

# Interference of neutron resonances and T-odd angular correlations in ternary fission

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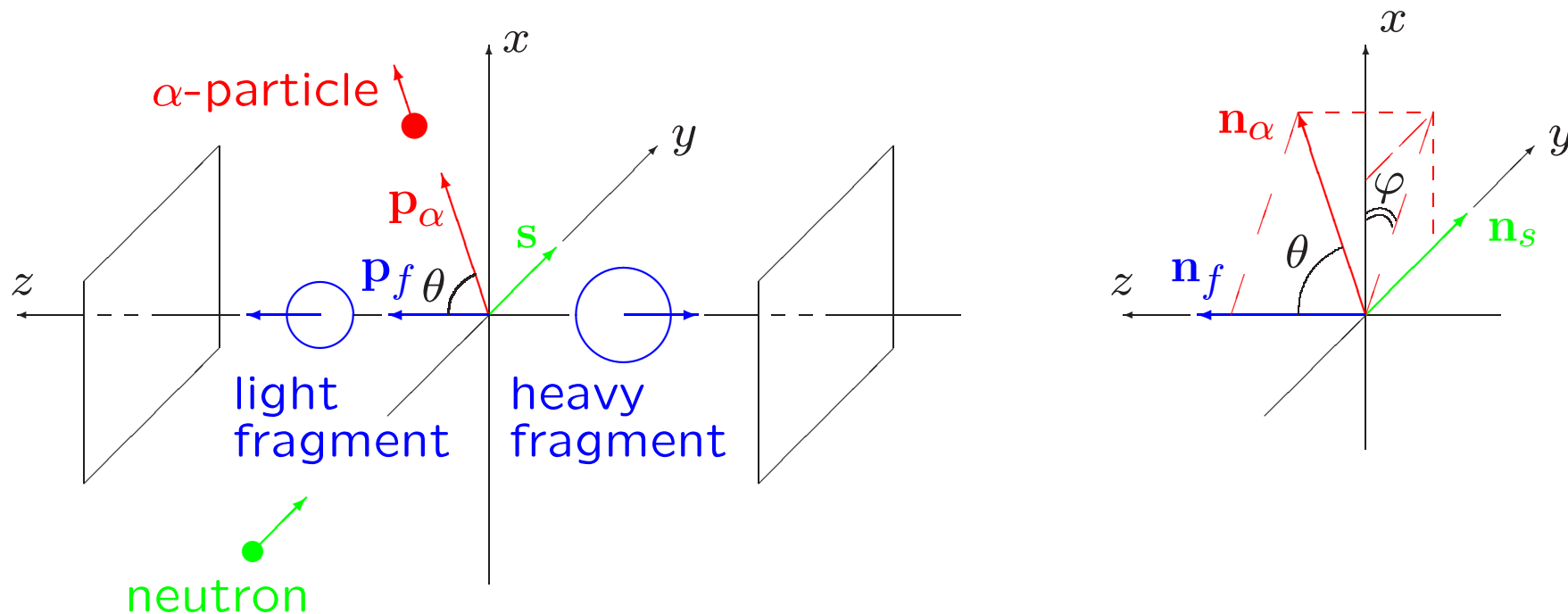
# T-odd angular correlations in neutron-induced ternary fission $n + {}^{233}\text{U}, {}^{235}\text{U}, {}^{239}\text{Pu}, {}^{241}\text{Pu}$ :

P. Jesinger et al., Nucl. Instr. Meth. Phys. Res. A. **440** 618 (2000)

F. Goennenwein et al., Phys. Lett. B. **652** 13 (2007)

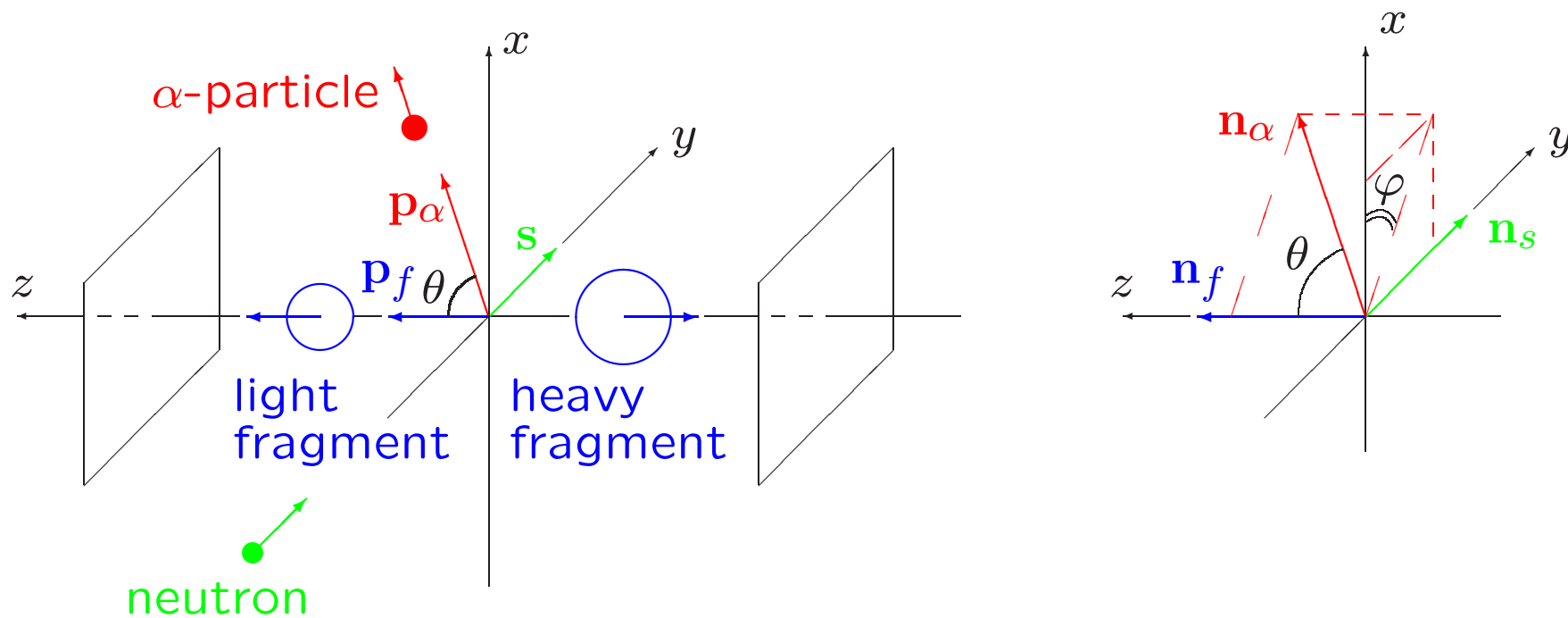
G.A.Petrov et al., Phys. At. Nucl., **71** 1149 (2008)

A.Gagarski et al., Proc. ISINN-21 (Alushta, 2013), p.85, Dubna, 2014



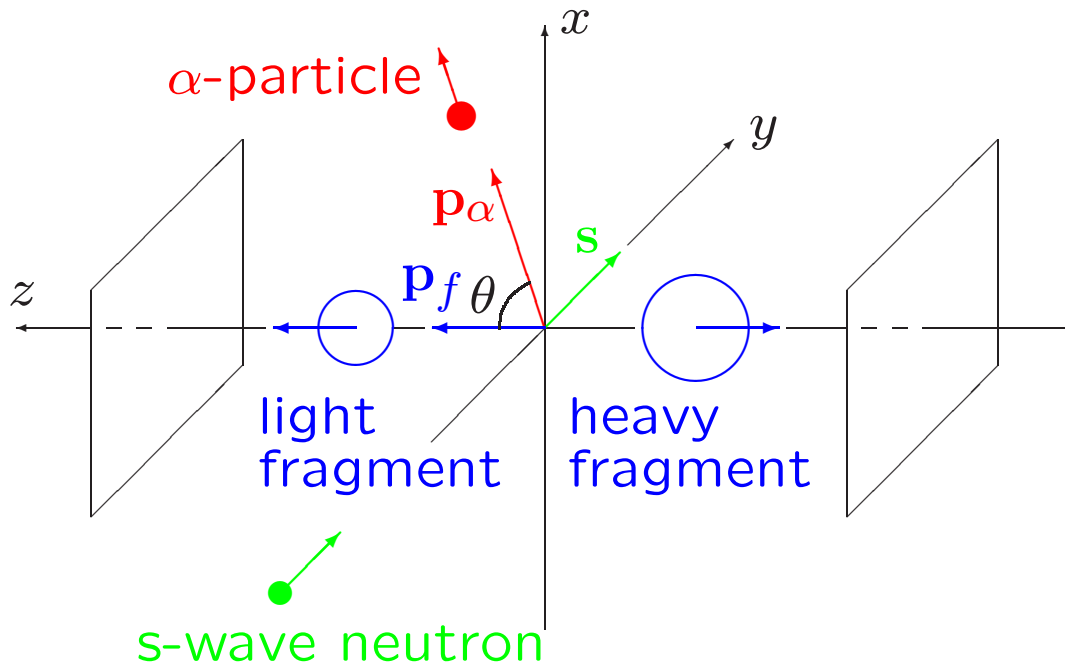
$$\frac{d\sigma}{d\Omega_f d\Omega_\alpha} = \frac{\sigma_0}{(4\pi)^2} + \dots + \underbrace{\sigma_T \left( \mathbf{n}_\alpha \left[ \mathbf{n}_s \times \mathbf{n}_f \right] \right)}_{\text{T-effect (2000)}} + \underbrace{\sigma_R \left( \mathbf{n}_\alpha \left[ \mathbf{n}_s \times \mathbf{n}_f \right] \right) \left( \mathbf{n}_\alpha \mathbf{n}_f \right)}_{\text{R-effect (2007)}} + \dots$$

Both effects  $\sim 10^{-3}$



Early analysis (model of a spin-orbital interaction):  
the  $\theta$ -dependance of asymmetry may be rather complex

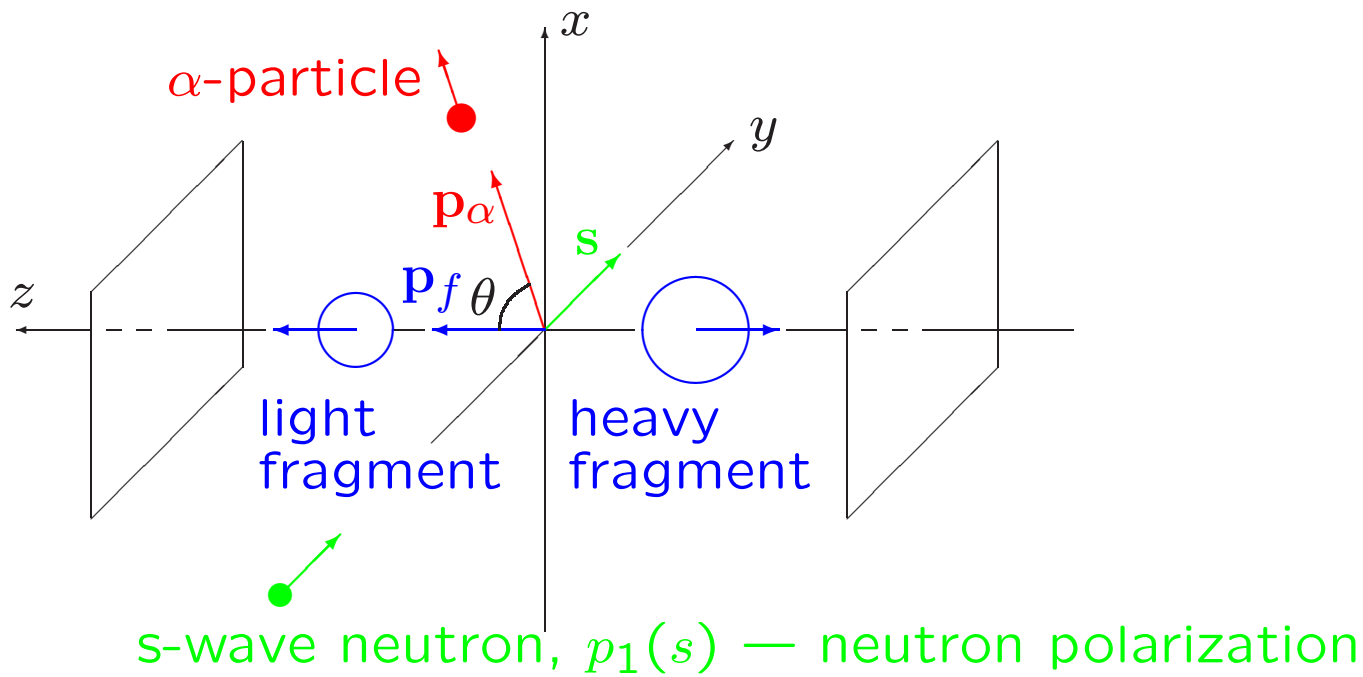
A.L.Barabanov. Proc. ISINN-9 (Dubna, 2001), p. 93, Dubna, 2002; arXiv: 0712.3543



s-wave neutron + nucleus (spin  $I$ )  $\Rightarrow$  compound nucleus  $J = I \pm \frac{1}{2}$

Generally:

$$\frac{d\sigma}{d\Omega_f d\Omega_\alpha} = \underbrace{\frac{d\sigma}{d\Omega_f d\Omega_\alpha}}_{J = I - 1/2} + \underbrace{\frac{d\sigma}{d\Omega_f d\Omega_\alpha}}_{J = I + 1/2} + \underbrace{\frac{d\sigma}{d\Omega_f d\Omega_\alpha}}_{J = I \pm 1/2, J' = I \mp 1/2}$$



Polarization of fissioning compound nucleus with spin  $J = I \pm \frac{1}{2}$ :

$$p_1(J) = \begin{cases} \frac{1}{3} p_1(s), & J = I - \frac{1}{2} \\ \frac{2I + 3}{3(2I + 1)} p_1(s), & J = I + \frac{1}{2}. \end{cases}$$

First description of the model:

A.L.Barabanov. ISINN-20 (Alushta, 2012); Phys. Part. Nucl. Lett. **10** 336 (2013)

Usual description of reactions (input wave + output waves):

$$\Psi = \Psi(n \rightarrow n) + \sum_{b \text{ (binary)}} \Psi(n \rightarrow b) + \sum_{t \text{ (ternary)}} \Psi(n \rightarrow t)$$

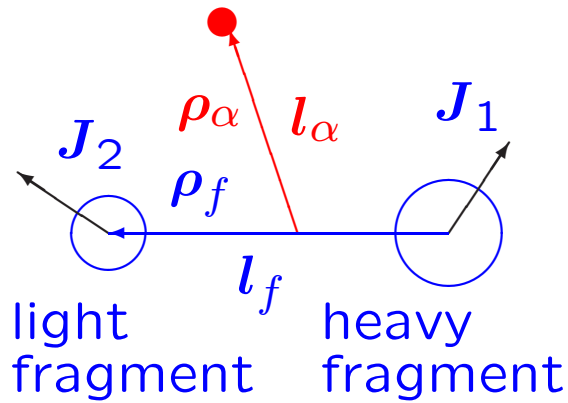
$$\Psi(n \rightarrow n) = \sum_{\lambda_n=(ljJM)} a(\lambda_n) \left( \psi_{\lambda_n}^{(-)} + \sum_{\lambda'_n} S_J(\lambda_n \rightarrow \lambda'_n) \psi_{\lambda'_n}^{(+)} \right)$$

$$\Psi(n \rightarrow b) = \sum_{\lambda_n=(ljJM)} a(\lambda_n) \sum_{\lambda_b=(LF)} S_J(\lambda_n \rightarrow \lambda_b) \psi_{\lambda_b}^{(+)}$$

$$\Psi(n \rightarrow t) = \sum_{\lambda_n=(ljJM)} a(\lambda_n) \sum_{\lambda_t=(l_f l_\alpha NLF)} S_J(\lambda_n \rightarrow \lambda_t) \psi_{\lambda_t}^{(+)}$$

Hyperspherical harmonics for three particles:

$\alpha$ -particle



$$\rho_f = \rho \sin \vartheta, \quad \rho_\alpha = \rho \cos \vartheta,$$

$$\lambda_t = (l_f, l_\alpha, N, L, F),$$

$$l_f + l_\alpha = L, \quad J_1 + J_2 = F, \quad L + F = J$$

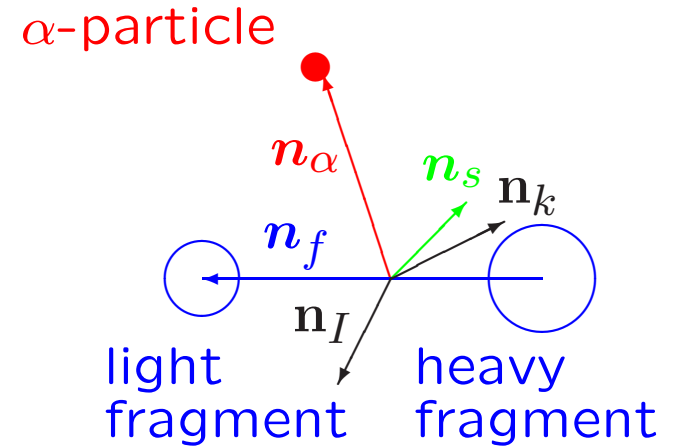
$$N = 2n + l_f + l_\alpha, \quad n = 0, 1, 2, \dots$$

$$\psi_{\lambda_t}^{(+)} = R_N^{(+)}(\rho) \sum_{\mu f} C_{L\mu Ff}^{JM} i^N \Phi_{NL\mu}^{l_f l_\alpha}(\vartheta, \Omega_f, \Omega_\alpha) \sum_{M_1 M_2} C_{J_1 M_1 J_2 M_2}^{Ff} \psi_{J_1 M_1} \psi_{J_2 M_2}$$

$$\Phi_{NL\mu}^{l_f l_\alpha}(\vartheta, \Omega_f, \Omega_\alpha) \sim (\sin \vartheta)^{l_f} (\cos \vartheta)^{l_\alpha} P_n^{l_f + \frac{1}{2} l_\alpha + \frac{1}{2}}(\cos 2\vartheta) \times$$

$$\times \sum_{m_f m_\alpha} C_{l_f m_f l_\alpha m_\alpha}^{L\mu} Y_{l_f m_f}(\Omega_f) Y_{l_\alpha m_\alpha}(\Omega_\alpha)$$

Differential cross section  
for ternary fission (general case):



$$\frac{d\sigma_t}{d\Omega_f d\Omega_\alpha} \sim \frac{\pi\lambda^2}{(4\pi)^2} \sum_{AB\Lambda_f\Lambda_\alpha QK} \tau'_{A0}(s) \tau'_{B0}(I) \times$$

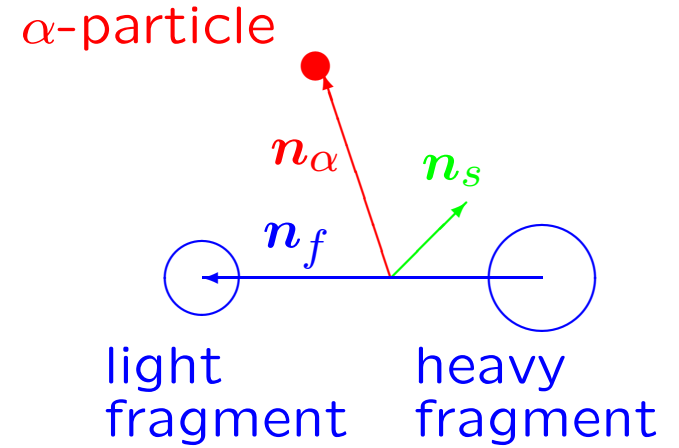
$$\times X_{\Lambda_f B \Lambda A}^{\Lambda_\alpha Q K} \times \phi_{\Lambda_f B \Lambda A}^{\Lambda_\alpha Q K}(\mathbf{n}_\alpha, \mathbf{n}_f, \mathbf{n}_I, \mathbf{n}_k, \mathbf{n}_s),$$

$$X_{\Lambda_f B \Lambda A}^{\Lambda_\alpha Q K} = \sum_{\lambda_n \lambda_t \lambda'_n \lambda'_t} Z_{\Lambda_f B \Lambda A}^{\Lambda_\alpha Q K}(\lambda_n, \lambda_t, \lambda'_n, \lambda'_t) S_{J'}^*(\lambda'_n \rightarrow \lambda'_t) S_J(\lambda_n \rightarrow \lambda_t),$$

$Z_{\Lambda_f B \Lambda A}^{\Lambda_\alpha Q K}(\lambda_n, \lambda_t, \lambda'_n, \lambda'_t) \sim$  the known functions of  $\vartheta$  and 3j-, 6j, 9j-symbols



Differential cross section  
for ternary fission (studied case):



$$\frac{d\sigma_t}{d\Omega_f d\Omega_\alpha} = \frac{\pi\lambda^2}{(4\pi)^2} \sum_{Q\Lambda_y\Lambda_x} \tau'_{Q0}(s) \times X_{Q\Lambda_f}^{\Lambda_\alpha} \times \phi_{Q\Lambda_f}^{\Lambda_\alpha}(\mathbf{n}_\alpha, \mathbf{n}_s, \mathbf{n}_f),$$

$$X_{Q\Lambda_f}^{\Lambda_\alpha} = \sum_{\lambda_t \lambda'_t} Z_{Q\Lambda_f}^{\Lambda_\alpha}(\lambda_t, \lambda'_t) S_{J'}^*(0 \frac{1}{2} \rightarrow \lambda'_t) S_J(0 \frac{1}{2} \rightarrow \lambda_t),$$

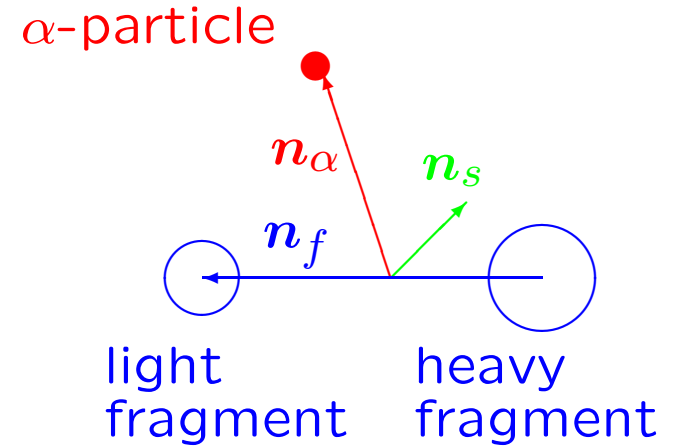
$Z_{Q\Lambda_f}^{\Lambda_\alpha}(\lambda_t, \lambda'_t) \sim$  the known functions of  $\vartheta$  and 3j-, 6j, 9j-symbols,

$$\phi_{Q\Lambda_f}^{\Lambda_\alpha}(\mathbf{n}_\alpha, \mathbf{n}_s, \mathbf{n}_f) = (4\pi)^{3/2} \sum_{\lambda_\alpha q \lambda_f} C_{Qq\Lambda_f\lambda_f}^{\Lambda_\alpha\lambda_\alpha} Y_{\Lambda_\alpha\lambda_\alpha}^*(\mathbf{n}_\alpha) Y_{Qq}(\mathbf{n}_s) Y_{\Lambda_f\lambda_f}(\mathbf{n}_f)$$

$$\phi_{11}^1(\mathbf{n}_\alpha, \mathbf{n}_s, \mathbf{n}_f) \sim (\mathbf{n}_\alpha [\mathbf{n}_s \times \mathbf{n}_f]) \Rightarrow \text{T-effect}$$

$$\phi_{12}^2(\mathbf{n}_\alpha, \mathbf{n}_s, \mathbf{n}_f) \sim (\mathbf{n}_\alpha [\mathbf{n}_s \times \mathbf{n}_f]) (\mathbf{n}_\alpha \mathbf{n}_f) \Rightarrow \text{R-effect}$$

Differential cross section  
for ternary fission (studied case):



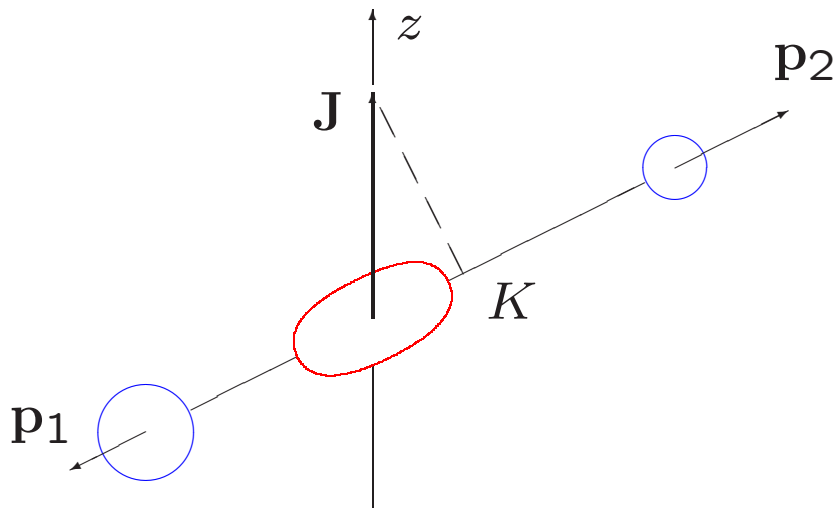
$$\frac{d\sigma_t}{d\Omega_f d\Omega_\alpha} = \frac{\pi\lambda^2}{(4\pi)^2} \sum_{Q\Lambda_y\Lambda_x} \tau'_{Q0}(s) \times X_{Q\Lambda_f}^{\Lambda_\alpha} \times \phi_{Q\Lambda_f}^{\Lambda_\alpha}(\mathbf{n}_\alpha, \mathbf{n}_s, \mathbf{n}_f)$$

The terms  $\phi_{Q\Lambda_f}^{\Lambda_\alpha}(\mathbf{n}_\alpha, \mathbf{n}_s, \mathbf{n}_f) \sim (\mathbf{n}_\alpha[\mathbf{n}_s \times \mathbf{n}_f]) \times \dots$  are due only to interference, because

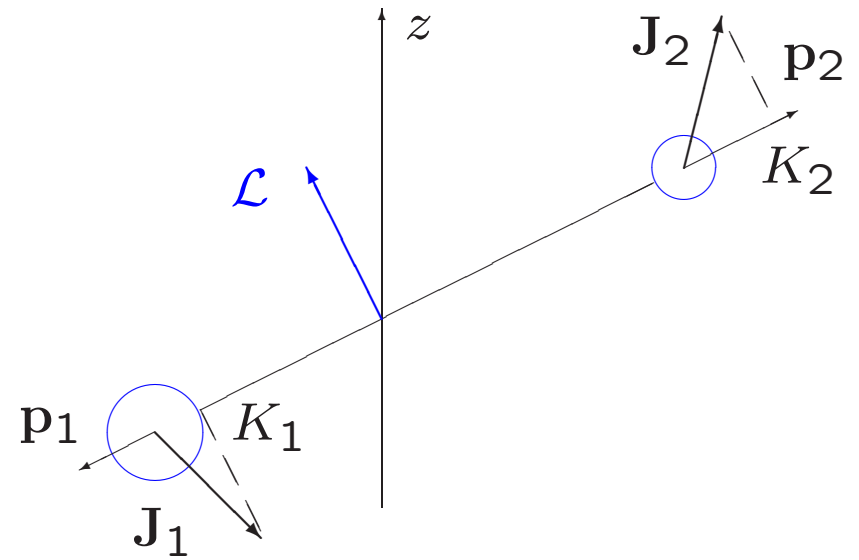
$$X_{Q\Lambda_f}^{\Lambda_\alpha} \sim \sum_{\lambda_t \lambda'_t} Z_{Q\Lambda_f}^{\Lambda_\alpha}(\lambda_t, \lambda'_t) \text{Im} \left( S_{J'}^*(0\frac{1}{2} \rightarrow \lambda'_t) S_J(0\frac{1}{2} \rightarrow \lambda_t) \right),$$

thus

$$X_{Q\Lambda_f}^{\Lambda_\alpha} \sim \sum_{\lambda_t \lambda'_t} Z_{Q\Lambda_f}^{\Lambda_\alpha}(\lambda_t, \lambda'_t) \text{Im} |S_J(0\frac{1}{2} \rightarrow \lambda_t)|^2 \equiv 0, \quad \text{if } J = J' \text{ and } \lambda_t = \lambda'_t.$$



A. Bohr, in *Proc. Int. Conf. on the Peaceful Uses of Atomic Energy, Geneva, 1955*



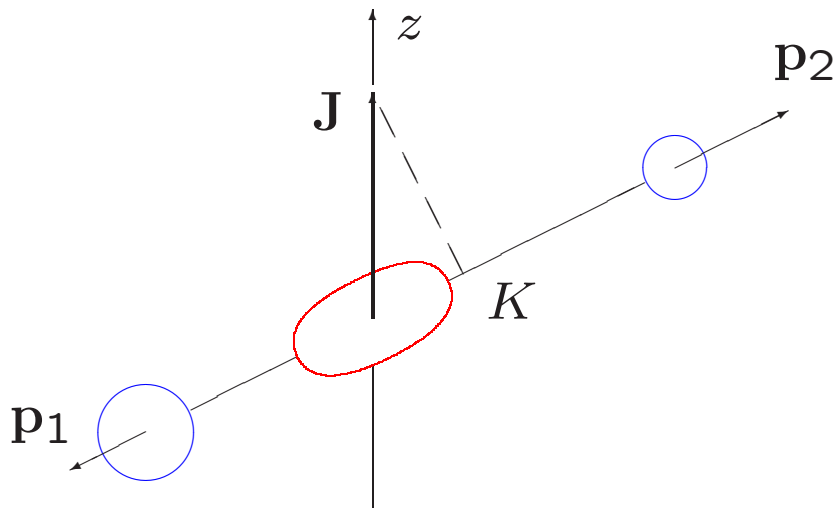
V.M. Strutinsky, *ZhETF* **30** 606 (1956)

Binary fission:

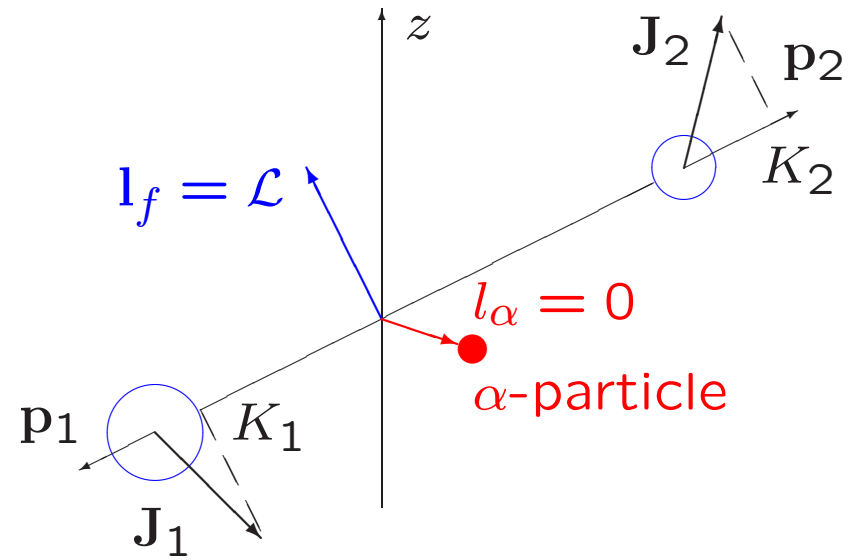
$$\begin{aligned} \mathbf{J} &\rightarrow \mathbf{J}_1 + \mathbf{J}_2 + \mathcal{L} = \mathbf{F} + \mathcal{L} \\ K &\rightarrow K_1 + K_2 \end{aligned}$$

Ternary fission:

$$\begin{aligned} \mathbf{J} &\rightarrow \mathbf{F} + \mathcal{L} \rightarrow \mathbf{F} + (\mathbf{l}_f = \mathcal{L}) + (\mathbf{l}_\alpha = 0) = \mathbf{F} + (\mathbf{L} = \mathcal{L}) \\ n &= 0 \rightarrow N = \mathcal{L} \end{aligned}$$



A. Bohr, in *Proc. Int. Conf. on the Peaceful Uses of Atomic Energy, Geneva, 1955*



V.M. Strutinsky, *ZhETF* **30** 606 (1956)

Binary fission:

$$\begin{aligned} \mathbf{J} &\rightarrow \mathbf{J}_1 + \mathbf{J}_2 + \mathcal{L} = \mathbf{F} + \mathcal{L} \\ K &\rightarrow K_1 + K_2 \end{aligned}$$

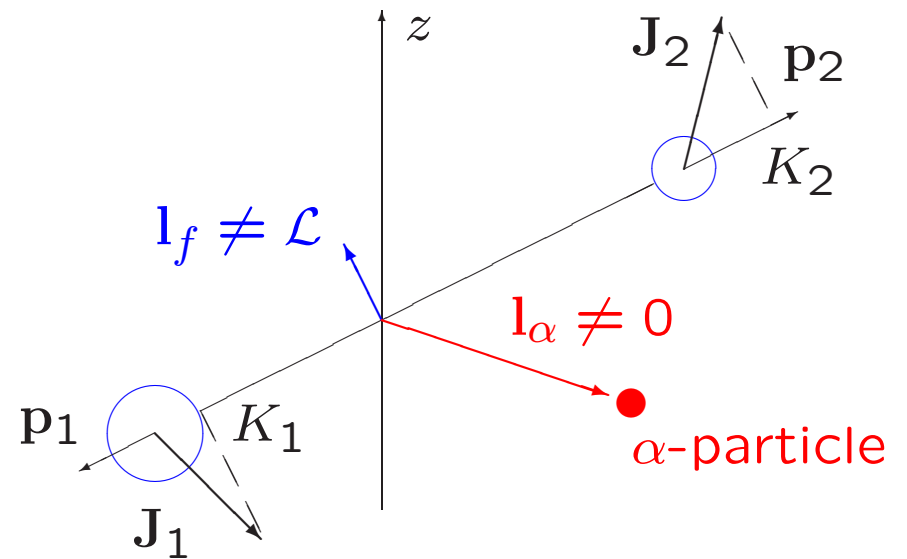
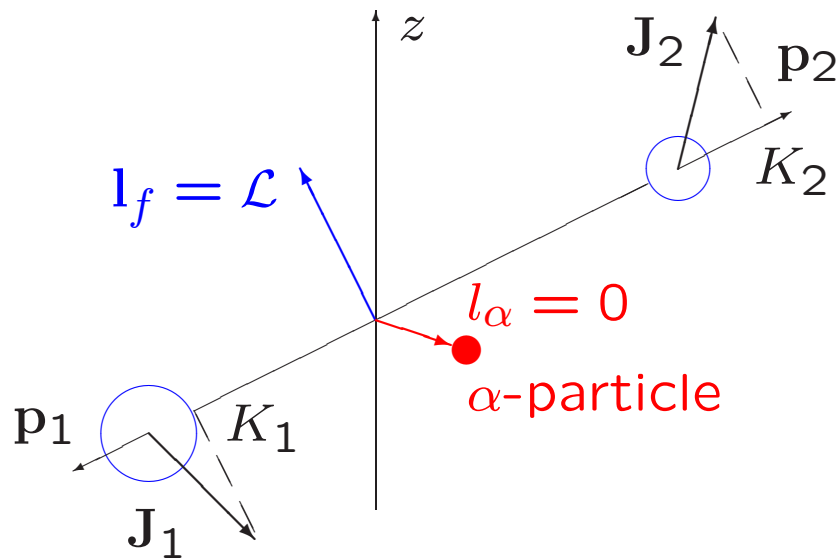
Ternary fission:

$$\begin{aligned} \mathbf{J} &\rightarrow \mathbf{F} + \mathcal{L} \rightarrow \mathbf{F} + (l_f = \mathcal{L}) + (l_\alpha = 0) = \mathbf{F} + (\mathbf{L} = \mathcal{L}) \\ n &= 0 \rightarrow N = \mathcal{L} \end{aligned}$$

# Coulomb mechanism of generation of T-odd angular correlations

$$S_J(\lambda_n \rightarrow \lambda_t) \sim S_J^b(\lambda_n \rightarrow LF) \times \underbrace{A(L \rightarrow l_f l_\alpha N)}_{\text{due to Coulomb repulsion}}$$

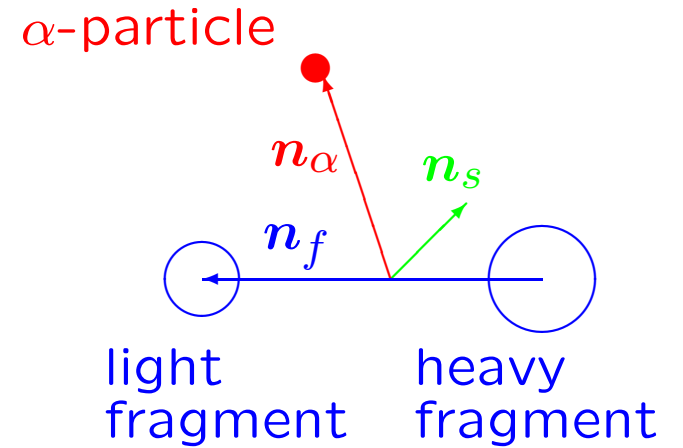
$$\sum_{l_f l_\alpha N} |A(L \rightarrow l_f l_\alpha N)|^2 = 1$$



Minimal model for  
T- and R-effects:

$L = 0$ , then

$$(l_f = 0, l_\alpha = 0) \Leftrightarrow (l_f = 1, l_\alpha = 1) \Leftrightarrow (l_f = 2, l_\alpha = 2) \Leftrightarrow \dots$$



$$\begin{aligned} \frac{d\sigma_t}{d\Omega_f d\Omega_\alpha} = & \frac{\sigma_0}{(4\pi)^2} + \dots \tau'_{10}(s) \times X_{Q=1\Lambda_f=1}^{\Lambda_\alpha=1} \times (\mathbf{n}_\alpha [\mathbf{n}_s \times \mathbf{n}_f]) + \\ & + \tau'_{10}(s) \times X_{Q=1\Lambda_f=2}^{\Lambda_\alpha=2} \times (\mathbf{n}_\alpha [\mathbf{n}_s \times \mathbf{n}_f]) (\mathbf{n}_\alpha \mathbf{n}_f) + \dots \end{aligned}$$

$$X_{Q=1\Lambda_f=1}^{\Lambda_\alpha=1} \sim \text{Im} \left( S_{J'}^* \left( 0 \frac{1}{2} \rightarrow l_f = 0 l_\alpha = 0 \right) S_J \left( 0 \frac{1}{2} \rightarrow l_f = 1 l_\alpha = 1 \right) \right)$$

$$X_{Q=1\Lambda_f=2}^{\Lambda_\alpha=2} \sim \text{Im} \left( S_{J'}^* \left( 0 \frac{1}{2} \rightarrow l_f = 1 l_\alpha = 1 \right) S_J \left( 0 \frac{1}{2} \rightarrow l_f = 1 l_\alpha = 1 \right) \right)$$

## Summary

1. T-odd angular correlations (both T- and R-effects) in ternary fission are due to quantum interference of exit states.
2. The exit states in ternary fission can be described in the formalism of hyperspherical harmonics.
3. In ternary fission the distribution of exit states over a wide range of quantum numbers seems to be provided by Coulomb repulsion of fragments and light charged particle ( $\alpha$ -particle).
4. Because Coulomb interaction is well known, the amplitude of exit states and effects of their interference, in principle, may be calculated.