

DISCRETENESS IN PARTICLE MASSES AND PARAMETERS OF THE STANDARD MODEL

or

Constituent Quarks in the Standard Model

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References:

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INTRODUCTION

- The Standard Model is a general theory of all interactions in which all three vector interactions (strong, weak and electromagnetic) are united in the representation:
- $$SU(3)_{\text{col}} \otimes SU(2)_L \otimes U(1)_Y$$
- Particle masses are parameters of the Standard Model. Y.Nambu suggested (1998) that empirical relations in particle masses could be used for the development of the Standard Model.
- Quantum chromodynamics (QCD) is the first of these three components. It deals with the strong interaction of colored quarks and gluons. It is simultaneously a theoretical base of nuclear physics.
- We discuss two aspects of the performed in PNPI analysis of nuclear effects of the general origin:
- 1) QCD-based hadronization of quarks into constituent quarks forming hadrons. It is connected with the gluon quark-dressing effect (C.D. Roberts *et al.*) which transforms the extremely low masses of light quarks into observed masses of baryons ($3M_q$) and mesons ($2M_q$). These quarks are described in NRCQM - Constituent Quark Model (C.Itoh, W.Plesas, L.Glozman). Initial masses of the baryon constituent quark in NRCQM $M_q=440 \text{ MeV}=m(\Xi)/3$ and $M_d=436 \text{ MeV}$ as well as meson constituent quark mass $M_q''=388 \text{ MeV}$ from a half of ρ -meson mass take part in the tuning effect in particle masses and nuclear binding energies and excitations (Fig.1-3).
- Simultaneously the same NRCQM parameters serve as the periods of a discreteness at larger mass scale--in masses of scalar and vector fields (which form with them the lepton ratio, Fig.4,5).
- 2) Separate important properties of empirically observed correlations in particle mass spectrum is connected with QED radiative correction of the type $\alpha/2\pi$ with $\alpha=137^{-1}$ and $\alpha_z=129^{-1}$.
- Initially this factor was noticed in nuclear nonstatistical effects, the recent data on masses are in favour for a common approach based on R.Feynman remarks: "The theories about the rest of physics are very similar to the theory of quantum electrodynamics: they all involve the interaction of spin $\frac{1}{2}$ objects with spin 1 objects. Why are .. the theories of physics so similar in their structure?".

Tuning effect in particle masses (low energies). Ia

- Particle masses including pion mass m_π , its β -decay parameter f_π , muon mass m_μ and neutron mass can be expressed as integers of the doubled value of the pion β -decay energy ($2\delta m_\pi - 2m_e$) which is close to $16m_e = \delta$ due to the relation 9:1 between the pion mass difference δm_π and the electron mass m_e (1968, Washington Conf., 1972 Conf. on Statistical Properties of Nuclei, Albany).
- The observed ratios are:
 - $(m_\mu + m_e)/2(\delta m_\pi - m_e) = 13.00$ The lepton ratio $L=207=13 \times 16 - 1$
 - $f_\pi=130.7 \text{ MeV (PDG)}/2(\delta m_\pi - m_e) = 16.01$
 - $(m_{\pi^{+-}} - m_e)/2(\delta m_\pi - m_e) = 17.03$
 - $\Delta M_\Delta/2(\delta m_\pi - m_e) = 18.02$
- equidistance in pseudoscalars $m_\mu/2(\delta m_\pi - m_e) = 50.1$
- neutron mass $m_n + m_e/2(\delta m_\pi - m_e) = 115.007$ Ratio $m_n/m_e=1838.6836605$, shift $\delta m_N/8$

The lepton ratio $L=m_\mu/m_e=206.77$ becomes the integer $207=9 \times 23=13 \times 16 - 1$ after a small QED radiative correction applied to me (it becomes $m_\mu/m_e(1-\alpha/2\pi)=207.01$). The well-known factor $\alpha/2\pi = 115.9 \times 10^{-5}$, the QED radiative correction to the magnetic moment of the electron (Schwinger term) coincides with the deviation of $\delta m_\pi = 4.5936(5) \text{ MeV}$ from $\Delta = 9m_e = 4.5990 \text{ MeV}$. It accounts the relative value $117(11) 10^{-5}$.

- Belokurov and Shirkov suggested (1991) that electron mass also contains this factor $\alpha/2\pi$.
- Earlier (1972) this factor was introduced from ratios in parameters of non-statistical effects (stable intervals) observed in low-lying excitations and in neutron resonances. Later it was found that this factor coincides empirically with the ratio of parameters of the Standard Model, masses of the second lepton and Z-boson: $m_\mu/m_Z = 115.9 \times 10^{-5}$.

Tuning effect in particle masses (low energy). Ib

Fig.1. Tests of Empirical Mass Formula $m=N \cdot 3m_e$ by R.Frosch (1967- 1991): deepest maximum in the mean root deviations corresponds to the period $3m_e$

- The experimental set of 47 masses was replaced by set of 47 random numbers... the probability for random masses to fit the $3m_e$ formula better than experimental masses is only 2×10^{-4}

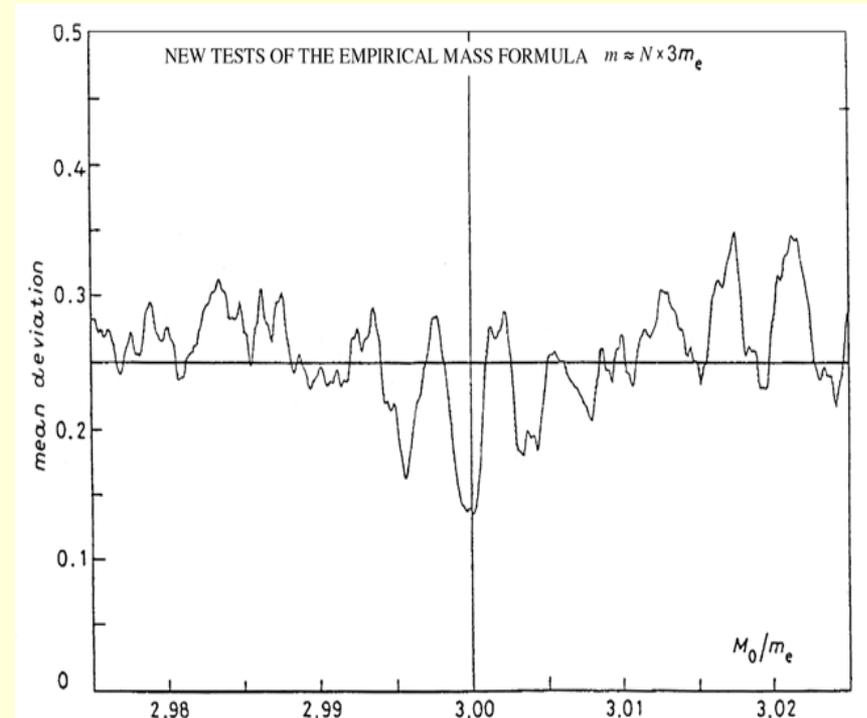


Table 1. Comparison of particle masses (PDG 2008) with the period $3m_e$ found by Frosch and with the parameter $16m_e = \delta$ close to the doubled value of pion's decay energy.

Neutron Δ -excitation is exactly $36\delta=294.3$ MeV.

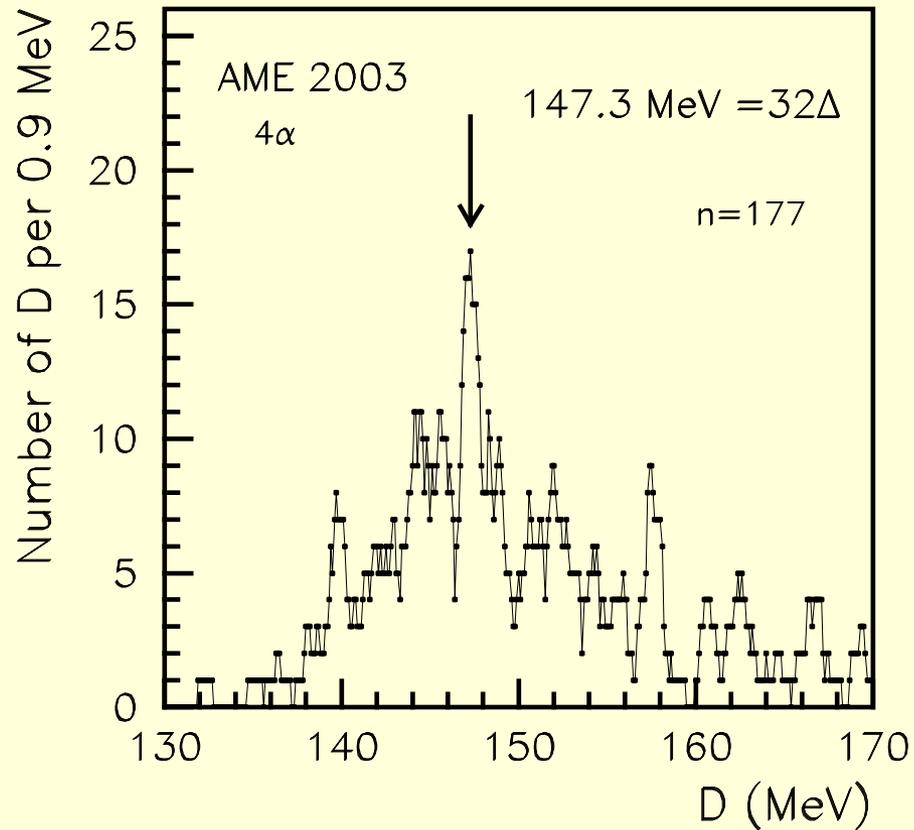
Difference of binding energies of light nuclei differing with four alpha cluster (8 protons and 8 neutrons) is close to $18\delta=147.2$ MeV (see Fig.2).

Stable interval 440 MeV = $3 \times 18\delta$ was noticed by R.Sternheimer (1964) and is presented in Fig.3 within two-dimensional presentation of particle masses as the period 16δ and $n\delta$.

Table 1. Tuning effect in particle masses (low energy). I c
Asterisk marks ratios $m_i/3m_e$ close to integer numbers noticed by R. Frosch.

Part.	m_i , MeV	$m_i/3m_e$	$N \cdot 16m_e$	N	$m_i - N \cdot 16m_e$	Comments
μ	105.658367(4)	68.92*	106.2878	13	-0.6294	-.0511-0.118
π^0	134.9766(6)	88,05*	138.9917	17	-4,0174	
π^\pm	139.5702(4)	91.04*		17	+0.5762	+0.511+0.065
η^0	547.853(24)	357.38	547.7908	67	0.06(2)	
Ω	782.65(12)	510.54	784.8943	96	-2.24(12)	
Φ	1019.46(2)	665.01*	1021.998	125	-2.54(2)	
K^\pm	493.677(16)	322.03*	490.5590	60	+3.118(16)	
P	938.2720(1)	612.05*	940.2380(1)	115	-1.9660	$-m_e - 9/8\delta m_N$
n Δ^0 -n	939.5654(1) 294.2(2)	612.89*	940.2380 294.3	115 36	$-m_e - 161.66(6)$ keV ≈ 0	$-m_e - 1/8\delta m_N$ $\Delta M \Delta$
Σ^0	1192.64(2)	777.98*	1193.693	146	-1.05(2)	-0.51·2=-1.02
Ξ^0	1314.86(20)	857.71*	1316.333	161	-1.47(20)	-0.51·3=-1.53

Tuning effect in nuclear binding energies. I d



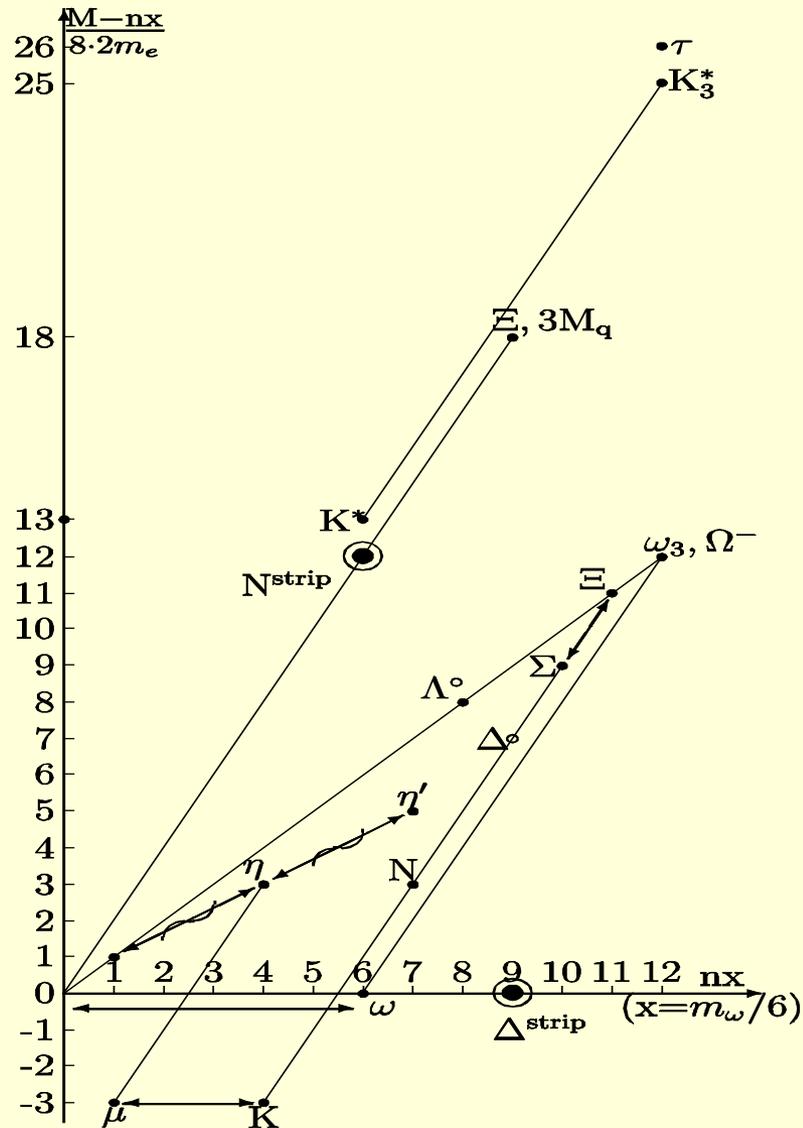
- ***Figure 2*** Observed intervals in nuclear binding energies of light nuclei ($Z \leq 26$) differing with 4α .

Tuning effect in nuclear binding energies. Ie

Table 2. Comparison of experimental ΔE_B (in keV) and theoretical estimates in magic nuclei ($N=82, N=20$) with $10\Delta = 45\varepsilon_0$ (${}^6\text{He}$ cluster, left part of the Table) and in light nuclei ${}^{39}\text{K}$ and ${}^{36}\text{K}$ differing with 4α (values $32\Delta = 18\delta = 144\varepsilon_0$). Small differences between observed energies and integer numbers of $\varepsilon_0=2m_e$ (Diff.) correspond to long-range correlations in binding energies.

	Z=55	${}^{137}\text{Cs}$		Z=57	${}^{139}\text{La}$	Z=58	${}^{138}\text{Ce}$	${}^{140}\text{Ce}$	${}^{39}\text{K}$	Z=19
N	80	82	78	80	82	78	80	82	20	17
ΔE_B	45946	45970	46018	45927	46024	46087	45997	45996	147160	147152
$N \cdot \varepsilon_0$		45990			45990			45990	147168	147168
Diff	-44	-20	28	-63	34	97	7	6	- 8(2)	- 16
FRDM	46620	46340	45950	46820	46970	45960	46850	47160	147450	145950
Diff.	630	350	-40	830	980	-30	860	1170	282	1220

Tuning effect in particle masses. I f
Fig.3. Different estimates of constituent quark masses



Part 2. Common approach to short/long distances, lepton ratio, QED corrections

The empirical fact consists in the coincidence of the lepton ratio $L=207=13 \times 16^{-1}$ to the ratios between masses of vector bosons $M_Z=91.188(2)$ GeV, $M_W=80.40(3)$ GeV and two commonly used estimates of the baryon/meson constituent quark masses:

- 1.) *Initial baryon constituent quark in QCD and NRCQM calculations (Fig. 4,5) coinciding with 1/3 part of the Ξ -- baryon mass $M_q=441$ MeV= $m_{\Xi}/3$.*

This parameter was earlier introduced by R.Sternheimer and P.Kropotkin as the stable interval in particle masses shown in Fig.3. Later it was realized as a constituent quark mass 441 MeV= $(3/2)(m_{\Delta}-m_N)=3 \times 147$ MeV with three-fold value of nucleon Δ -excitation .

The ratio with the vector Z-boson mass is close to $L=207$: namely, $M_Z/441$ MeV= 206.8 it results in $M_Z/207=440.5$ MeV= M_q close to $M_H/18 \cdot 16=436$ MeV and $M_d=436$ MeV
close to $3 \cdot 18 \cdot 16 m_e - 9 m_e$

- 2.) *Standard estimate of meson constituent quark mass from 1/2 part of rho-meson mass -- a single vector meson with parallel quark spins $M''_q=m_{\rho}/2=775.5(4)$ MeV/2 = $387.8(2)$ MeV.*

The ratio with the W-boson mass is close to $L=207$: namely, $M_W/(m_{\rho}/2)=207.3$ it results in $M_W/207=388.4$ MeV= M''_q

close to $3 \cdot 16 \cdot 16 m_e - 9 m_e$

Introduction of the QED correction $\alpha/2\pi$ into mass relations is based on empirical fact that masses forming the common lepton ratio $L= M_Z/M_q = m_{\mu}/m_e$ are in the accurately known ratio $m_{\mu}/M_Z = 115.9 \cdot 10^{-5}$ coinciding with $\alpha/2\pi=115.9 \cdot 10^{-5}$. The second ratio $m_e : (M_q=441$ MeV= $3 \cdot 147.2$ MeV) is a part of the same ratio based on the recently measured scalar field mass $M_H=125.5(6)$ GeV. Namely,

$$147.2 \text{ MeV} : 125.5 \text{ GeV} = 117 \cdot 10^{-5} \text{ and } (m_e/3=170.3 \text{ keV}) : 147.2 \text{ MeV} = 115.7 \cdot 10^{-5}$$

Confirmed stable nuclear intervals 161 keV and the pion mass are in the ratio $\alpha/2\pi$.

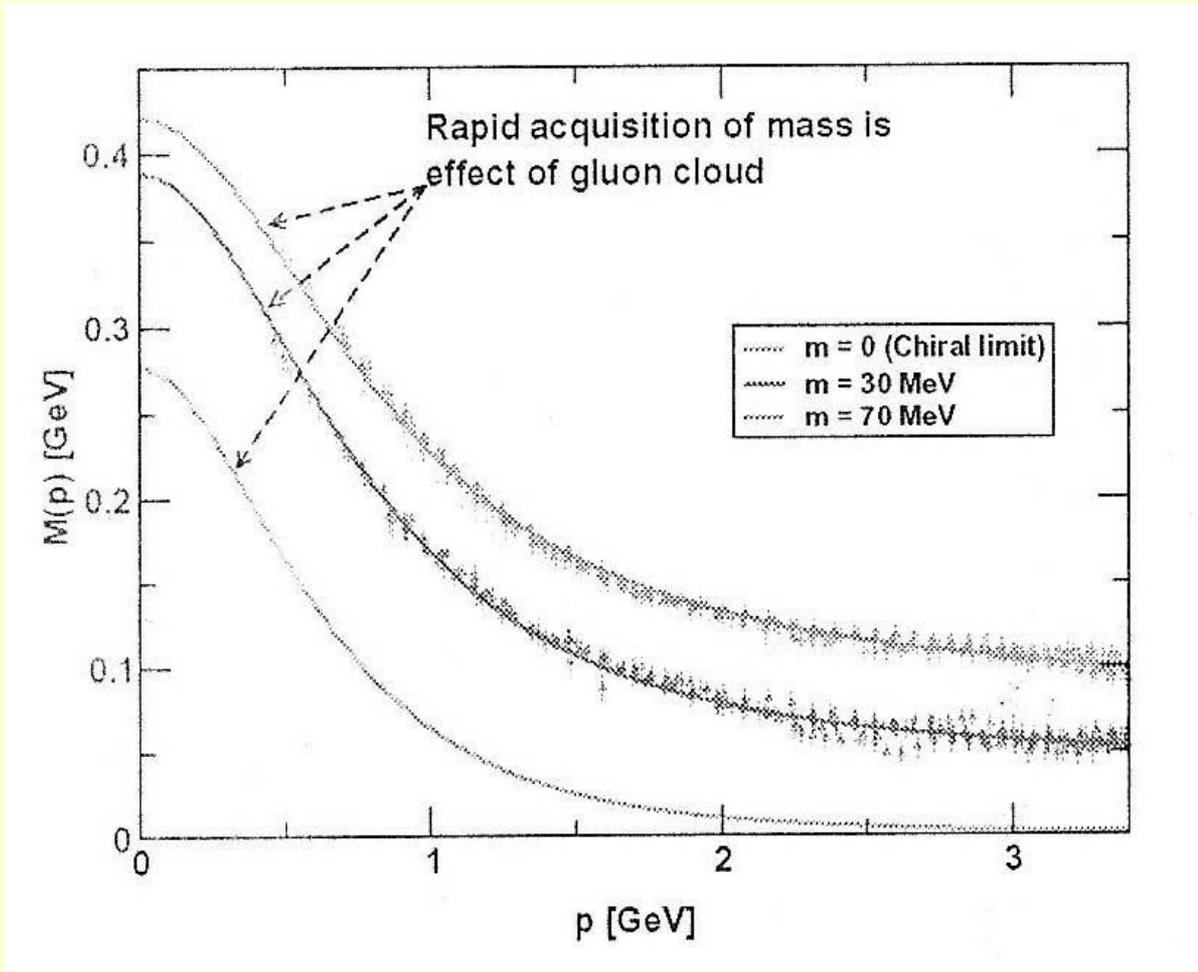
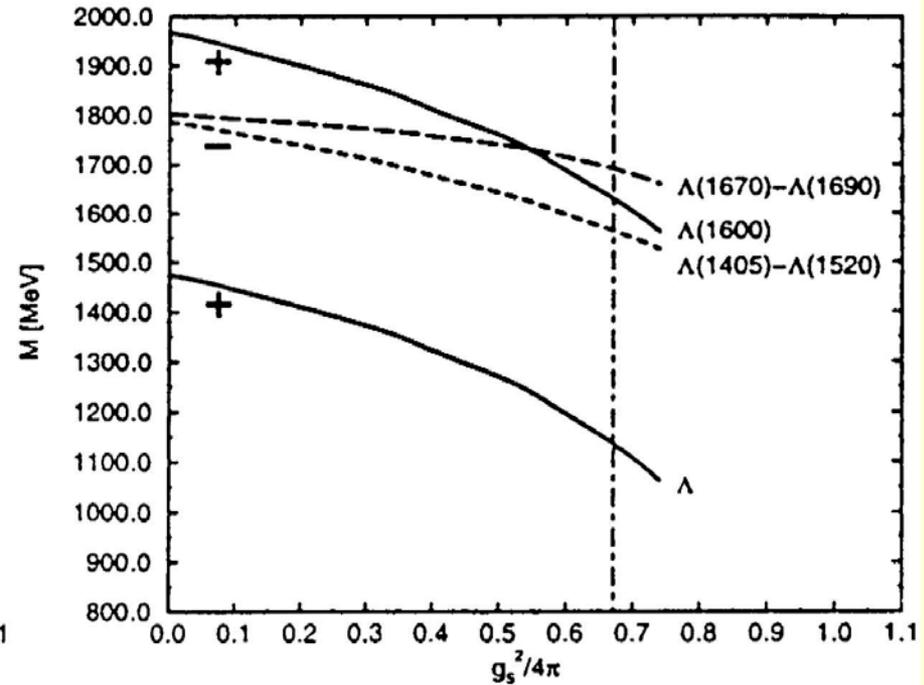
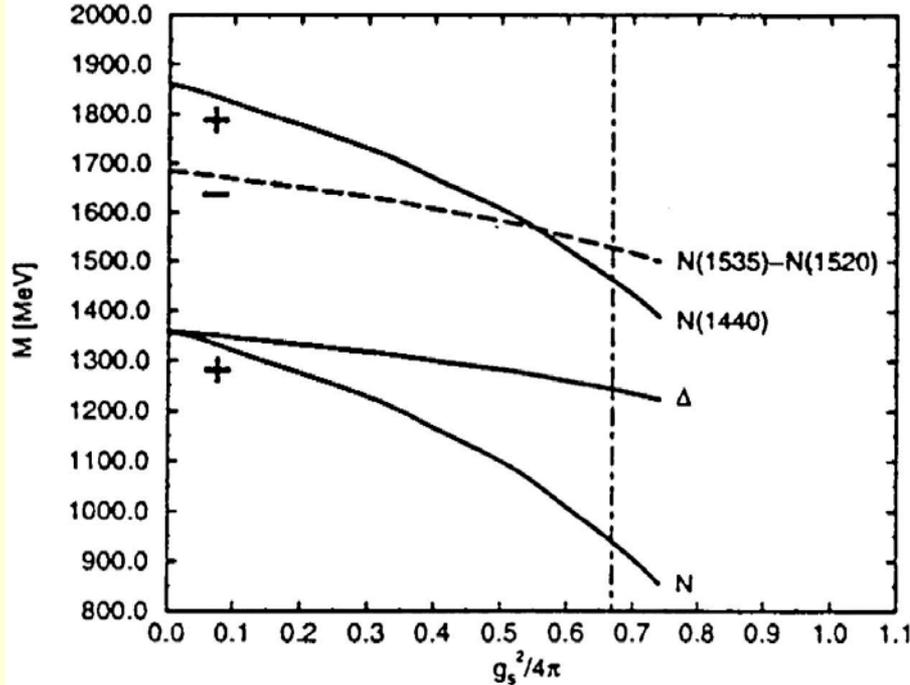


Fig.4. (C.D. Roberts et al.). QCD gluon-quark-dressing effect calculated with Dyson-Schwinger Equation, initial masses m ; constituent quark mass arises from a cloud of low-momentum gluons attaching themselves to the current-quark; this is chiral symmetry breaking: nonperturbative effect that generates a quark mass from nothing even at $m=0$ (bottom).



- Fig.5 (L.Glozman *et al.*). Calculation of nonstrange baryon (left) and lambda-baryon masses as a function of interaction strength within Goldstone Boson Exchange interaction Constituent Quark Model; initial baryon mass $1350 \text{ MeV} = 3 \cdot 450 \text{ MeV} = 3M_q$ is marked as bottom “+” on left vertical axis.

Common approach to short/long distances, lepton ratio, QED corrections (continuation) 2

Table 3. Parameters of tuning effects in particle masses (three upper parts with $x = -1,0,1$) and in nuclear data ($x = 1,2$) by expression $(n \cdot 16m_e(\alpha/2\pi)^X) \cdot m$ with QED parameter $\alpha = 137^{-1}$. Higgs boson and pion masses, stable nuclear intervals and shift in neutron mass $n\delta - m_n - m_e = 161$ keV, the nucleon Δ - excitation and $m_e/3$ related as $\alpha/2\pi$ are marked (bold).

Values M'_H and M_{L3} correspond to unconfirmed mass grouping found at CERN.

x	m	$n=1$	$n=13$	$n=16$	$n=17$	$n=18$
-1	3/2			$m_t=171.2$		
GeV	1	$7.0=\delta^0$	$M_Z=91.188$	$M'_H=115$		$M_H=125.5$
	1/2			$M_{L3}=58$		
0	1	$16m_e=\delta=8.176\text{MeV}$	$m_\mu=105.658$	$f_\pi=130.7$	$m_\pi - m_e=139.06$ keV	$(m_\Delta - m_N)/2=147$
MeV	1 3	$2\Delta - \varepsilon_0$	$106=\Delta E$	$130=\Delta E_B$ $M_q''=m_p/2$	$140=\Delta E_B$	$147.2=\Delta E_B$ $M_q=441=3\Delta M_\Delta$
1 keV	1 3 8	9.5 keV	123 keV		$n\delta - m_n - m_e=161.65(6)$ $\delta m_N=1293.34(1)$	$170=m_e/3$ $m_e=510.99891$
2 eV	1 2	11 eV 22 eV	143 eV		187 eV 375 eV	

Common approach to short/long distances, lepton ratio, QED corrections. 3

We have found out that using the exactly known neutron/electron mass ratio and observed proximity of the neutron mass $m_n=939.5654(1)\text{MeV}$ to the integer number of 115 of the common period $16m_e$ (and $-m_e$), one can determine the deviation of neutron mass from the integer numbers of m_e as equal to $161.65(6)\text{keV}$. This value coincides with $1/8$ of $\delta m_N =$ nucleon mass difference 1293.3keV and simultaneously is the stable interval of 161keV in nuclear excitations connected with the one-pion exchange dynamics. It reflects a common QCD - based hadronization in particle masses and in nuclear binding energies and excitations.

Small deviations (about 10keV) observed between integers of common period $2m_e$ and experimental differences of binding energies in nuclei close to shells $Z, N=20, N=82$ and the stable character of intervals $D=161\text{keV}$ and $D=1293\text{keV}=8\cdot 161\text{keV}$ in nuclei where the one-pion dynamics (tensor forces) are expected, confirm the long-range correlation between the neutron mass and the electron rest mass.

Ratio of the shift/interval to the pion mass (without m_e) accounts $116.2\cdot 10^{-5}$ close to $\alpha/2\pi=115.9\cdot 10^{-5}$, while the ratio of the same pion mass to $m_e/3$ accounts $122.5\cdot 10^{-5}$ which is close to QED correction $\alpha_z/2\pi=123.1\cdot 10^{-5}$ at the short distance about $1/M_z$. The ratio between QED parameters $(137/129)=1.062$ coincides with $17/16=1.063$ and is close to $18/17=1.059$ and $(m_\pi-m_e)/(f_\pi=130.7\text{MeV})=1.064$. As a result, the value $m_e/3$ at the bottom of column $n=18$ is connected with both principal mass sequences starting with the scalar mass M_H (top right) and the top-quark mass m_t (top center).

Electron rest mass as a common parameter of the Standard Model. 4

We interpret observed discreteness in the tuning effect in particle masses at low energies with the period $\delta=16m_e=8.176$ MeV as the distinguished role the value $m_e=3\cdot(m_e/3)$ where the value $m_e/3$ is the low part of the main sequence in Table 3 started with the scalar mass and 147 MeV $=(1/3)M_q$. Parameter δ is the doubled value of the pion beta decay energy $\delta m_\pi - m_e$. This difference can be represented with the following expression:

$$\delta m_\pi - m_e = 9m_e - m_e = \frac{m_e}{3} + 8\left(\frac{m_e}{3}\right) - \frac{m_e}{3} = \delta / 2 = 8m_e$$

The additional shift $-m_e$ in neutron mass relative to integer number δ in the common tuning effect permits to suggest that this value can be associated with the known fact that baryon is composed out of three constituent quarks. If $m_e/3$ is connected somehow with fermion properties (spin $1/2$) then the presence of number 8 in representation of the charge splitting $9m_e$ could be considered as an indication of the symmetry at short distances with the generation of the new principal quantum number $(1 + 8 \text{ minus } 1)$.

We suggest an interconnection between the discreteness in the spectrum and the symmetry in the system (this time a symmetry in fermion space).

Common approach to short/long distances, lepton ratio, QED corrections. 5.

Table 4. Empirically observed proximity of the lepton ratio $L=16 \cdot 13 - 1$ to the numbers of fermions in the central field (see numbers of electrons or nucleons in other quantum systems).

Numbers 16 in the upper line (and 1 in the bottom) correspond to $16M_q = \delta^0 = 2\Delta^0 - 2M_q$ with the value $\Delta^0 \approx m_b = 4$ GeV close to the bottom quark mass.

Number	1	16	16-13-1	16-16	16-18
Particle	m_e	δ	m_μ	f_π	ΔM_Δ
Particle	M_q	δ^0	M_z	$(2/3) \cdot m_t$	M_H
Ratio	$116 \cdot 10^{-5}$		$116 \cdot 10^{-5}$	$116 \cdot 10^{-5}$	$116 \cdot 10^{-5}$
Number	(1/16)	(1)	12+1	16	18
States	$1s^4_{1/2}$		$1s^4_{1/2},$ $1p^8_{3/2}, 1p_{1/2}$	$1s^4_{1/2},$ $1p^8_{3/2}, 1p^4_{1/2}$	
Comments			a hole in 16-13	filled subshells	

Conclusions

- In these work we used new data on the scalar mass 125.5 GeV and the recently obtained CODATA ratio m_n/m_e to find out an explanation of the systematic appearance of nuclear mass/energy intervals close to mass differences of the nucleon, leptons and the pion. Confirmation of this tuning effect in nuclear data together with some empirical observations on parameters of the Standard Model could be considered as the reflection of the common dynamics of hadronization in particle masses and nuclear data.
- Presence of tuning effects in nuclear excitations and nuclear binding energies is confirmed with new analysis of data from PNPI compilations (Vols. I/19, I/22, I/25 LB Springer) based on the role of one-pion-exchange.
- Relation $(n \cdot 16m_e - m_e - m_n)/\delta m_N = 1/8.001(2)$ and the similar long-range correlations in data could be checked with the new data.
- Relation between observed stable nuclear intervals and particle masses can be considered in connection with recently obtained mass of the scalar field, the ratios $m_\mu/M_Z = \alpha/2\pi$ and $(1/3m_e)/M_H = (\alpha/2\pi)^2$. There is a possibility that they are reflections of the fundamental relations between SM-parameters (which were mentioned as a future “super-duper” model by R. Feynman).
- Observed analogy between tuning effects in particle masses and in nuclear data should be theoretically based on QCD as a part of Standard Model. It is in line with Y.Nambu suggestion about the role of empirical relations in particle masses for the SM development.
- Scientific potential of nuclear physics can be connected with a fundamental role of QCD in the Standard Model and with the role of QED parameters.