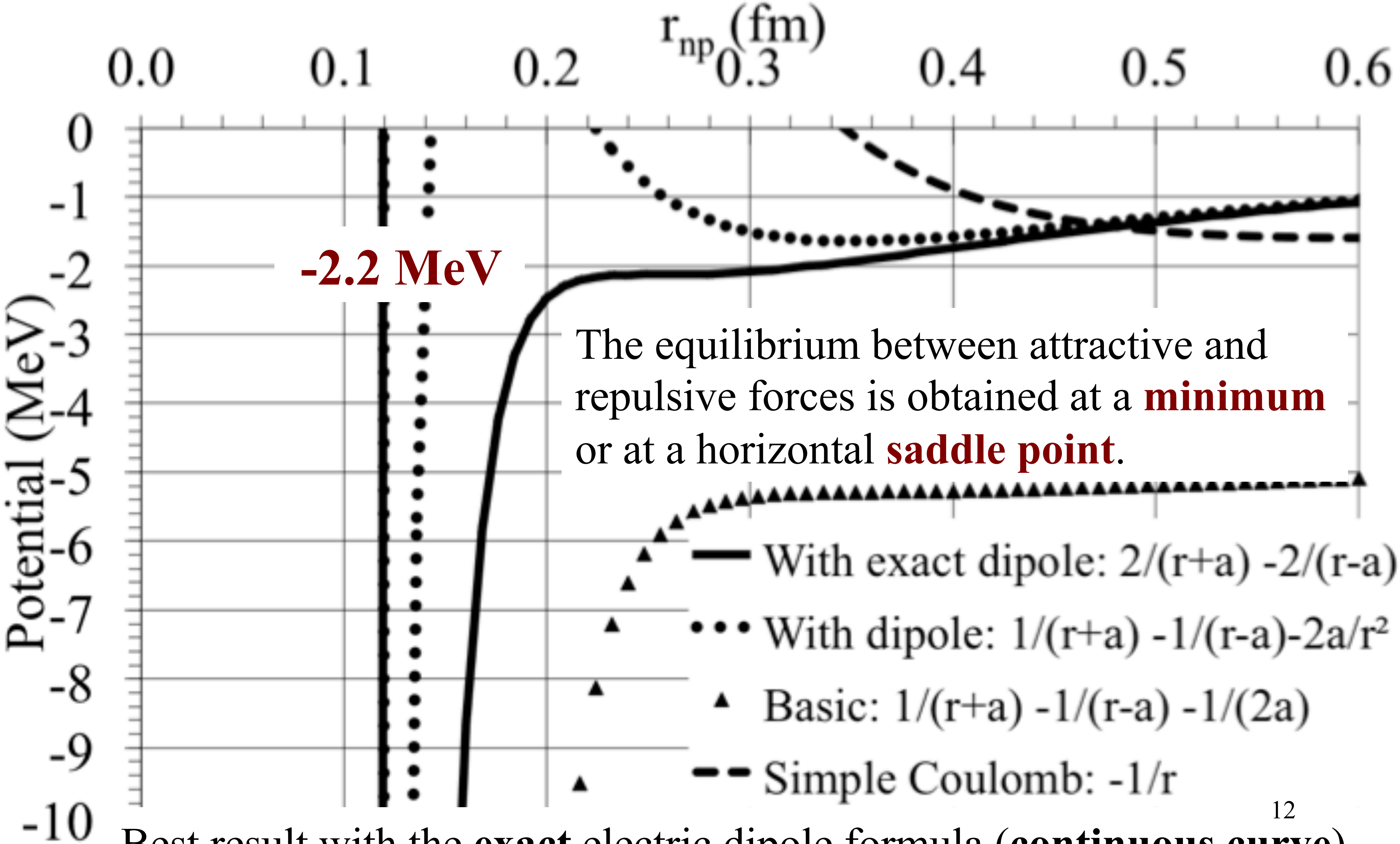
or, numerically:

$$U_{em}^{^2H} = 2\left[1.442\left(\frac{1}{r_{np}+a} - \frac{1}{r_{np}-a}\right) + \frac{0.0849}{r_{np}^3}\right]$$

Only fundamental laws and constants.

No fit, empirical parameter or cutoff.

Theoretical proton-neutron electromagnetic potentials



The equilibrium between attractive and repulsive forces is obtained at a **minimum** or at a horizontal **saddle point**.

Best result with the **exact** electric dipole formula (**continuous curve**).

Proton-neutron Schrödinger equation

With the Coulomb-Maxwell potential V, the ^2H Schrödinger equation writes:

$$\frac{\hbar^2}{2m}\Delta\psi + \left[U_{em}^{^2H} - V(r,a)\right]\psi = 0$$

With the simplest exponential well $\psi = e^{-r/b}$ and the Coulomb-Maxwell electromagnetic potential, we obtain:

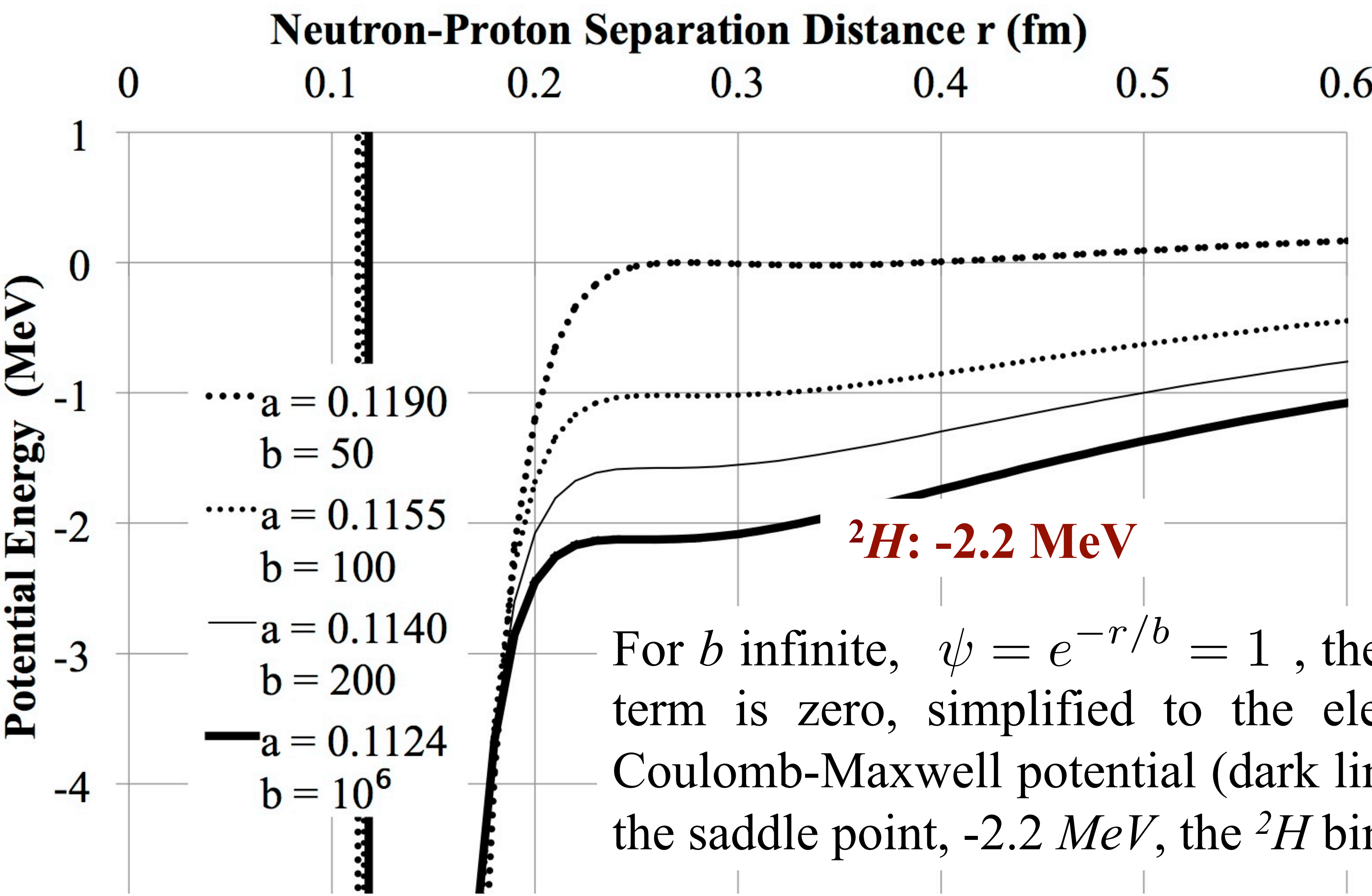
$$\left[\frac{\hbar^2}{2m}\left(\frac{1}{b^2} - \frac{2}{rb}\right) + U_{em}^{^2H} - \frac{e^2}{4\pi\epsilon_0}\left(\frac{2}{r+a} - \frac{2}{r-a}\right) - \frac{\mu_0}{4\pi}\left(\frac{2|\mu_n\mu_p|}{r^3}\right)\right] e^{-r/b} = 0$$

Dividing the Schrödinger equation by $e^{-r/b}$, we obtain the classical electromagnetic ^2H Coulomb-Maxwell potential except for the first term.

$$\frac{\hbar^2}{2m}\left(\frac{1}{b^2} - \frac{2}{rb}\right) + U_{em}^{^2H} - \frac{e^2}{4\pi\epsilon_0}\left(\frac{2}{r+a} - \frac{2}{r-a}\right) - \frac{\mu_0}{4\pi}\left(\frac{2|\mu_n\mu_p|}{r^3}\right) = 0$$

Proton-neutron Schrödinger equation

$$U_{em}^{^2H} = -\frac{\hbar^2}{2m}\left(\frac{1}{b^2} - \frac{2}{rb}\right) + \frac{e^2}{4\pi\epsilon_0}\left(\frac{2}{r+a} - \frac{2}{r-a}\right) + \frac{\mu_0}{4\pi}\left(\frac{2|\mu_n\mu_p|}{r^3}\right)$$

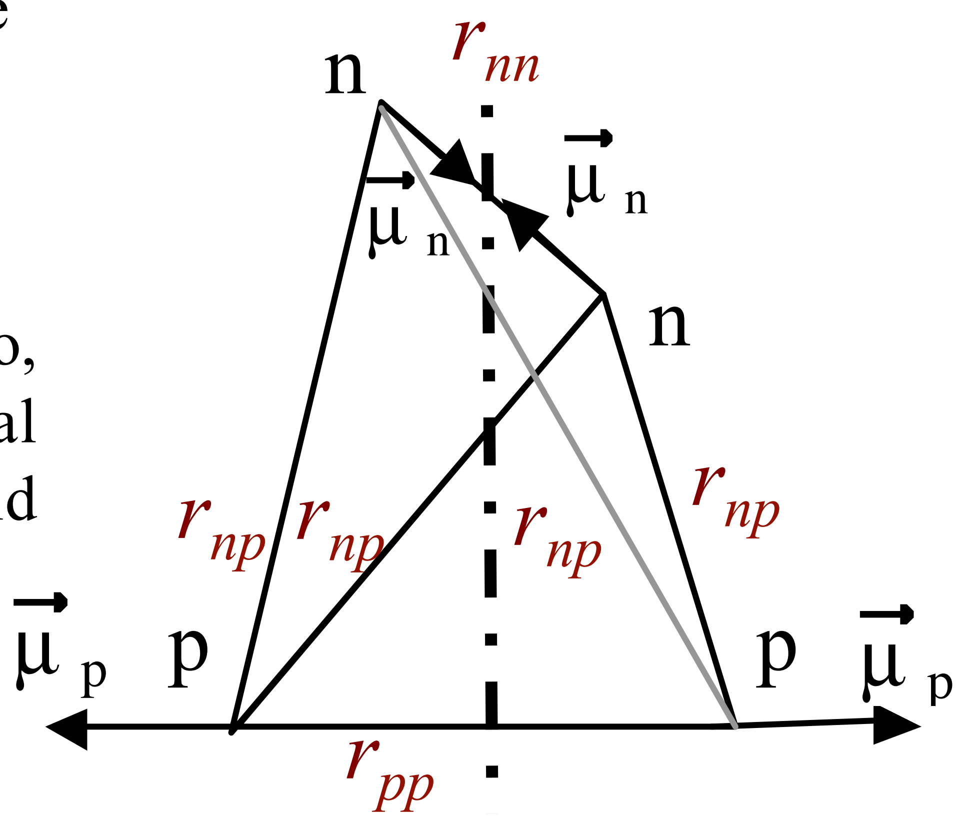


For b infinite, $\psi = e^{-r/b} = 1$, the Schrödinger term is zero, simplified to the electromagnetic Coulomb-Maxwell potential (dark line), giving, at the saddle point, -2.2 *MeV*, the ^2H binding energy.

α particle structure

^4He, with 2 protons + 2 neutrons, may be considered in a first approximation as a **regular tetrahedron** with 60° angles.

The magnetic moment of ^4He being zero, the **magnetic moments** of identical nucleons have to be **collinear** and **oppositely paired**.



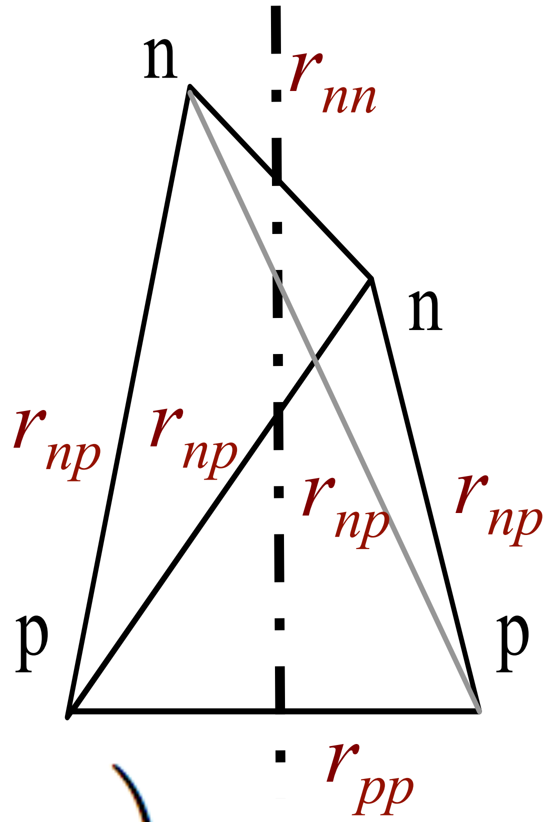
The other nucleon-nucleon interactions, being each ¼ of the total proton-neutron interactions are neglected provisionally.

^4He electric potential energy

(*only proton-neutron interactions considered*)

Per nucleon (= per bond in ^{4}He)

- **2** protons interacting with each neutron, the electric potential is **twice** that of ^{2}H/A
- ^4He having one proton-neutron bond, twice the ^{2}H binding energy per nucleon, the electric potential is **again doubbled**
- The electric potential of ^4He is thus **4 times larger** than that of ^2H



$$U_e^{^4He}/A = \boxed{4} \times \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r_{np}+a} - \frac{1}{r_{np}-a}\right)$$

^4He magnetic potential energy

(*only proton-neutron interactions considered*)

Per nucleon (= per bond in ^{4}He)

- **2** protons interacting with each neutron: the magnetic potential is **twice** that of ^{2}H/A.
- The **magnetic potential of one bond** is given by the Maxwell formula of the magnetic dipole-dipole interaction energy:

$$U_m = \frac{\mu_0}{4\pi r_{np}^3}\left[\vec{\mu}_n \bullet \vec{\mu}_p - \frac{3\left(\vec{\mu}_n \bullet \vec{r}_{np}\right)\left(\vec{\mu}_p \bullet \vec{r}_{np}\right)}{r_{np}^2}\right]$$

The proton and neutron magnetic moments being perpendicular, the first term is zero. Being **inclined** at 60° and 120° with respect to the edges of the tetrahedron, one has $-(-\frac{1}{2}\times\frac{1}{2}) = +\frac{1}{4}$.

- The magnetic part of the ^4He binding energy per nucleon is thus **2×3×¼** = **3/2** times larger than that of ^{2}H/A:

$$U_m^{^4He}/A = \boxed{\frac{3}{2}}\left(\frac{\mu_0|\mu_n\mu_p|}{4\pi r_{np}^3}\right)$$

^2H and ^4He potential energies:

(*only proton-neutron interactions considered,*
***per nucleon**, analytical formulas*)

The ^2H potential energy per nucleon is **½ of its bond energy**:

$$U_{em}^{^2H}/A = \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r_{np}+a} - \frac{1}{r_{np}-a}\right) + \frac{\mu_0|\mu_n\mu_p|}{4\pi r_{np}^3}$$

The ^4He electric potential energy/A is **4 times** that of ^2H.
The ^4He magnetic potential energy/A is **3/2 times** that of ^2H:

$$U_{em}^{^4He}/A = \boxed{4}\times \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r_{np}+a} - \frac{1}{r_{np}-a}\right) + \boxed{\frac{3}{2}}\left(\frac{\mu_0|\mu_n\mu_p|}{4\pi r_{np}^3}\right)$$

^{2}H and ^{4}He potential energies:

(*only proton-neutron interactions,*
per nucleon, numerical formulas)

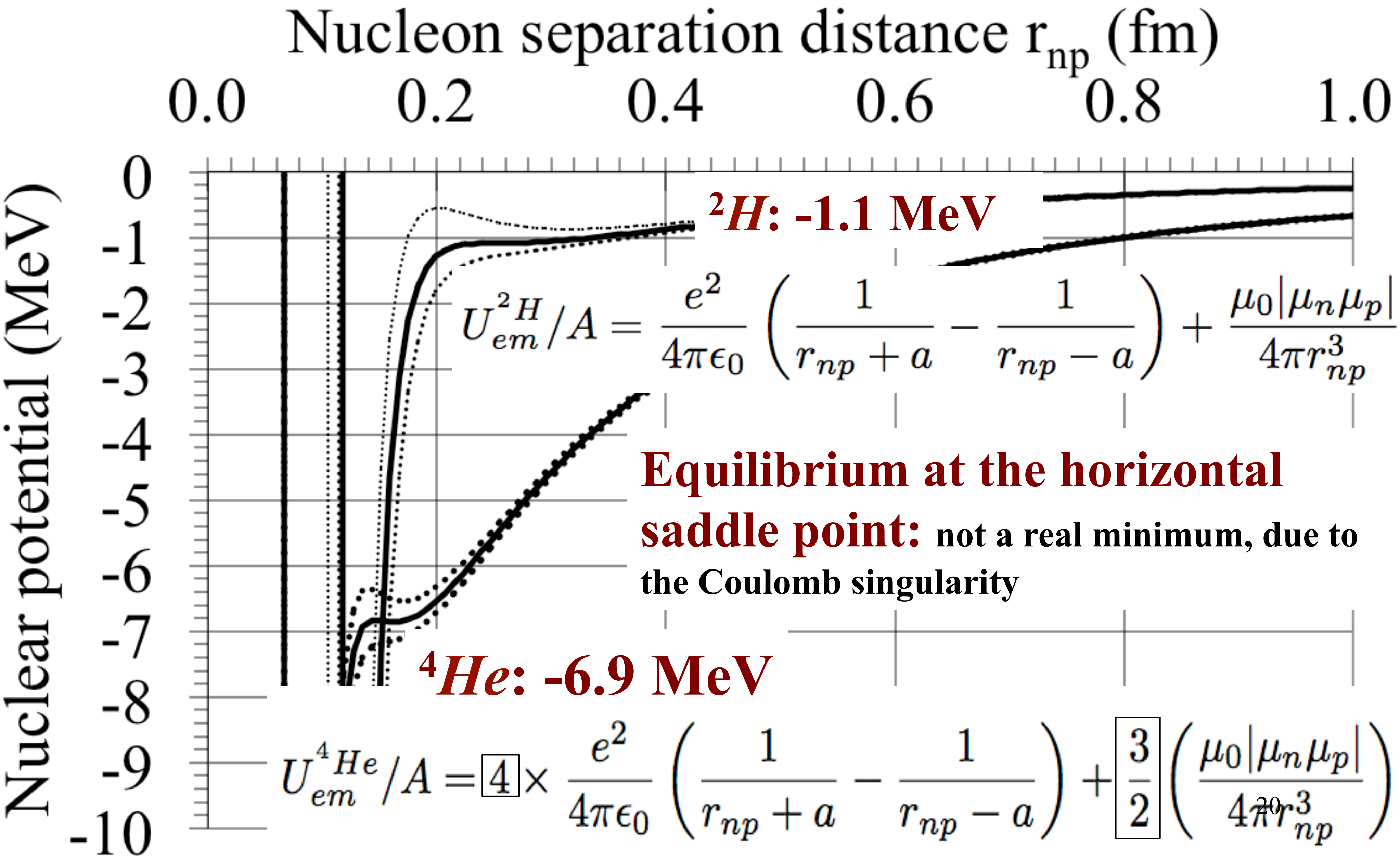
The $^{2}\boldsymbol{H}$ potential energy **per nucleon** is **½** of its bond energy:

$$U_{em}^{^2H}/A = 1.442\left(\frac{1}{r_{np}+a} - \frac{1}{r_{np}-a}\right) + \frac{0.0849}{r_{np}^3}$$

The $^{4}\boldsymbol{He}$ electric potential energy **per nucleon** is **4 times** that of $^{2}\boldsymbol{H}$. The magnetic moments being perpendicular and **inclined** at 60°, the magnetic potential energy is **1.5** that of $^{2}\boldsymbol{H}$:

$$U_{em}^{^4He}/A = 5.76\left(\frac{1}{r_{np}+a} - \frac{1}{r_{np}-a}\right) + \frac{0.1274}{r_{np}^3}$$

^2H and ^4He potential energies.



^2H: -1.1 MeV

$$U_{em}^{^2H}/A = \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r_{np}+a} - \frac{1}{r_{np}-a}\right) + \frac{\mu_0|\mu_n\mu_p|}{4\pi r_{np}^3}$$

Equilibrium at the horizontal saddle point: **not a real minimum, due to the Coulomb singularity**

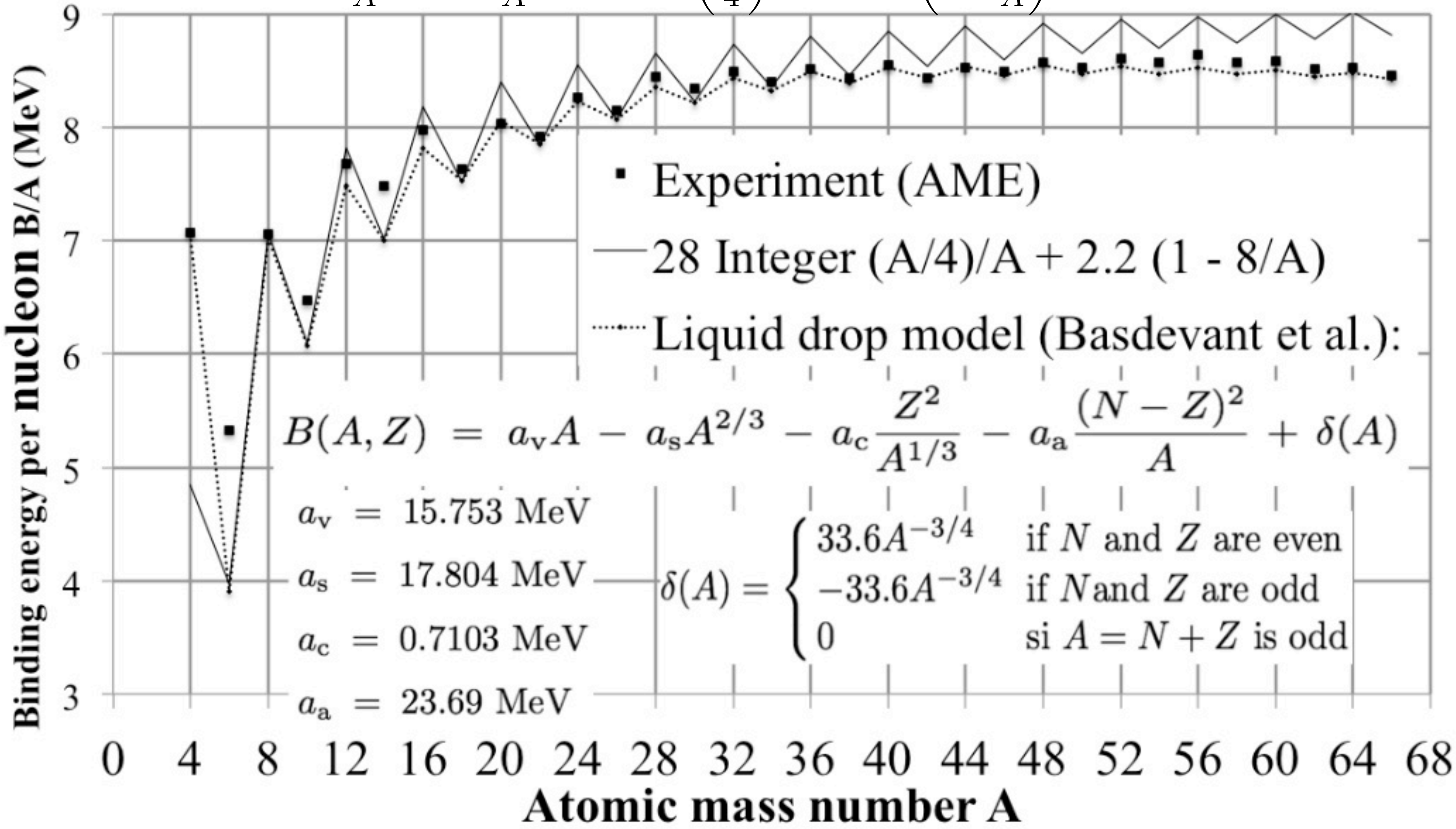
^4He: -6.9 MeV

$$U_{em}^{^4He}/A = \boxed{4}\times\frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r_{np}+a} - \frac{1}{r_{np}-a}\right) + \boxed{\frac{3}{2}}\left(\frac{\mu_0|\mu_n\mu_p|}{4\pi r_{np}^3}\right)$$

Pairing effect of N=Z nuclei

*Calculated as proton-neutron bonds between **α** particles*:

$$\frac{B\,(A)}{A} = \frac{B(^4He)}{A} Integer\left(\frac{A}{4}\right) + B(^2H)\left(1-\frac{8}{A}\right)\ MeV$$



Fundamental nuclear constants

Electric:

$$\frac{e^2}{4\pi\epsilon_0 R_P} = \alpha m_p c^2 = 6.846\ 901\ 65\ MeV$$

Magnetic:

$$\frac{\mu_0 |\mu_n \mu_p|}{4\pi R_P^3} = \alpha m_p c^2 \frac{|g_n g_p|}{16} = 9.147\ 871\ 896\ MeV$$

$\boldsymbol{R_P}$: proton Compton radius, 0.210 308 910 fm ≈ r_p / 4

$\boldsymbol{r_p}$: proton radius

$\boldsymbol{m_p}$: proton mass

$\boldsymbol{\alpha}$: fine structure constant

$\boldsymbol{g_p}$: proton Landé factor

$\boldsymbol{g_n}$: neutron Landé factor

H, ^{2}*H*, ^{4}*He*, ^{58}*Fe* energies compared

(absolute values)

- ***H* atom** binding energy: $13.6\ eV \approx \frac{1}{2}\alpha^2 m_e c^2$ **1**

Binding energy per nucleon:

- **^{2}*H* :** $1.11\ MeV \approx \frac{1}{6}\alpha m_p c^2$ **80.000**
- **^{4}*He* :** $7.07\ MeV \approx 6.85\ MeV = \alpha m_p c^2$ **500.000**
- **^{58}*Fe*:** $8.79\ MeV = 1.28\ \alpha m_p c^2$ **650.000**

Nuclear to chemical energy ratio order of magnitude:

$$\frac{m_p}{\alpha m_e} = 250,000$$

$\boldsymbol{m_p}$: proton mass
$\boldsymbol{m_e}$: electron mass
$\boldsymbol{\alpha}$: fine structure constant

Discussion

- Strong force (unknown but strength 1!)
- Magic numbers (numerology)
- Charge independence (neutron has +e,-e)
- Quark (virtual, unobservable, undetectable)

} **Doubtful**

- Schrödinger equation (no kinetic energy: useless)
- **With the bare application of electric and magnetic forces with fundamental constants only, the binding energies of ^{2}H and ^{4}He nuclei have been calculated** without fitting, cutoff, input parameter…

The quantitative adequation (no fitting) between theory and experiment proves the electromagnetic nature of the nuclear energy.

Спасибо за внимание
Thank you for your attention