Effect of Shape of Nuclear Level Density and Gamma-Ray Strength on Gamma-Ray Spectra

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INTRODUCTION

A nuclear level density (NLD) is one of the most crucial ingredients for a reliable theoretical analysis and prediction of the nuclear reaction observables (cross sections, spectra, angular distributions, abundance of elements in the Universe, and other) within statistical models.

Average probabilities for γ - transitions can be described through the use of radiative (gamma-ray, photon) strength functions (RSF)

RSF and NLD are important ingredient of statistical theory of nuclear reactions. Calculations of observed characteristics of nuclear reactions are as a rule time consuming, and for decreasing in computing time, simple closed-form expressions are preferable in evaluation of gamma-ray strengths and NLD.

Evaluation of the effect of collective states of nucleons on NLD is ambiguous and requires a correct theoretical description.

COLLECTIVE ENHANCEMENT (VARIATION) FACTOR

$$K_{coll} = \rho / \rho_{int}$$

ρ , ρ_{int} - **NLD with and without allowance for collective states**

$$\rho(U, A) = \Phi\{Z(\alpha^*, \beta^*)\}$$

$$Z(\alpha, \beta) = \mathrm{Tr}\left[\exp(\alpha \hat{A} - \beta H)\right] \text{- partition function}$$

Equation of thermodynamic state:

$$\begin{cases} A = \partial \ln Z / \partial \alpha \,|_{\alpha^*, \beta^*}, \\ E = -\partial \ln Z / \partial \beta \,|_{\alpha^*, \beta^*} = U + E_0 \end{cases} \equiv> \begin{pmatrix} \alpha^* \\ \beta^* \end{pmatrix} = \vec{\Psi}_Z \begin{pmatrix} A \\ U \end{pmatrix}$$

$T = 1/\beta^*$, $\mu = \alpha^*/\beta^*$ **- the temperature and chemical potential**

$$Z(\alpha, \beta) = Z_{int}(\alpha, \beta) \Delta Z(\beta)$$

$$\rho_{int}(U, A) = \Phi\{Z_{int}(\alpha^*_{int}, \beta^*_{int})\}$$

$$\begin{pmatrix} \alpha^*_{int} \\ \beta^*_{int} \end{pmatrix} = \vec{\Psi}_{Z_{int}} \begin{pmatrix} A \\ U \end{pmatrix} \quad => \quad T_{int} = 1/\beta^*_{int}$$

$$T_{int} \neq T$$

NLD WITHIN ENHANCED GENERALIZED SUPERFLUID MODEL (EGSM or EMPIRE GLOBAL SPECIFIC MODEL)

$$\rho(U,J) = \bar{\rho}_{EM}(U,J) \cdot K_{EM}(T_{\text{int}})$$

$\bar{\rho}_{EM}(U,J)$ - **INTRINSIC NLD WITH ROTATIONAL ENHANCEMENT**

K_{EM} - **LDM VIBRATIONAL ENHANCEMENT FACTOR WITH DAMPING** q_{vib}

$$K_{EM} = K_{LDM} \cdot (1 - q_{vib}) + q_{vib}$$

$$K_{LDM} \equiv Z_{boz}(\{n(\omega_L)\}) = \exp\left[C_3 A^{2/3} \cdot T_{\text{int}}^{4/3}\right], \quad \omega_L \Rightarrow \text{Liquid Drop Model}$$

$$q_{vib} = 1/[1 + \exp\{(T_{1/2} - T_{\text{int}})/DT\}]$$

$$\bar{\rho}_{EM}(U,J) = \Xi\{\tilde{a}, \tilde{\delta}_{shift}\}$$

$\tilde{a}$ -asymptotic value of a-parameter of NLD

$\tilde{\delta}_{shift}$ -additional shift of excitation energy to adjust cumulative sum N_{cum} of experimental discrete levels

Herman M., Capote R.,et al. Nucl. Data Sheets 108 (2007) 2655; EMPIRE-3.1, available online at http://www.nndc.bnl.gov/empire/

Comparison of nuclear level density with experimental data

MODIFIED EGSM

$$\rho(U,J)=\bar{\rho}_{EM}(U,J)\cdot K_{vibr}$$

Vibrational enhancement factors with damping

Boson partition function with damped occupation numbers(DN)

Ignatyuk A.V., Weil J.L., Raman S., Kahane S. PRC 47 (1993) 1504

$$K_{vibr} \equiv Z_{boz}(\{\bar{n}_L\}) = \exp\left(\bar{S} - \bar{U}/T_{\text{int}}\right) \equiv K_{DN}(T_{\text{int}})$$

$$\bar{S} = \sum_L (2L+1)\left[(1+\bar{n}_L)\ln(1+\bar{n}_L) - \bar{n}_L \ln \bar{n}_L\right], \qquad \bar{U} = \sum_L (2L+1)\hbar\omega_L \bar{n}_L,$$

$$\bar{n}_L = \frac{\exp\left[-\Gamma_L/(2\hbar\omega_L)\right]}{\exp(\hbar\omega_L/T_{\text{int}})-1}, \qquad \Gamma_L = C\cdot\left[(\hbar\omega_L)^2 + 4\pi^2 T_{\text{int}}^2\right], \qquad L = 2,\ 3$$

Boson partition function with average occupation numbers

$$\Delta Z(\tau) = \frac{Z_{bos}(\{<n(\omega_L,\tau)>\})}{Z_{bos}(\{<n(\tilde{\omega}_L,\tau)>\})} \equiv \frac{\prod_L (1+<n(\omega_L,\tau)>)^{2L+1}}{\prod_L (1+<n(\tilde{\omega}_L,\tau)>)^{2L+1}}$$

$$<n(\omega_L,\tau)> = \frac{1}{T_p}\int_0^{T_p} n(\omega_L)\exp(-\Gamma_L t)dt = \frac{1-\exp(-2\pi\Gamma_L/\hbar\omega_L)}{\exp(\hbar\omega_L/\tau)-1}\frac{\hbar\omega_L}{2\pi\Gamma_L}$$

$$L^{\pi} = 2^{+},\ 3^{-};\ \hbar\tilde{\omega}_{2^{+}} = \hbar\omega_{shell}/2,\ \hbar\tilde{\omega}_{3^{-}} = \hbar\omega_{shell},\ \hbar\omega_{shell} = 41A^{-1/3}$$

BAN approximation without allowance for changing temperature

$$K_{vib} \equiv K_{BAN}(T_{\text{int}}) = \Delta Z(\tau = T_{\text{int}})$$

BAN approximation with allowance for changing temperature

$$Z_{\text{int}}(\alpha,\beta) \equiv Z_{BSFG}(\alpha,\beta);\ \ Z'(\alpha,\beta) = Z_{BSFG}(\alpha,\beta)\cdot\Delta Z(\beta = 1/\tau)$$

$$K_{vib} \equiv K_{BANT}(T = T_{\text{int}} + \delta T_{BSFG}) = \frac{\rho}{\rho_{\text{int}}} = \frac{\Phi\{Z'(\alpha^{*},\beta^{*}=1/T)\}}{\Phi\{Z_{BSFG}(\alpha^{*}_{\text{int}},\beta^{*}_{\text{int}}=1/T_{\text{int}})\}}$$

Plujko V.A. et al., Int.J.of Mod.Phys.E 16, 570 (2007); Phys. Atom. Nucl. 70, 1643 (2007)

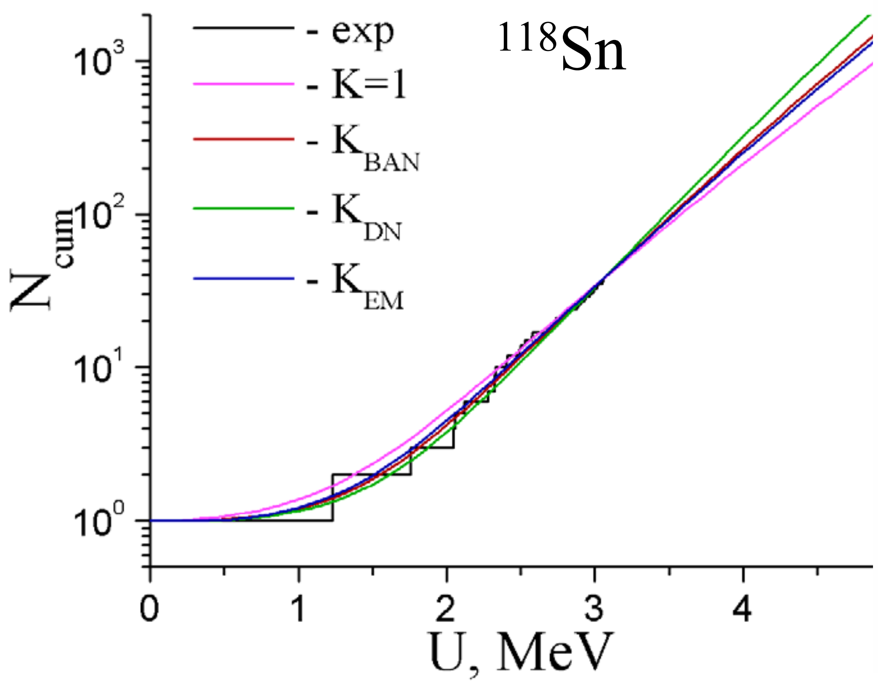
Modified EGSM

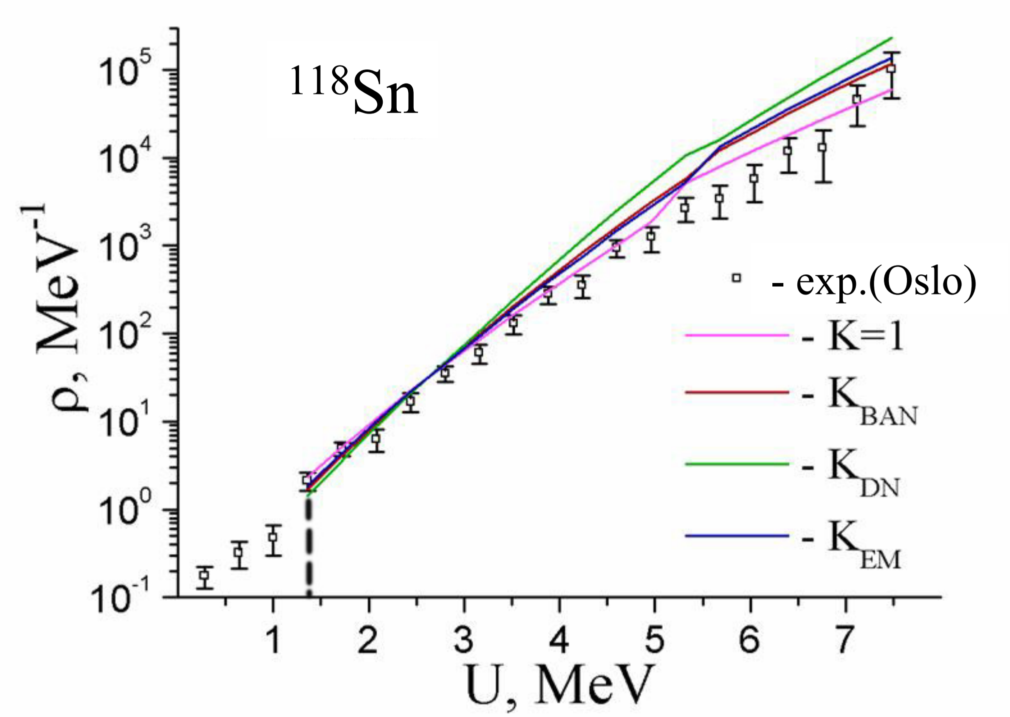
$$\rho(U,J) = \bar{\rho}_{EM}(U,J)\cdot K_{vibr} = \tilde{\Xi}\{\tilde{a},\tilde{\delta}_{shift}\}$$

Determination of $\tilde{a},\ \tilde{\delta}_{shift}$

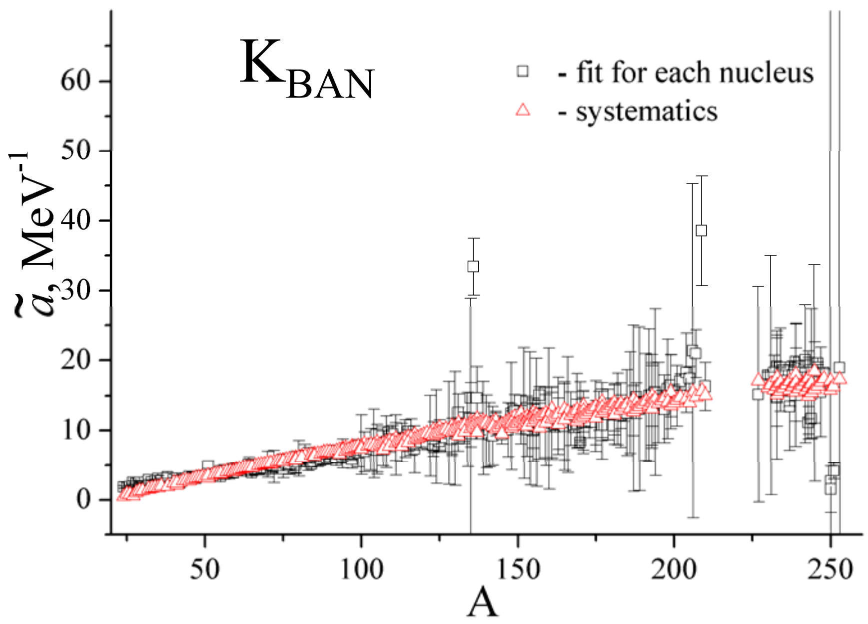
$\tilde{a}$ was found from fitting $D_0^{(theor)} = 1/\rho_{theor}(U=S_n)$ to $D_0^{(\exp)}$ with $\tilde{\delta}_{shift} = 0$

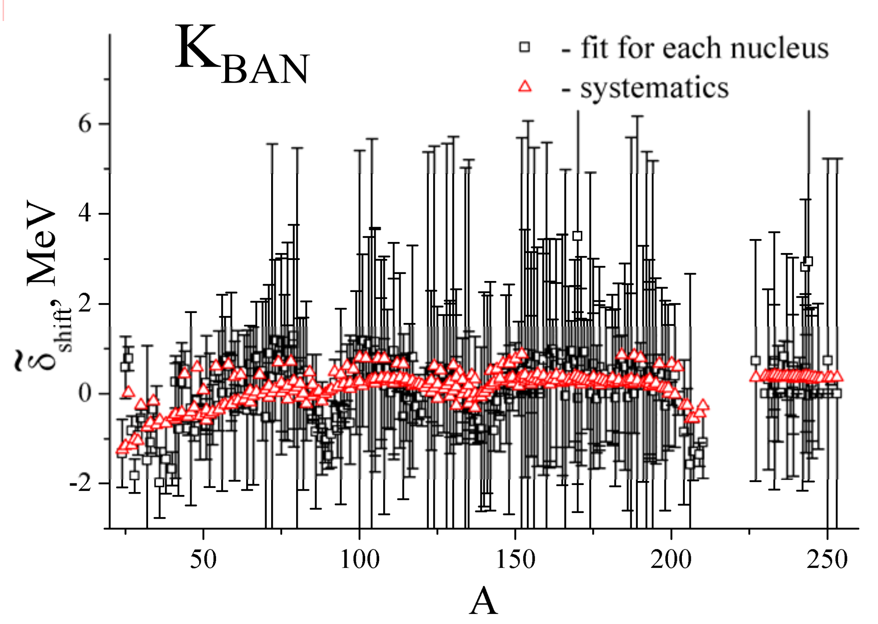
$\tilde{\delta}_{shift}$ => from fit $N_{cum}^{\exp}$ to $N_{cum} = \int_0^{U_{cum}} \sum_{J,\pi}\rho(U,J,\pi)\,\mathrm{d}U$ with previously determined $\tilde{a}$





Systematics of $\tilde{a}$ and $\tilde{\delta}_{shift}$





$$\tilde{a}(A,I) = 0.527(6)\,A(1+4.50(6)\,I^2) \;-1.17(2)\,A^{2/3}(1+13.3(2)I^2)$$
$$-0.0447(6)Z^2/A^{1/3}\;\;(MeV^{-1}),\qquad I=(N-Z)/A$$

$$\tilde{\delta}_{shift}(A,I) = 1.37(2)\,(1-11.7(5)I^2)\;+0.00000003(1)A(1+76503(23974)I^2)$$
$$-0.697(9)E_{2_1^+}\quad (MeV),$$

Contributions of vibrational enhancement in continuum range

Comparisons of ratios of chi-square deviations

$$\Delta\chi^2 \equiv \sum_{i=1}^{N_{nucl}} \chi_i^2(K_{vibr}) / \sum_{i=1}^{N_{nucl}} \chi_i^2(K_{vibr}=1)$$

$$\chi_i^2(K_{vibr}) = \sum_{j=1}^{n_i} (\rho_{theor,i}(U_j) - \rho_{\exp,i}(U_j))^2 / n_i,$$

n_i – number of experimental data for nucleus i, N_{nucl} – number of nuclei

Data	$\Delta\chi^2$			
	K_{EM}	K_{DN}	K_{BAN}	K_{BANT}
Dubna	**4.0**	**1.7**	**1.5**	**1.6**
Obninsk	**1.5**	**5.5**	**0.9**	**0.6**
Oslo	**1**	**0.9**	**1.0**	**0.9**
average	**2.2**	**2.7**	**1.1**	**1.0**

$$K_{BAN}(T_{\text{int}})$$

$$K_{BANT}(T = T_{\text{int}} + \delta T_{BSFG})$$

$$K_{vibr}(S_n) <\sim 2 \div 5 \qquad (A \sim 100)$$

Dubna - Sukhovoj A.M.et al., in Proc. of the ISINN-15, Dubna, May 2007, 92 (2007).
Obninsk –Zhuravlev B.V., IAEA, INDC(NDS)-0554, Distr. G+NM, (2009).
Oslo - Agvaanluvsan U. et al., Phys. Rev.C 70, 054611 (2004); http://ocl.uio.no/compilation/.

Radiative strength functions (RSF=PSF=GSF)

$$\vec{f}_{\alpha\lambda}(E_\gamma) = F_{\alpha\lambda}\left(E_\gamma, T_i\right), \qquad \overleftarrow{f}_{\alpha\lambda}(E_\gamma) = F_{\alpha\lambda}(E_\gamma, T_f), \qquad T_f = \varphi(T_i, E_\gamma)$$

$$\sigma_{E1}(E_\gamma) = 3E_\gamma\left(\pi\hbar c\right)^2 \vec{f}_{E1}\left(E_\gamma\right)$$

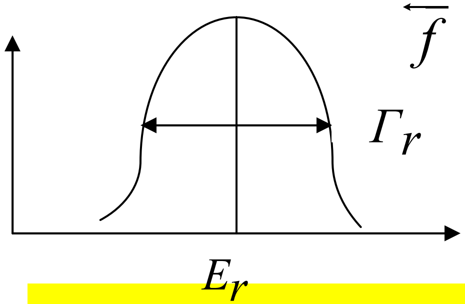
$$\frac{d\Gamma_{E1}\left(E_\gamma\right)}{dE_\gamma} = 3\,E_\gamma^{\,3}\ \overleftarrow{f}_{E1}\left(E_\gamma\right)\frac{\rho_f\left(U_f = U_i - E_\gamma\right)}{\rho_i\left(U_i\right)}$$

$$T_{E1}(E_\gamma) \sim 2\pi E_\gamma^3 \overleftarrow{f}_{E1}(E_\gamma)$$

Different RSF closed-forms

Standard Lorentzian (SLO)

[D. Brink. PhD Thesis(1955); P. Axel. PR 126(1962)]



$$\overleftarrow{f} = \overrightarrow{f} \sim \frac{E_\gamma \Gamma_r^2}{(E_\gamma^2 - E_r^2)^2 + E_\gamma \Gamma_r^2} \underset{E_\gamma \to 0}{\Rightarrow} 0$$

$$\Gamma_r = const \neq \varphi(E_\gamma) \sim 5 MeV \; (T = 0)$$

Enhanced Generalized Lorentzian (EGLO)

[J. Kopecky , M. Uhl, PRC47(1993)] based on
S. Kadmensky, V. Markushev, W.Furman, Sov.J.N.Phys 37(1983)]

$$\overleftarrow{f} = \frac{E_\gamma \Gamma(E_\gamma, T_f)}{(E_\gamma^2 - E_r^2)^2 + E_\gamma^2 \Gamma_\gamma^2(E_\gamma, T_f)} + \frac{0.7\Gamma(E_\gamma = 0, T_i)}{E_r^3}$$

$$\overleftarrow{f} \Rightarrow const \neq 0 \;\; [E_\gamma \to 0]$$

$$T_f = \sqrt{\frac{U - E_\gamma}{a}};$$

$$\Gamma(E_\gamma, T_f) = \Gamma_r \frac{E_\gamma^2 + 4\pi T_f^2}{E_\gamma^2} \cdot K(E_\gamma)$$

→ Infinite fermi- liquid (two-body dissipation)

$K(E_\gamma) \rightarrow$ empirical factor from fitting exp. data

Generalized Fermi liquid (GFL) model

(extended to GDR energies of gamma- rays)

[Mughabghab&Dunford]

$$\vec{f} = \overleftarrow{f} = 8.674 \cdot 10^{-8} \cdot \sigma_r \Gamma_r \frac{K_{GFL} \cdot E_r \Gamma_m}{\left(E_\gamma^2 - E_r^2\right) + K_{GFL}\left[\Gamma_m E_\gamma\right]^2}$$

$$\Gamma_m = \Gamma_{coll}\left(E_\gamma, T_f\right) + \Gamma_{dq}\left(E_\gamma\right)$$

$$\Gamma_{coll} \equiv C_{coll}\left(E_\gamma^2 + 4\pi^2 T_f^2\right) \qquad K_{GFL} = 0.63$$

”Fragmentation” component

$$\Gamma_{dq}\left(E_\gamma\right) = C_{dq} E_\gamma \left|\overline{\beta}_2\right| \sqrt{1 + \frac{E_2^+}{E_\gamma}}$$

Extension of expression for GDR damping via coupling with surface vibrations (J.Le Tournie, 1964,1965)

General expression for gamma-decay RSF within modified Lorentzian (MLO)

(Plujko et al)

$$\overleftarrow{f}\left(E_\gamma, T_f\right) = 8.674 \cdot 10^{-8} \frac{1}{1-\exp\left(-E_\gamma / T_f\right)} s\left(\omega = \frac{E_\gamma}{\hbar}, T_f\right), \quad \text{MeV}^{-3}$$

$$s\left(\omega, T_f\right) = -\operatorname{Im} \chi\left(\omega, T_f\right) / \pi$$

PECULARITIES

Presence of low-energy enhancement factor

$$N_{1p1h} \equiv \frac{1}{\hbar\omega} \int d\varepsilon_1 d\varepsilon_2 f_0\left(\varepsilon_1\right)\left(1 - f_0\left(\varepsilon_2\right)\right) \delta\left(\varepsilon_1 - \varepsilon_2 + \hbar\omega\right) = \frac{1}{1-\exp\left(-E_\gamma / T_f\right)} = (E_\gamma \to 0) = \frac{T_i}{E_\gamma} \gg 1$$

Non-zero limit at $E_\gamma \to 0$

$$\overleftarrow{f}_{E1}(E_\gamma = 0, T_f = T_i) \sim \cdot T_i \cdot \operatorname{Im}\Phi_{E1}(+0) \neq 0, \quad \operatorname{Im}\Phi_{XL}(\omega) \equiv \operatorname{Im}\chi_{XL}(\omega) / \omega$$

Energy-dependent width within MLO4

$$\Gamma_{r,j} = a_1 \cdot E_{r,j} + a_2 \cdot |\beta_2| \cdot E_{r,j} \cdot \gamma_j - \text{systematics}$$

$$\Gamma_j(E_\gamma, T) = b_j \cdot \{a_1 \cdot [E_\gamma + U(T)] + a_2 \cdot |\beta_2| \cdot E_{r,j} \cdot \gamma_j\}$$

$$\gamma_j = \begin{cases} 1, & \text{sph. nucl.} \\ \left(R_0 / R_j\right)^{1.6}, & \text{def. nucl. [B.Bush, Y.Alhassid, NPA 531 (1991) 27]} \end{cases}$$

$$|\beta_2| = \sqrt{1224 A^{-7/3} / E_{2_1^+}}$$ **- dynamical deformation**

[L. Esser, U. Neuneyer, R. F. Casten, P. von Brentano, PRC 55(1997) 206]

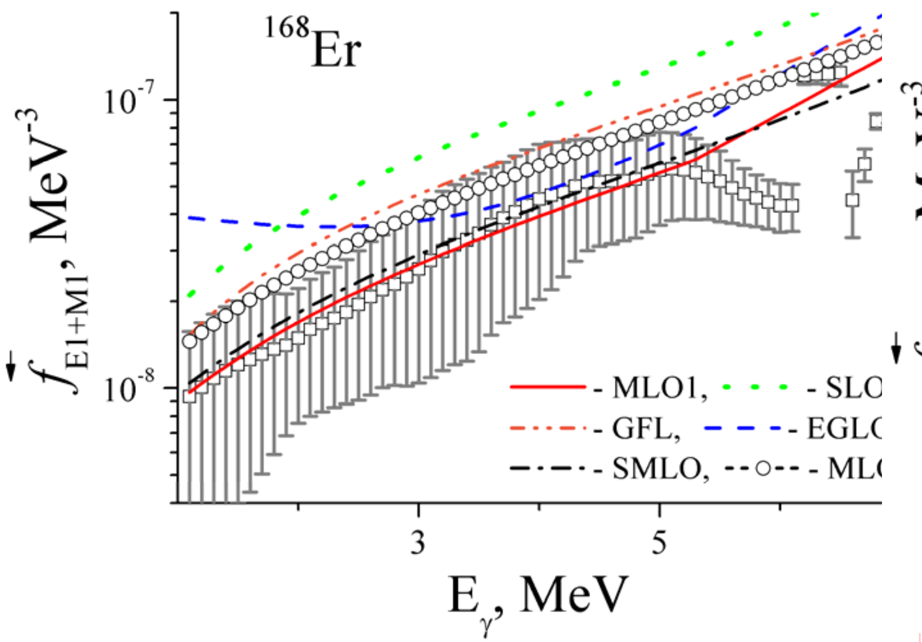
$E_{2_1^+}$ **from experimental data-base (RIPL3) or systematics by**

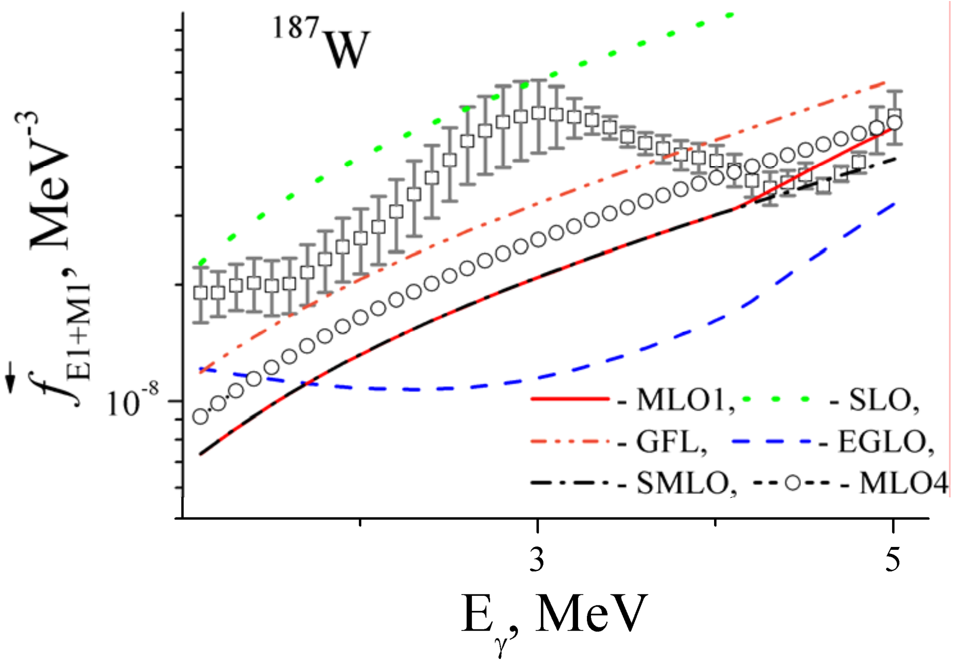
S.Hilaire, S.Goriely, NPA 779(2006)63:

$$E_{2_1^+} = 65 A^{-5/6} / (1 + 0.05 E_{shell})$$

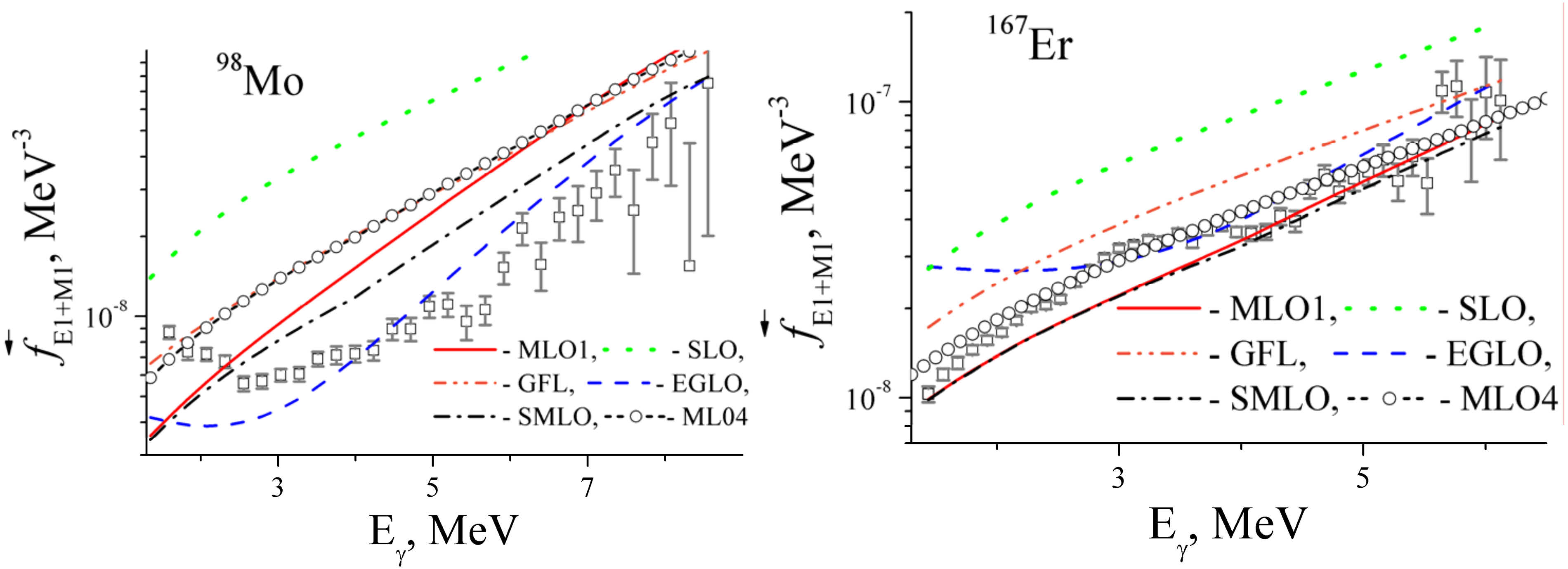
SLO, EGLO, GFL, MLO1-3, SMLO: $\beta_{2,eff} = \varphi(Q_2[\{\beta_j\}])$

Comparisons of gamma-decay RSF





Comparisons of gamma-decay strength functions for ^{168}Er **and** ^{187}W **:**
$U = S_n$**; experimental data -- A.M. Sukhovoj et al. Izvestiya RAN. Seriya Fiz. 69, 641 (2005); A.M. Sukhovoj et al. in Proc. of the XV Int. Seminar on Interaction of Neutrons with Nuclei. (Dubna, May 2007), 92 (2007).**



Comparisons of gamma-decay strength functions for ^{98}Mo **and** ^{167}Er **: experimental data --** E. Melby, M. Guttormsen, et al., Phys. Rev.C. 63, 044309 (2001); U. Agvaanluvsan, A. Schiller, et al., Phys. Rev.C. 70, 054611 (2004); http://www.mn.uio.no/fysikk/english/research/about/infrastructure/OCL/compilation/

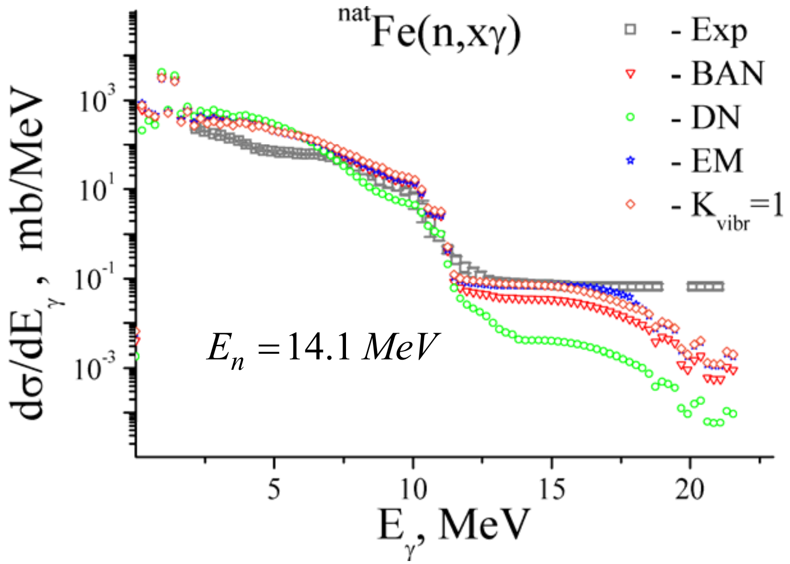
$$\overleftarrow{f}_{aver}(E_\gamma) = \begin{cases} \dfrac{1}{S_n - 4} \displaystyle\int_4^{S_n} \overleftarrow{f}(E_\gamma, U_f = U_i - E_\gamma) dU_i, & 1 < E_\gamma \le 4, \\ \dfrac{1}{S_n - E_\gamma} \displaystyle\int_{E_\gamma}^{S_n} \overleftarrow{f}(E_\gamma, U_f = U_i - E_\gamma) dU_i, & 4 < E_\gamma \le S_n, \end{cases}$$

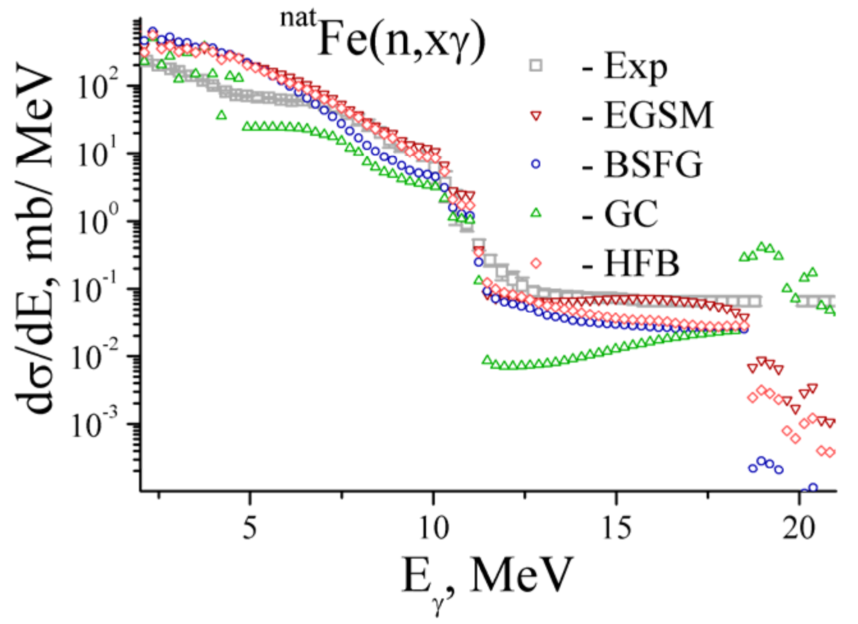
The average $\sum_{i=1}^{n}\left(\chi_i^2(model)/\chi_i^2(\mathrm{SLO})\right)/n$ ratio of chi-square deviations of the theoretical RSF of γ-decay from experimental data. n - number of nuclei

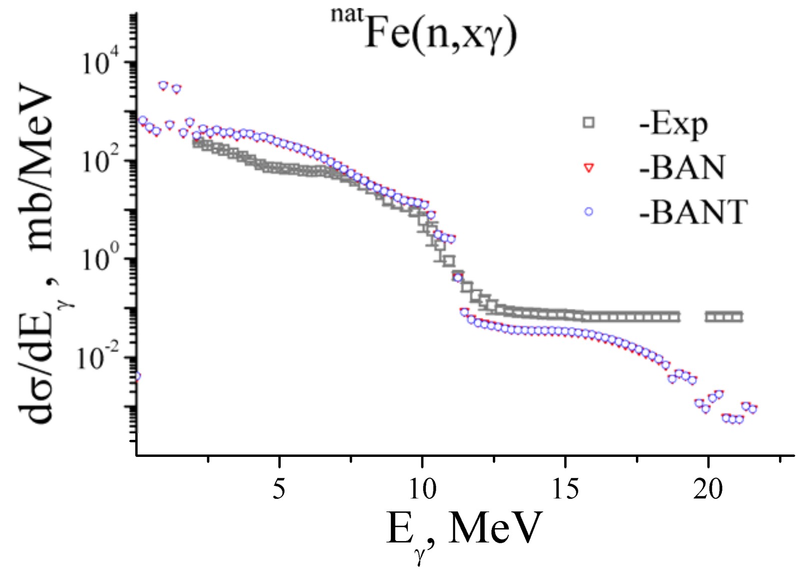
Exp.Data	n	Model				
		EGLO	GFL	MLO1	SMLO	MLO4
[1]	38	1,22	0,91	0,98	1,01	0,89
[2]	41	0,18	0,17	0,11	0,11	0,13
[3]	7	2,22	2,11	1,16	1,71	1,20
[4]	53	9,38	2,76	8,75	13,81	6,97

1. A.M. Sukhovoj et al. Izvestiya RAN. Seriya Fiz. **69**, 641 (2005); A.M. Sukhovoj et al. in Proc. of the XV Int. Seminar on Interaction of Neutrons with Nuclei. (Dubna, May 2007), 92 (2007).
2. E. Melby, M. Guttormsen, et al., Phys. Rev.C. **63**, 044309 (2001); U. Agvaanluvsan, A. Schiller, et al., Phys. Rev.C. **70**, 054611 (2004); http://www.mn.uio.no/fysikk/english/research/about/infrastructure/OCL/compilation/
3. R. Schwengner, G. Rusev, et al., Phys. Rev. C. 78 (2008) 064314; Phys. Rev. C. **81**, 034319 (2010)
4. J. Kopecky Handbook for calculations of nuclear reaction data. Reference Input Parameter Library (RIPL), Tech. Rep. IAEA-TECDOC-1034, Ch. 6, 1998; directory "Gamma" on the RIPL-1 web site at http://www-nds.iaea.or.at/ripl/

Effect of vibrational states and RSF forms on gamma-ray spectra and excitation functions

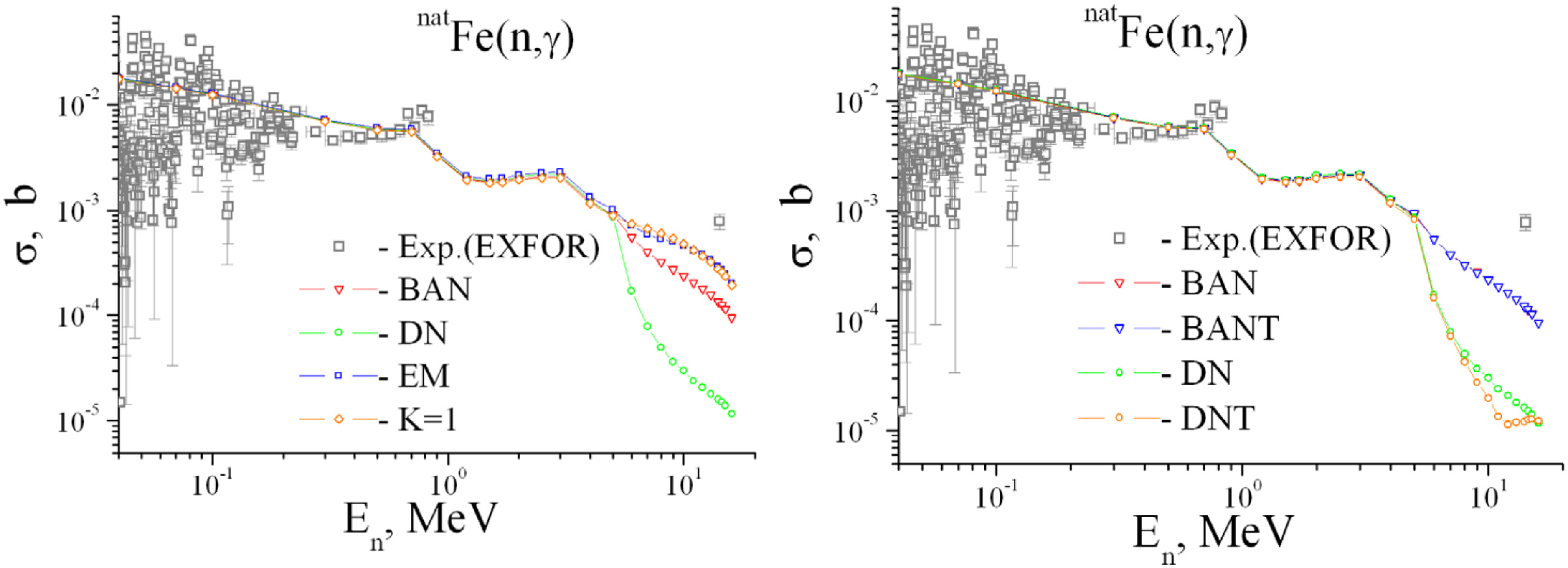






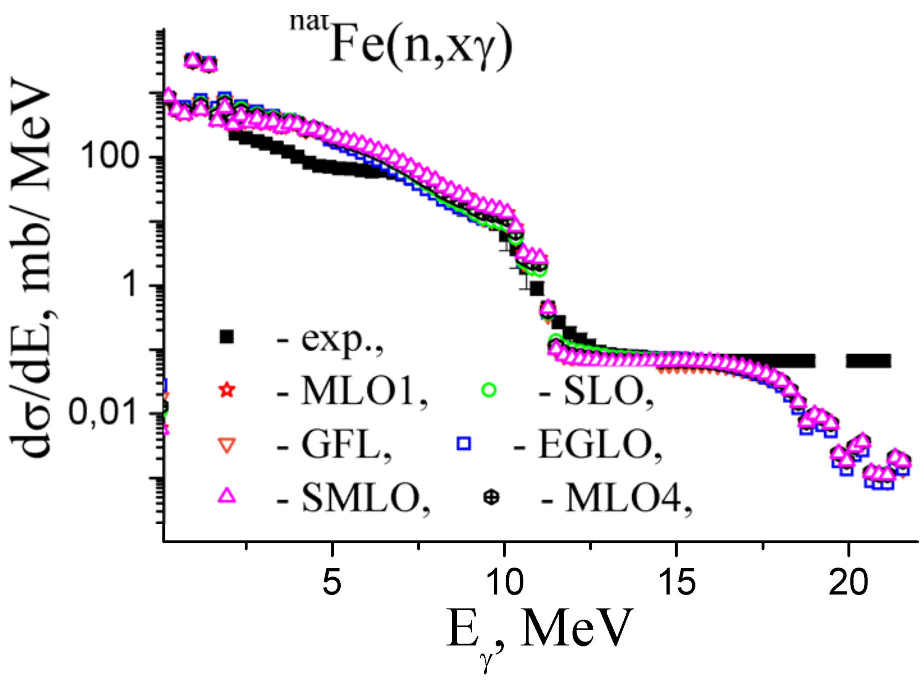
The scatter of gamma-ray spectra calculated within EGSM for intrinsic and rotational level densities with different K_{vibr} are the same order as scatter of the spectra calculated using different models of NLD

Calculations - EMPIRE 3.1; $E_n = 14.1\ (MeV)$
Experimental data - Bondar V.M., et al. // Proc. of the ISINN-18, Dubna, May 26-29, 2010, 135 (2011)

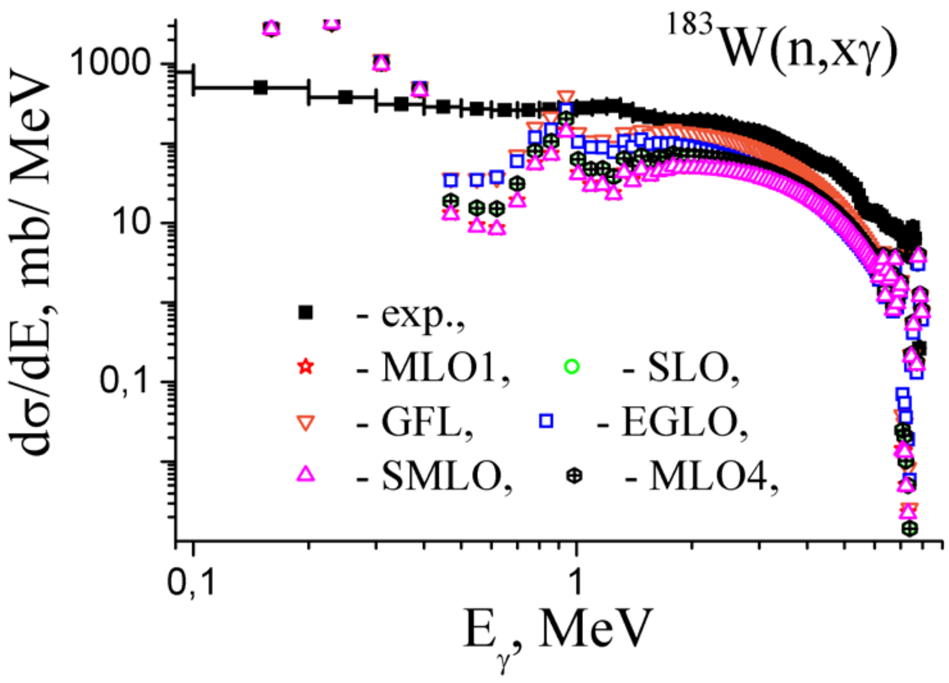


Calculations- EMPIRE 3.1; experimental data - EXFOR

Results of calculations of excitation functions and gamma-gay spectra are sensitive to vibrational enhancement



(*a*)

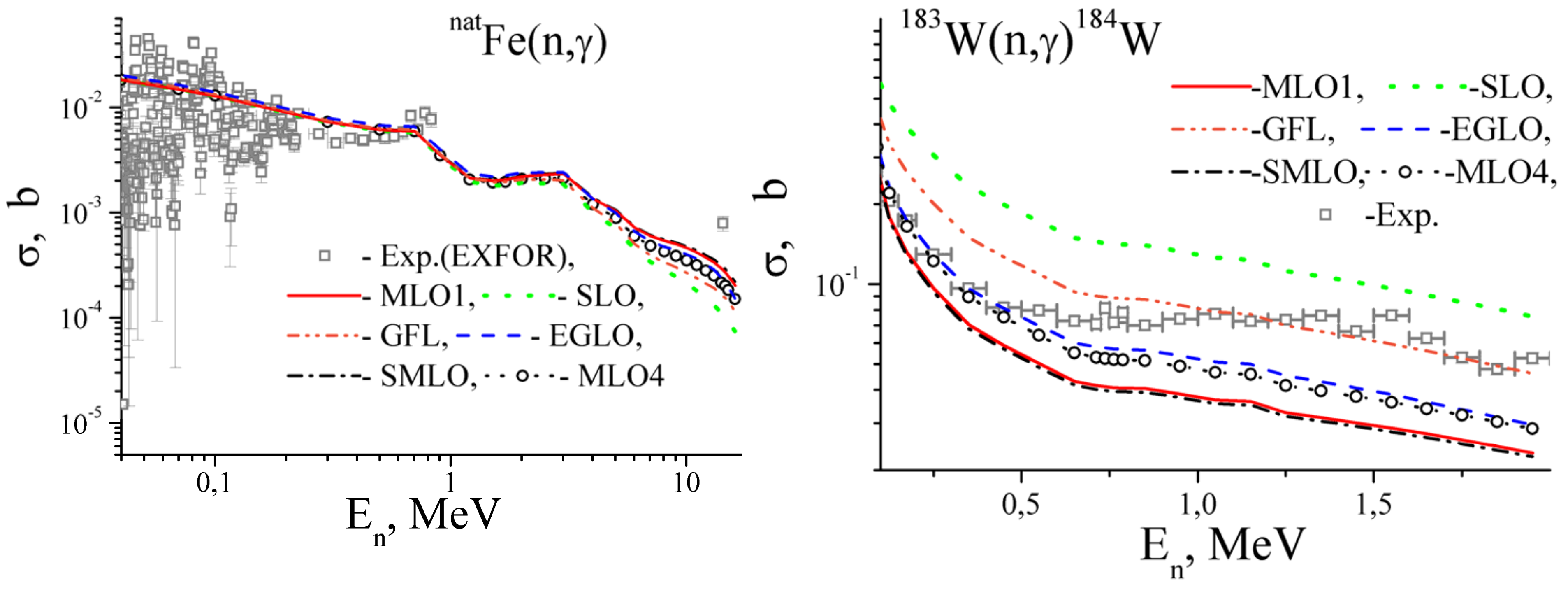


(*b*)

Gamma-ray spectra from $^{nat}Fe(n, x\gamma)$ and $^{183}W(n, x\gamma)$ reactions calculated with EMPIRE code using different models for the RSF. The experimental data are taken from [1] for panel *a* ($E_n = 14.1 \quad MeV$) and from [2] for panel *b* ($E_n = 0.5 \quad MeV$)

1.V.M. Bondar et al., in Proc. of the 18th Int. Sem.onInter. of Neutrons with Nuclei "Neutron Spectroscopy, Nuclear structure, Related topics" (Dubna, May, 2010). (2011) 135

2. J.Voignier et al., J. Nucl. Science and Engineering, **112**, 87 **(**1992)



(*a*) (*b*)

The excitation functions of $^{nat}Fe(n,\gamma)$ and $^{183}W(n,\gamma)^{184}W$ reactions using different RSF models. The experimental data are taken from EXFOR data library [1] for panel *a* and from [2] for panel *b*.

1. Experimental Nuclear Reaction Data (EXFOR); http://www.nndc.bnl.gov/exfor/exfor00.htm
2. R.L.Macklin, D.M. Drake, E.D. Arthur, J. Nucl. Science and Engineering **84**, 98 (1983)

SUMMARY

- NLD within modified EGSM (EGSM for shape of intrinsic and rotational level densities with different vibrational enhancement factors) was studied.

- For modified EGSM, approximation of boson partition function with average occupation numbers (BAN) can be considered as the most appropriate approach for calculation of the vibrational enhancement factor and within BAN

$$K_{vibr}(S_n) <\sim 2 \div 5 \qquad (A \sim 100)$$

- Rather reliable simple description of E1 gamma-decay strength can be obtained by the use of models with dependence of line spreading on gamma-ray&excitation energies. It seems that the MLO4 is best candidate for good overall description of the RSF.

- Shape and values of excitation function and gamma-ray spectra are sensitive to choice of vibrational enhancement factor of NLD and shape of RSF.

- The scatter of gamma-ray spectra calculated within EGSM for intrinsic and rotational NLD and different K_{vibr} are the same order as scatter of the spectra calculated using different NLD models.

- To better understand role of the temperature and energy dependence of the RSF, experimental data are necessary as functions of gamma-ray and excitation energies, especially at low gamma-ray energy.

***V.A.Plujko, O.M. Gorbachenko, B.M.Bondar et al* , Nucl. Data Sheets** (in press);
***V.A.Plujko, O.M. Gorbachenko, E.P. Rovenskykh et al* , Nucl. Data Sheets** (in press);
***R.Capote et al* , Nucl. Data Sheets 110 (2009) 310; http://www-nds.iaea.or.at/ripl3/;**
***V.A.Plujko, R.Capote, O.M. Gorbachenko*, At.Data Nucl.Data Tables 97(2011) 567;**
***V.A.Plujko, R.Capote, V.M.Bondar, O.M. Gorbachenko, J. Kor. Phys.Soc.* 59(2011) 1514**
***V.A.Plujko et al, Nucl. Phys. At.Energy 13(2012)341;* http:/jnpae.kinr.kiev.ua**

THANK YOU FOR YOUR ATTENTION !!!

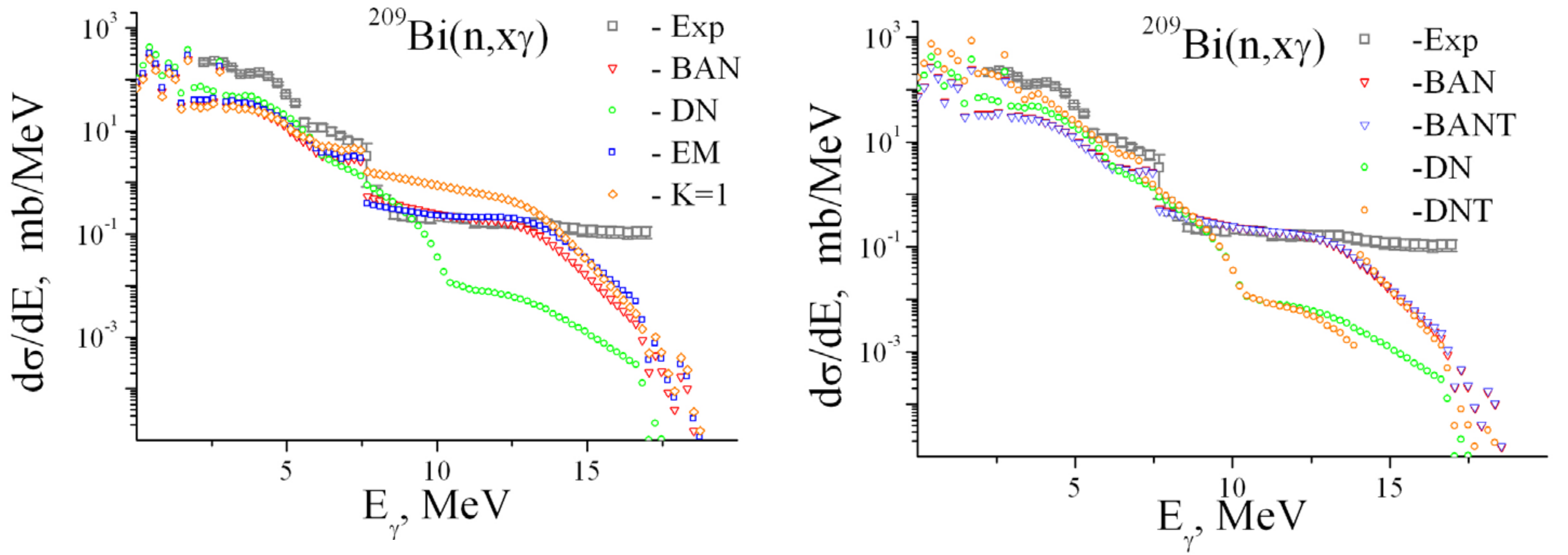


Fig. Dependence of $d\sigma/dE$ on gamma-ray energy for ^{209}Bi$(n,x\gamma)$ reaction.
Calculation was made with the use of the EMPIRE 3.1 code at $E_n = 14.1\,(MeV)$
Experimental data are taken from: [Bondar V.M., Gorbachenko O.M., Kadenko I.M., Leshchenko B.Yu., Onischuk Yu.M., Plujko V.A. // Nuclear Physics and Atomic Energy. – Vol.11 №3. (2010) 246-251.]

The temperature change by the vibrational states can effect on gamma-gay spectra

Compound nucleus reaction for *b* channel

$$\sigma_b(E,J,\pi) = \sigma_a(E,J,\pi)\frac{\Gamma_b(E,J,\pi)}{\sum_c \Gamma_c(E,J,\pi)}$$

A particle decay width have the form:

$$\Gamma_c(E,J,\pi) = \frac{1}{2\pi\rho_{CN}(E,J,\pi)}\times$$

$$\times\sum_{J'=0}^{\infty}\sum_{\pi'}\sum_{j=J'-J}^{J'+J}\int_{0}^{E-B_c}\rho_c(E',J',\pi')\times T_c^{l,j}(E-B_c-E')dE'$$

$T_c^{l,j}(E-B_c-E')$ - transmission coefficient for the particle c having channel energy $\varepsilon = E - B_c - E'$ and orbital angular momentum l

$\rho_c(E',J',\pi')$ - nuclear level density

For Gamma-ray emission: $T_{XL}(\varepsilon_\gamma) = 2\pi E_\gamma^{(2L+1)} f_{XL}(E_\gamma)$

$f_{XL}(E_\gamma)$ - radiative strength function of radiation *X(E,M)* with multipolarity *L*

Effect of the collective states on nuclear level density

The collective states can strongly effect on the level density, specifically, at low excitation energies. The simplest method to estimate effect of the vibrational states on level densities is calculation of collective enhancement factor

$$K = \rho / \rho_0$$

ρ , ρ_0 - level densities with and without allowing for collective states

Collective enhancement factor

$$K = K_{vibr} \cdot K_{rot}$$

Problems with estimation of K

1. *Uncertainties in value of K_{vibr}*

Closed-form approach

Ignatyuk A.V. Statistical properties of excited nuclei. 1983; Yad. Fiz. 21(1975) 20 ; Izv.AN SSSR 38 (1974) 2612 (RPA approach);
Ignatyuk A.V., Weil J.L., Raman S., Kahane S. PRC. 47 (1993) 1504

$$K_{vibr}(S_n) \sim 15 \div 30$$

$$(A \sim 100); \quad RIPL-2,3$$

Microscopic calculations within quasiparticle-quasiphonon model

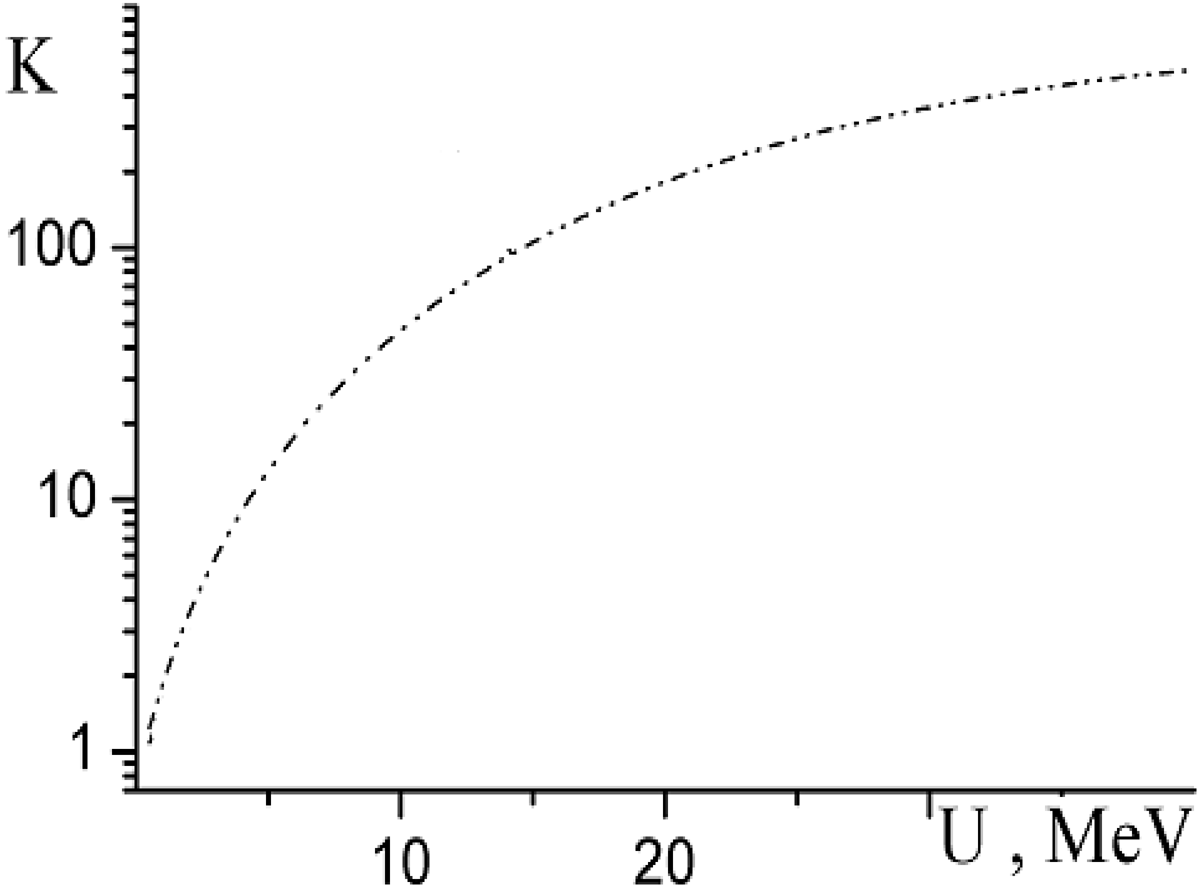
Soloviev V.G., Stoyanov Ch., Vdovin A.I. NPA224 (1974) 411; Voronov V.V., Malov L.A., Soloviev V.G. Yad.Fiz. 21 (1975) 40; Malov L.A., Soloviev V.G., Voronov V.V. Phys.Lett. B55 (1975) 17; Vdovin A.I., Voronov V.V., Malov L.A. Soloviev V.G., Stoyanov Ch. Fiz.El.Chas.At.Yad. 7 (1976) 952

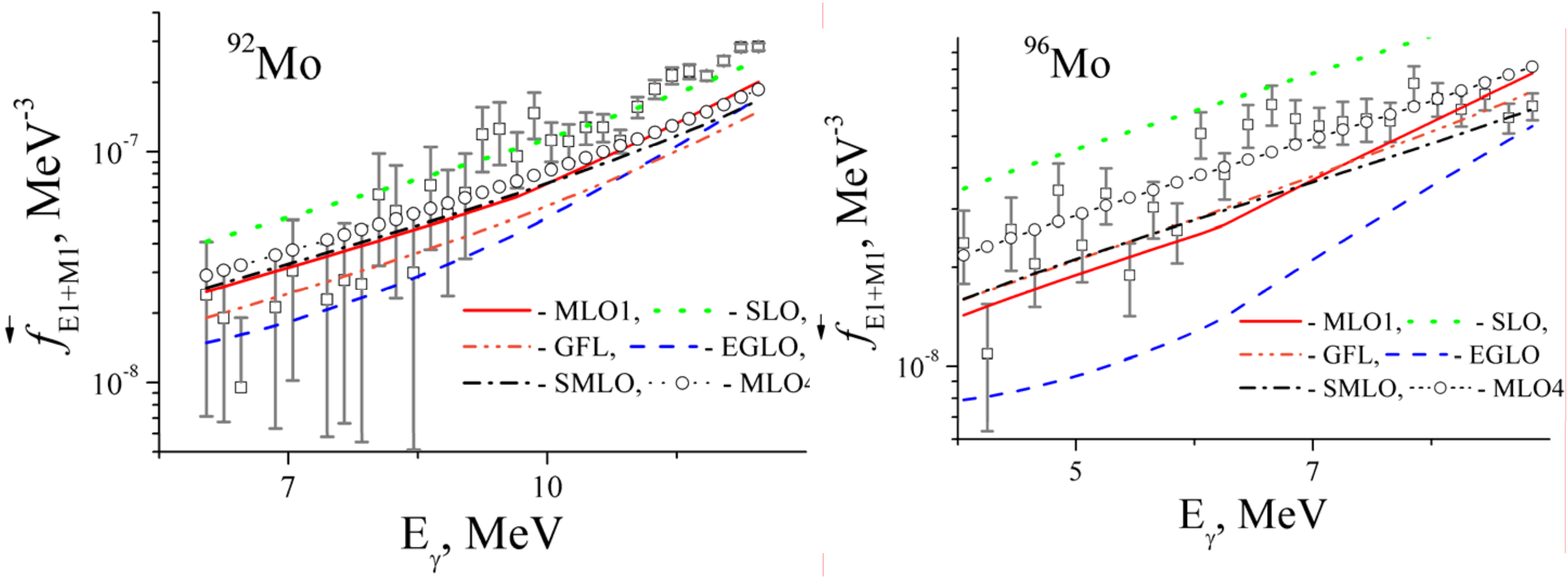
$$K_{vibr}(S_n) \sim 2 \div 5 \qquad (A \sim 100)$$

$$K_{rot}(S_n) \approx J_{\perp}T \sim 100 \qquad (A \sim 100)$$

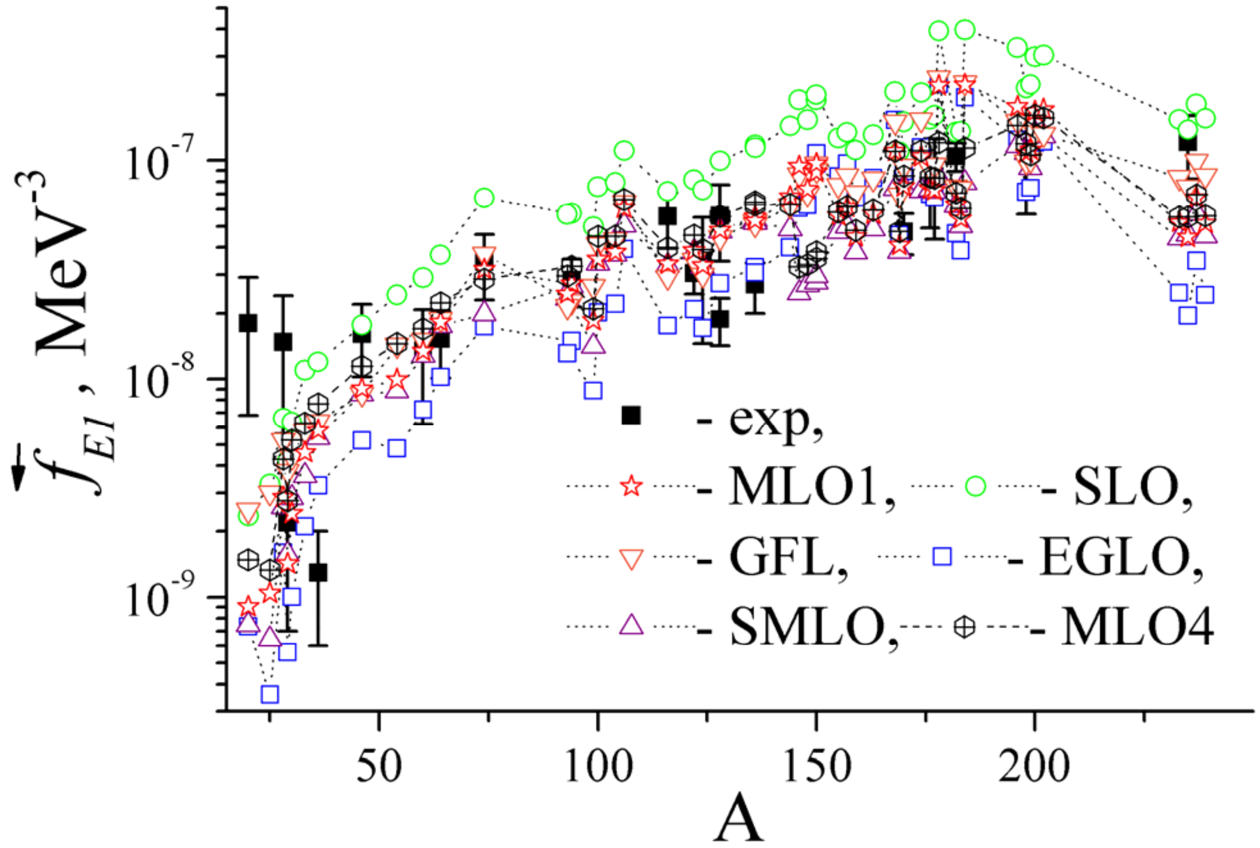
2. Energy dependence

Unrealistic dependence of vibrational enhancement factor
on excitation energy without collective state damping





Comparisons of gamma-decay strength functions for ^{92}Mo and ^{96}Mo :
$U = S_n$ **;experimental data -- R. Schwengner, G. Rusev, et al., Phys. Rev. C. 78 (2008) 064314; Phys. Rev. C. 81, 034319 (2010)**



The gamma-decay strength functions within different RSF models: $U = S_n$. Experimental data are taken from J. Kopecky Handbook for calculations of nuclear reaction data. Reference Input Parameter Library (RIPL), Tech. Rep. IAEA-TECDOC-1034, Ch. 6, 1998; directory "Gamma" on the RIPL-1 web site at http://www-nds.iaea.or.at/ripl/

Average probabilities for γ - transitions can be described through the use of radiative (gamma-ray, photon) strength functions (RSF)

RSF are important ingredient of statistical theory of nuclear reactions. Calculations of observed characteristics of nuclear reactions are as a rule time consuming, and for decreasing in computing time, simple closed-form expressions are preferable in evaluation of gamma-ray strengths

The goal of the studies was to develop the theory supported practical method for the calculation of E1 radiative strength function both for γ-decay between c-c states and photoabsorption, to study shape of γ - strength and to extract GDR parameters with uncertainty estimations