



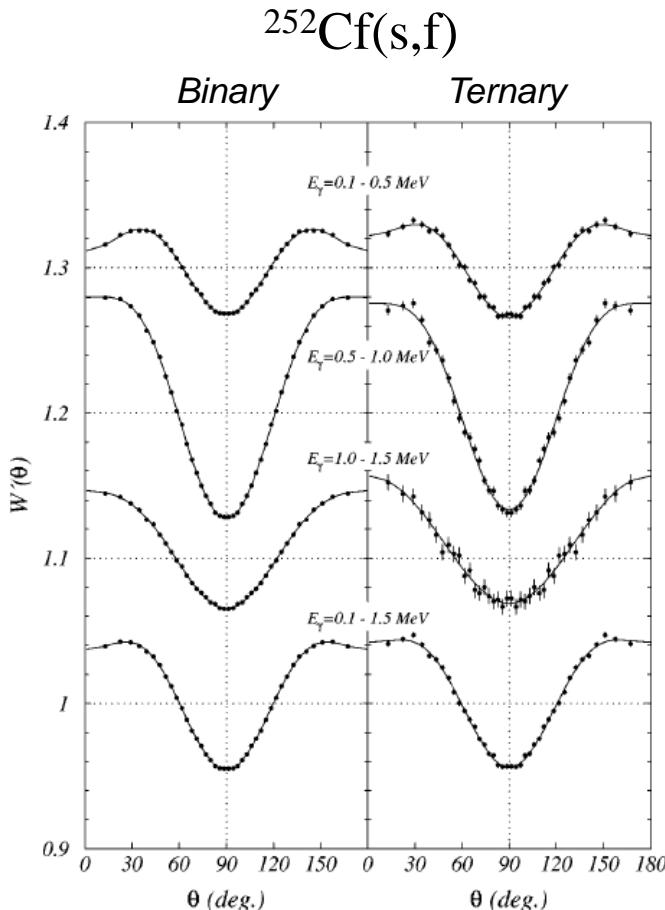
# ANISOTROPY OF PROMPT NEUTRON'S EMISSION FROM THE FISSION FRAGMENTS

I.S. Guseva

NRC “Kurchatov Institute”, Petersburg Nuclear Physics Institute, Gatchina, Russia

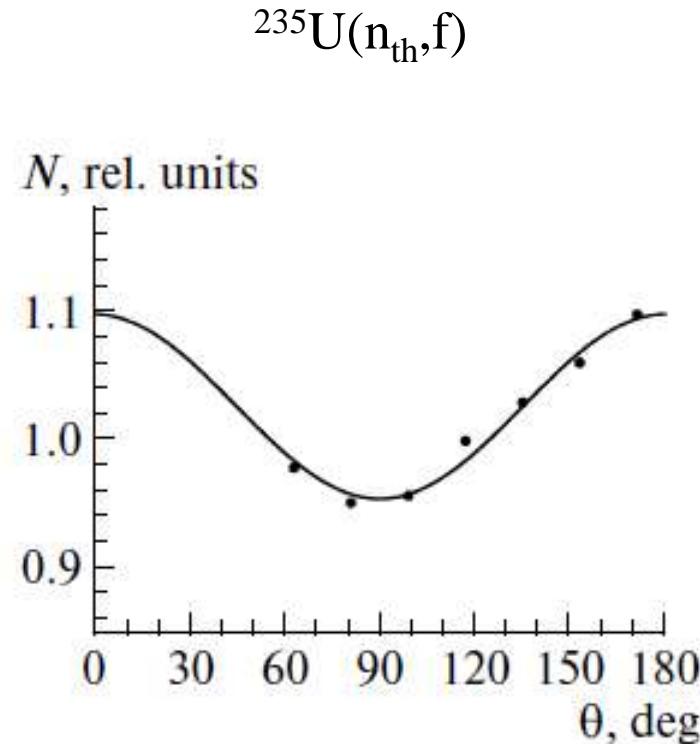
# Anisotropy of $\gamma$ -rays in spontaneous and neutron-induced fission

Yu. N. Kopach et. al, Phys.Rev.Lett., v.82, № 2, 303 (1999)



Angular distributions  $W'(\theta)$  of  $\gamma$ -rays with respect to the fragment motion for binary (left side) and  $\alpha$ -accompanied (right side) fission of  $^{252}\text{Cf}$ , for different  $\gamma$ -ray energy intervals

G. V. Valsky, A. M. Gagarski, I. S. Guseva, et. al  
Bull. of the RAS: Physics, v.74, №6, 767 (2010)



Angular distribution  $W(\theta)$  of  $\gamma$ -rays with respect to the fragment motion direction for the reaction  $^{235}\text{U}(n_{\text{th}}, f)$

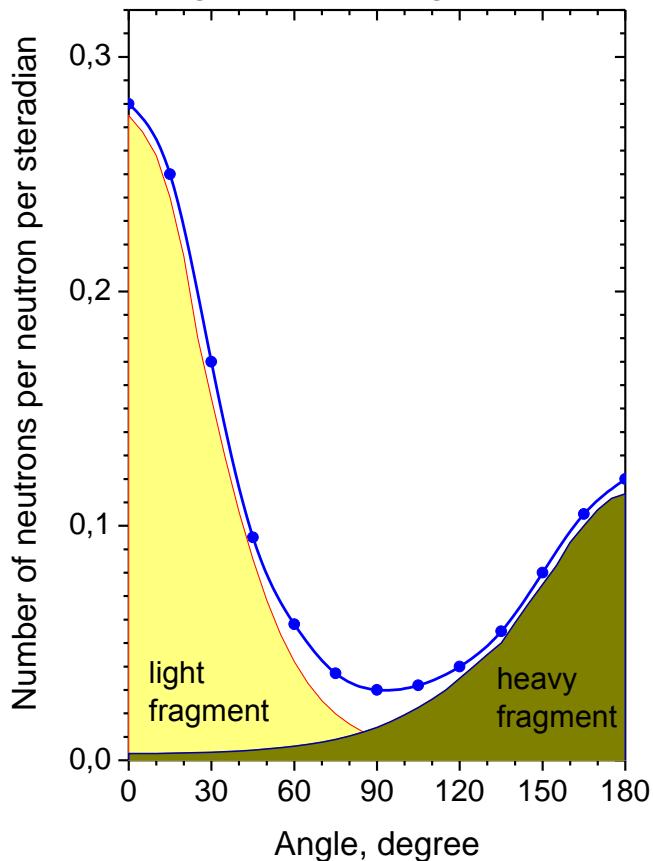
## Gamma-rays

Since the fragment velocity is significantly less than the speed of the light ( $\beta=v_{FF}/c \leq 0.05$ ), so the anisotropy of the emission of gamma rays from a stationary fission fragment is close to that observed in the laboratory system.

According to V. Strutinski a reason for such anisotropy can be the presence of a large angular momentum of the primary fission fragment. This fragment spin can appear at the scission point and correlated with the direction of fragment motion.

## Angular distribution of neutrons for the reaction $^{235}\text{U}(\text{n}_{\text{th}}, \text{f})$

K. Skarsvag and K. Bergheim (1963)



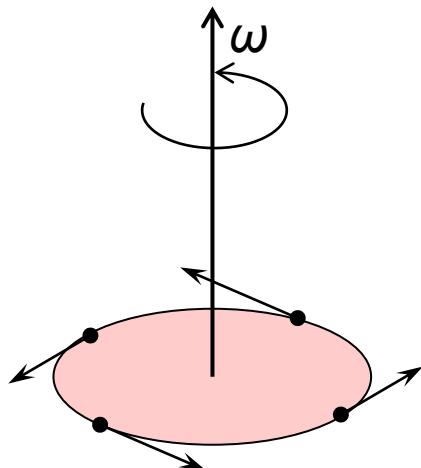
## Neutrons

The average energy of emitted neutrons in the fragment center-of-mass is comparable to kinetic energy per nucleon for a moving fragment. This leads to a large kinematic effect, which does not allow so easy to observe the influence of similar anisotropy in the rest frame of fragment on neutron-fragment correlation.

$$N(0^\circ) : N(180^\circ) : N(90^\circ) = 9:4:1$$

# Angular distribution of evaporated neutrons in the CM system of fission fragments

$$W(\theta) \propto 1 + A \cdot \cos^2(\theta)$$



T. Ericson and V. Strutinski  
(quasi-classical approach, 1958)

$$A = \frac{\mu \cdot \omega^2 \cdot R}{2 \cdot T}$$

$\mu$  – the mass of the emitted particle  
 $\omega$  – the angular velocity of the fragment  
 $R$  – the fragment's radius  
 $T$  – the nuclear temperature

# Statistical model calculation of the angular distribution of evaporated neutrons from fission fragments (A.Gavron, 1976)

$$W(\theta) = \sum_{lm} P_{lm} |Y_{lm}(\theta, \varphi)|^2$$

$$P_{lm} \propto \sum_{J_f} \int_0^{E^* - B_n} \rho_{J_f}(E^* - B_n - \varepsilon) \cdot T_l(\varepsilon) \cdot \left| C_{J_f M_f l m}^{J_i M_i} \right|^2 d\varepsilon$$

$E^*$  – the excitation energy

$B_n$  – the neutron binding energy

$\varepsilon$  – the energy of evaporated neutron

$\rho_{J_f}(E^* - B_n - \varepsilon)$  – the level density

$T_l(\varepsilon)$  – the neutron transmission coefficient

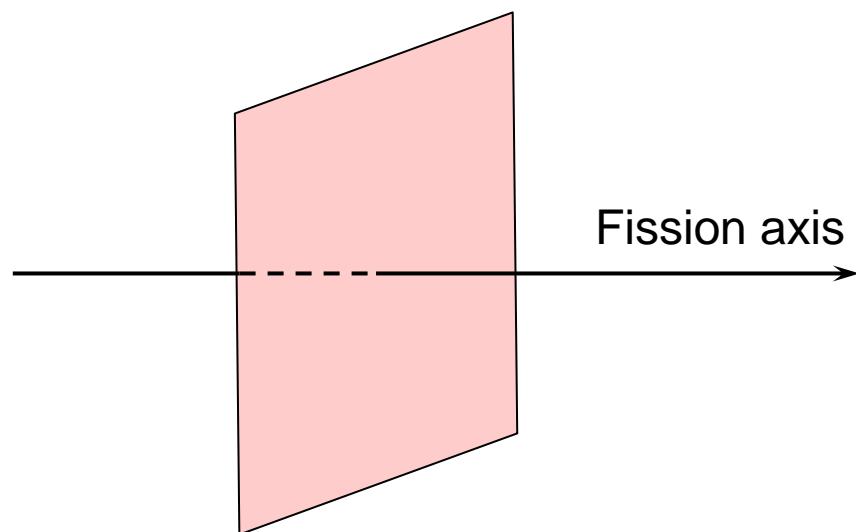
$C_{J_f M_f l m}^{J_i M_i}$  – the coefficient of vector coupling

$J_i, J_f$  – initial and final angular momenta of fragment

$M_i, M_f$  – their projections on the fission axis

$l, m$  – the neutron orbital momentum and its projection

Initial  $M=0$  according to assumption that the angular momentum of primary fragments is aligned in a plain perpendicular to the direction of fission  
(J.B. Wilhelmy, E. Cheifetz, R. C. Jared, S. G. Thompson, H. R. Bowman, and J. O. Rasmussen, Phys. Rev. C 5, 2041 (1972)).



# Initial spin distribution of fission fragments

1) "standard" form:

$$\rho_J = (2J+1) \cdot \exp(-(J+0.5)^2 / B^2)$$

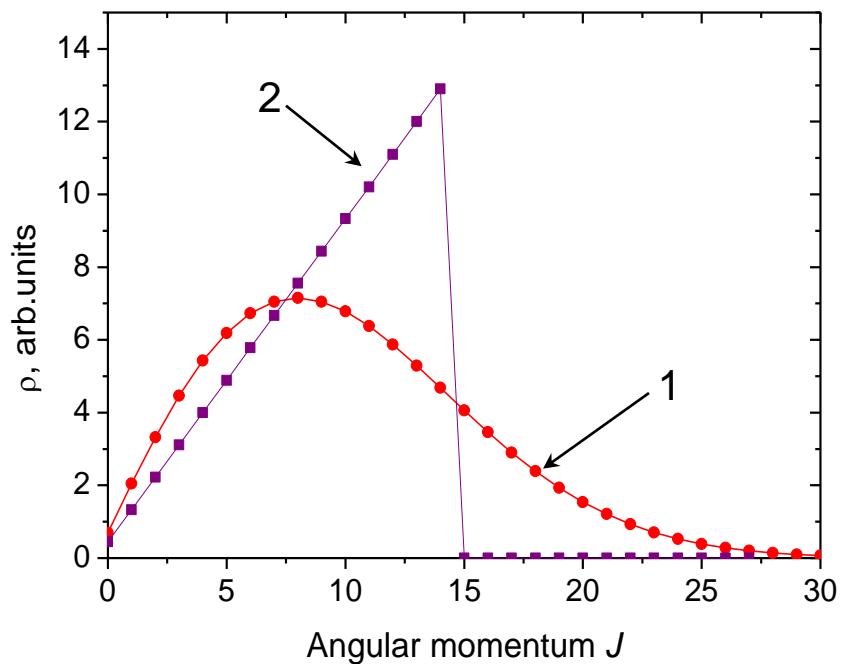
2)  $\rho_J = (2J+1) \quad J \leq J_{\max}$

$$\rho_J = 0 \quad J > J_{\max}$$

This 'exotic' shape of spin distribution was proposed by S. G. Kadmenski (Yad. Fiz. Vol. 69, 2005)

1)  $B = 12 \quad \langle J \rangle \approx 9.5$

2)  $J_{\max} = 14 \quad \langle J \rangle \approx 9.5$



# Neutron spectra in CMS

$^{252}Cf$ , light fragment ( $A=109$ )

$$\rho \propto \exp\left(2\sqrt{aE^*}\right)$$

$a$  – parameter of level density

$E_0^*$  – the excitation energy directly after scission

$$E_i^* = E_{i-1}^* - B_n - \varepsilon_i$$

$B_n$  – the neutron binding energy

$\varepsilon_i$  – the energy of evaporated neutron

$i$  – serial number of neutron evaporation

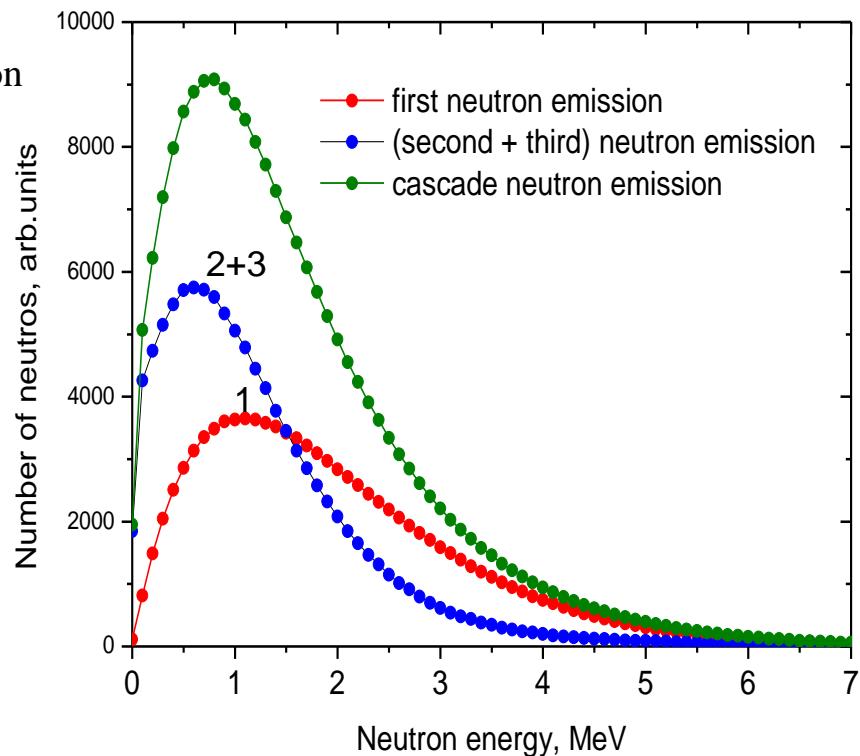
$\langle \nu \rangle$  – the average neutron multiplicity

$$\langle \nu \rangle \Rightarrow \langle E_0^* \rangle$$

$^{252}Cf$ , heavy fragment ( $A=143$ )

$$\langle \nu_h \rangle = 1.71 \Rightarrow \langle E_0^* \rangle = 13.7 \text{ MeV}$$

$$B_n = 6 \text{ MeV} \quad \langle \nu_l \rangle = 2.05 \Rightarrow \langle E_0^* \rangle = 19.3 \text{ MeV}$$



## Neutron transmission coefficients $T_l(E_n)$

$$T_l = \frac{4xXv_l}{X^2 + (2xX + x^2 v'_l)v_l}$$

$$v_l \equiv \frac{1}{G_l^2(R) + F_l^2(R)} \quad v'_l \equiv \frac{1}{k^2} \left[ \left( \frac{dG_l}{dr} \right)^2 + \left( \frac{dF_l}{dr} \right)^2 \right]$$

$$F_l(r) = \sqrt{\frac{\pi kr}{2}} J_{l+1/2}(kr) \quad J_p(z) - \text{Bessel's function}$$

$$G_l(r) = \sqrt{\frac{\pi kr}{2}} N_{l+1/2}(kr) \quad N_p(z) - \text{Neuman's function}$$

$$x \equiv kR$$

$k$  – the wave number outside of nucleus

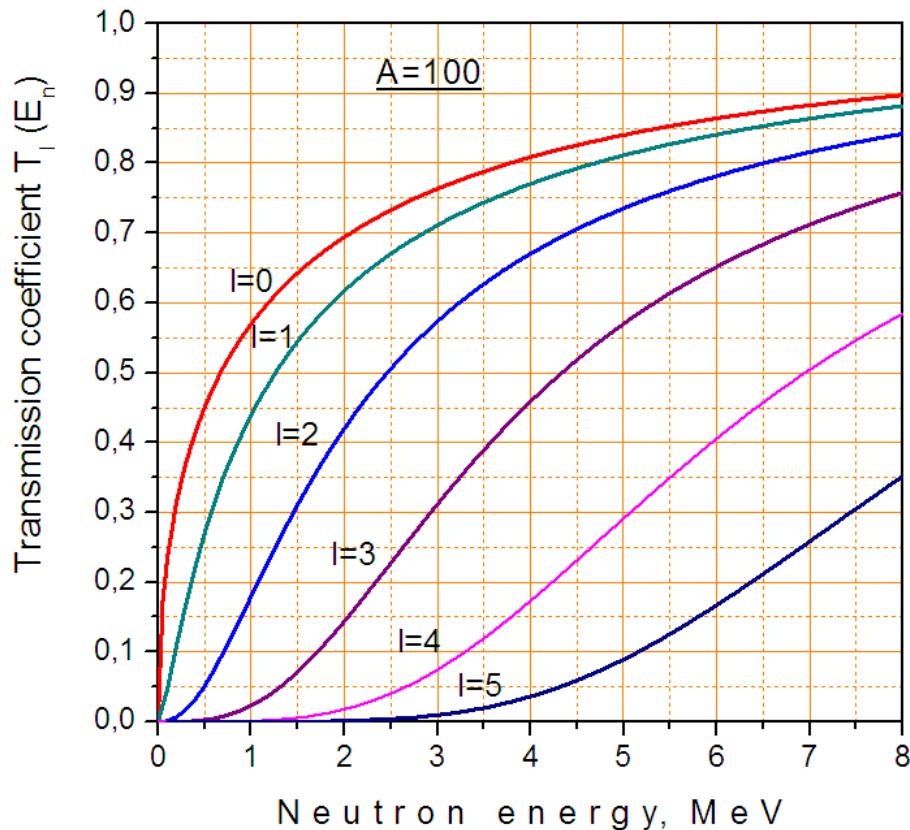
$$X \equiv KR$$

$K$  - the wave number inside of nucleus

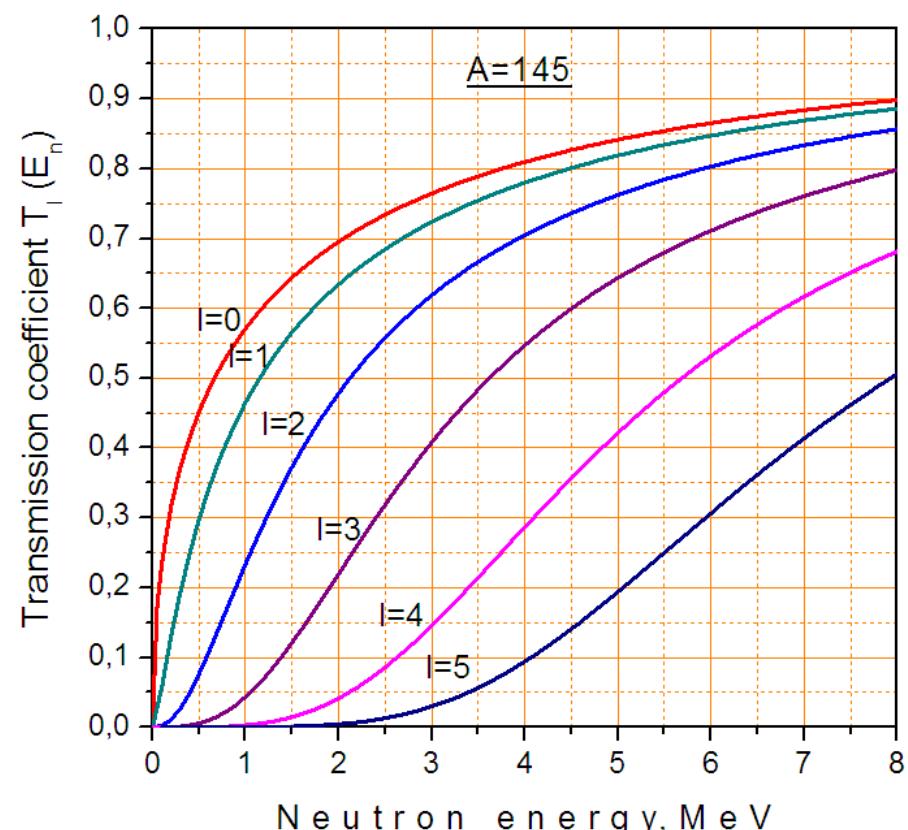
$R$  - the nuclear radius

# Neutron transmission coefficients

For light fragment region

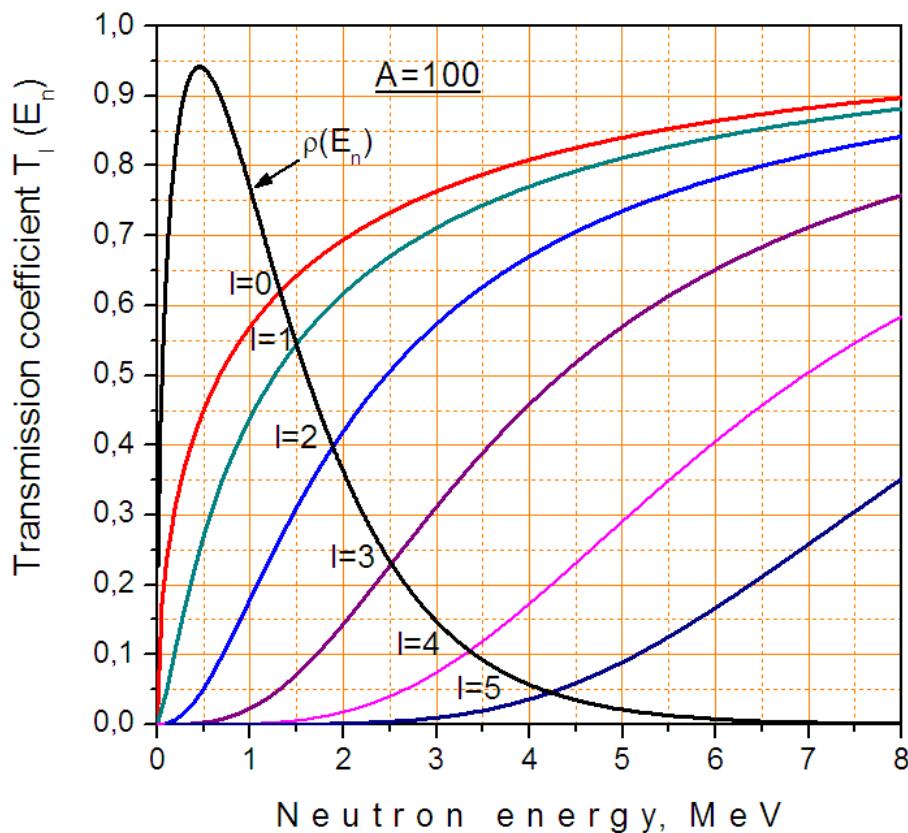


For heavy fragment region

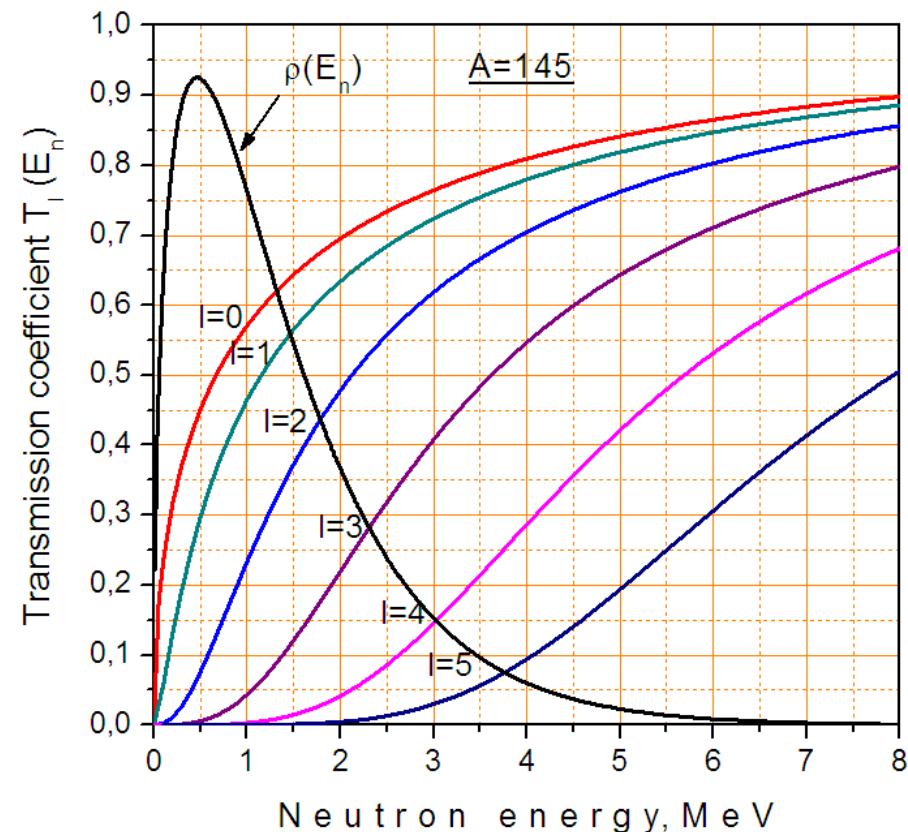


# Neutron transmission coefficients

For light fragment region



For heavy fragment region



# Monte-Carlo calculations

## for neutron emission anisotropy in CM system

Examples of calculated matrices for  
light fragment  
(total number of events 20000000)

Matrix for the first neutron emission

1\m	-5	-4	-3	-2	-1	0	1	2	3	4	5
0	0	0	0	0	0	3943478	0	0	0	0	0
1	0	0	0	0	1017334	1116038	1018705	0	0	0	0
2	0	0	0	316613	400702	433940	401021	316112	0	0	0
3	0	0	76890	107306	131195	139910	130871	107342	76851	0	0
4	0	13708	20065	26270	30871	32795	31358	26358	19889	13443	0
5	2144	3272	4570	5688	6511	6907	6541	5715	4388	3158	2041

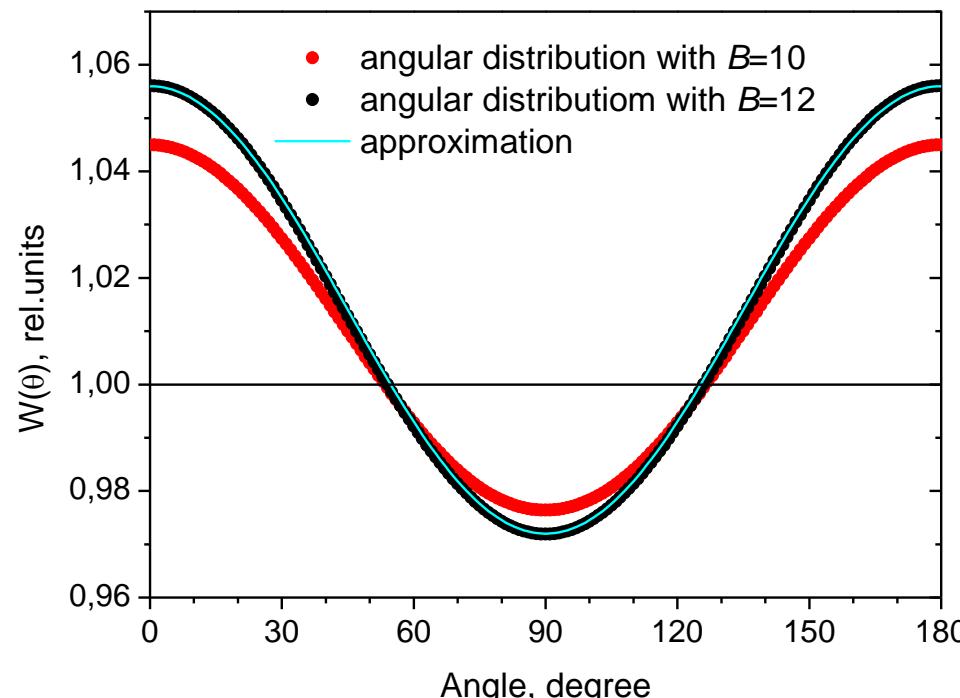
Matrix for the second neutron emission

1\m	-5	-4	-3	-2	-1	0	1	2	3	4	5
0	0	0	0	0	0	4758196	0	0	0	0	0
1	0	0	0	0	1072972	1168318	1074081	0	0	0	0
2	0	0	0	250017	309894	334271	310547	249457	0	0	0
3	0	0	38894	52329	63355	66942	62701	52386	38641	0	0
4	0	3627	5347	6981	8209	8623	8006	6896	5410	3862	0
5	300	441	569	765	893	937	902	801	628	456	271

# The calculated angular distribution relative to the fission axis for different values of initial fragment spin

$$\text{Spin distribution: } \rho = (2J+1) \cdot \exp(-(J+0.5)^2/B^2)$$

These curves can be well approximated by the expression:  $W(\theta) \sim 1 + P_2(\cos^2 \theta)$ , where  $P_2$  is Legendre polynomial of the second degree. Such mathematical expression allows us to have the same number of evaporated neutrons for different value of anisotropy parameter. This is why the Legendre polynomial in case of analytical description usually used.



## The relation between two different definitions of anisotropy

$$\varphi(\mu_{cm}) = 1 + bP_2(\mu_{cm}) = \\ 1 + b \cdot (3\cos^2(\theta) - 1)/2$$

$$\varphi(\mu_{cm}) \sim 1 + A_{nf} \cdot \cos^2(\theta) \\ A_{nf} = \frac{N(0^\circ) - N(90^\circ)}{N(90^\circ)}$$

Here  $\theta$  is the angle between fission axis and neutron motion direction.

$$\mu_{cm} = \cos(\theta)$$

The anisotropy “ $A_{nf}$ ” is possible to use during the Monte-Carlo calculations. In process of analytical transformations it is necessary to use the anisotropy “ $b$ ” because this form of the angular distribution saves normalization.

Relations between “ $b$ ” and “ $A_{nf}$ ”:

$$A_{nf} = \frac{3b}{2 - b}$$

$$b = \frac{2A_{nf}}{3 + A_{nf}}$$

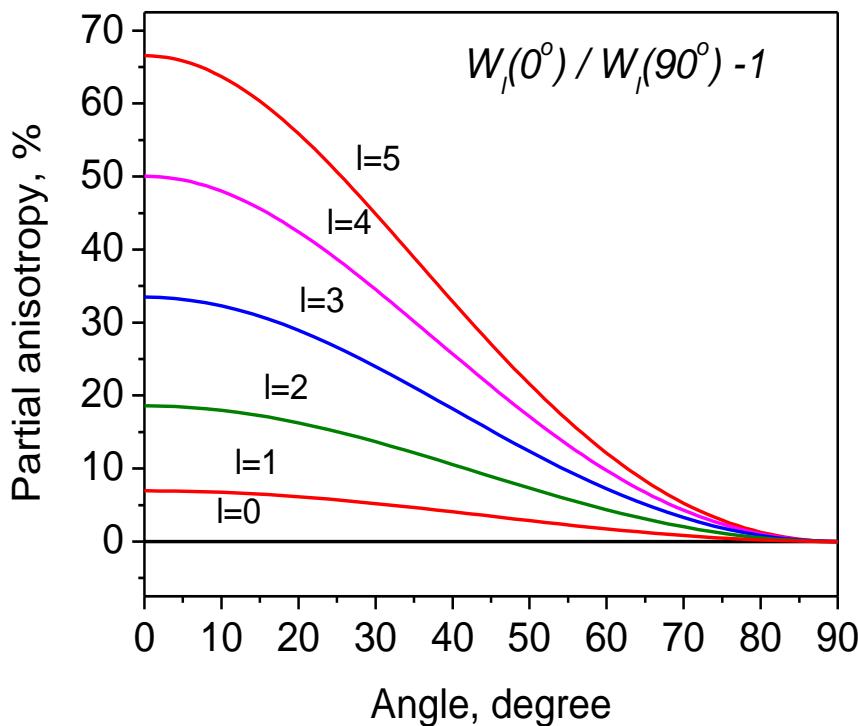
# Anisotropy in CMS

$$W_l(\theta) = \sum_m P_{lm} |Y_{lm}(\theta, \varphi)|^2$$

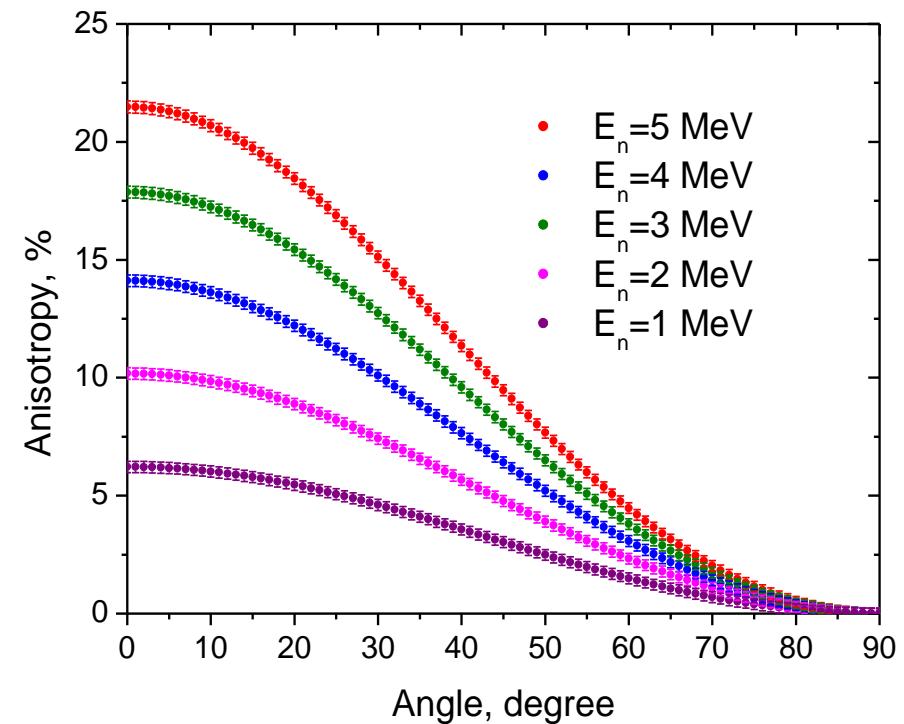
For these calculations  $\sum_m P_{lm} = Const$

$$A_{nf} = [W(0^\circ) - W(90^\circ)] / W(90^\circ)$$

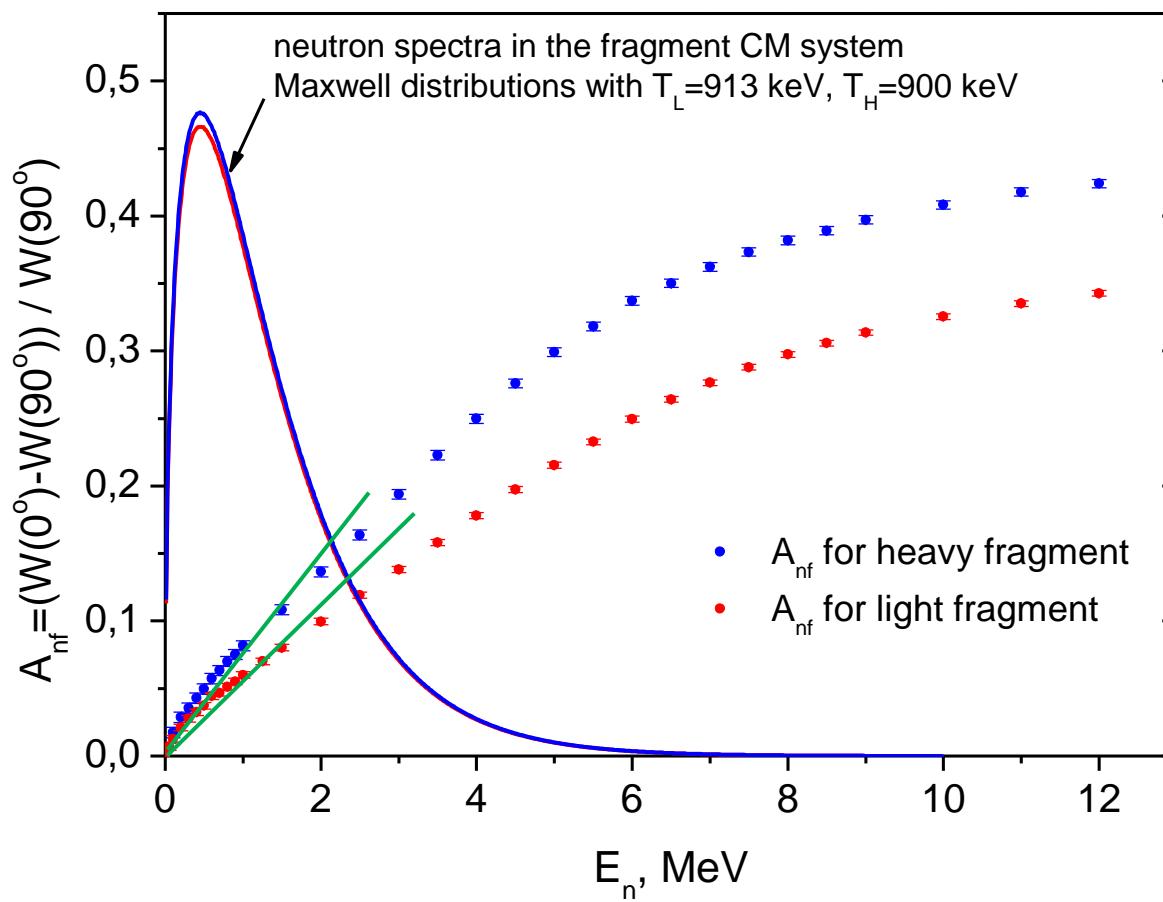
Angular anisotropy as a function of the orbital momentum of evaporated neutrons



Energy dependence of the angular anisotropy in CMS



# The energy dependence of neutron emission anisotropy $A_{nf}$ for light and heavy fission fragments (CM system) relative to the fission axis



# The experiment for n-f angular correlation in $^{252}\text{Cf}$

(A.Vorobiev et al. EPJ Web of Conf. 8, 03004 (2010))

The angular distribution of neutron emission in the fragment CMS

$$W(E_n, \theta_{cm}) = 1 + b \cdot P_2(\cos(\theta_{cm}))$$

It was used the linear dependence of the coefficient  $b$  versus neutron energy:

$$b = A_2 \cdot E_n$$

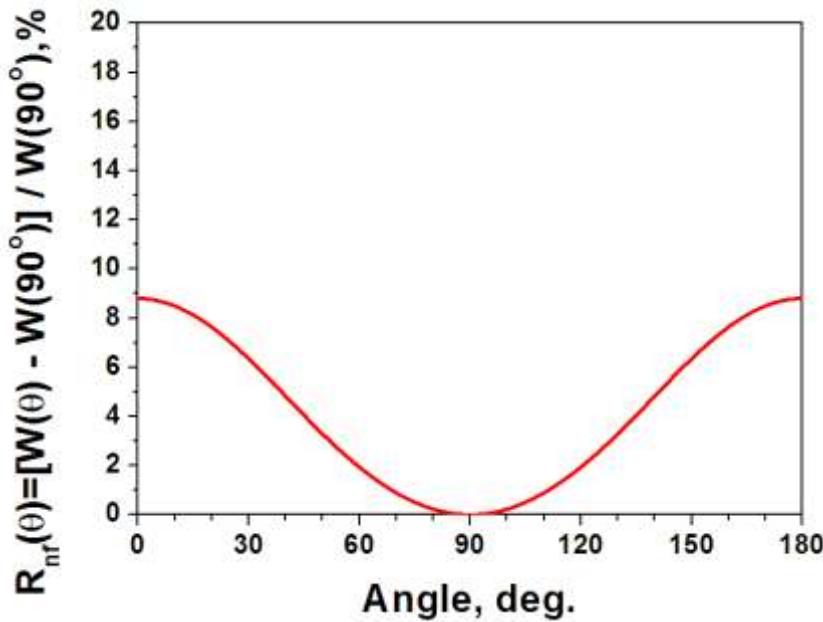
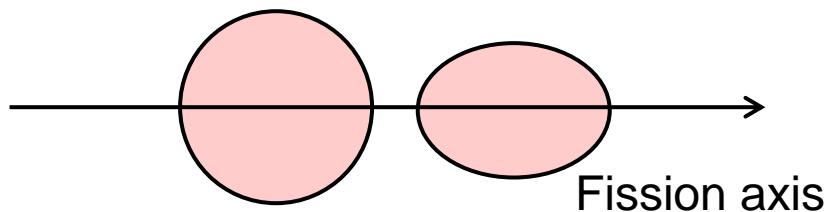
The best agreement of calculated curves with experimental data for neutron-fragment correlations obtained by A. Vorobiev gives the value  $A_2=0.04$ . It means that for averaged neutron energy 1.36 MeV in the fragment system anisotropy  $A_{nf}$  is equal:

$$A_{nf}(E_{cm}) = \frac{3b}{2-b} = \frac{3 \cdot A_2 \cdot E_n}{2 - A_2 \cdot E_n} = 0.084$$

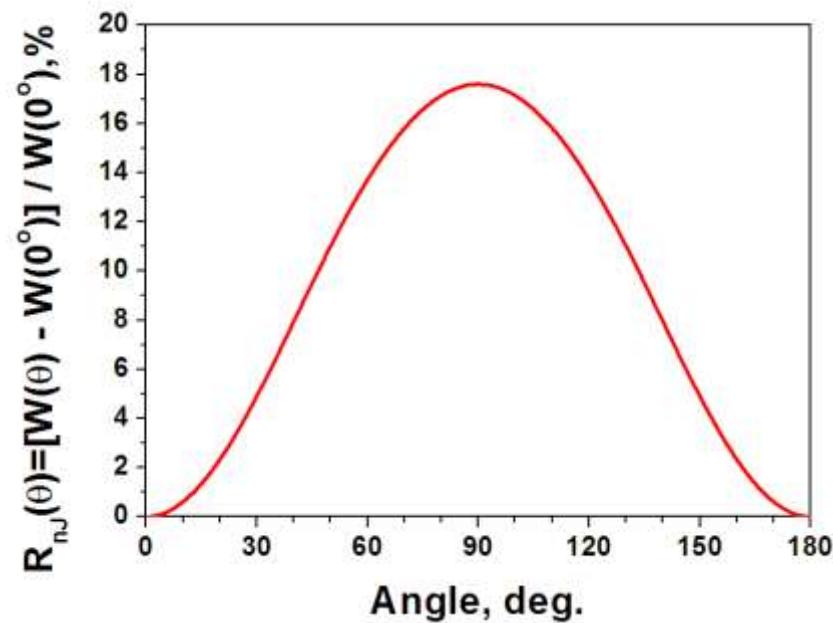
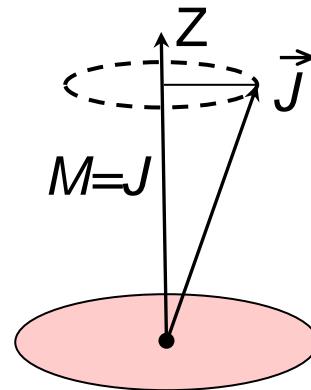
what is very close to the anisotropy calculated with respect to the fission axis on the base of a large initial fragment spin.

# The relation between two types of anisotropy

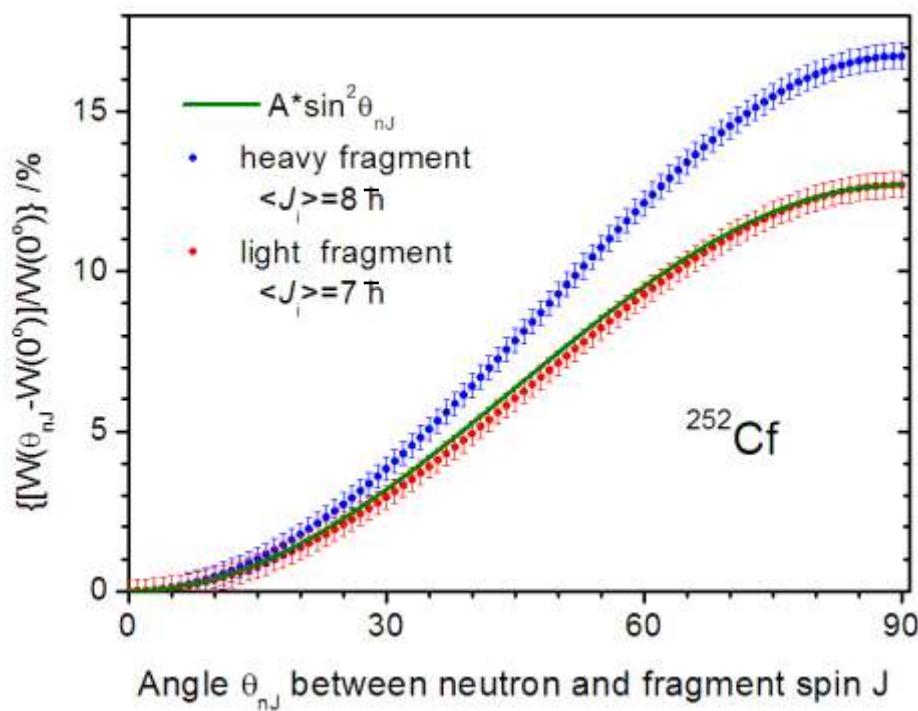
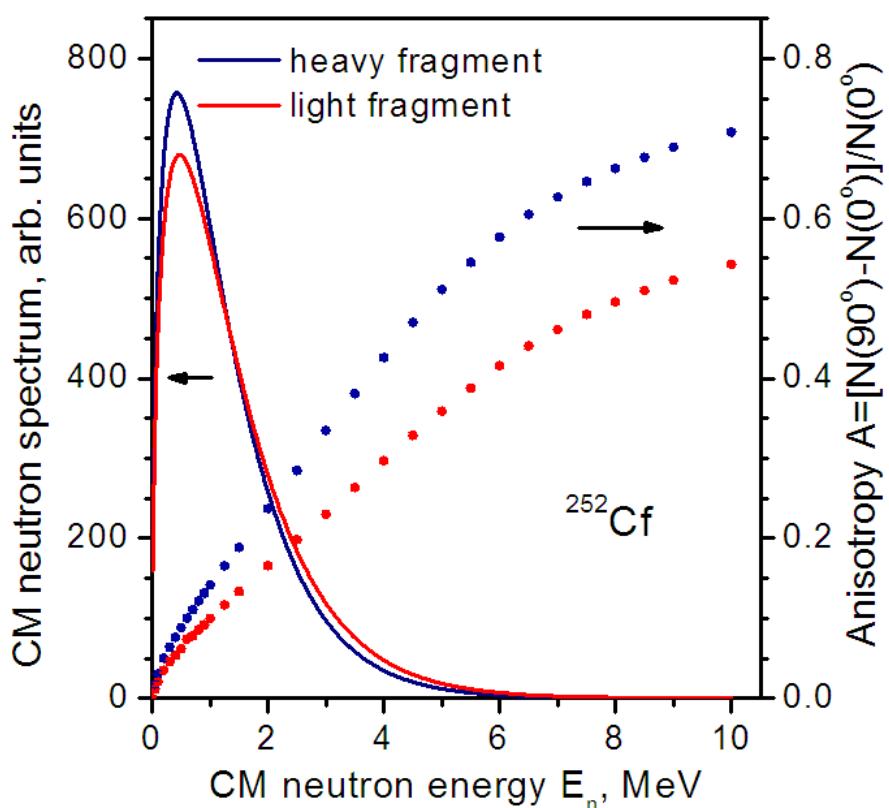
Anisotropy with respect to fission axis



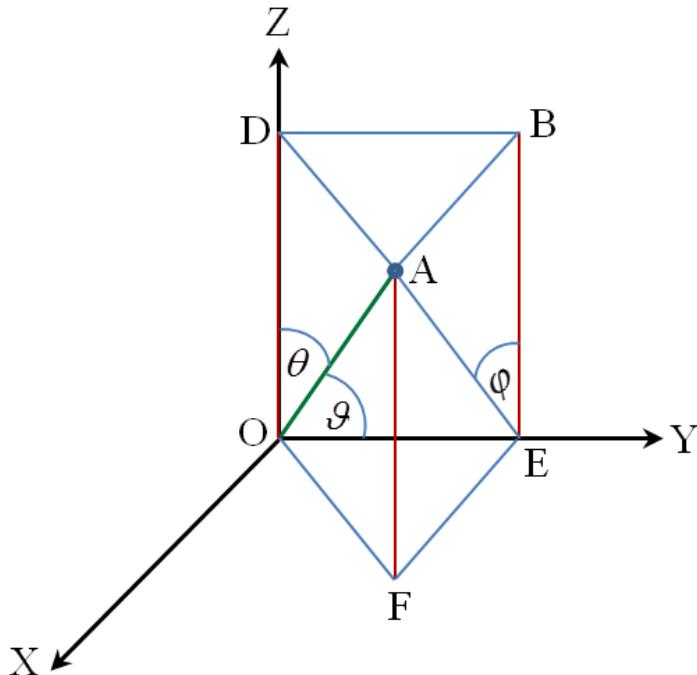
Anisotropy versus Z axis



# The energy and angular dependence of neutron emission anisotropy $A_{nJ}$ for light and heavy fission fragments (in CM system) relative to fragment spin orientation



# The relation between two types of anisotropy



$\angle AOD = \theta$  is the polar angle of neutron evaporation relative to the axis corresponding to  $M = J$ .

$\angle AOE = \vartheta$  is the polar angle of neutron evaporation relative to the fission axis.

$\angle AOB = \varphi$  is the azimuthal angle of neutron evaporation relative to the fission axis.

$$\cos \theta = \sin \vartheta \cdot \cos \varphi$$

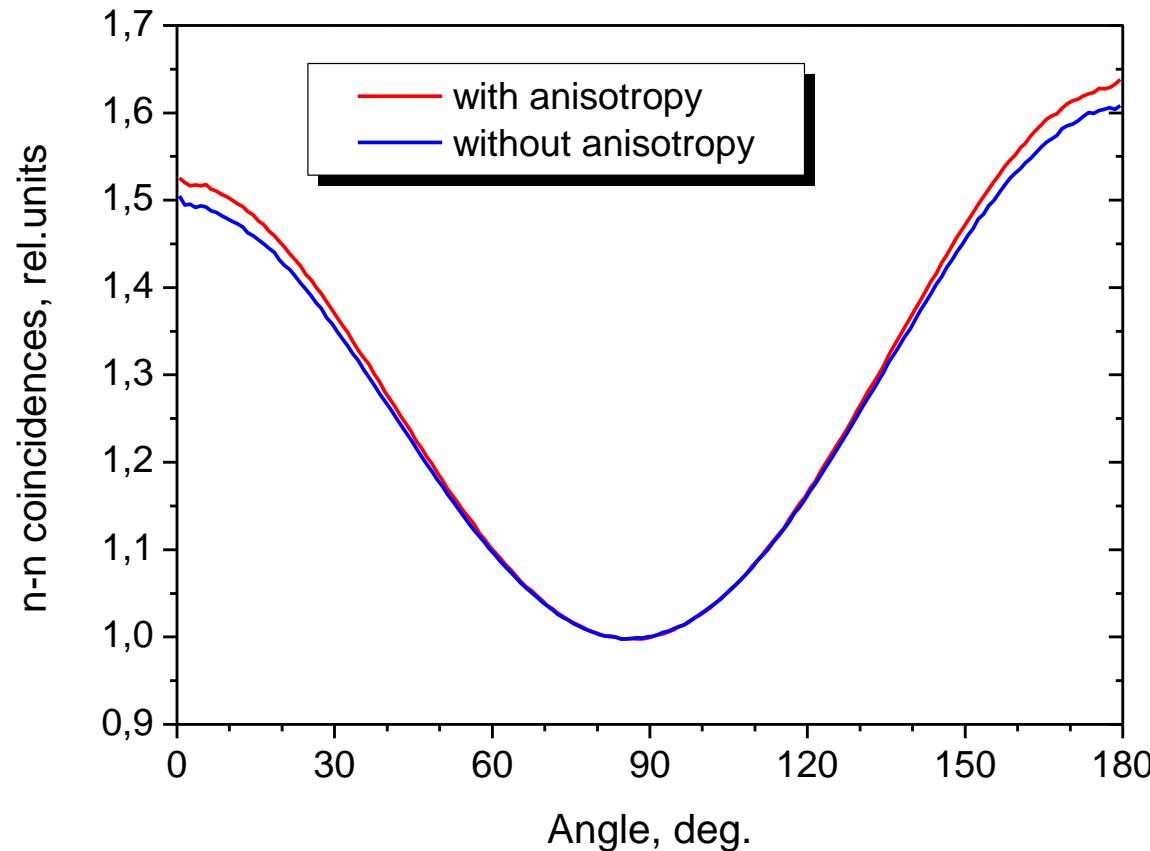
The angular distribution of emitted neutrons relative to the fission axis can be obtained due to the averaging of angular distributions with respect to the axis with  $M=J$  over all its positions at the plane perpendicular to the fission axis.

$$\frac{1}{b-a} \int_{a=0}^{b=2\pi} \cos^2 \varphi \cdot d\varphi = \frac{1}{2\pi} \left( \left( \frac{\varphi}{2} + \frac{\sin \varphi \cdot \cos \varphi}{2} \right) \Big|_0^\pi \right) = \frac{1}{2}$$

$$\cos^2 \theta = \frac{1}{2} \sin^2 \vartheta$$

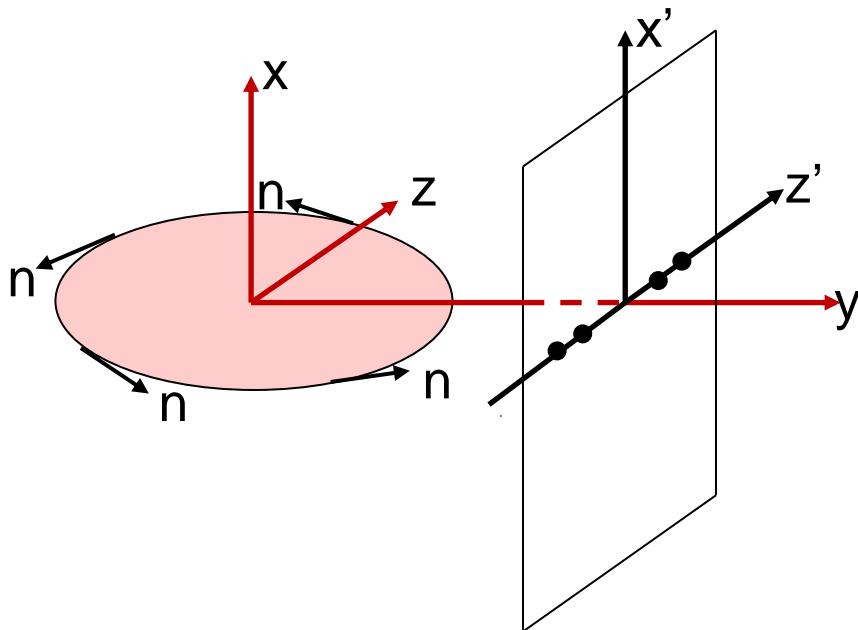
# The calculated curves for n-n coincidences

The inclusion into analysis of the neutron-neutron correlations of such kind anisotropy slightly changes the shape of the curve. This curve becomes steeper, forcing to use additional (1÷2)% of scission neutron component for the experimental data description (it means, isotropic component in laboratory system).

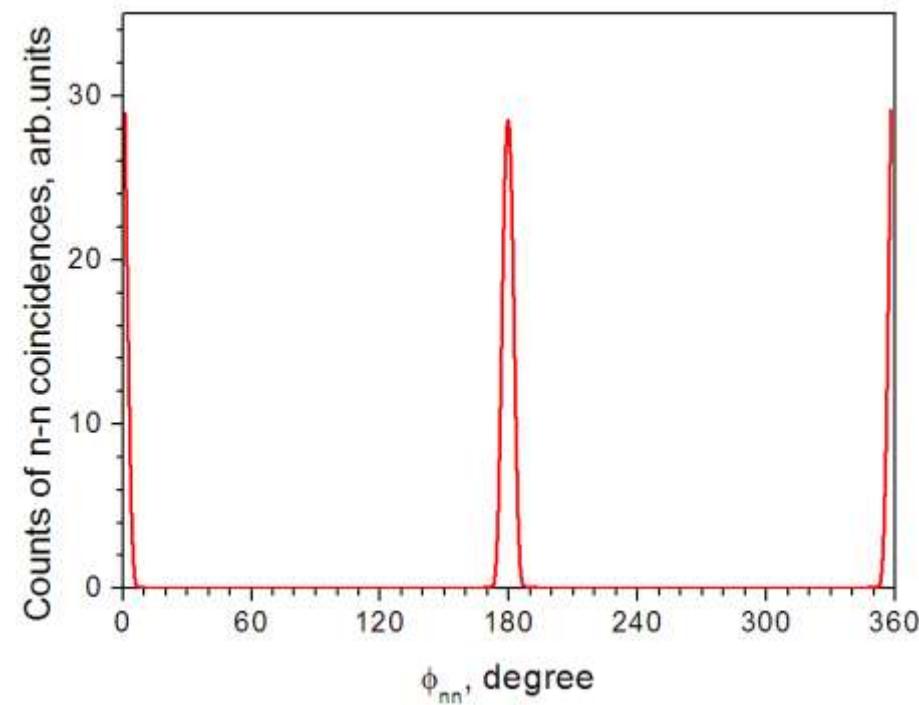


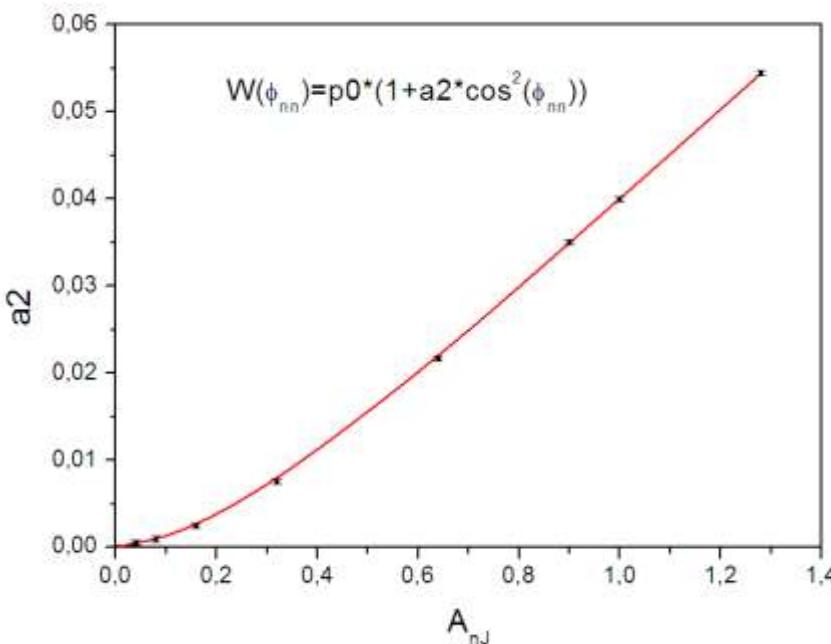
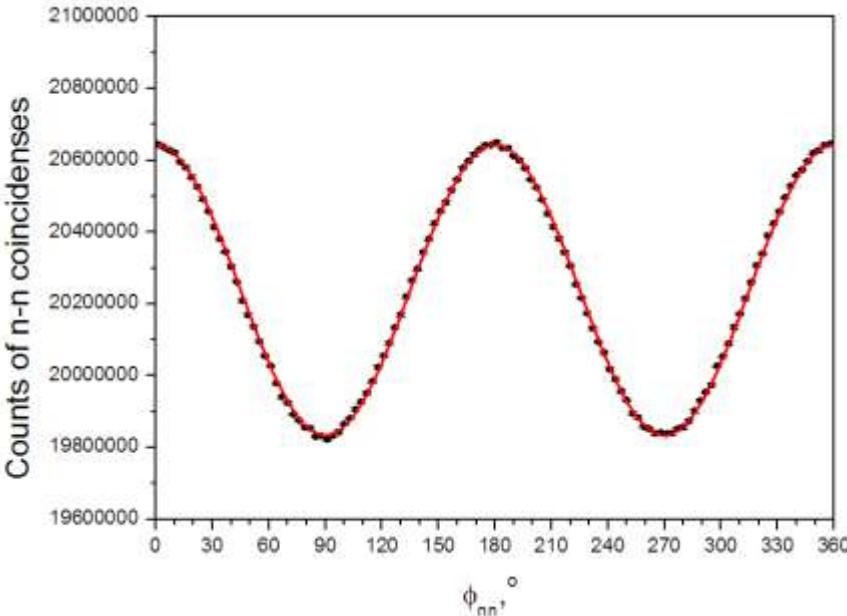
The CORA experiment was proposed by F. Goennenwein and undertaken to demonstrate the existence of the CMs anisotropy. Here was applied new approach, which is more sensitive to this effect.

Sketch of CORA experiment



The result of calculations for CORA experiment  
(naive version)





This figure shows the counts of n-n coincidences in the plane perpendicular to the fission axis.

The simulation was performed with  $A_{nJ}=1$ . In this case CORA parameter  $a2 \approx 0.04$ . But the estimation gives the average value of anisotropy  $A_{nJ} \approx 0.16$ . This leads to smaller  $a2$  and makes the measurement more complicated.

The dependence of the parameter  $a2$  versus the average anisotropy  $A_{nJ}$  of neutron emission in the CM system of fragment.



Thank you for your attention!