Neutron diffraction by a moving grating: theory and experiment

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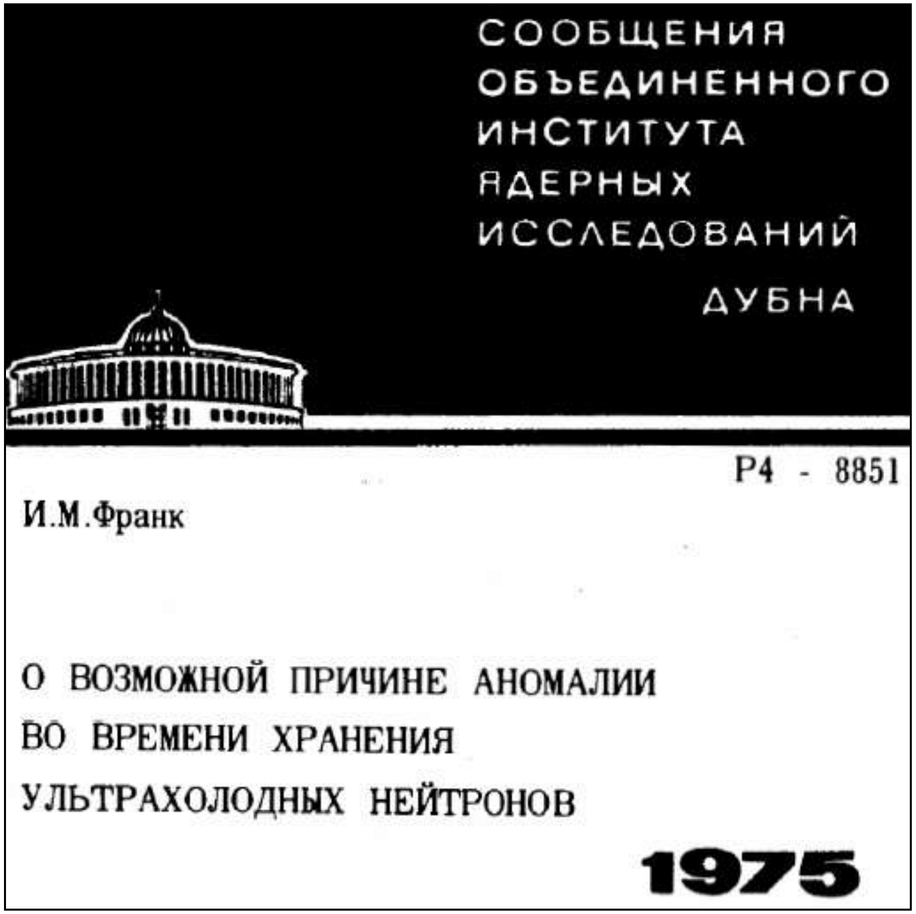
Outline

- *Prehistory*
- *Moving grating as a non stationary phenomenon. (kinematic theory)*
- *First observation of the energy splitting*
- *Application: focusing in time*
- *Application: gravity experiments*
- *Found problems*
- *Progress in theory I – improved kinematic approach*
- *Progress in theory II– dynamical theory of diffraction*
- *New experimental approach to the measurement of diffraction spectra*
- *Conclusion*



СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

Р4 - 8851

И.М.Франк

О ВОЗМОЖНОЙ ПРИЧИНЕ АНОМАЛИИ
ВО ВРЕМЕНИ ХРАНЕНИЯ
УЛЬТРАХОЛОДНЫХ НЕЙТРОНОВ

1975

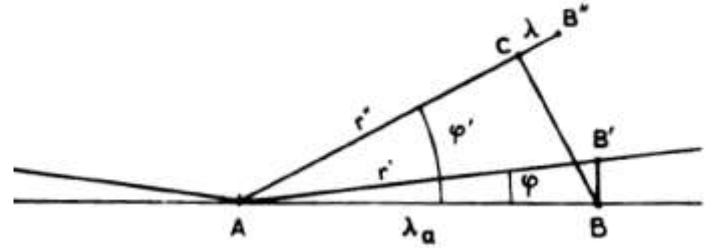


Рис. 2

UCN diffraction by the Raleigh waves results in changing of the normal components of the k-vector

I.M. Frank. On the possible issue of the anomaly in UCN storage

VOLUME 58, NUMBER 26 PHYSICAL REVIEW LETTERS 29 JUNE 1987

Neutron Diffraction by Surface Acoustic Waves

W. A. Hamilton, A. G. Klein, and G. I. Opat
School of Physics, University of Melbourne, Parkville, Victoria 3052, Australia

and

P. A. Timmins
Institut Laue-Langevin, 38042 Grenoble Cedex, France
(Received 3 March 1987)

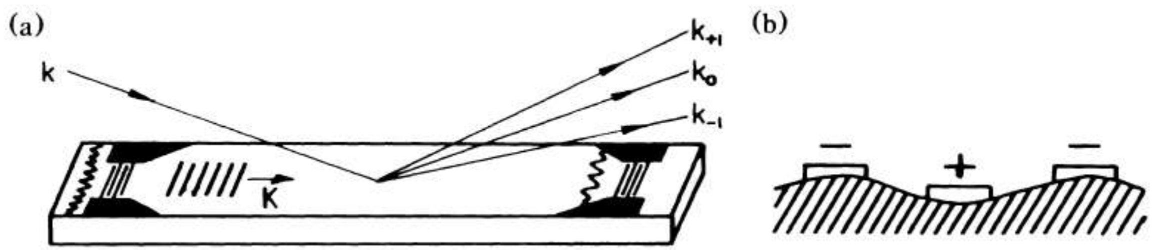
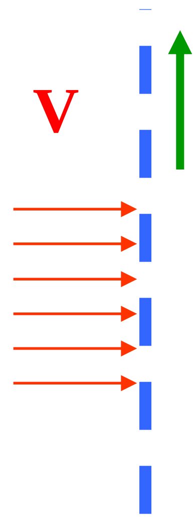


FIG. 1. (a) Schematic of neutron "grating" diffraction by surface acoustic waves. (SAW propagation sense $s = +1$.) (b) Instantaneous profile of the interdigitated electrode structure showing the applied voltage and the deformation of the piezoelectric surface.

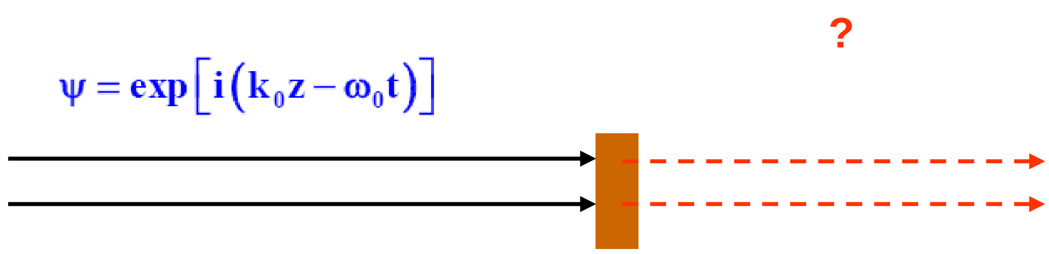
Matter wave diffraction by a moving grating. Formulation of the problem

$$\psi = \exp\left[i\left(k_0 z - \omega_0 t\right)\right]$$

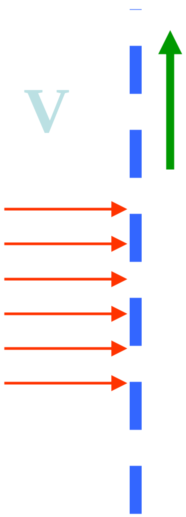


V.G. Nosov and A.I.Frank , 1991

$$\psi = \exp\left[i\left(k_0 z - \omega_0 t\right)\right]$$



Neutron diffraction by a moving grating.
Formulation of the problem



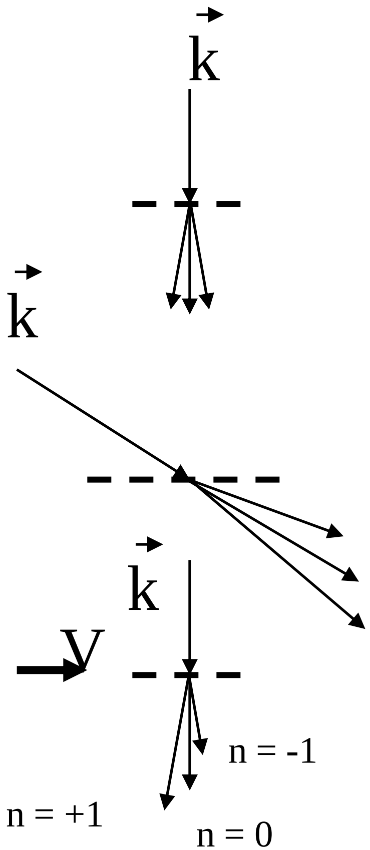
$$\psi = \exp\left[i\left(k_0 z - \omega_0 t\right)\right]$$

$$\psi = \frac{1}{2}\exp\left[i\left(kx - \omega t\right)\right] + \frac{1}{\pi}\sum_{s=-\infty}^{\infty}\frac{1}{2s-1}\exp\left[i\left(k_s x - \omega_s t\right)\right]$$

$$\omega_s = \omega + \frac{\pi(2s-1)}{T} \qquad k_s = \left(\frac{2m\omega_s}{\hbar}\right)^{1/2}$$

Elementary (kinematic) theory.

A.Frank, V.Nosov, 1994



1. **Solving the diffraction problem in a moving system.**

2. **Galilean transformation of the wave function.**

$$\Psi(z,y,t)=\sum_j a_j \exp[i(k_j z+q_j y-\omega_j t)] \qquad (k_0 L \ll 1)$$

$$\boxed{a_j=\frac{1}{d}\int_0^L T(x)\exp(-iq_j x)dx} \qquad q_j=j\cdot\left(2\pi/d\right)=jq_0$$

$$\omega_j=\omega_0+j\Omega \qquad k_j\cong k_0\left(1+j\frac{\Omega}{\omega_0}\right)^{\frac{1}{2}} \qquad j=0,\ \pm1,\ \pm2....$$

$$\Omega=\frac{2\pi}{T}=2\pi f=2\pi\left(\frac{V}{d}\right)$$

d – space period of a grating

Main peculiarity of the kinematic approach

1. Neglecting by the grating form (1D grating)

2. Amplitudes of the diffraction orders don't depend on the grating velocity

For the phase π - modulation

$$T(x) = \begin{cases} 1 & \text{if} \quad 0 < y < d/2 \\ \exp(i\pi) & \text{if} \quad d/2 < y < d \end{cases}$$

$$a_j = 0, \quad j = 2s$$

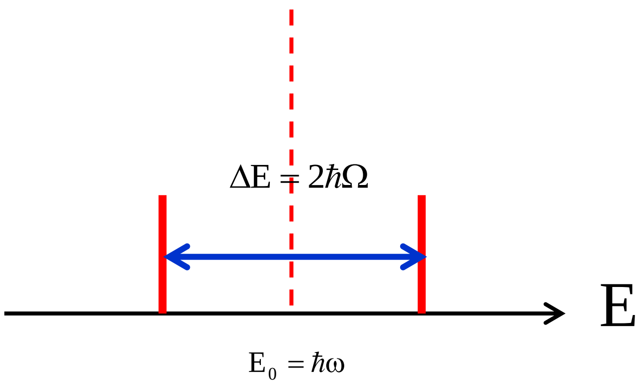
For zero and even orders

$$a_j = \frac{2}{i\pi j}, \quad j = 2s - 1$$

For odd orders

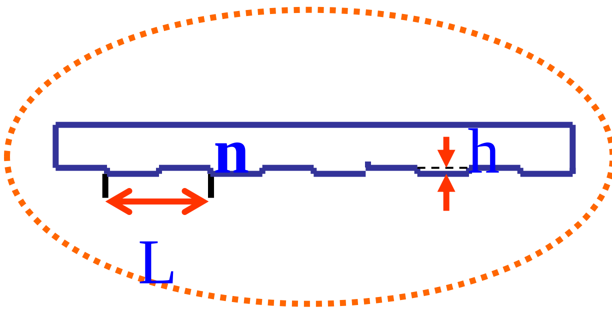
$$|a_{\pm 1}|^2 = \frac{4}{\pi^2} = 0.405$$

$$\Omega = 2\pi\left(\frac{V}{d}\right)$$

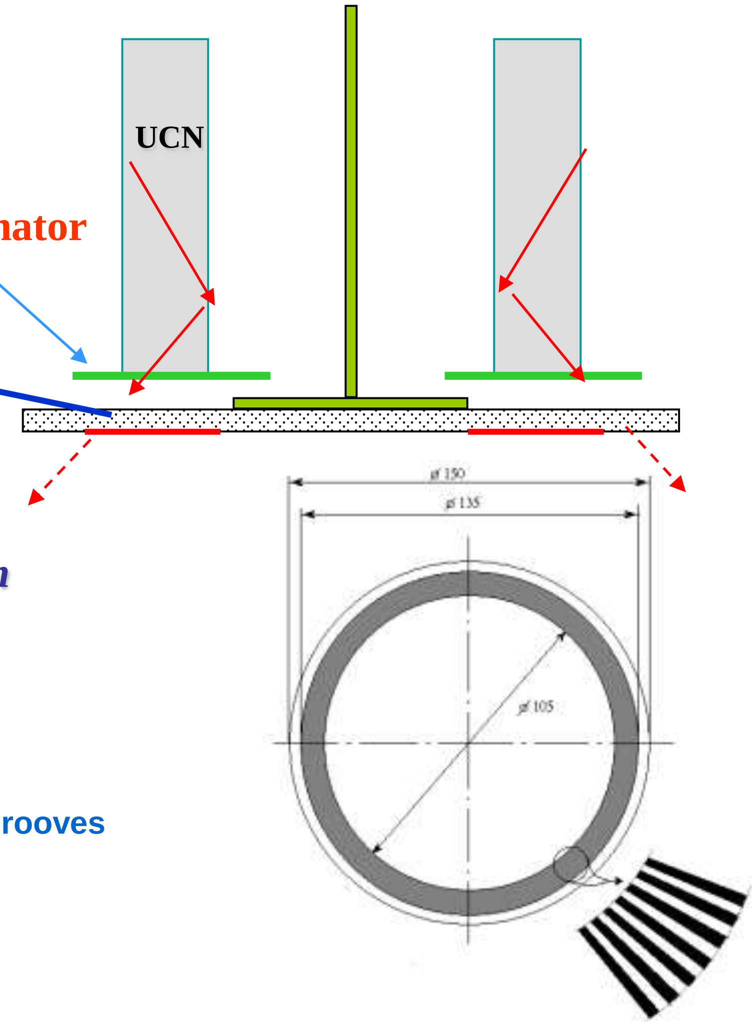


Experimental realization - rotating grating

Phase π -grating

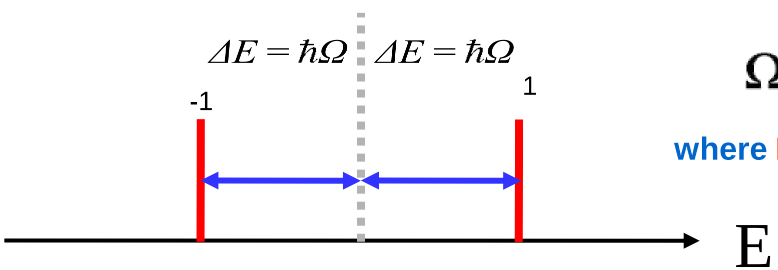


Monochromator



$$\boxed{\Delta\varphi = k(n-1)h = \pi}$$

h = 0.14 mkm

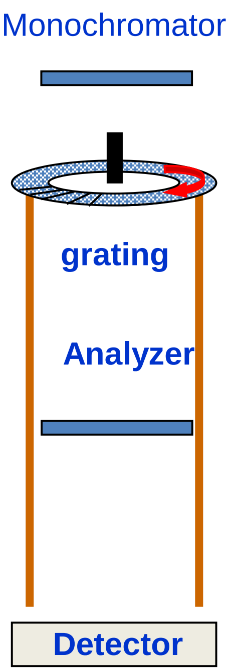


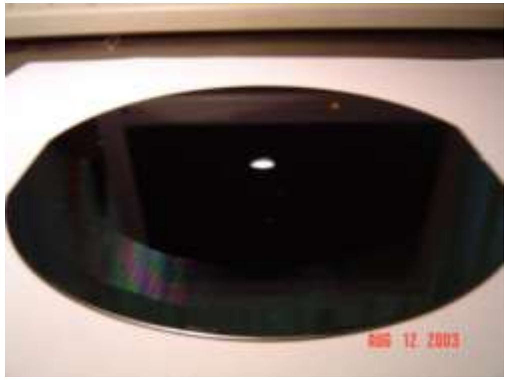
$$\Omega = 2\pi f N$$

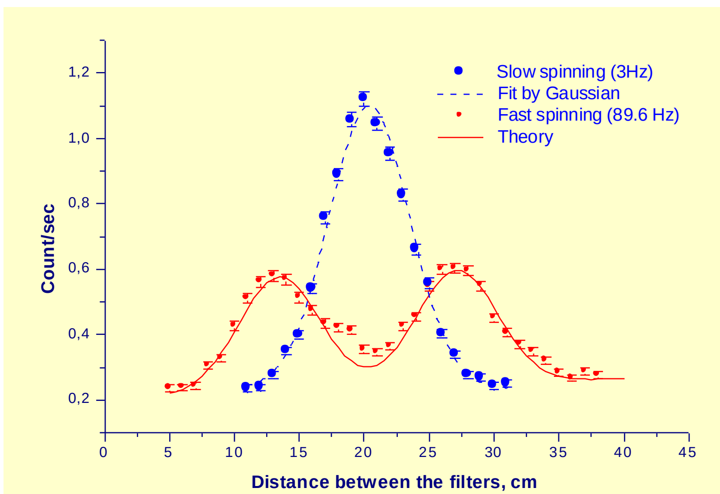
where N is number of grooves

First experimental results

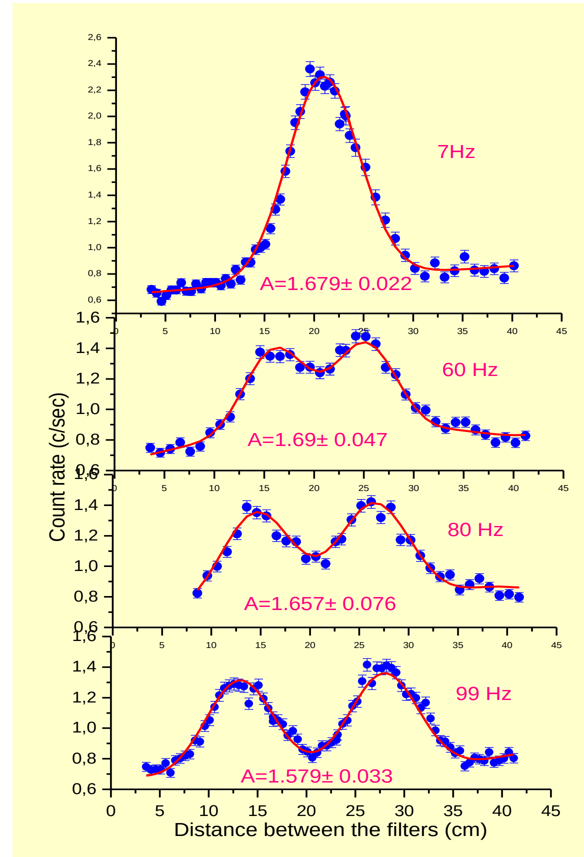
Angular period of grating 0.3325mrad (20μ at the middle diameter)







Splitting of the spectrum



$$\left|\mathbf{a}_1\right|^2_{\mathbf{th}} = 0.405$$

$$\left|\mathbf{a}_1\right|^2_{\mathbf{exp}} = 0.383(8)$$

A.I.Frank et al. ILL annual report 2001
Phys.Lett.A 311 (2003) 6

A.I.Frank et al.Jetp Lett, 81 (2005) 427

Pulse source and UCN pumping in a trap

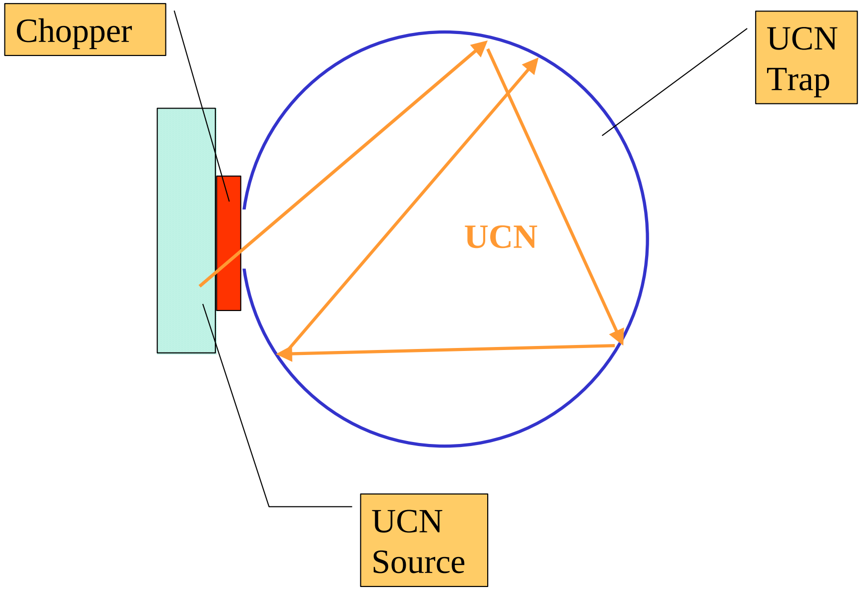
1915-1973

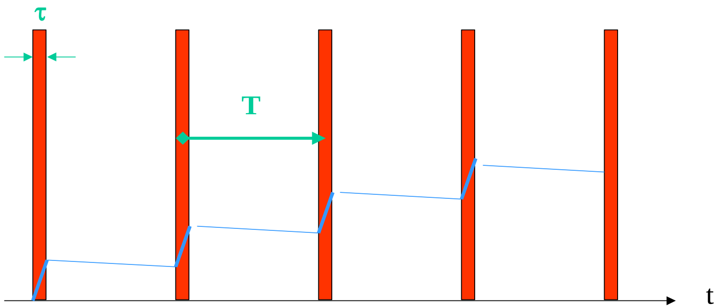
$$G = 1 + \frac{1 - \frac{\tau_1}{T}}{\frac{\tau_1}{T} + \frac{\Sigma\mu}{S}}$$

$G \to 10^2 \div 10^3$

$\tau_1 > \tau$ - chopper opening time
S – active convertor area
Σ -- area of the trap
μ- probability of the UCN lost

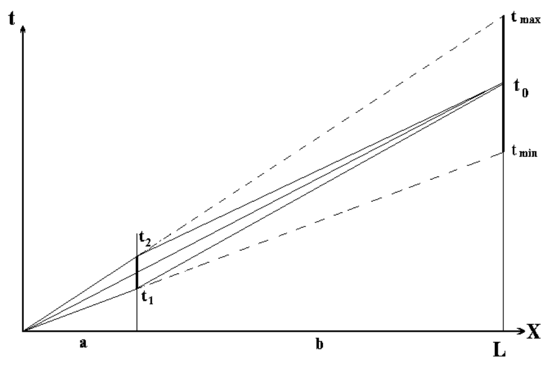
F.L. Shapiro, 1972

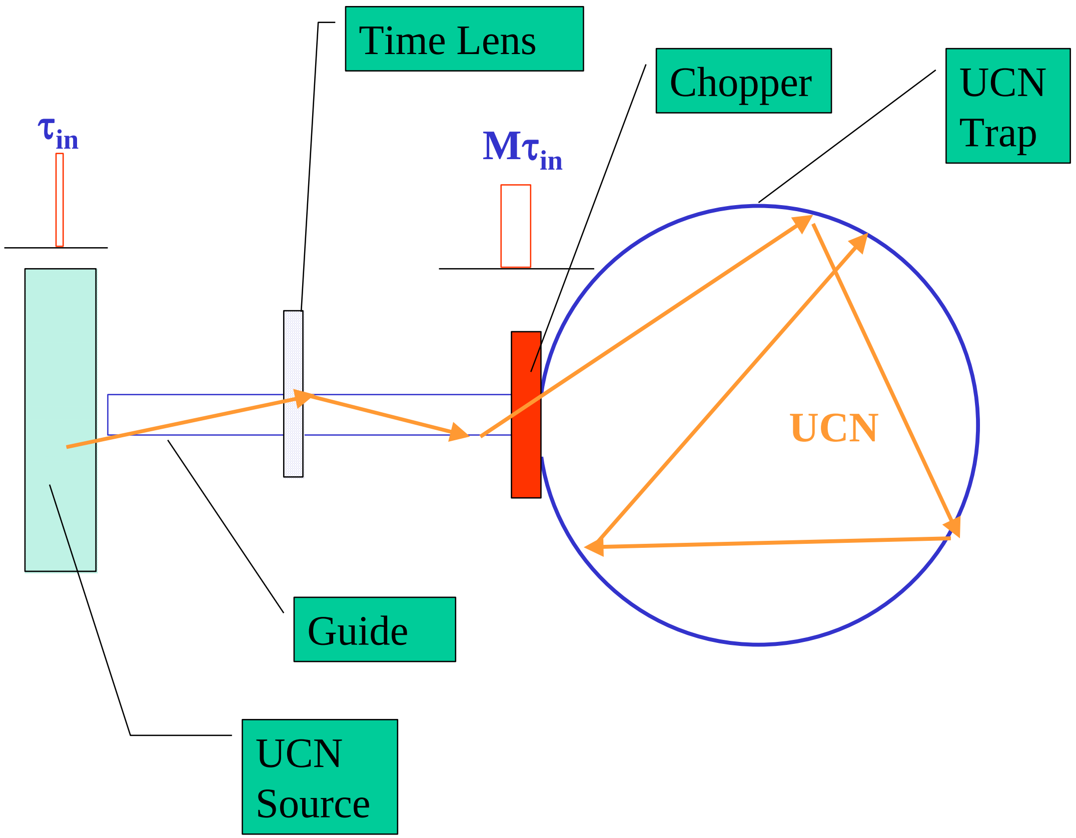




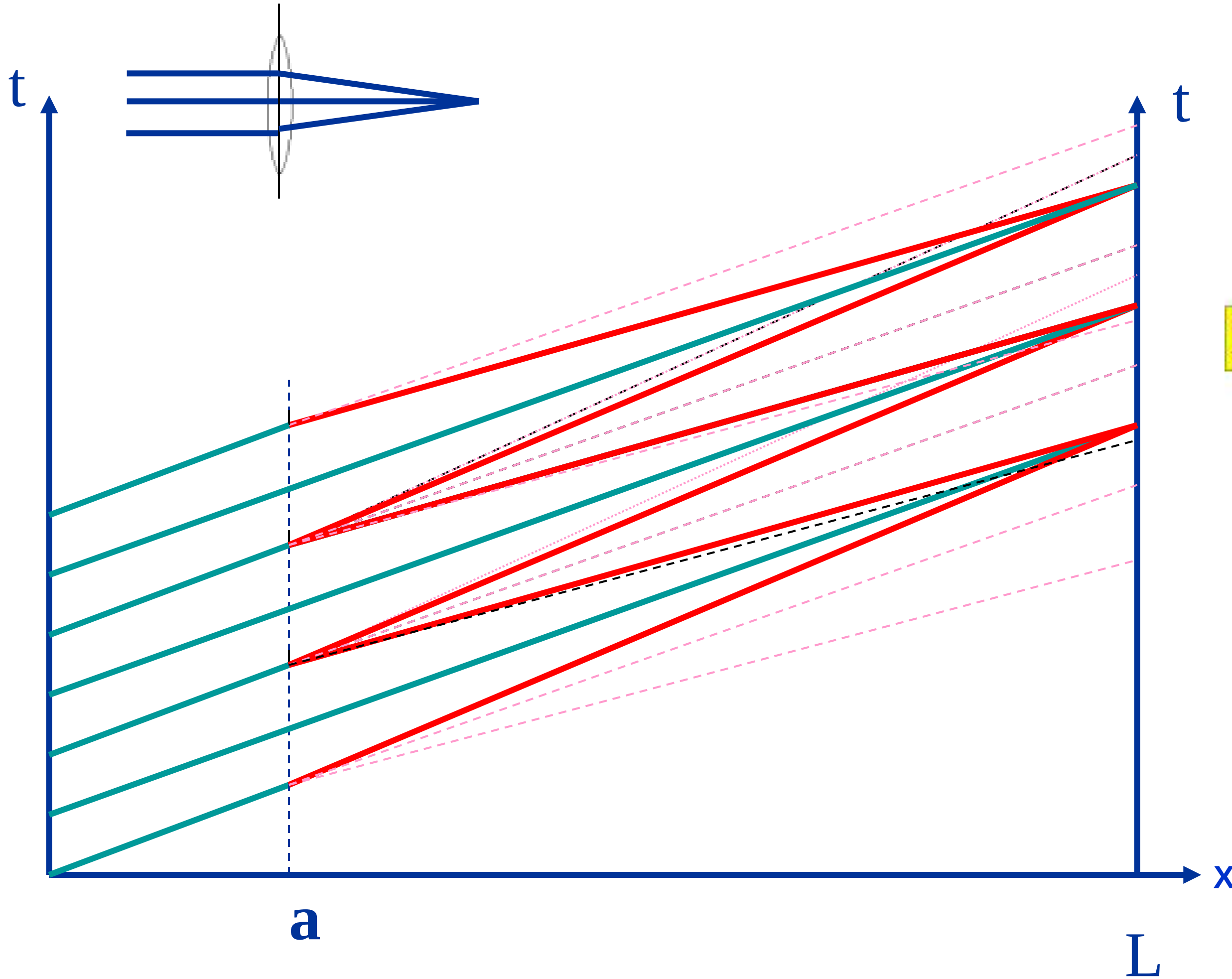
Pumping option of the pulsed source – time lens

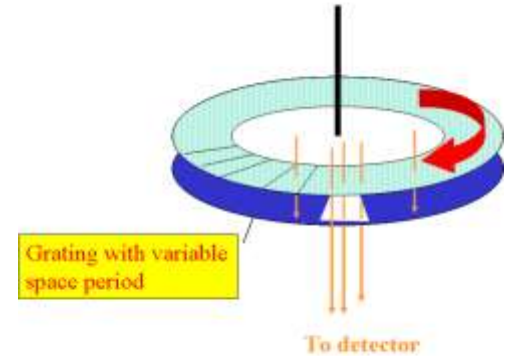
A.I.Frank and R.Gähler. Phys. of Atomic Nuclei, 63, (2000) 545

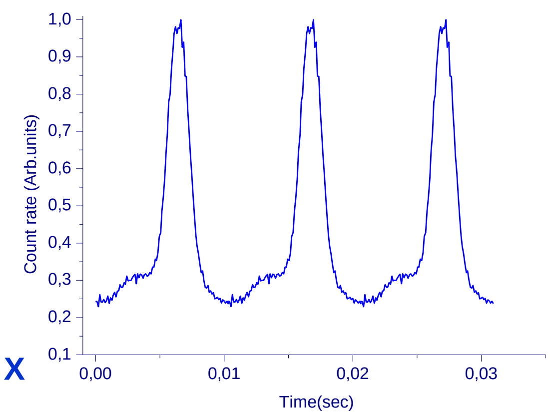




Neutron focusing in time

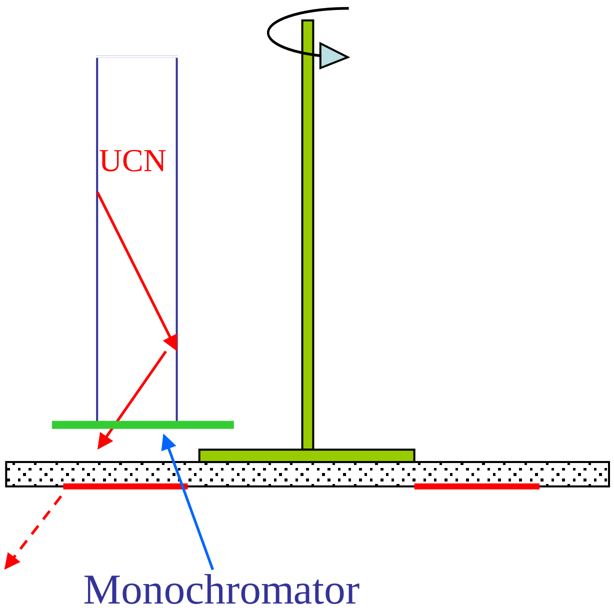


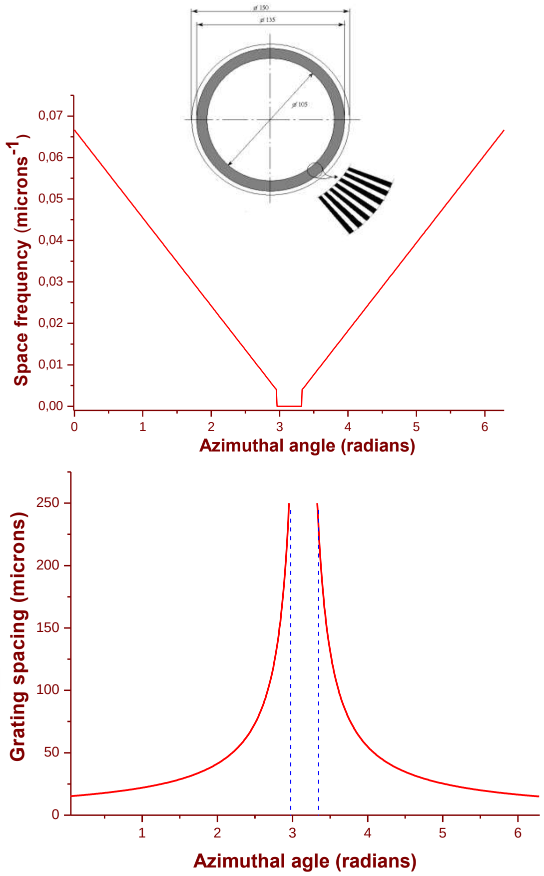


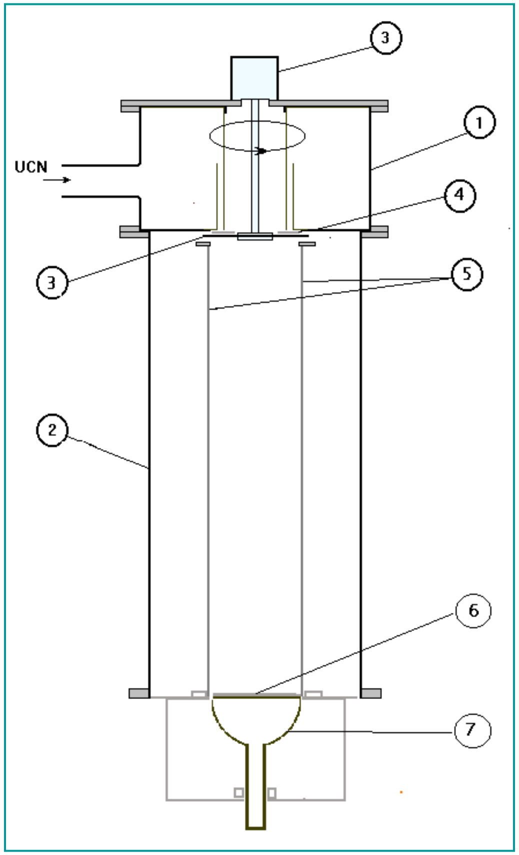


A.I.Frank and R.Gähler. Phys. of Atomic Nuclei, 63, (2000) 545

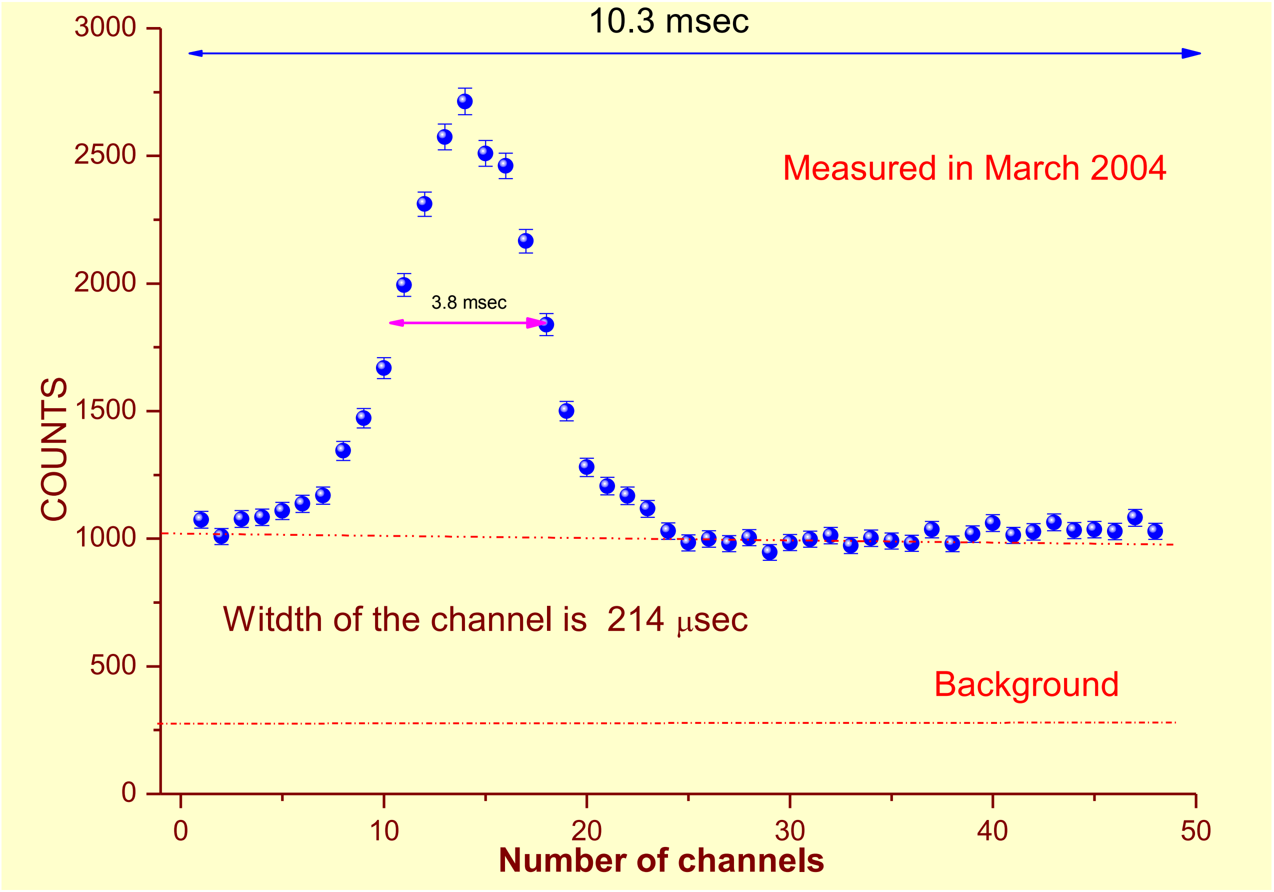
Rotating grating as a time lens





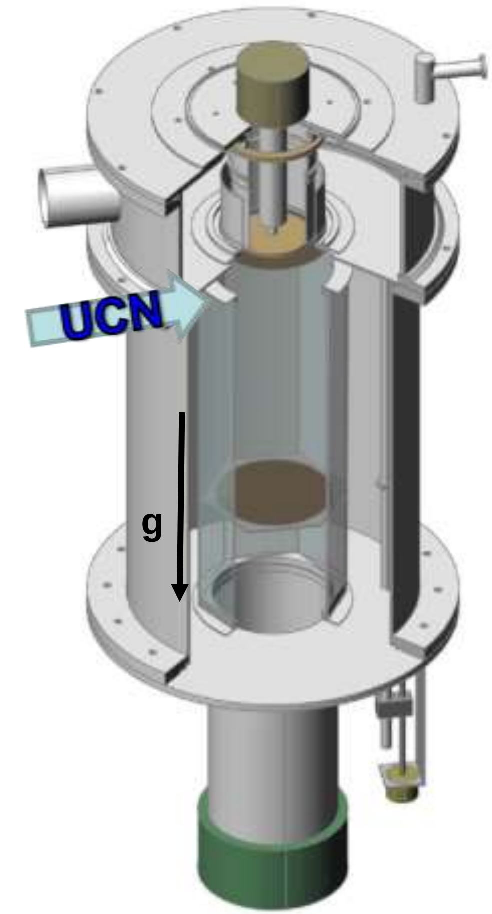


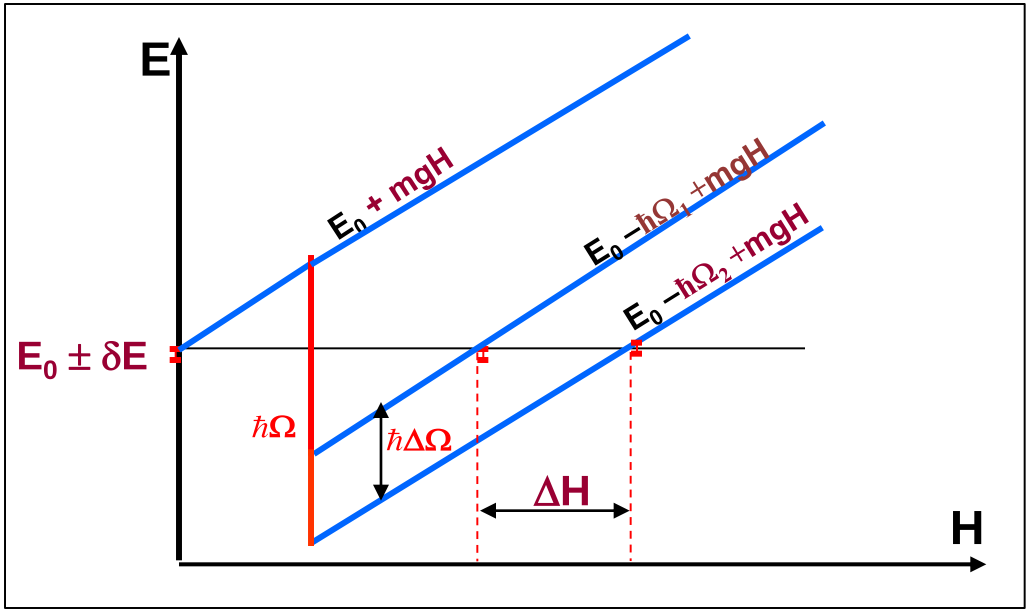
Time lens is working!



A. I. Frank, P. Geltenbort, G. V. Kulin et al. JETP Lett. 78, (2003) 188
S.N. Balashov, I.V. Bondarenko, A.I. Frank et al, Physica B, 350 (2004) 246

Test of the weak equivalence principle for neutrons (2006)





The idea was to compare the change of energy *mgH* with energy *ħΩ* transferred to neutron by a moving grating

Frank A.I., Masalovich S.V., Nosov V.G. (ISINN-12). E3-2004-169, 215, Dubna, (2004)

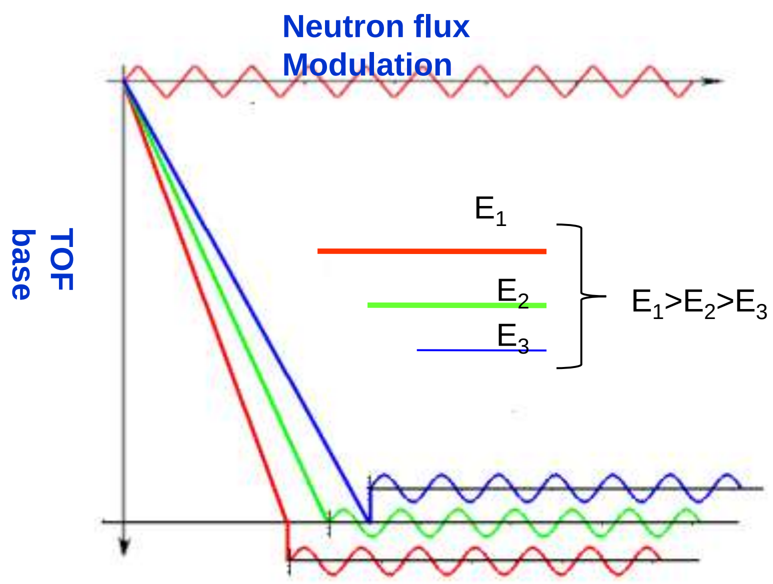
$$\boxed{m_g g_n = \hbar \frac{\Delta\Omega}{\Delta H}}$$

$$\boxed{1 - \frac{m_g g_n}{m_n g_{loc}} = (1.8 \pm 2.1) \cdot 10^{-3}}$$

A.I. Frank, P. Geltenbort, M. Jentschel, et al. JETP Letters, 86, 225 (2007)

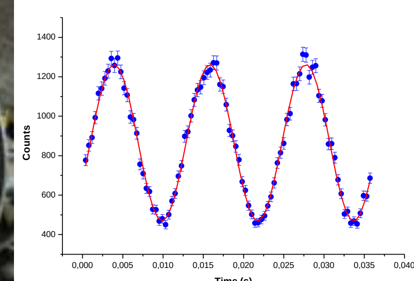
New experiment for the test of weak equivalence principle for neutron (started in 2010)

Comparing the energy $m_g g_n H$ with energy $\hbar\Omega$ as before.



Count rate oscillations

$$\varphi = 2\pi f t$$

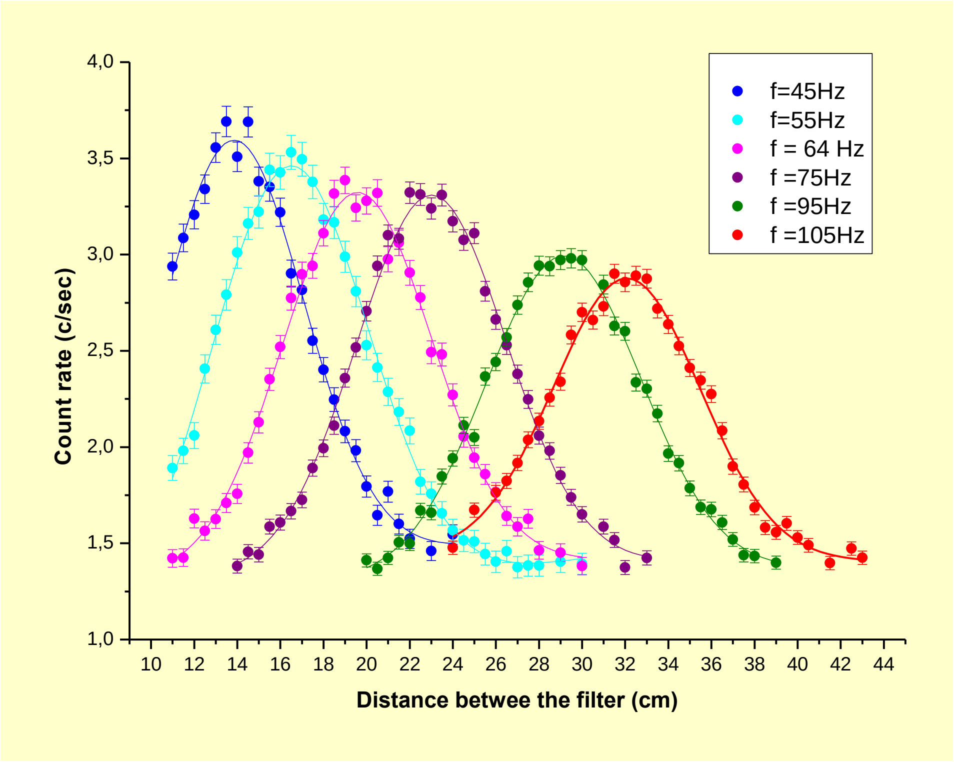


Combination of Neutron Interference Filters with peculiar TOF spectrometry

G.V. Kulin et al. NIM A 792 (2015) 38-46

Contradictions with kinematic theory (2006-2012)

1. **Intensity of the first order DEPENDS on the velocity of the grating**
2. **The first order line is remarkably wider than initial spectrum**
3. **The presence of the zero diffraction order was detected**

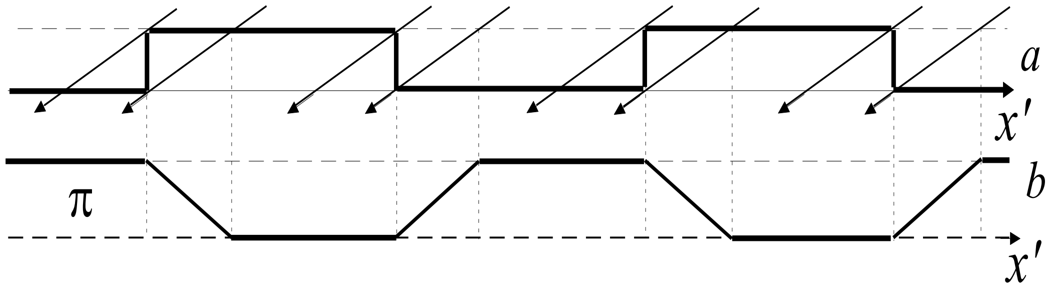


Angular period of grating 0.0831mrad (5μ at the middle diameter)

Scanning curves of the -1 diffraction order measured at different frequencies of the grating rotation

Next step in a theory was made in 2004
It was not taken into account at planning the experiment of 2010-12

A.I. Frank et al. JINR Communication P3-2004-207



$$a_j = \frac{1}{d}\int_0^L T(x)\exp(-iq_j x)dx$$

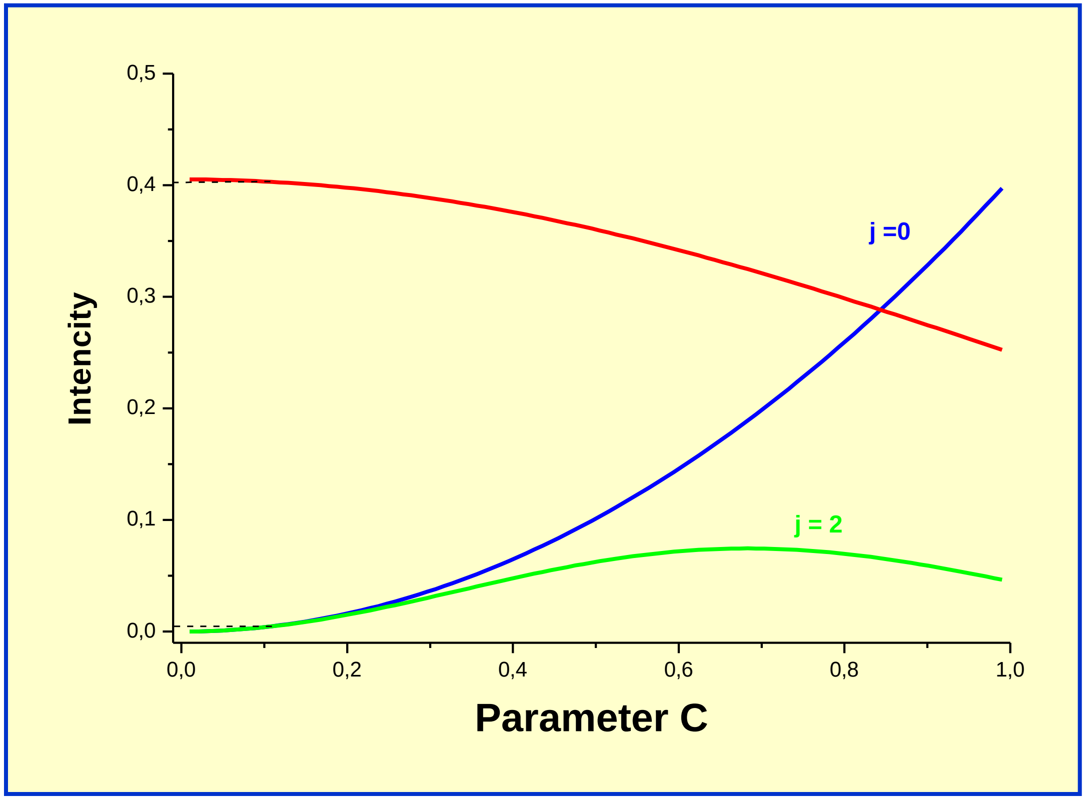
$$T(x) = \begin{cases} e^{i\frac{\pi}{cd}x} & \text{if } 0 < x < cd \\ e^{i\pi} & \text{if } cd < x < d \\ -e^{i\frac{\pi}{c}\left(1-\frac{x}{d}\right)} & \text{if } d < x < d+cd \\ 1 & \text{if } d+cd < x < 2d \end{cases}$$

$$a_j = \begin{cases} cB_j & \text{if } \ j = 2s \\ -B/j & \text{if } \ j = 2s-1 \end{cases}$$

$$B_j = \frac{1+\exp(-i\pi jc)}{i\pi(1-j^2c^2)},$$

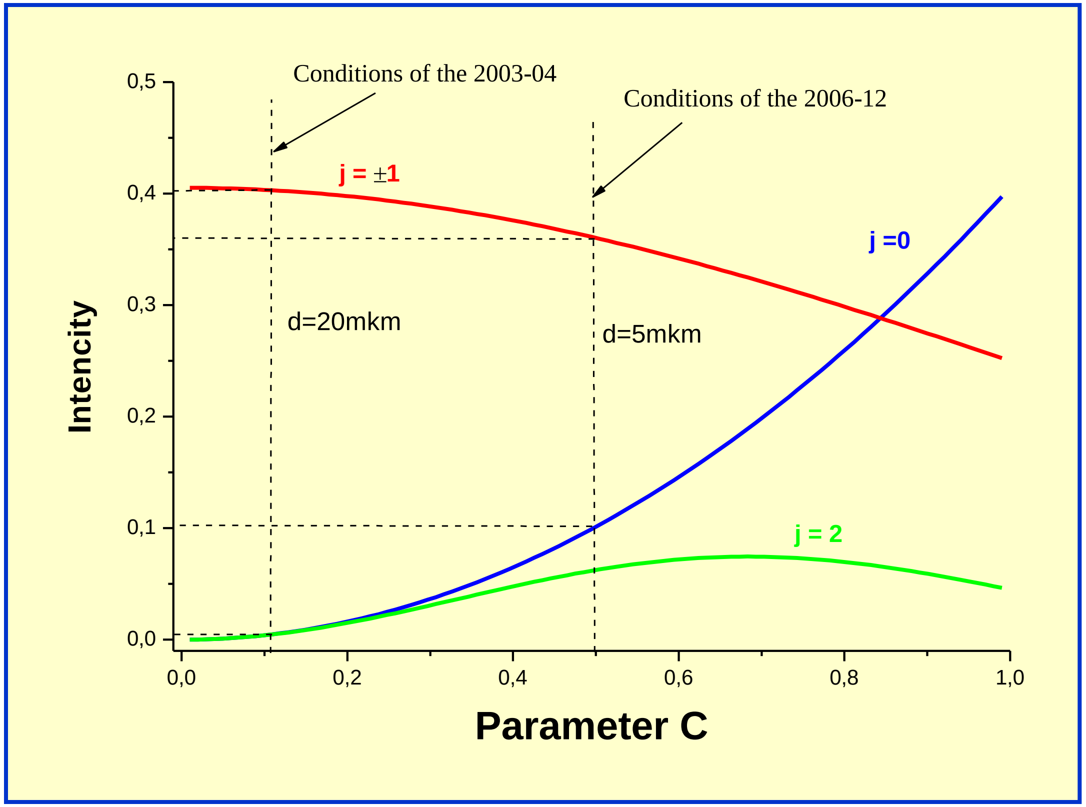
$$c = \frac{2hV}{dV_0}$$

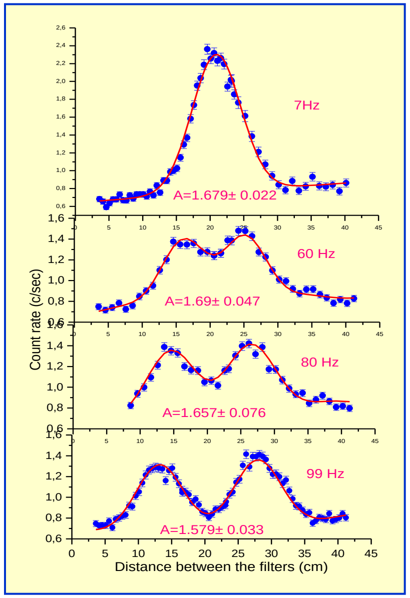
Improved kinematic theory for the moving 3D grating



$$c = \frac{2hV}{dV_0}$$

Improved kinematic theory for the moving 3D grating





$$c = \frac{2hV}{dV_0}$$

1. **Intensities of the diffraction waves depends on the grating velocity**
2. **All waves are independent**
3. **“Naive” combination of geometric and wave optics**

New stage of the investigation of neutron diffraction by a moving grating.

1. **Theory: Multiwave dynamic approach.**

V.A. Bushuev, A.I. Frank and V.G. Kulin. arXiv:1502.04751v1 [physics.optics] (to be published in JETP)

2. Experiment: TOF Fourier spectroscopy of UCN's

Proposed in : V.G. Kulin, D.V. Kustov, A.I. Frank et al. JINR Communication, P3-2014 -48

G.Kulin (next presentation)

Multiwave dynamic theory

Moving reference

$$\boxed{\Delta\psi(\mathbf{r})+[k^2-\chi(\mathbf{r})]\psi(\mathbf{r})=0}$$

$$\chi(\mathbf{r})=4\pi N(\mathbf{r})b(\mathbf{r})$$

$$\boxed{\chi(x)=\sum_{n=-\infty}^{\infty}\chi_n\exp(iq_nx)} \qquad (0\le z\le h) \qquad \chi_n=\tfrac{1}{d}\int_0^d\chi(x)\exp(-iq_nx)dx$$

$$q_n=n\cdot\left(2\pi/d\right)=nq_0$$

$$\boxed{\Psi'(x',z)=\sum_{m=-\infty}^{\infty}\Psi_m(z)\exp[i(g_{mx}x'+g_{0z}z)]}$$

Sum of the Bloch waves

$$g_{mx}=k_{0x}-k_V+q_m, \qquad k_v=\frac{mV}{\hbar}, \qquad g_{0z}=(k_{0z}^2-\chi_0)^{1/2}$$

Multiwave dynamic theory

$$\boxed{\frac{d^2\Psi_m}{dz^2} + 2iq_{0z}\frac{d\Psi_m}{dz} - \alpha_m\Psi_m - \sum_{n\neq 0}\chi_n\Psi_{m-n} = 0}$$

where $\alpha_m = q_m[q_m - 2(k_V - k_{0x})]$

Border conditions $\psi_0(z=0) = A_0, \quad \psi_{m\neq 0}(z=0) = 0$

Multiwave dynamic theory

$$\boxed{\cancel{\frac{d^2\Psi_m}{dz^2}} + 2iq_{0z}\frac{d\Psi_m}{dz} - \alpha_m\Psi_m - \sum_{n\neq 0}\chi_n\Psi_{m-n} = 0}$$

$|R_F|^2 \approx$ 1-2 %

where $\alpha_m = q_m[q_m - 2(k_V - k_{0x})]$

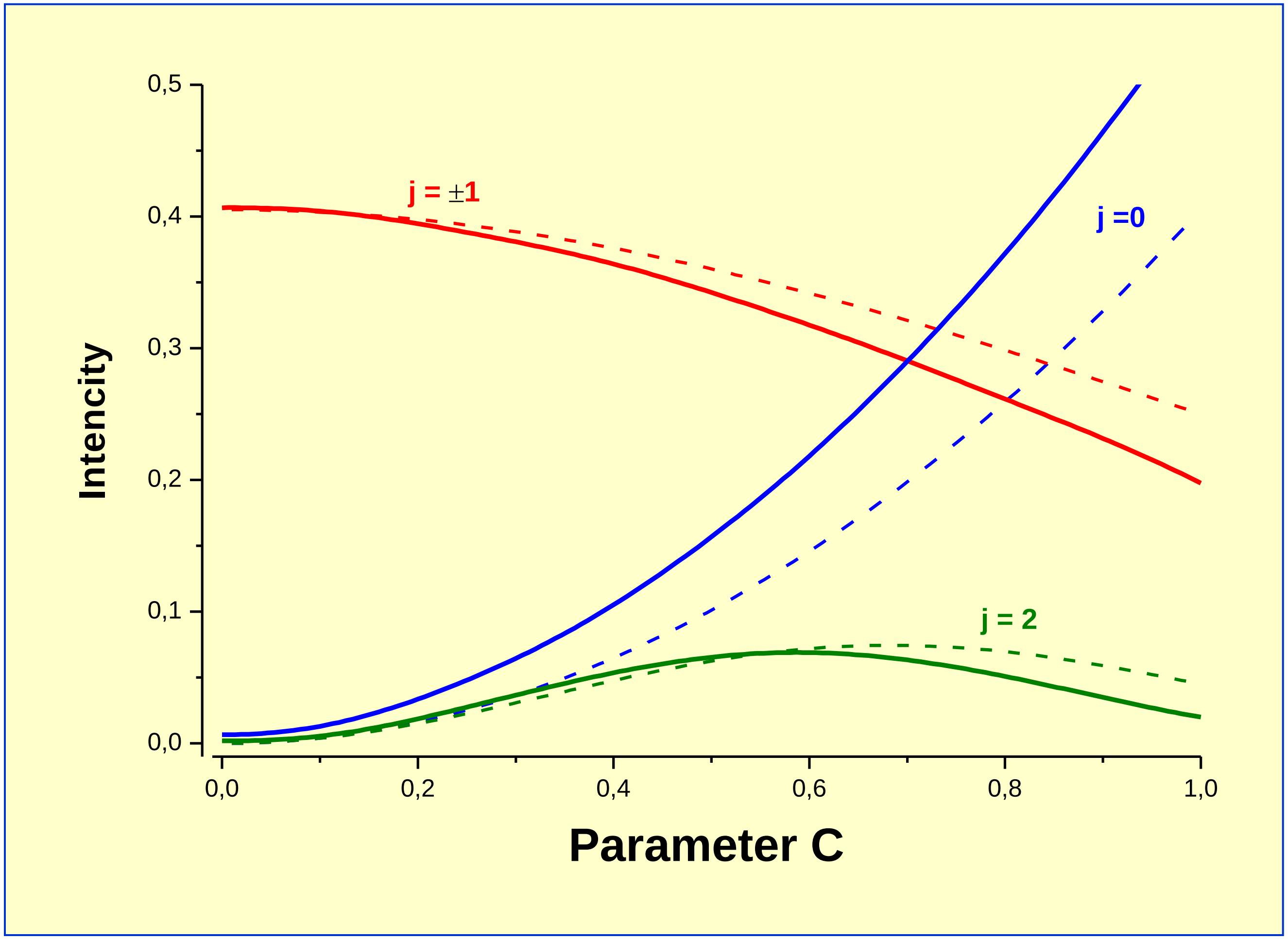
Border conditions $\psi_0(z=0) = A_0, \quad \psi_{m\neq 0}(z=0) = 0$

$$\boxed{\frac{d\Psi_m}{dz} = -i\gamma_m\Psi_m - i\sum_{n\neq 0}\beta_n\Psi_{m-n}}$$

$$q_n = n\cdot\left(2\pi/d\right) = nq_0$$

$$\boxed{\gamma_m = \alpha_m/2g_{0z}, \quad \beta_n = \chi_n/2g_{0z}}$$

$$g_{0z} = (k_{0z}^2 - \chi_0)^{1/2}$$



$$c = \frac{2hV}{dV_0}$$

Solid lines –dynamic theory, dash lines - improved kinematics theory

TOF Fourier mode of the UCN spectrometer (November 2014)

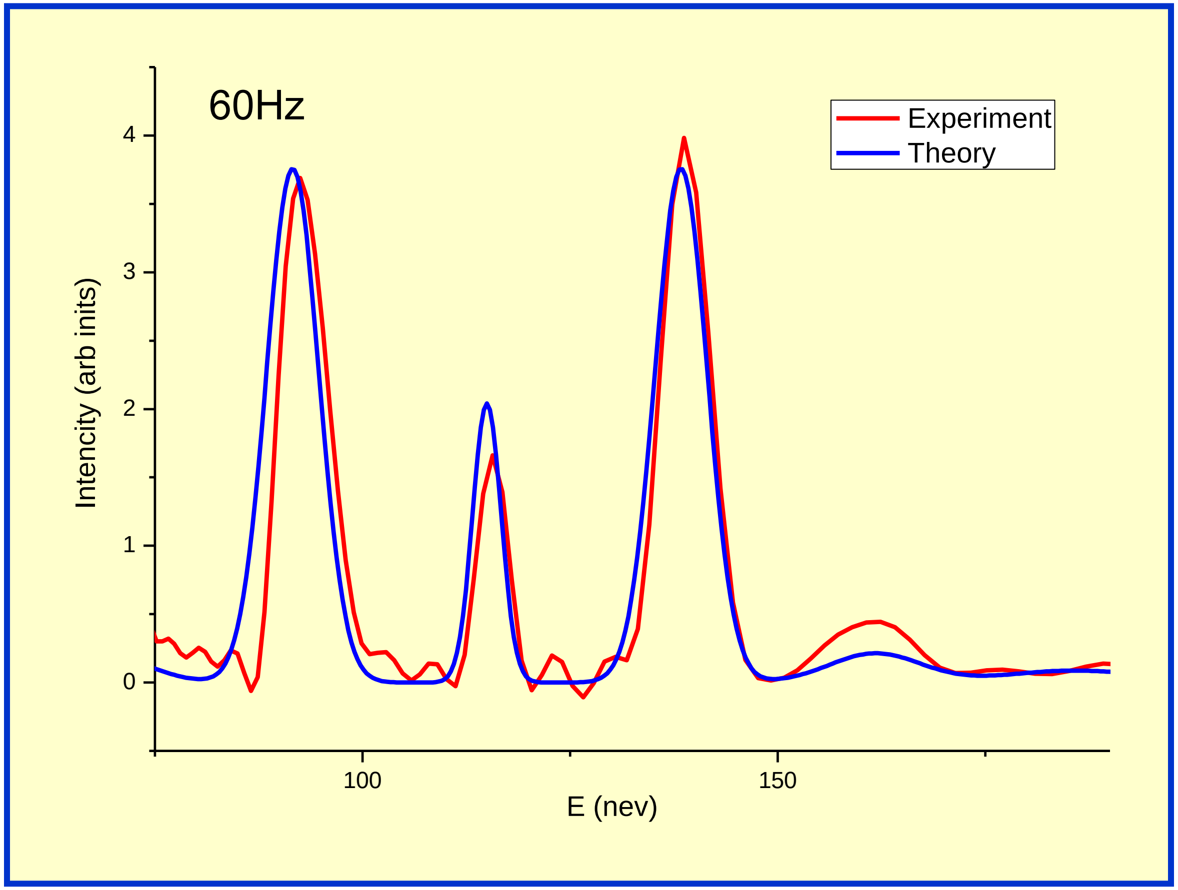
I(t) – **time of flight spectra**

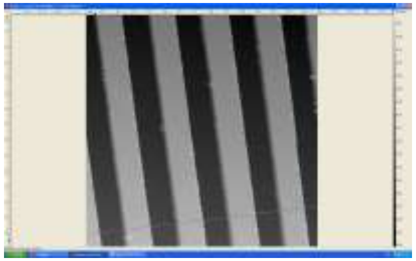
$$\mathbf{I(t)} = \int_0^\infty \mathbf{R}(\omega)\sin\left[\omega t - \varphi(\omega)\right]d\omega$$ **- Fourier expand**

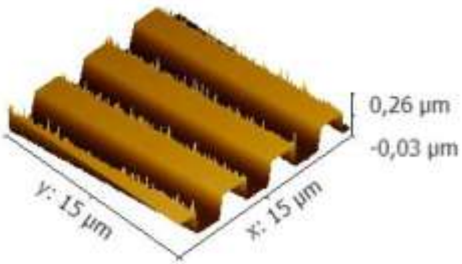
$$\mathbf{Z(t)} = \int_0^\infty \mathbf{I(t')}\sin\left[\omega(t - t')\right]dt'$$ **Fourier harmonic (measured)**

G.Kulin (next presentation)

Recent results: TOF spectra and comparing with theory (December 2014)







Non-destroyed
Atomic Force Microscope Inspection

Angular period of grating 0.0665mrad (4µ at the middle diameter)

Conclusion

1. ***Diffraction grating which is moving across the direction of neutron propagation acts as a nonstationary device. That results in the appearance of line spectrum.***
2. ***The phenomenon was firstly observed in 2001***
3. ***It may be (and was) used for precise change of the neutron energy. That was demonstrated in the experiment for neutron focusing in time and gravity experiments with UCN.***
4. ***Dynamical theory of UCN diffraction by a grating satisfactorily describes experimental data but the precision of these last is rather pure yet.***
5. ***We plan to continue experimental investigations of the phenomena using recently tested TOF Fourier method of UCN spectrometry***

Many thanks to my co-authors

I.V. Bondarenko, S.V.Goryunov, G.V.Kulin, D.Kustov.

S.N. Balashov, S.V.Masalovich, V.G.Nosov S.N.Strepetov

V. A. Bushuev

P. Geltenbort, P. Høghøj, M. Jentschel

Thank you for your attention!