What is coherence length?

A property of a beam,
or the size of the wave-packet
of an individual particle?
How to check?

Ignatovich V.K.
FLNP JINR

Neutron is not a plain wave because of the life time

A. Steyerl introduced the individual coherence length

$$l_n = \sqrt{\hbar\tau/m} \approx 7.5\ \text{mm}$$

The neutron can be described by the de Broglie's wave-packet, which can explain UCN anomaly, if its size is

$$l_n = 10^5/k_c = 10^5\lambda_c/2\pi \approx 1.5\ \text{mm}$$

However the size can decrease with energy like

$$l_n = 10^5/k = 10^5\lambda/2\pi$$

How to check it?

And why de Broglie's instead a gauss wave-packet?

I propose 2 experiments to measure the size of the de Broglie's wave packet

$$\psi_{\mathrm{dB}}(q,\mathbf{k},\mathbf{r},t)=\sqrt{\frac{q}{2\pi}}\exp(i\mathbf{k}\mathbf{r}-i\omega t)\frac{\exp\left(-q\left|\mathbf{r}-\mathbf{k}t\right|\right)}{\left|\mathbf{r}-\mathbf{k}t\right|}$$

,

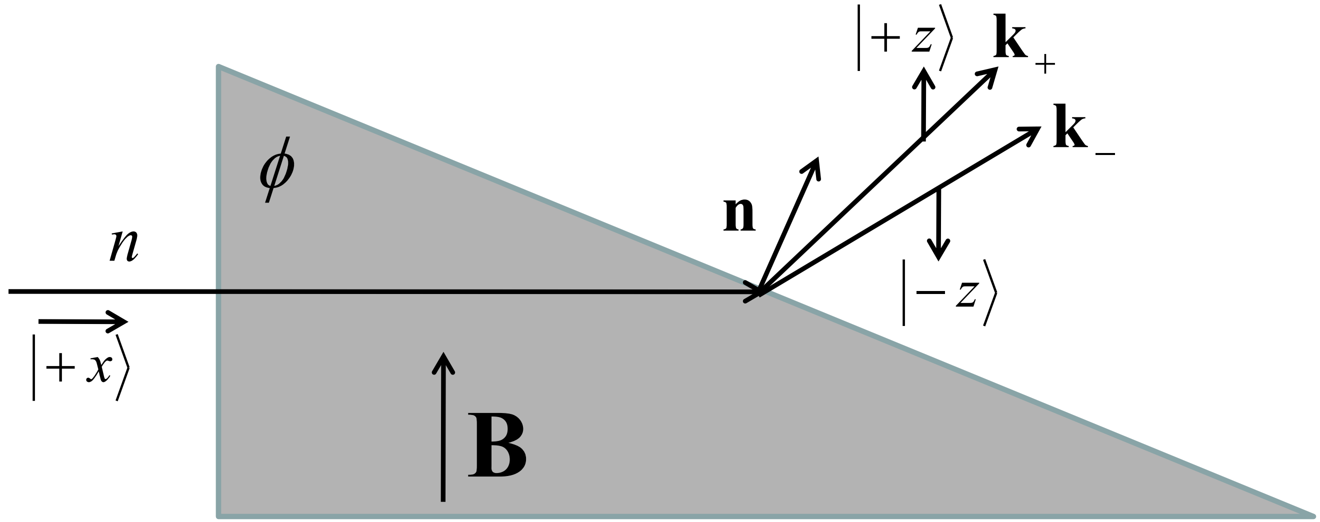
$$\omega=\frac{k^2-q^2}{2}$$

$$l=\frac{1}{q}$$

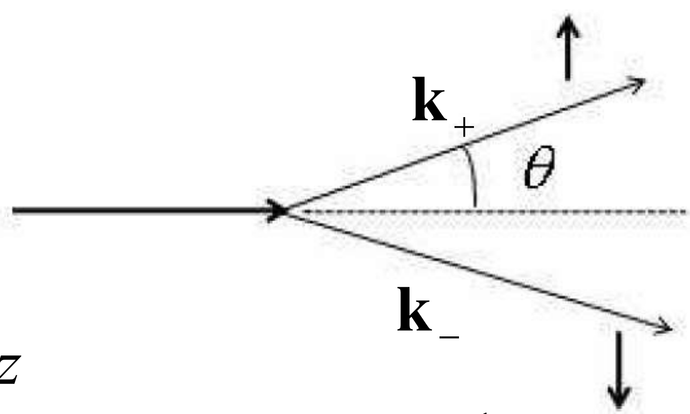
$$\psi_{\mathrm{dB}}(q,\mathbf{k},\mathbf{r},t)=\sqrt{\frac{q}{2\pi}}\exp(i\mathbf{k}\mathbf{r}-i\omega t)\frac{4\pi}{(2\pi)^3}\int d^3p\,\frac{\exp\left(i\mathbf{p}\left(\mathbf{r}-\mathbf{k}t\right)\right)}{p^2+q^2}$$

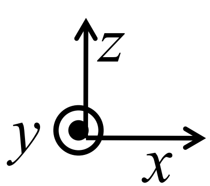
$$\left(i\frac{\partial}{\partial t}+\frac{\Delta}{2}\right)\psi_{\mathrm{dB}}(q,\mathbf{k},\mathbf{r},t)=-\sqrt{2\pi q}\exp(i(\omega+q^2)t)\delta(\mathbf{r}-\mathbf{k}t)$$

Split coherently a neutron into 2 components with equal speeds



The simplified scheme

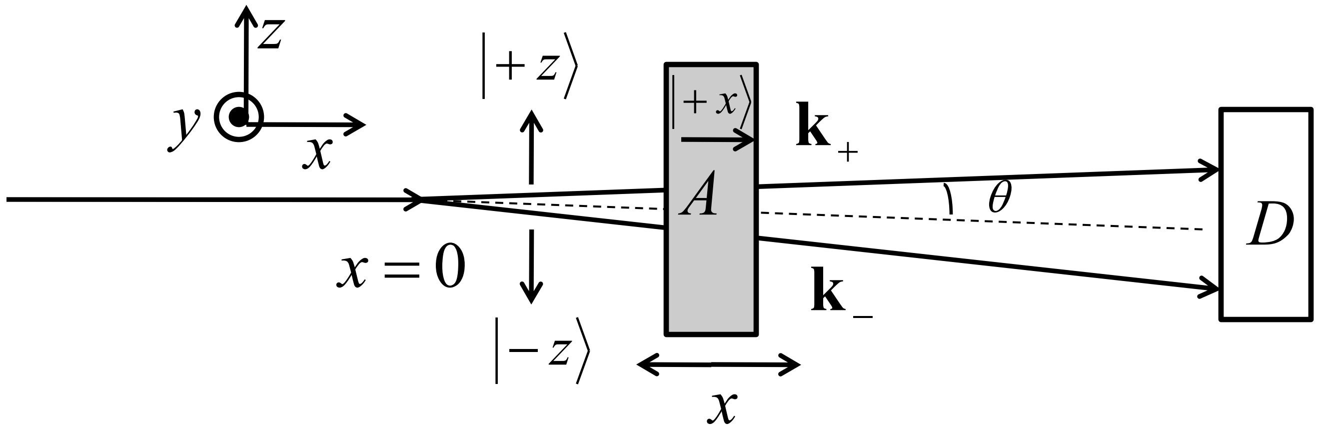




$$\mathbf{k}_\pm = (k_x, 0, \pm k_z)$$

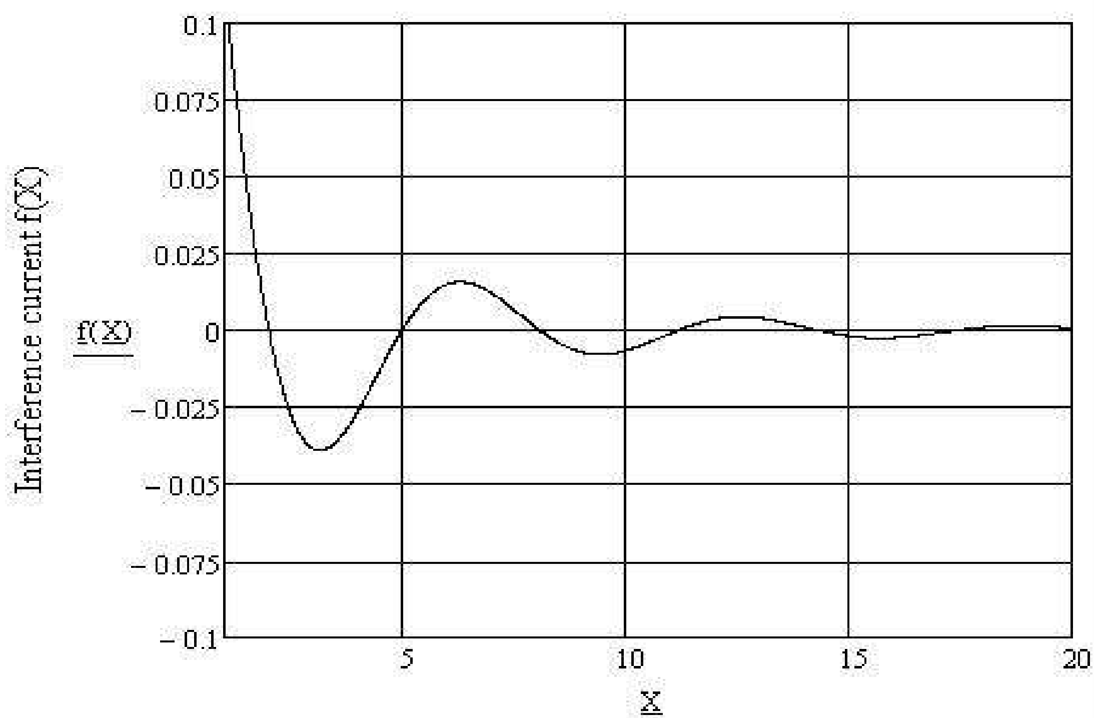
$$\theta = \frac{k_z}{k_x} = \frac{\Delta u}{4k^2}\tan\phi$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\psi_{\mathrm{dB}}(\mathbf{k}_+, \mathbf{r}, t)|+z\rangle + \psi_{\mathrm{dB}}(\mathbf{k}_-, \mathbf{r}, t)|-z\rangle\right)$$



Analyzer transmits only |+x> polarization

Let's move analyzer away along x and see the transmission

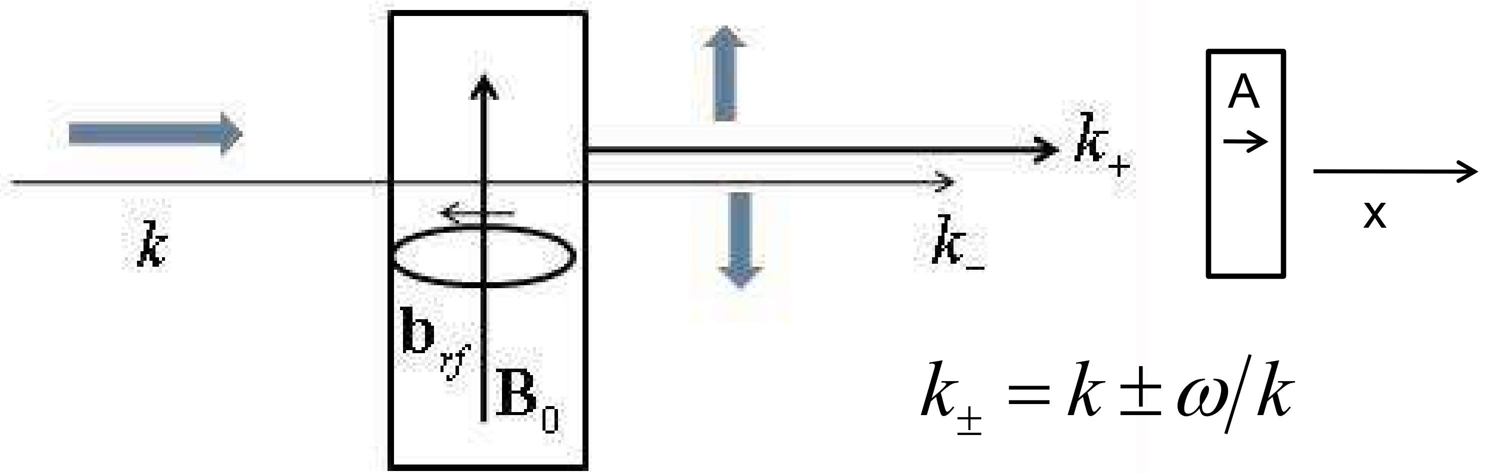


$$X = 2kx\theta^2$$

$$\theta = \frac{k_z}{k_x} \approx \frac{k_z}{k}$$

$$x = \frac{X\lambda}{4\pi\theta^2}$$

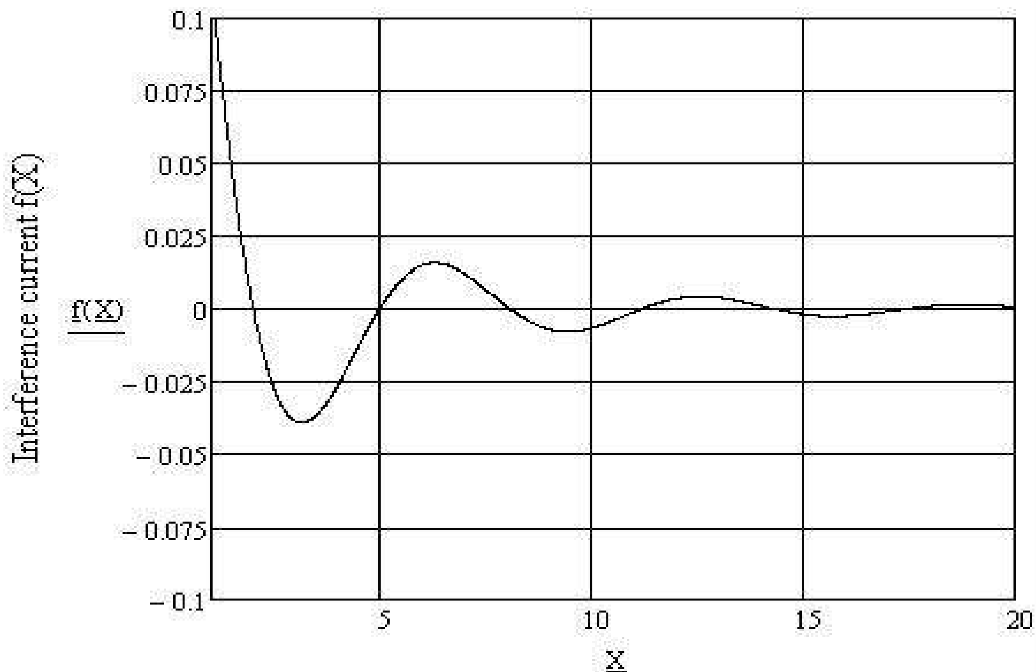
The second type of splitting with rf spin-flipper



$$k_{\pm} = k \pm \omega / k$$

$$\mathbf{b}_{rf} = b_{rf}(\cos 2\omega t, \sin 2\omega t, 0)$$

$$\delta = \Delta k / k = \omega / k^2$$



$$X = 2kx\delta^2$$

$$x = \frac{X\lambda}{4\pi\delta^2}$$

Analyzer transmits only |+x> polarization

$$\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left(\psi_{\mathrm{dB}}(\mathbf{k}_+, \mathbf{r}, t)\left|+z\right\rangle + \psi_{\mathrm{dB}}(\mathbf{k}_-, \mathbf{r}, t)\left|-z\right\rangle\right)$$

$$\left|\pm z\right\rangle = \frac{1}{\sqrt{2}}\left(\left|+x\right\rangle \pm \left|-x\right\rangle\right) \qquad \left\langle +x \| \Psi\right\rangle = \frac{1}{2}\left(\psi_{\mathrm{dB}}(\mathbf{k}_+, \mathbf{r}, t) + \psi_{\mathrm{dB}}(\mathbf{k}_-, \mathbf{r}, t)\right)$$

$$I(x) = k_x \int dydzdt \left|\left\langle +x \| \Psi\right\rangle\right|^2 = \left(I_+ + I_- + I_\pm\right)/4$$

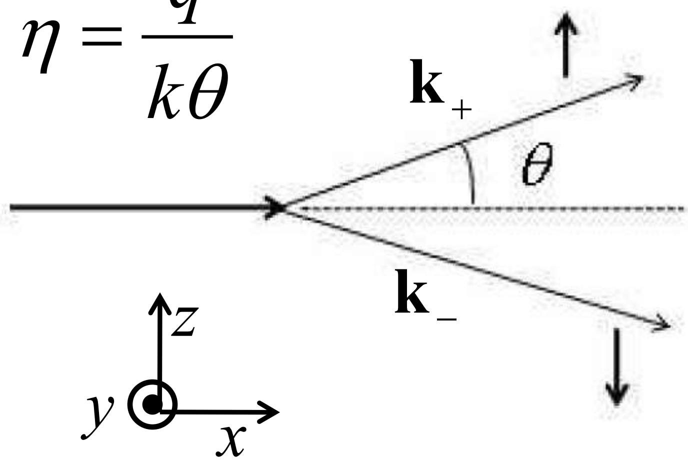
$$I_\pm(x) = 2k_x \int dydzdt \cos(2k_z z)\left|\psi_{\mathrm{dB}}(\mathbf{k}_+, \mathbf{r}, t)\psi_{\mathrm{dB}}(\mathbf{k}_-, \mathbf{r}, t)\right|$$

$$J_\pm = \frac{q(4\pi)^2}{(2\pi)^7} \int \frac{k_x d^3 p d^3 p' dydzdt}{\left(p^2 + q^2\right)\left(p'^2 + q^2\right)} e^{i\mathbf{p}(\mathbf{r} - \mathbf{k}_+ t) + i\mathbf{p}'(\mathbf{r} - \mathbf{k}_+ t) \pm 2k_z z} =$$

$$= \frac{qk_x}{\pi^2} \int \frac{d^3 p d^3 p' \delta(p_y + p'_y)}{\left(p^2 + q^2\right)\left(p'^2 + q^2\right)} e^{i(p_x + p'_x)x} \delta(p_z + p'_z \pm 2k_z) \delta\left(\mathbf{p}\mathbf{k}_+ + \mathbf{p}'\mathbf{k}_-\right) \Rightarrow$$

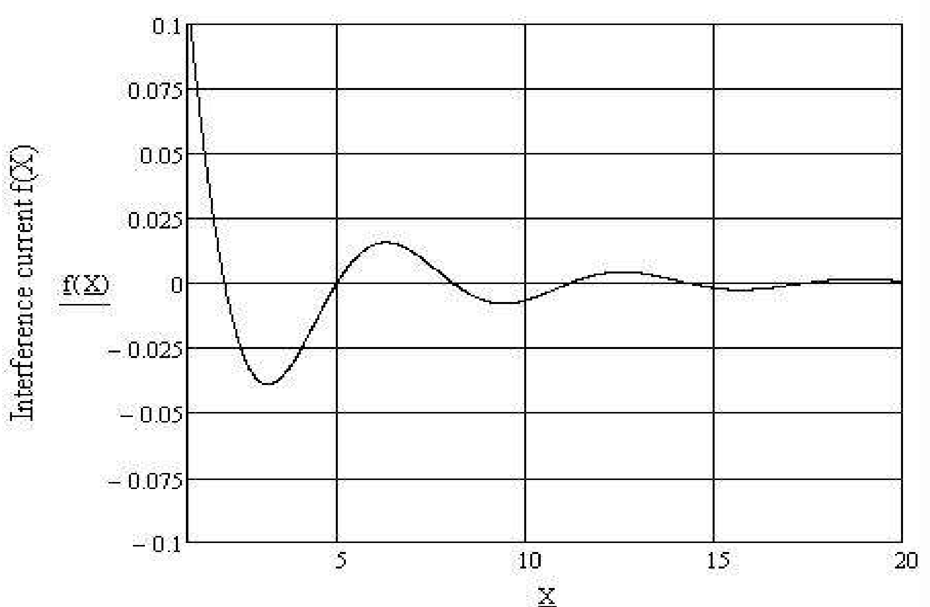
$$I_{\pm}(x) \equiv f(X) = 2\eta \int_0^1 ds \cos(sX) \frac{\exp\left(-X\sqrt{1-s^2+\eta^2}\right)}{\sqrt{1-s^2+\eta^2}}$$

$$X = 2kx\theta^2 \qquad \eta = \frac{q}{k\theta}$$



$$\theta = \frac{k_z}{k_x} \approx \frac{k_z}{k}$$

What are the difficulties?



To see oscillations we need small η

$$q = 10^{-5} k$$

$$\theta = 10^{-4} \Rightarrow \eta = \frac{q}{k\theta} = 0.1$$

$$x = \frac{X\lambda}{4\pi\theta^2}$$

to be macroscopic requires

$$\lambda = 100 \text{ Å}$$

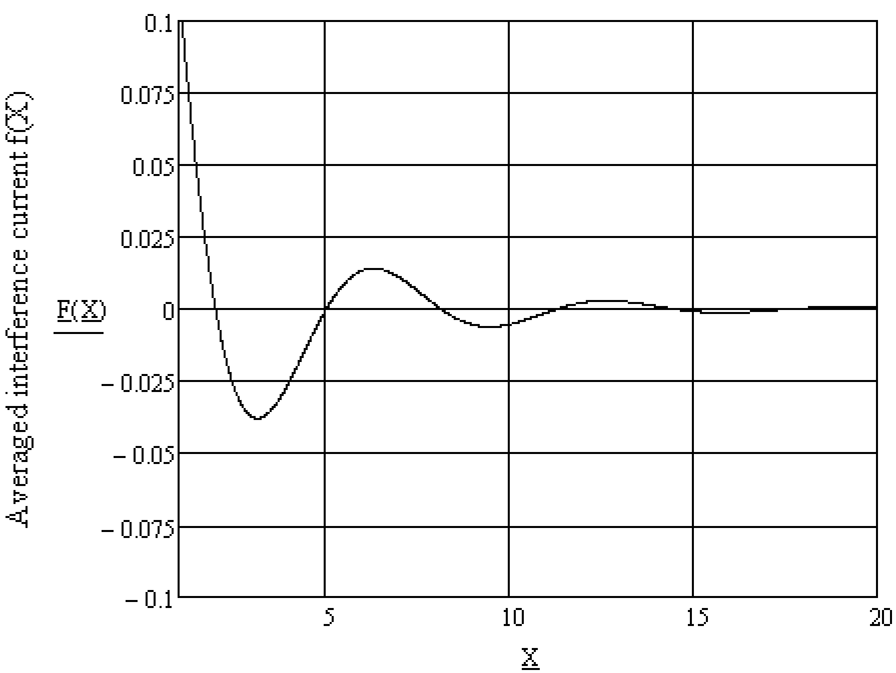
Spectral averaging

$$w(k) = \frac{1}{\sqrt{\pi}\Delta} \exp\left(-\frac{(k-1)^2}{\Delta^2}\right)$$

$$k = k/k_0$$

$$\Delta = \Delta/k_0$$

$$F(X) = \int dk w(k) 2\eta \int_0^1 ds \cos(skX) \frac{\exp\left(-kX\sqrt{1-s^2+\eta^2}\right)}{\sqrt{1-s^2+\eta^2}}$$



$$\Delta = 0.1$$

$$X = 2xk_0\theta^2$$

An alternative

$$q = 10^{-5} k_{\mathrm{Cu}} \qquad\qquad q = 10^{-5} k_{\mathrm{Cu}}$$

$$\eta = q/k\theta = 10^{-2} \Rightarrow E = 1\,\mathrm{meV},\, \theta = 10^{-5}$$

$$\Delta u \approx 10^{-8},\, \tan\phi = 4 \Rightarrow \theta = \frac{\Delta u}{4k^2}\tan\phi \approx 10^{-5}$$

$$x = \frac{X\lambda}{4\pi\theta^2} \qquad X = 1,\, \theta = 10^{-5} \Rightarrow x = \frac{X\lambda}{4\pi\theta^2} \approx 1\,\mathrm{m}$$

$$\lambda = 10\,\mathring{\mathrm{A}}$$

If we use a gauss wave-packet

$$\psi = \psi_1 + \psi_2 = \int d^3k \exp\left(-\frac{(\mathbf{k}-\mathbf{k}_+)^2}{4\Delta k^2} \right) e^{i\mathbf{kr}-i\omega t} + \int d^3k \exp\left(-\frac{(\mathbf{k}-\mathbf{k}_-)^2}{4\Delta k^2} \right) e^{i\mathbf{kr'}-i\omega t}$$

$$f(\Delta x) = 2\,\mathrm{Re}\left(\int dtdydz\ \psi_1^*(x,t)\psi_2(x',t) \right) = C\int dk \exp\left(-\frac{(k-k_x)^2}{2\Delta k^2} \right) \cos(k\Delta x)$$

$$f(\Delta x) = \cos(k_x \Delta x)\exp\left(-(\Delta x)^2 \Delta k^2\right) \qquad \Delta x \propto \lambda$$

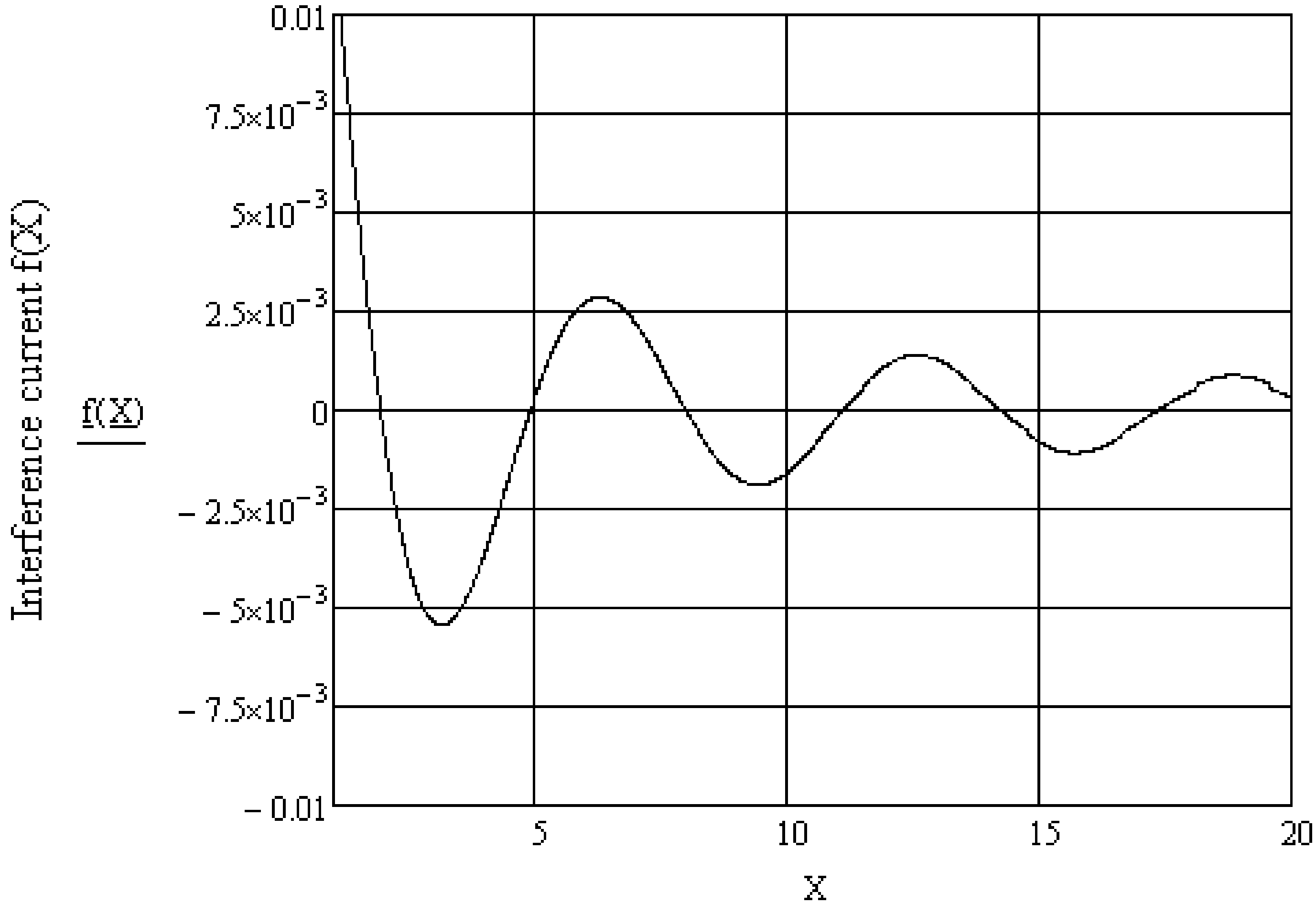
If we have simple plain waves

$$f(\Delta x) = 2\,\mathrm{Re}\,\psi_1^*(x,t)\psi_2(x',t) = 2\cos(2kz\theta) = 2\cos(X = 2kx\theta^2)$$

$$x = X\frac{\lambda}{4\pi\theta^2} \qquad \lambda = 1.8\,\mathring{\mathrm{A}},\ \theta = 10^{-5} \Rightarrow x = 10X \text{ cm}$$

In this case there is no extinction

Thanks



$$\eta = 0.01$$

$$X = 1, \theta = 10^{-5} \Rightarrow x = \frac{X\lambda}{4\pi\theta^2} \approx 1\,\mathrm{m}$$

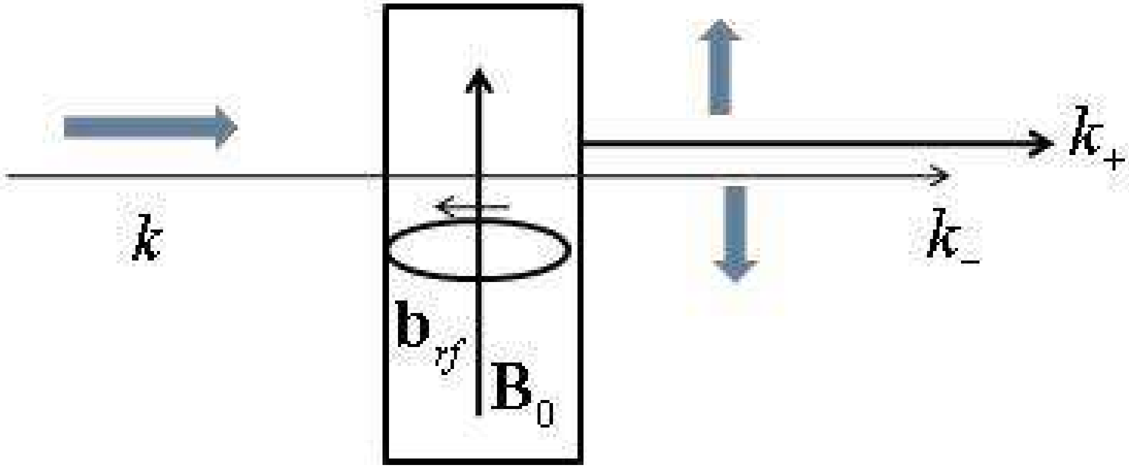
В случае когда скорости различаются и по величине и направлению

$$f(X) = 2\eta \int_0^1 ds \cos(sX) \frac{\exp\left(-X\sqrt{1-s^2+\eta^2}\right)}{\sqrt{1-s^2+\eta^2}}$$

$$\mu^2 = \frac{\delta^2 + k_z^2}{k^2}$$

И основную роль играет наибольший параметр

Расщепление на 2 компоненты с разными но параллельными скоростями

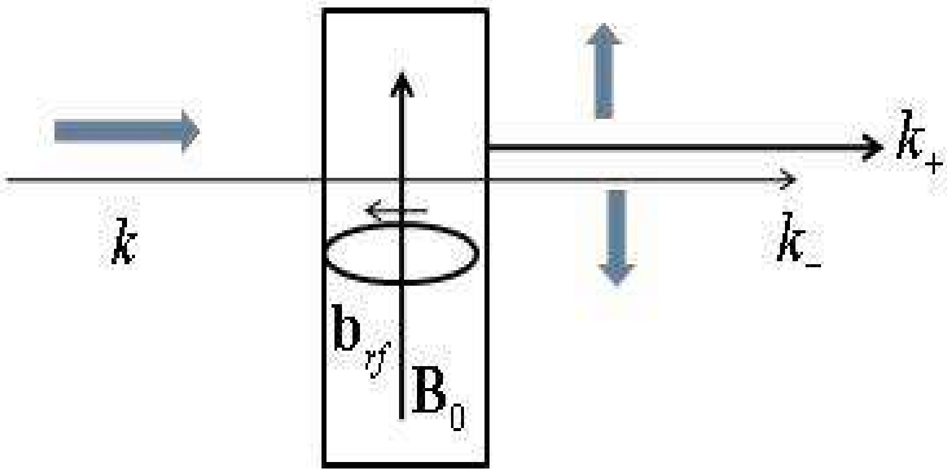


$$I_{\pm}(x) = 2\bar{k}\int dydzdt\cos(2\Delta k_x x - 2\bar{k}\Delta k_x)\left|\psi_{\mathrm{dB}}(\mathbf{k}_+,\mathbf{r},t)\psi_{\mathrm{dB}}(\mathbf{k}_-,\mathbf{r},t)\right|$$

$$J_{\pm} = \frac{q(4\pi)^2}{(2\pi)^7}\int\frac{k_x d^3p d^3p' dydzdt}{\left(p^2+q^2\right)\left(p'^2+q^2\right)}e^{i\mathbf{p}(\mathbf{r}-\mathbf{k}_+t)+i\mathbf{p}'(\mathbf{r}-\mathbf{k}_+t)\pm 2\Delta k_x x\mp 2\bar{k}\Delta k_x t}$$

$$I_{\pm}(x) \equiv f(X) = 2\eta \int_0^1 ds \cos(sX) \frac{\exp\left(-X\sqrt{1-s^2+\eta^2}\right)}{\sqrt{1-s^2+\eta^2}}$$

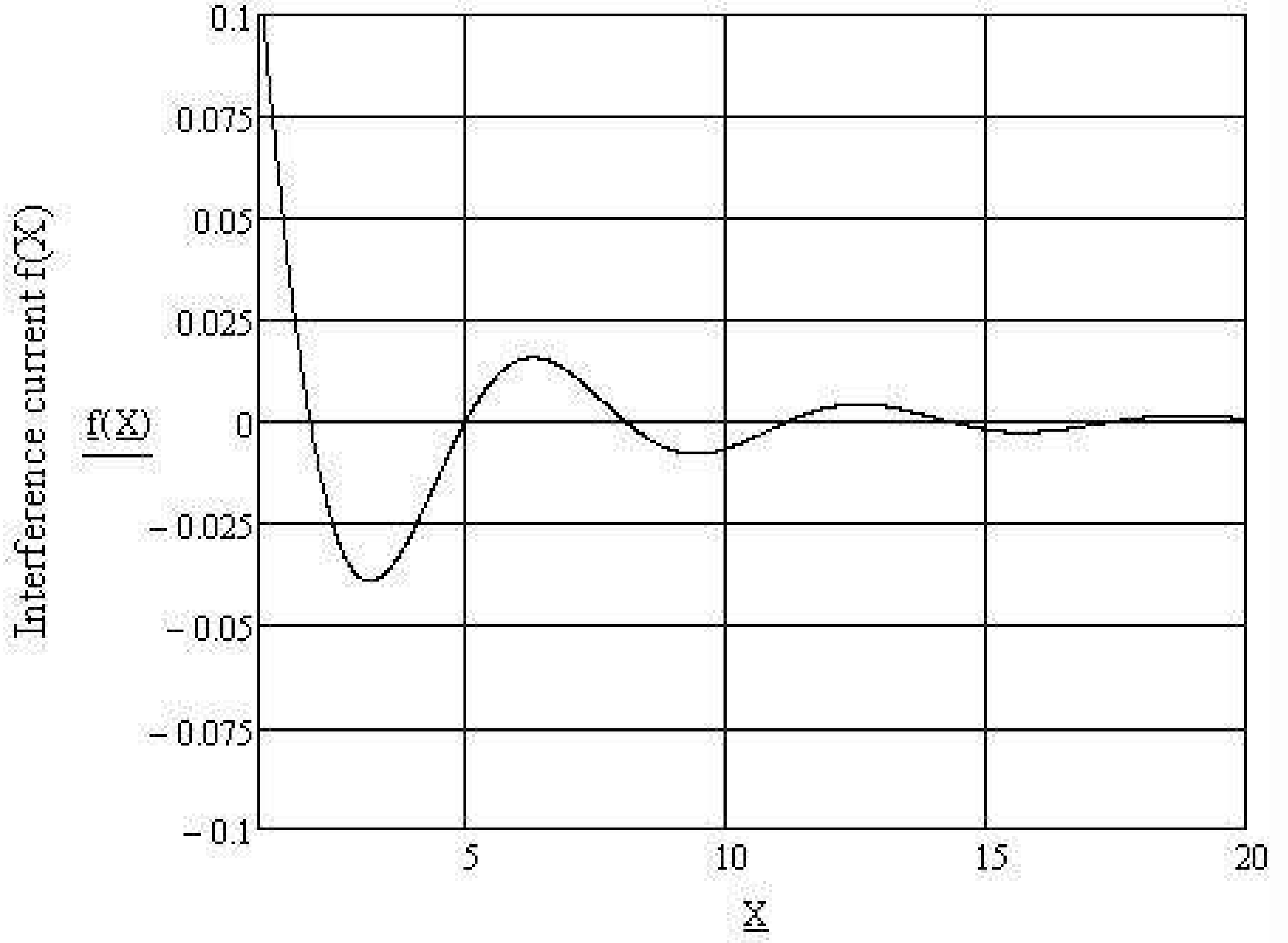
$$X = 2kx\delta^2 \qquad \eta = \frac{q}{k\delta}$$



$$\delta = \frac{k_+ - k_-}{k_+ + k_-} = \frac{\Delta k_x}{\bar{k}}$$

$$x = \frac{X\lambda}{4\pi\delta^2} \qquad q = 10^{-5}k$$

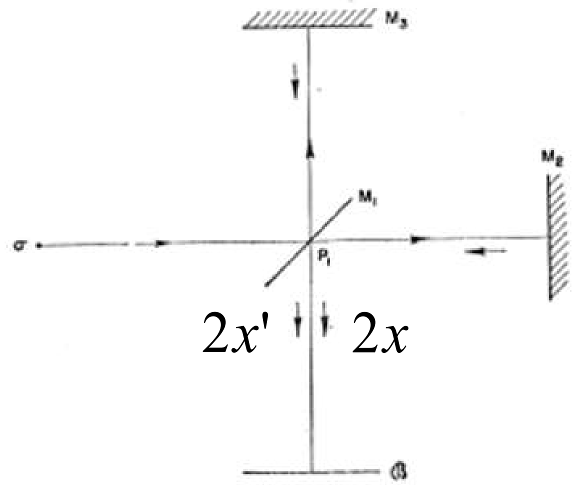
$$\delta = 10^{-4} \Rightarrow \eta = 0.1$$



$\eta = 0.1$

$$X = 1, \delta = 10^{-4} \Rightarrow x = \frac{\lambda}{4\pi} 10^{8} = 10 \text{ cm} \Rightarrow 100 \ \overset{0}{\text{A}} \Rightarrow v = 40 \text{ m/s}$$

Everyone knows that uncertainty Δk in beam preparation creates coherence length l_c=1/Δk. It is possible to see it!

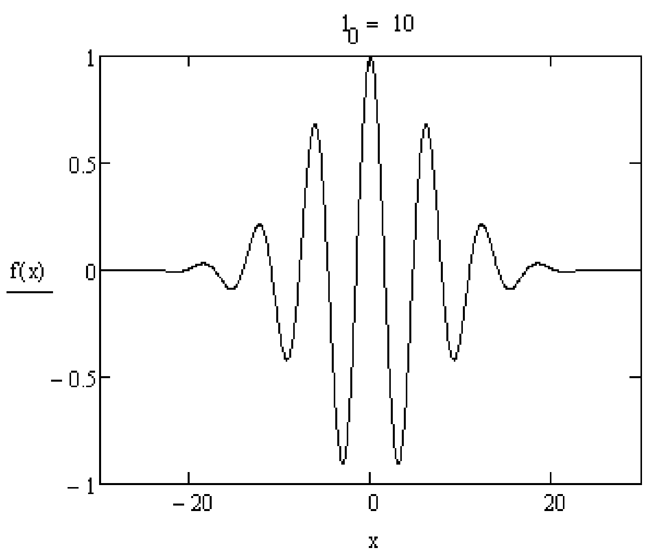


$$\psi = \psi_1 + \psi_2 = \int dk \exp\left(-\frac{(k-k_0)^2}{4\Delta k^2}\right) e^{ikx - i\omega t} +$$

$$+ \int dk \exp\left(-\frac{(k-k_0)^2}{4\Delta k^2}\right) e^{ikx' - i\omega t}$$

$$\int dt |\psi|^2 = \int dt |\psi_1|^2 + \int dt |\psi_2|^2 + \int dt \, \mathrm{Re}(\psi_1^* \psi_2)$$

$$f(\Delta x) = 2\,\mathrm{Re}\left(\int dt\, \psi_1^*(x,t)\psi_2(x',t)\right) = 4\pi \int dk \exp\left(-\frac{(k-k_0)^2}{2\Delta k^2}\right)\cos(k\Delta x)$$



$$f(x) = \cos(x)\exp\left(-x^2/l_0^2\right)$$

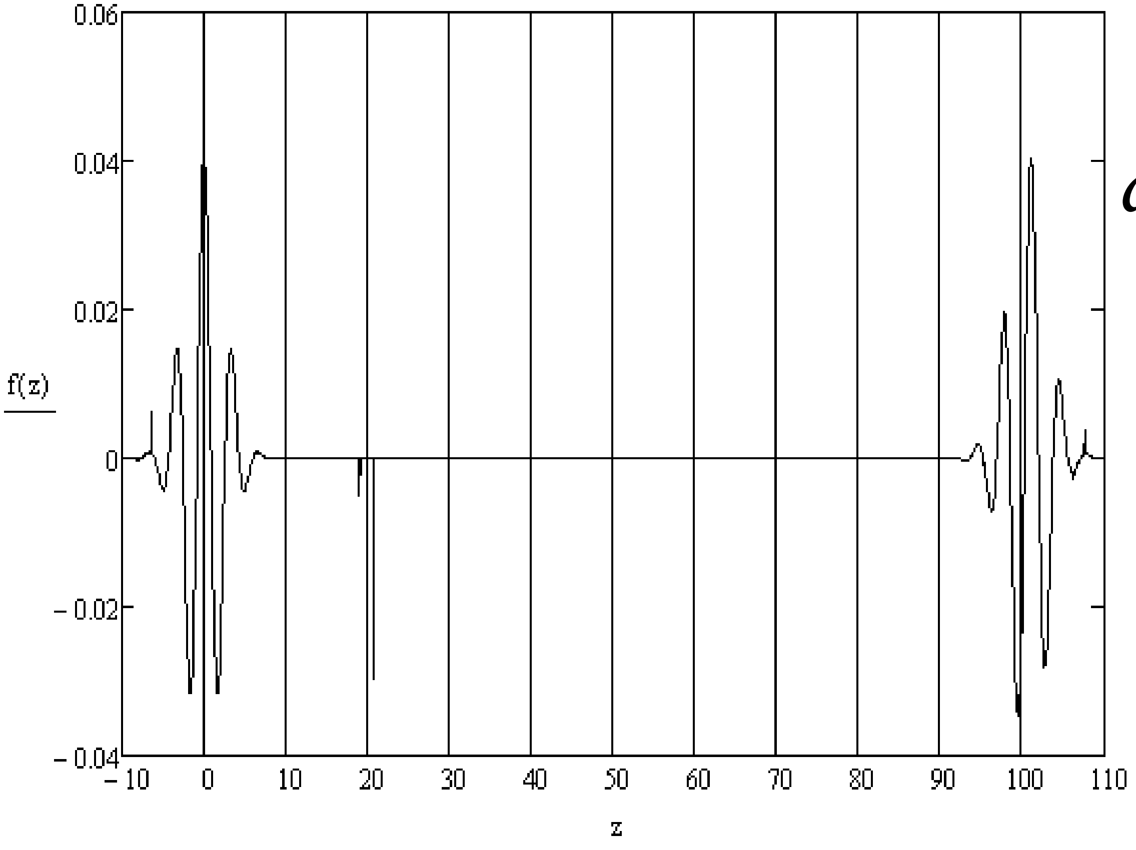
$$l_0 = 10 \qquad x = k_0 \Delta x \qquad l_0 = k_0/\Delta k$$

$$l_c = l_0/k_0 = 1/\Delta k$$

Take a Si plate and neutrons instead of the light then

$$r(k)=\frac{k-\sqrt{k^2-u}}{k+\sqrt{k^2-u}} \qquad R(k)=r(k)\frac{1-e^{2id\sqrt{k^2-u}}}{1-r(k)^2 e^{2id\sqrt{k^2-u}}} \qquad u=10^{-5}k^2$$

$$f(z)=c\int \exp\left(-\frac{(k-1)^2}{2\Delta k^2}\right)\mathrm{Re}\left(\exp(-2ikz)R(k)\right)$$



$$k_0=k_T \qquad dk_T=100$$

$$d=\frac{dk_T}{k_T}=\frac{180}{2\pi}\approx 30\ \overset{0}{\mathrm{A}}$$

$$\frac{\Delta k}{k_T}=0.2 \qquad l_0=\frac{k_T}{\Delta k}=5$$

$$l_c=\frac{l_0}{k_T}=\frac{5*1.8}{2\pi}=1.5\ \overset{0}{\mathrm{A}}$$

For light

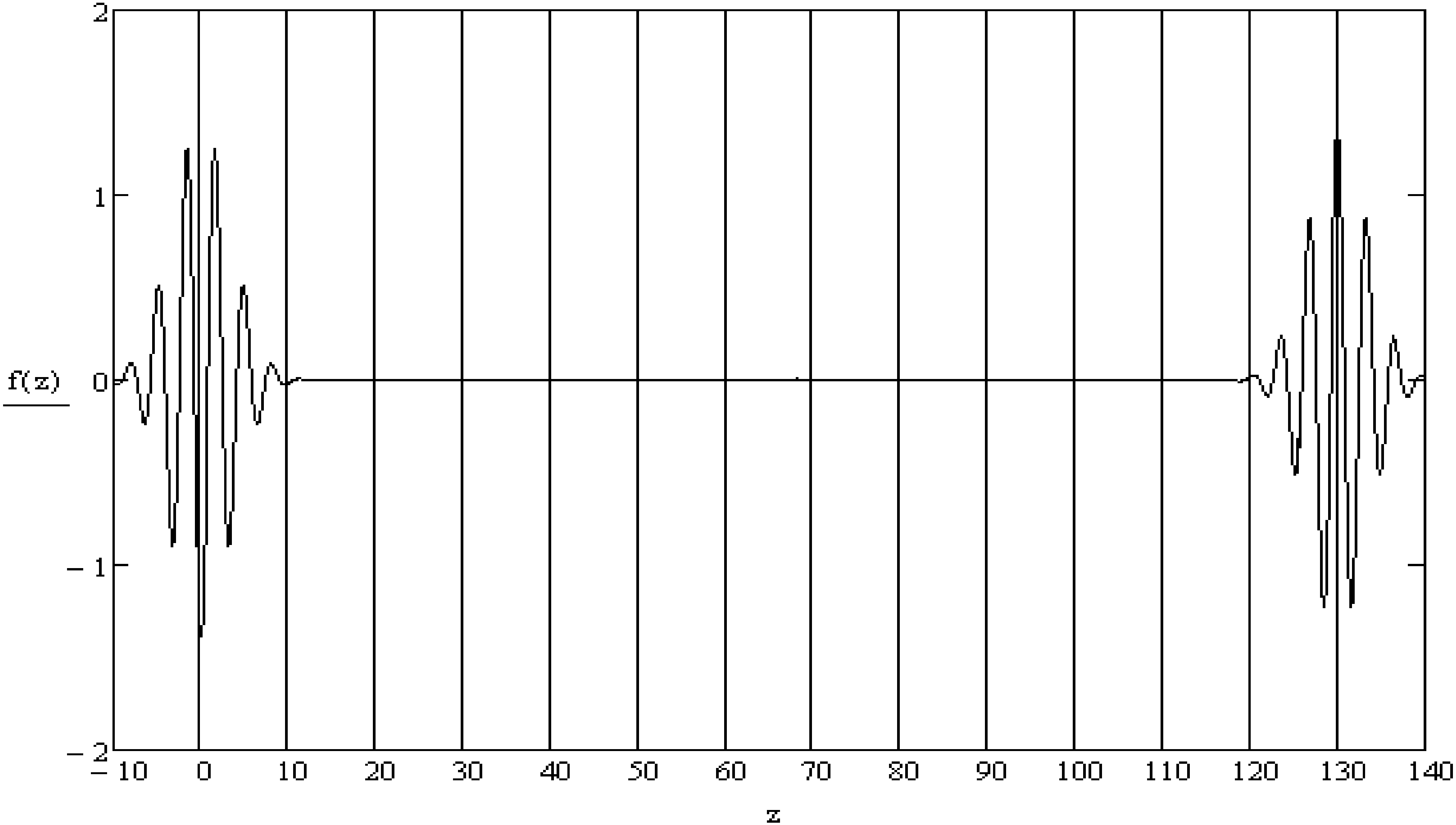
$$r = \frac{1-n}{1+n}$$

$n = 1.3$

$k' = nk$

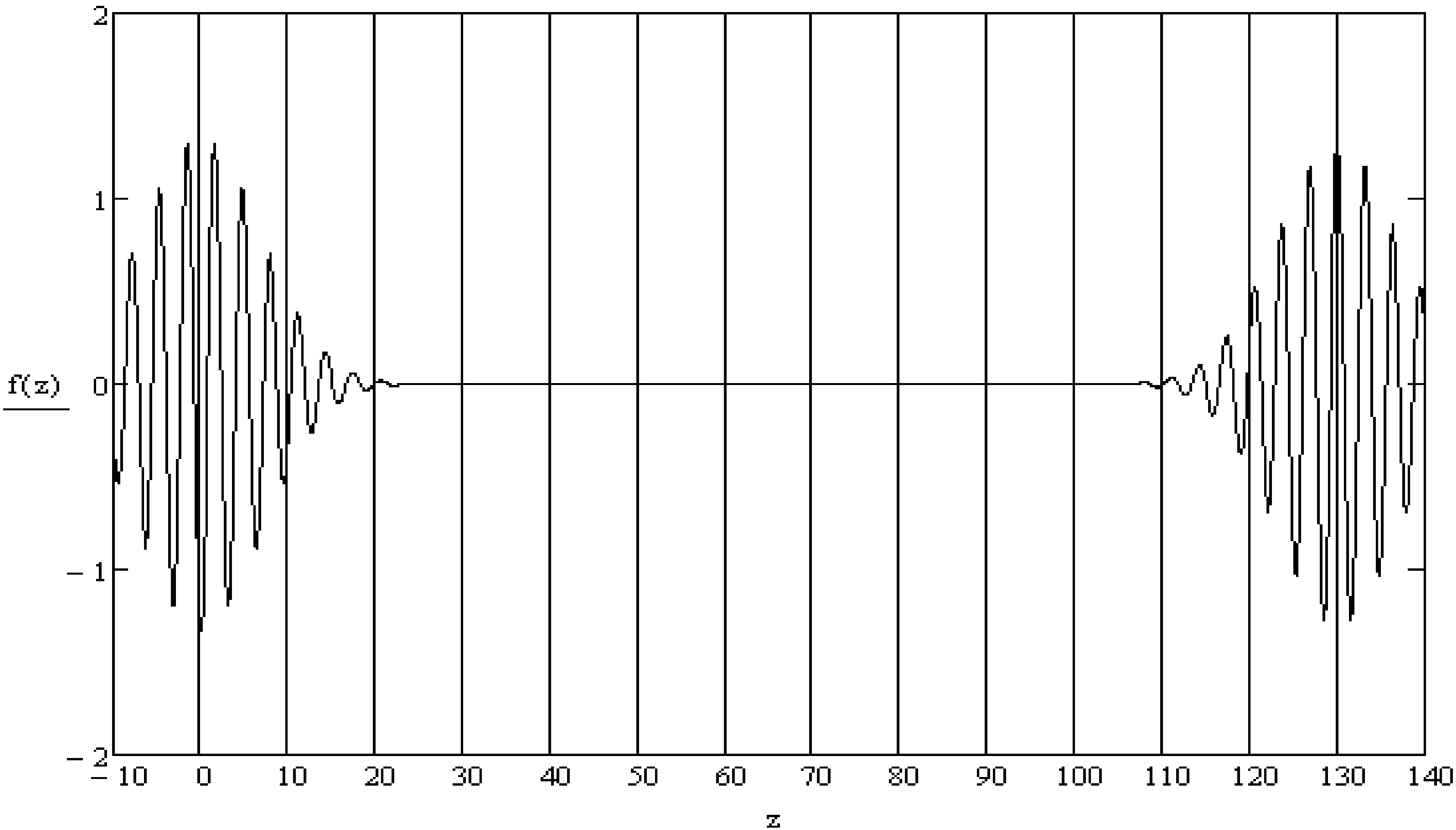
$$R(k) = r\,\frac{1-\exp(2idnk)}{1-r^2\exp(2idnk)}$$

$\Delta k = 0.2 \quad l_c = 1/\Delta k = 5 \qquad nd = 130$

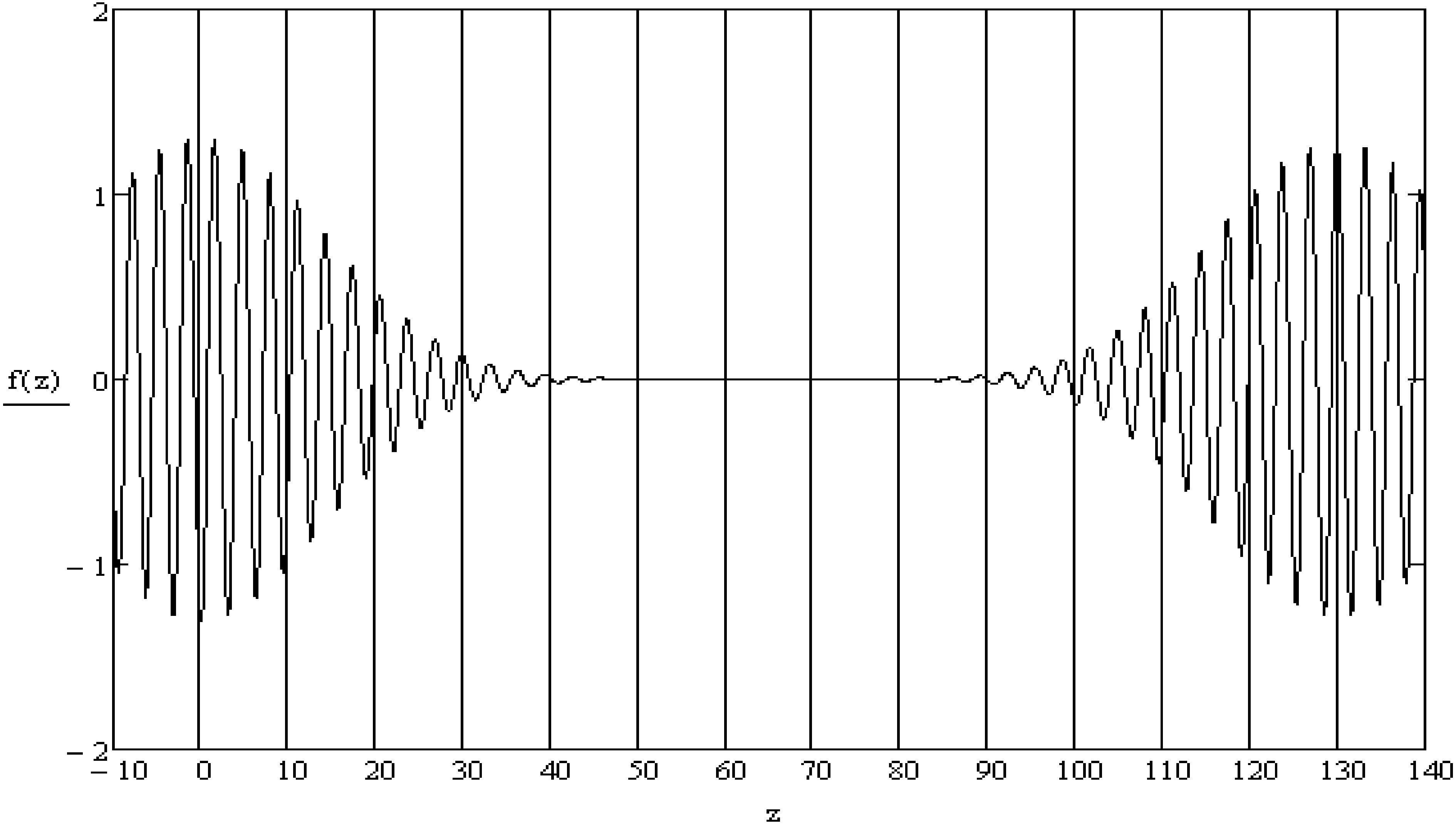


$\Delta k = 0.1$

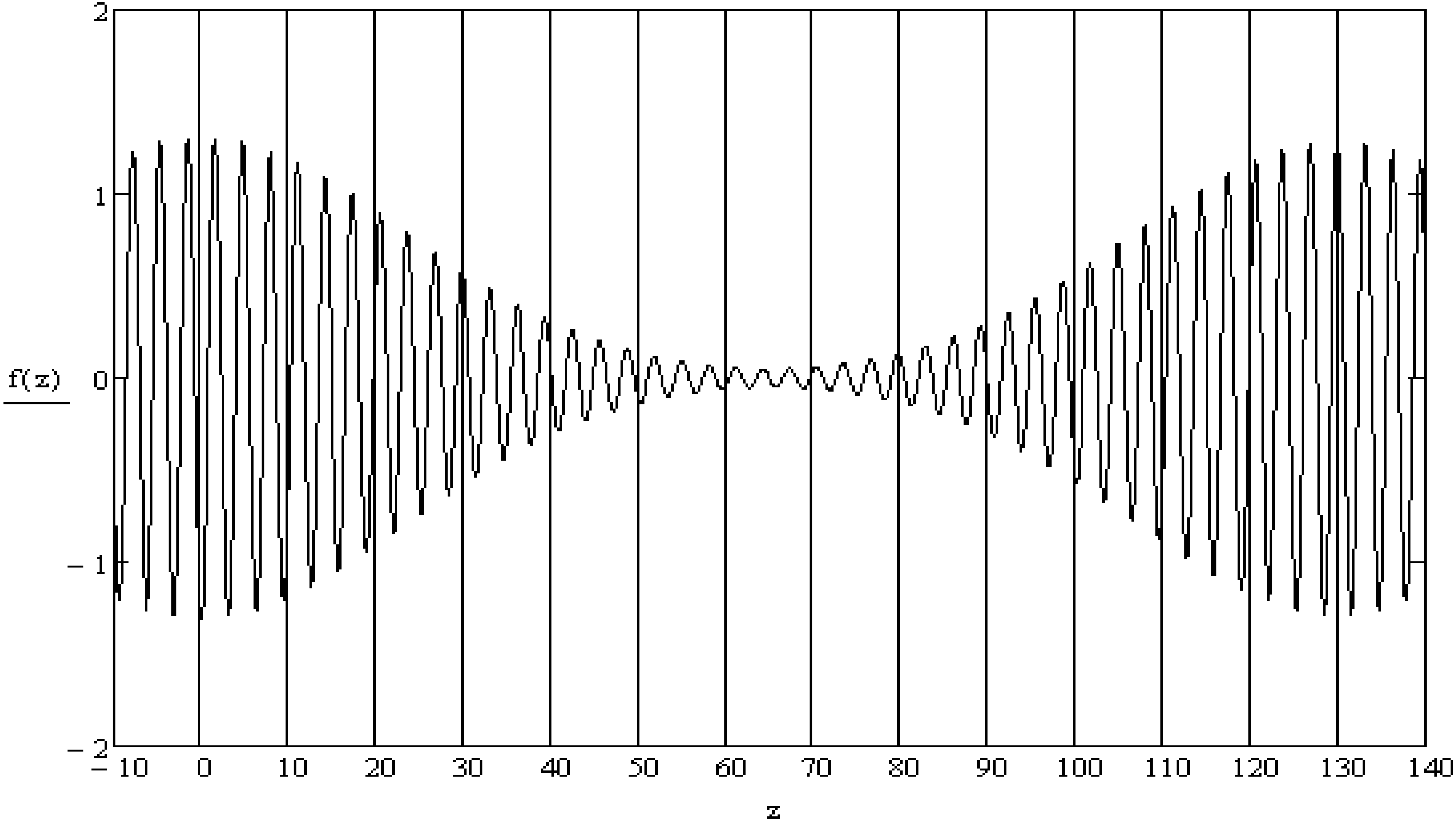
$l_c = 1/\Delta k = 10$



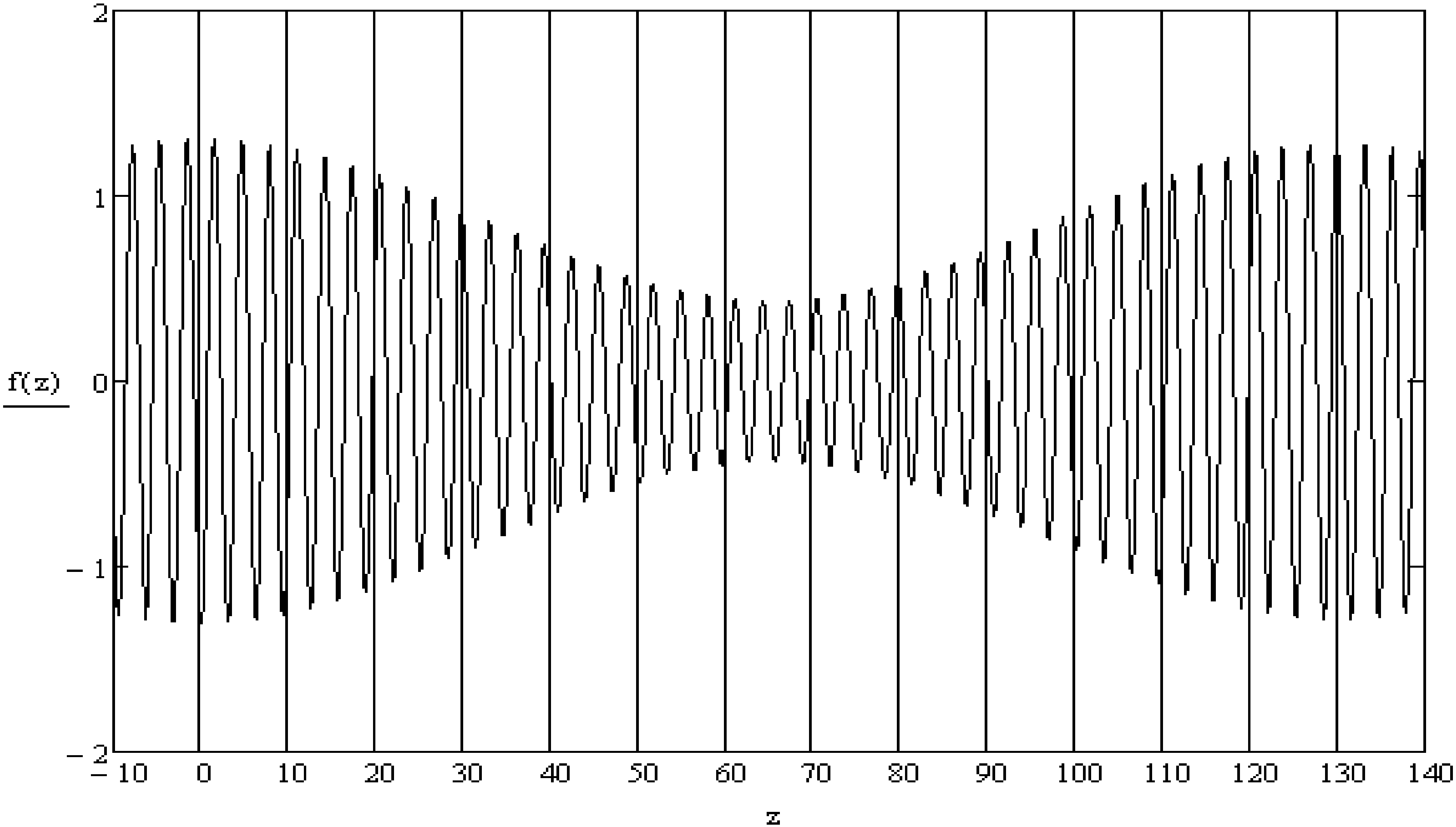
$$\Delta k = 0.05 \qquad l_c = 1/\Delta k = 20$$



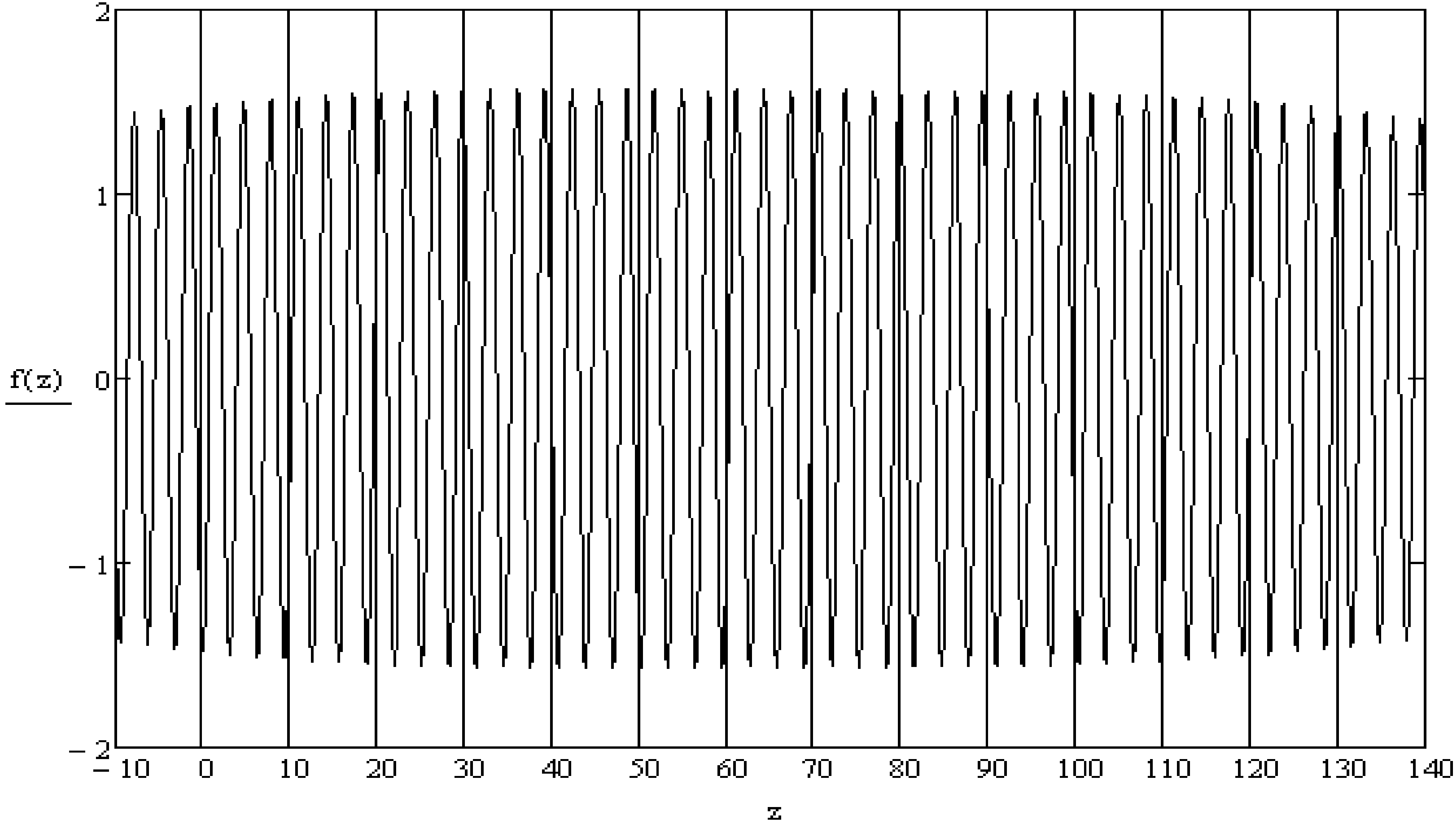
$\Delta k = 0.03$ $l_c = 1/\Delta k = 33$



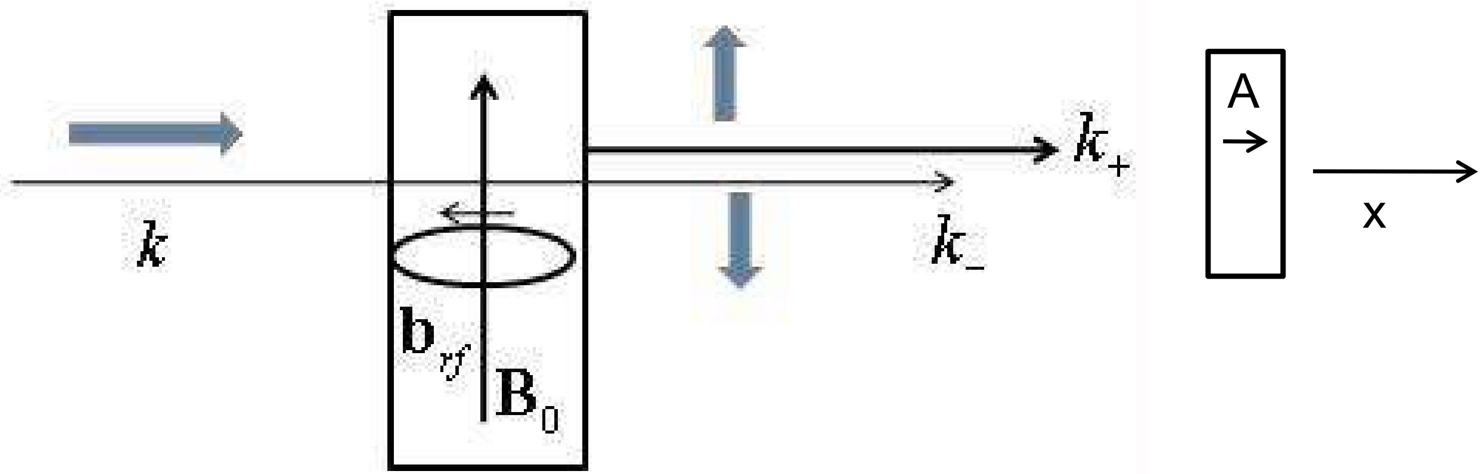
$\Delta k = 0.02 \qquad l_c = 1/\Delta k = 50$



$\Delta k = 0.01 \qquad l_c = 1/\Delta k = 100$



The second type of splitting with rf spin-flipper



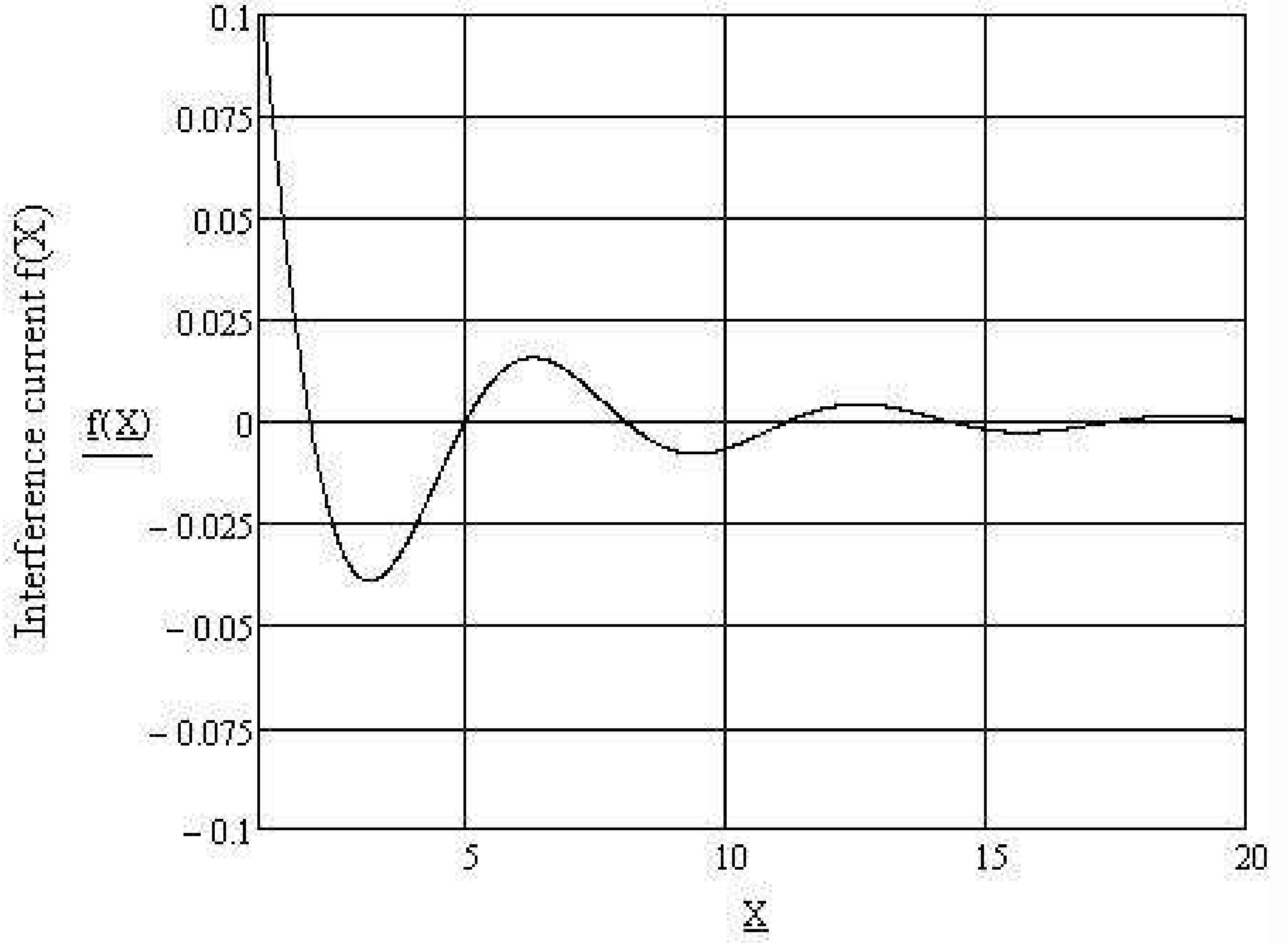
$$\mathbf{b}_{rf} = b_{rf}\left(\cos 2\omega t, \sin 2\omega t, 0\right)$$

$$k_{\pm} = k \pm \omega/k$$

$$\delta = \Delta k/k = \omega/k^2 \qquad x = \frac{X\lambda}{4\pi\delta^2}$$

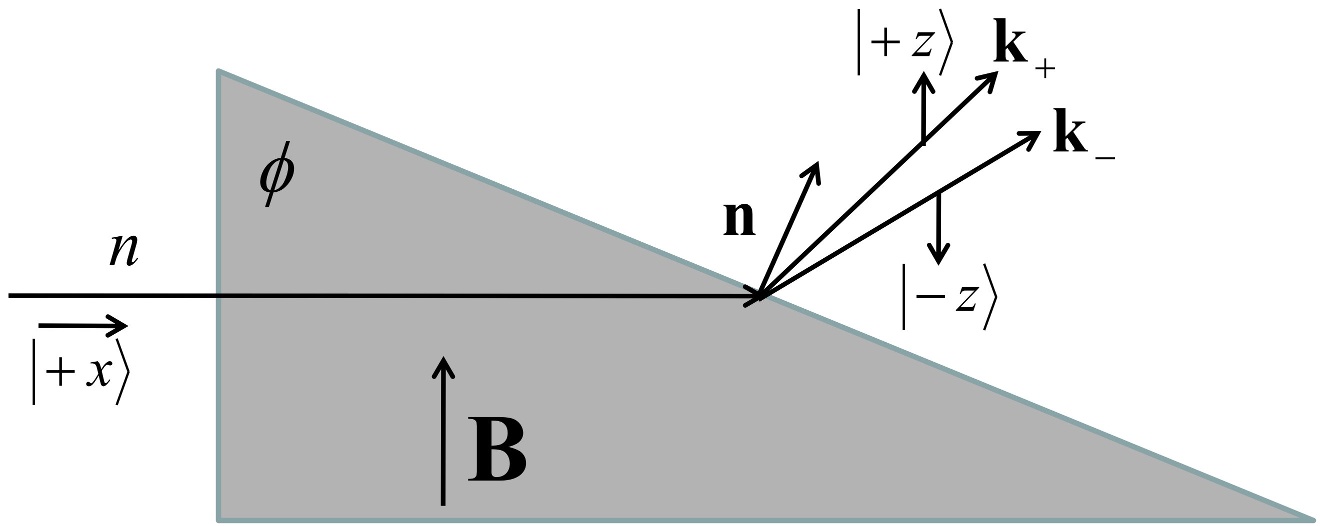
$$X = 1, \delta = 10^{-4} \Rightarrow x = \frac{\lambda}{4\pi}10^8 = 10\text{ cm} \Rightarrow 100\ \overset{0}{\text{A}} \Rightarrow v = 40\text{ m/s}$$

$$\delta = 10^{-4}, E = 10^{-5} \Rightarrow \omega = 1\,\text{MHz}$$



$\eta = 0.1$

$$X = 1, \theta = 10^{-4} \Rightarrow x = \frac{\lambda}{4\pi} 10^{8} = 10 \text{ cm} \Rightarrow 100 \overset{0}{\text{A}} \Rightarrow v = 40 \text{ m/s}$$



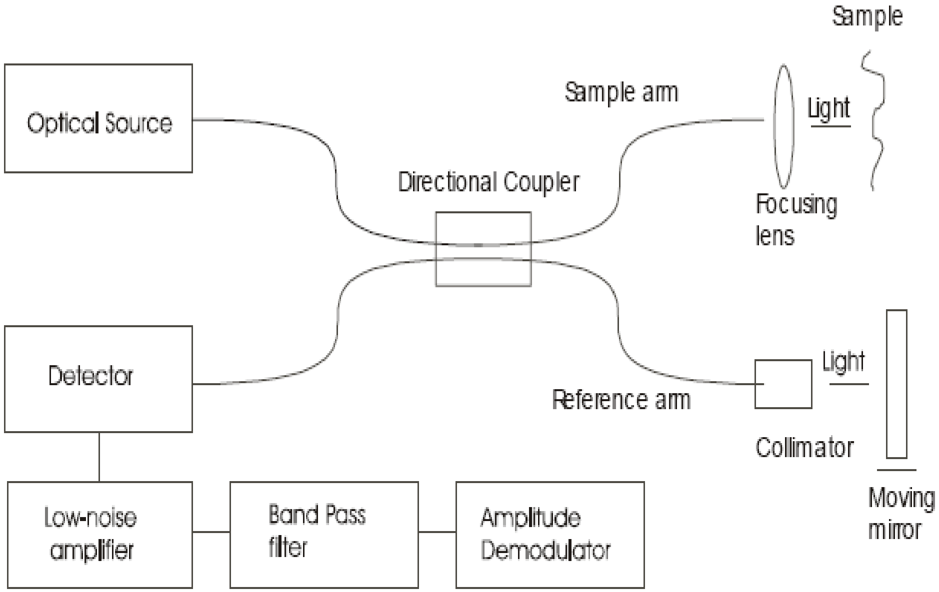
$$\theta^2 = \frac{(\mathbf{k}_+ - \mathbf{k}_-)^2}{(\mathbf{k}_+ + \mathbf{k}_-)^2}$$

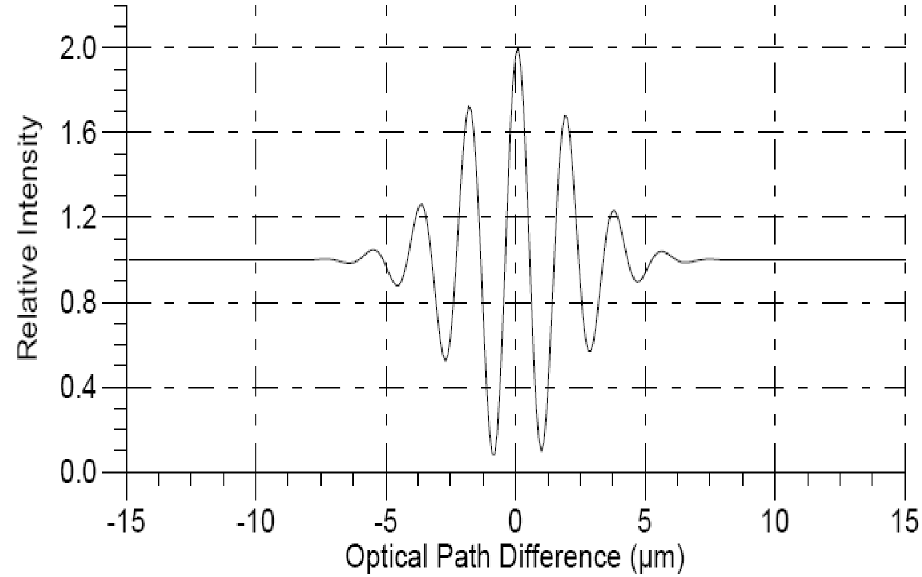
$$\left(\sqrt{(k^2-u_1)\cos^2\phi + u_1} - \sqrt{(k^2-u_2)\cos^2\phi + u_2}\right)^2 + \left(\sqrt{k^2-u_1} - \sqrt{k^2-u_2}\right)^2 \sin^2\phi = 4k_z^2$$

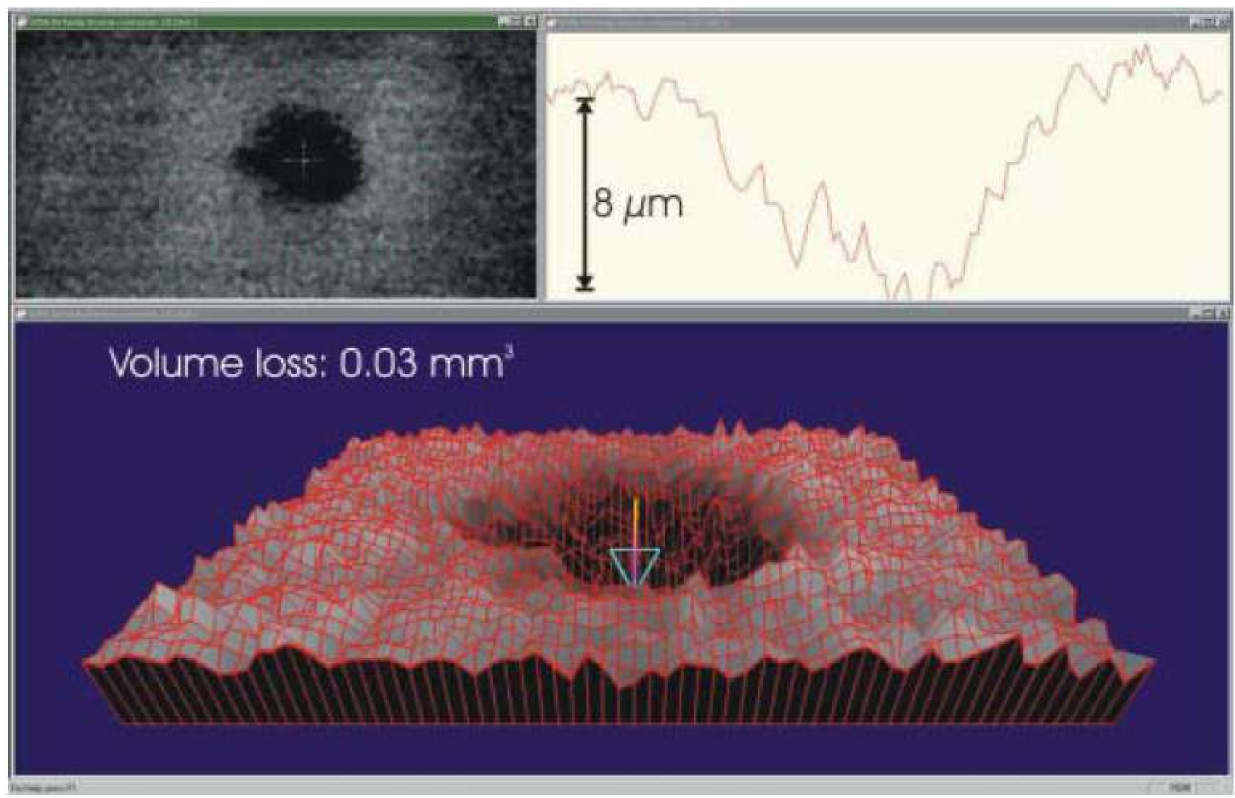
$$\left(\sqrt{k^2\cos^2\phi + u_1\sin^2\phi} + \sqrt{k^2\cos^2\phi + u_2\sin^2\phi}\right)^2 + \left(\sqrt{k^2-u_1} + \sqrt{k^2-u_2}\right)^2 \sin^2\phi = 4k_x^2$$

$$\frac{\left(\sqrt{k^2+u_1\tan^2\phi} - \sqrt{k^2+u_2\tan^2\phi}\right)^2 + \left(\sqrt{k^2-u_1} - \sqrt{k^2-u_2}\right)^2 \tan^2\phi}{\left(\sqrt{k^2+u_1\tan^2\phi} + \sqrt{k^2+u_2\tan^2\phi}\right)^2 + \left(\sqrt{k^2-u_1} + \sqrt{k^2-u_2}\right)^2 \tan^2\phi} = \theta^2$$

$$\frac{(\Delta u)^2(\tan^4\phi + \tan^2\phi)}{(4k^2)^2(1+\tan^2\phi)} = \theta^2 \qquad \theta = \frac{\Delta u}{4k^2}\tan\phi \approx 10^{-4} \qquad E = 10^{-5}, \theta = 10^{-4} \Rightarrow \Delta u \approx 10^{-8}$$







a surface of 10 mm X 10 mm

M.L.Dufour et al
Industrial Materials Institute
Boucherville, Quebec, Canada

Измерение параметров глаза