On the calculation of angular anisotropy of fragments from fission of nuclei by neutrons with energies up to 200 MeV

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A.S. Vorobyev et al. JETP Letters 102, 203 (2015): ^{232}Th, ^{238}U, ^{235}U

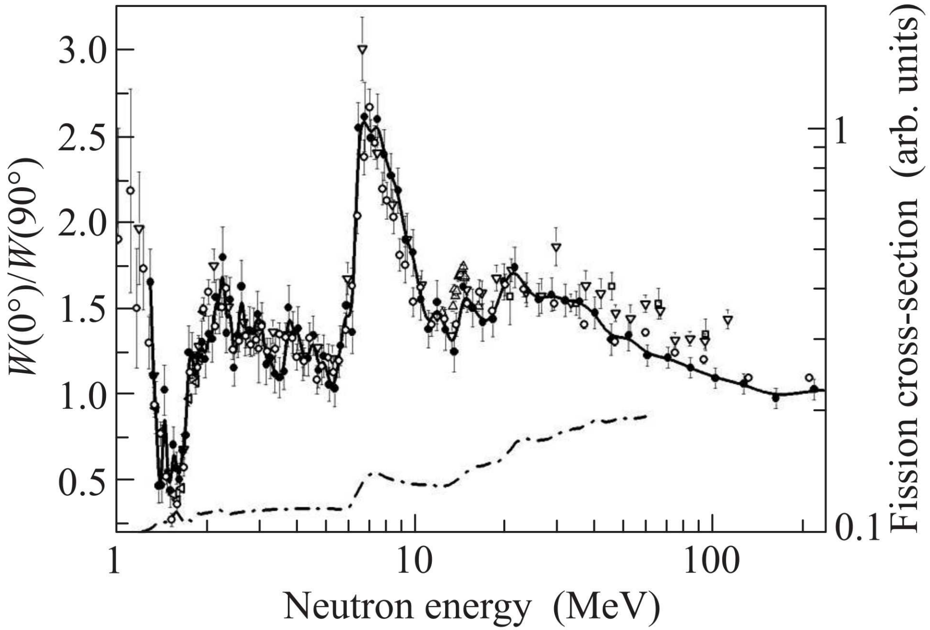


Fig. 3. Anisotropy of fission fragments of ^{232}Th: ▽ – [6], ◁ – [19], △ – [11], □ – [7], ○ – [8], ● – present data, -●- – fission cross-section [9]

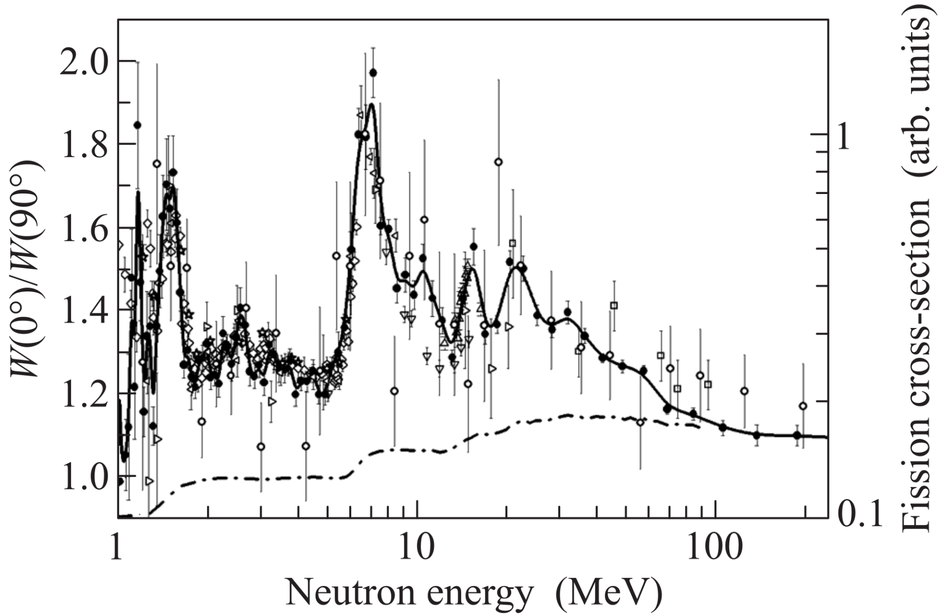


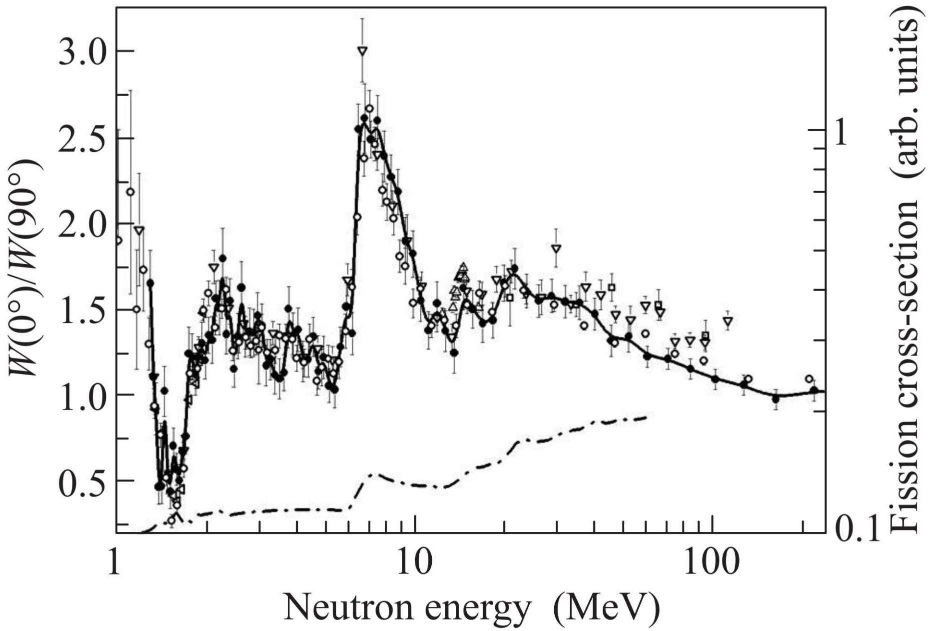
Fig. 5. Anisotropy of fission fragments of ^{238}U: ▽ – [12], ◁ – [14], ▷ – [16], △ – [11], ♢ – [17], ★ – [18], □ – [7], ○ – [8], ● – present data, -●- – fission cross-section [9]

Previous data for $E_n > 20$ MeV:

[7] I.V. Ryzhov et al. Nucl. Phys. A760, 19 (2005): quasi-monochromatic neutron beam

[8] L.S. Leong. PhD Thesis, CERN-Thesis-2013-254 (2013): n_TOF

[6] D. Tarrio et al. Nuclear Data Sheets, 119, 35 (2014): n_TOF



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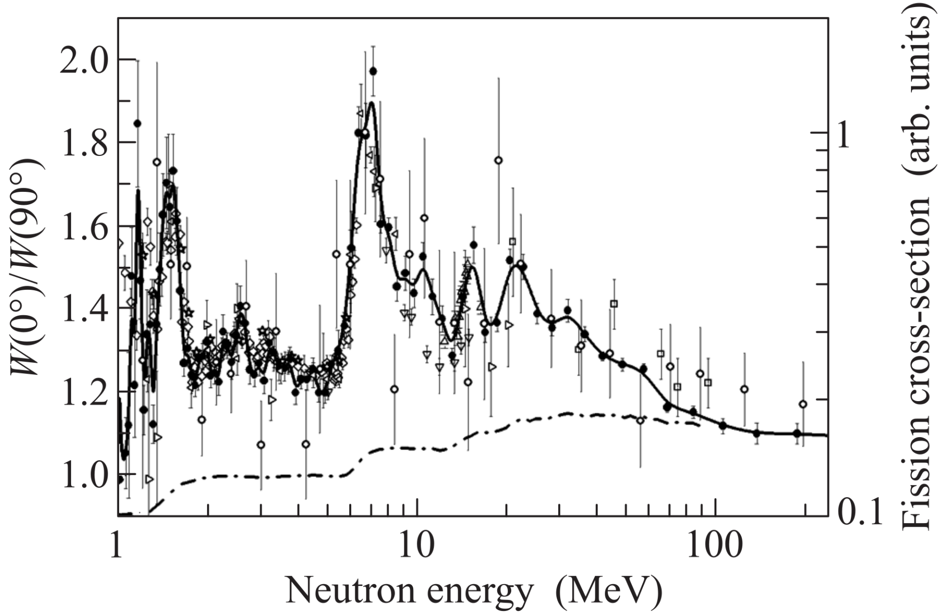

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Main results for ^{232}Th and ^{238}U:

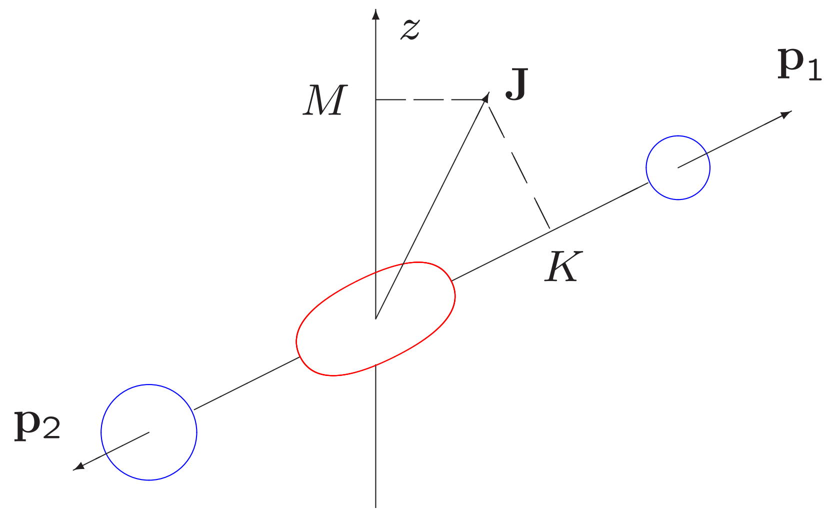
— an agreement with all previous data for $E_n < 20$ MeV,

— some disagreements with [6] and [7] for $E_n = 20 - 200$ MeV,

— the measured angular anisotropy is lower than the theoretical prediction from [7].

Angular distributions and
transition states on the fission barrier
(A. Bohr, 1955):

1) non-uniformity by M,
2) non-uniformity by K.



$$\Psi_J \sim \sum_M a_M(J) \sum_K g^{JK} \Phi_K(\tau) D^J_{MK}(\mathbf{n}_f)$$

$$\frac{dw(\mathbf{n}_f)}{d\Omega} \sim \int |\Psi_J|^2 \, d\tau \sim \sum_Q (2Q+1) \underbrace{\left(\sum_M C^{JM}_{JMQ0} |a_M(J)|^2\right)}_{\substack{|||\\ \tau_{Q0}(J)}} \underbrace{\left(\sum_K C^{JK}_{JKQ0} |g^{JK}|^2\right)}_{\substack{|||\\ b_Q(J) = \langle b_Q(J,K)\rangle}} P_Q(\cos\theta),$$

$\tau_{Q0}(J)$ — spin-tensor of orientation, $b_Q(J)$ — parameter of anisotropy, e.g.

$$b_2(J,K) \sim \tfrac{3K^2}{J(J+1)} - 1 \quad = \quad -1 \quad -0.75 \quad 0 \quad 1.25$$

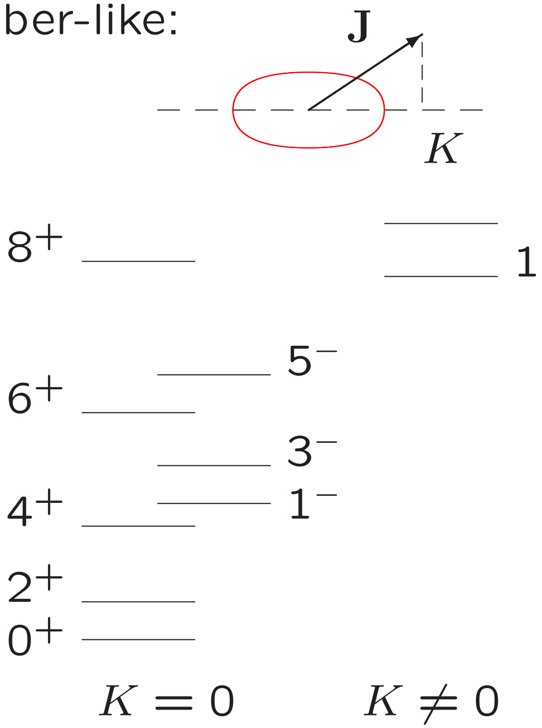
$$J = 3, \quad K \quad = \quad 0 \quad 1 \quad 2 \quad 3$$

Fission fragment's angular anisotropy is of interest both from academic and applied points of view

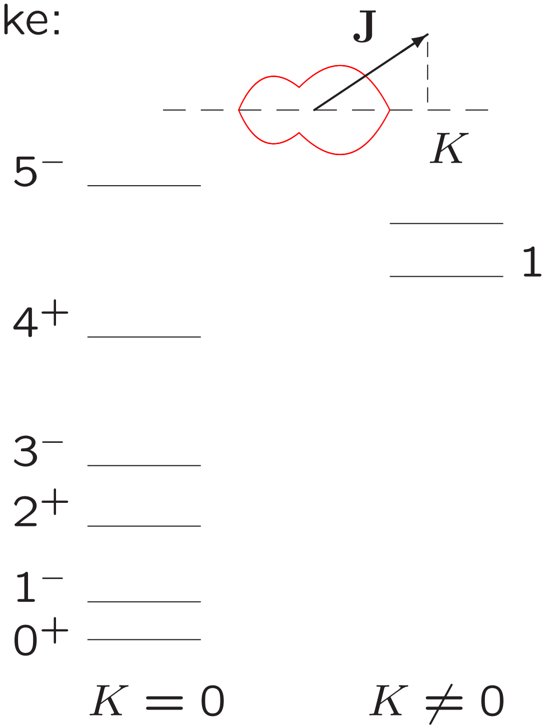
1) Sensitivity to the transition states at the fission barriers (to the symmetry of the nucleus on the barrier).

A. Bohr, 1955 (simplified model):

cucumber-like:

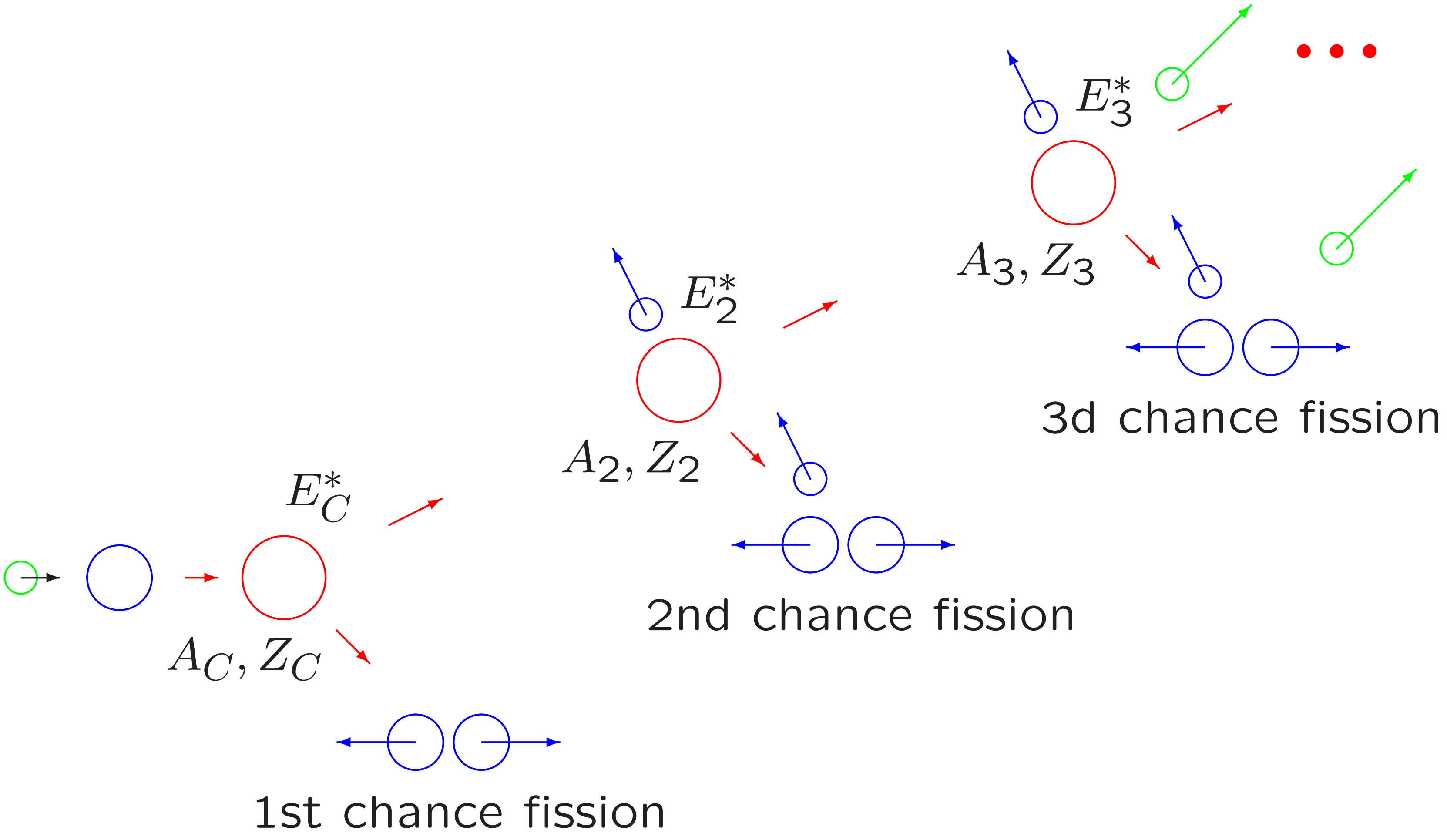


pear-like:



The lower is the nuclear excitation energy the greater is the angular anisotropy.

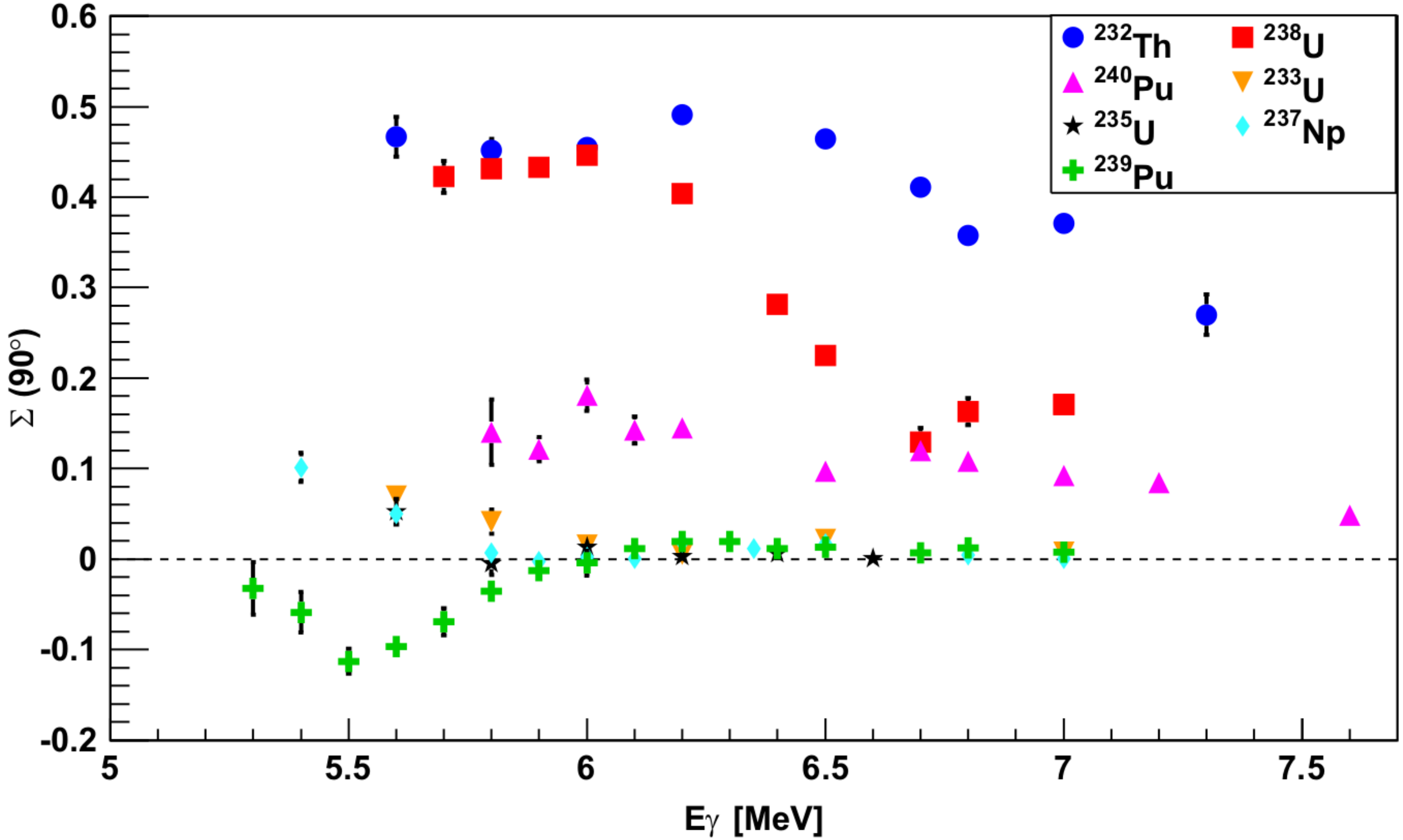
2) Sensitivity to multichance fission, i.e. to fission after neutron (or proton . . .) emission . . . An instrument to study nuclear cascades (branchings, transition probabilities, equilibrium and non-equilibrium processes) . . .



particle: γ, n, p, d, t=^{3}H, h=^{3}He, α, . . .

3) Sensitivity to the fissioning isotopes . . . The prompt neutron’s angular anisotropy may be used:

J.M.Mueller, M.W.Ahmed, H.R.Weller. A novel method to assay special nuclear materials by measuring prompt neutrons from polarized photofission. — NIMA, 2014, v. 754, p. 57вЂ“62.



TALYS-1.8

New Edition December 26, 2015

A nuclear reaction program

Talys is a computer code system for the analysis and prediction of nuclear reactions.

The basic objective is the simulation of nuclear reactions that involve neutrons, photons, protons, deuterons, tritons, 3He- and alpha-particles, in the 1 keV – 200 MeV energy range and for target nuclides of mass 12 and heavier.

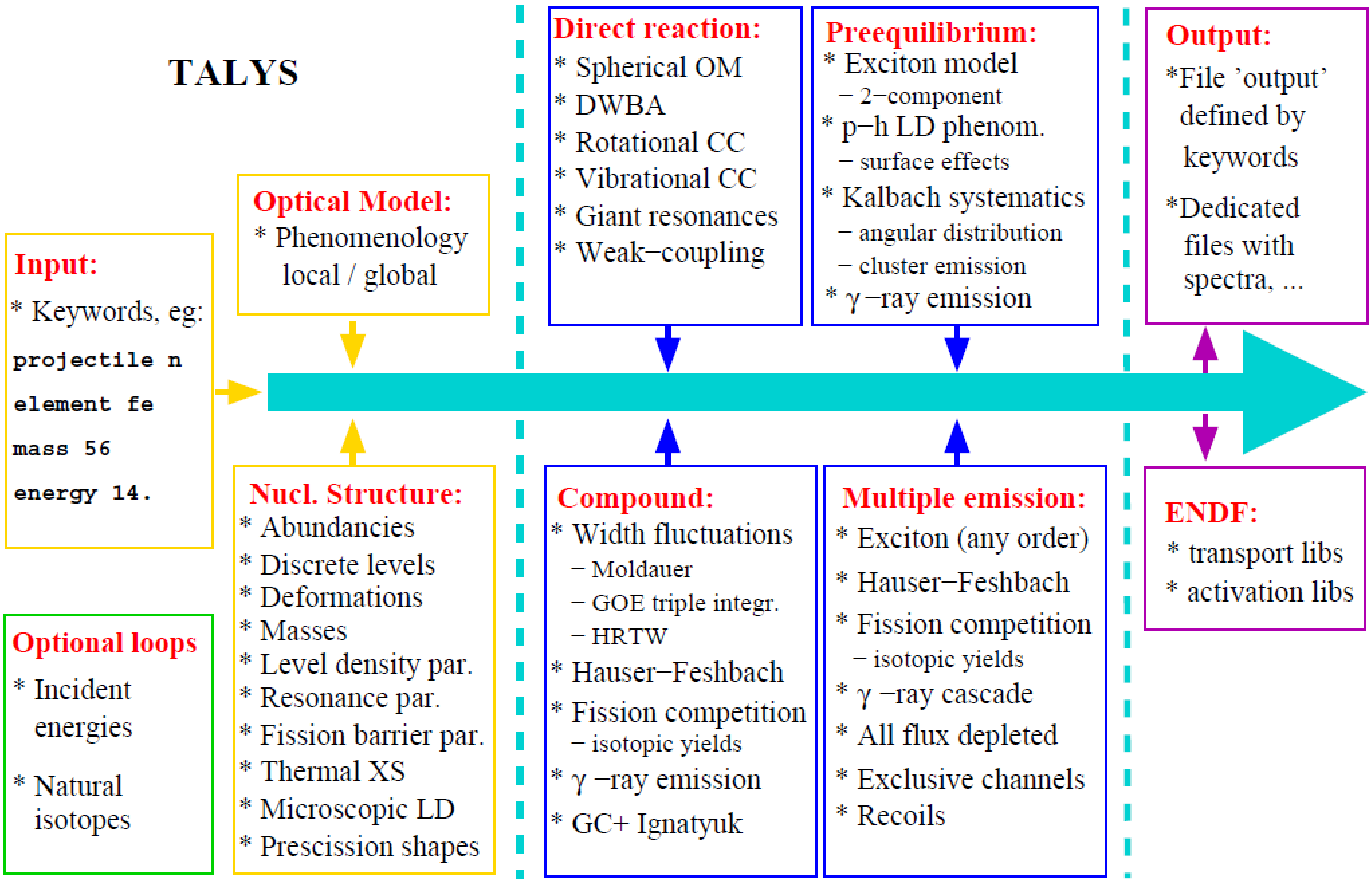
Free use, open software, always under development: from TALYS-1.0 — December 2007 to TALYS-1.8 — December 2015.

User Manual

More than 300 subroutines, more than 100 000 lines (commands), more than 500 pages in the Manual.

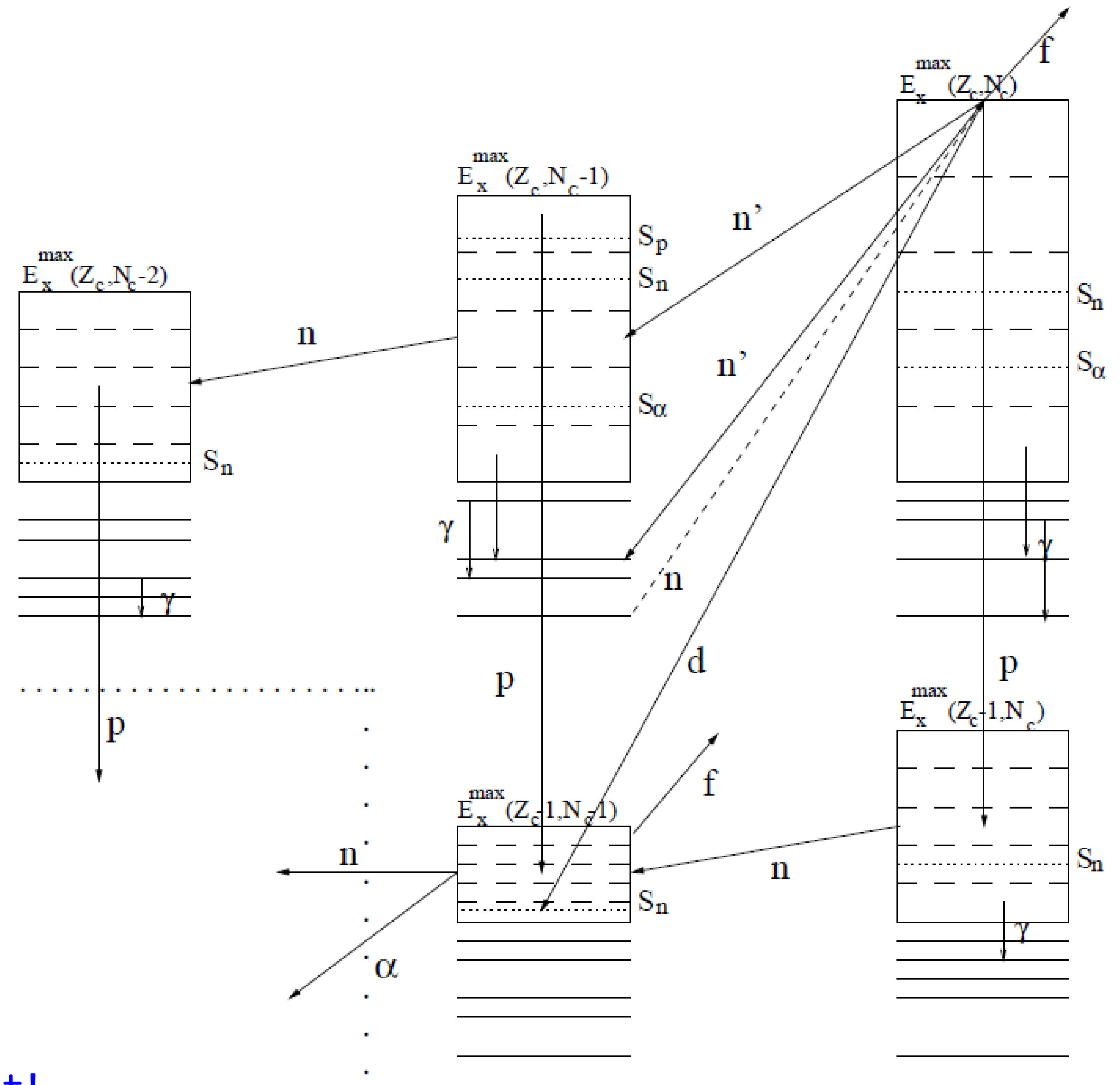
Arjan Koning
Stephane Hilaire
Stephane Goriely

Completely integrated optical model and coupled-channels calculations by the ECIS-06 code,



TALYS
Input:
* Keywords, eg:
projectile n
element fe
mass 56
energy 14.
Optional loops
* Incident energies
* Natural isotopes
Optical Model:
* Phenomenology local / global
Nucl. Structure:
* Abundancies
* Discrete levels
* Deformations
* Masses
* Level density par.
* Resonance par.
* Fission barrier par.
* Thermal XS
* Microscopic LD
* Prescission shapes
Direct reaction:
* Spherical OM
* DWBA
* Rotational CC
* Vibrational CC
* Giant resonances
* Weak−coupling
Compound:
* Width fluctuations
− Moldauer
− GOE triple integr.
− HRTW
* Hauser−Feshbach
* Fission competition
− isotopic yields
* γ −ray emission
* GC+ Ignatyuk
Preequilibrium:
* Exciton model
− 2−component
* p−h LD phenom.
− surface effects
* Kalbach systematics
− angular distribution
− cluster emission
* γ −ray emission
Multiple emission:
* Exciton (any order)
* Hauser−Feshbach
* Fission competition
− isotopic yields
* γ −ray cascade
* All flux depleted
* Exclusive channels
* Recoils
Output:
*File ’output’ defined by keywords
*Dedicated files with spectra, ...
ENDF:
* transport libs
* activation libs

All partial cross sections can be found, due to



the calculation of
all transition probabilities:

$w(i \to i')$, where

$i \equiv (Z_i, N_i, E_i^*, J_i, \pi_i)$

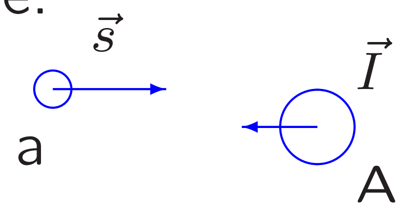
But!

— angular distributions — only for the first step reaction: a+A $\to$ C$\to$ b + B

— angular distribution for fission fragments (even for the first step or first chance) can not be calculated!

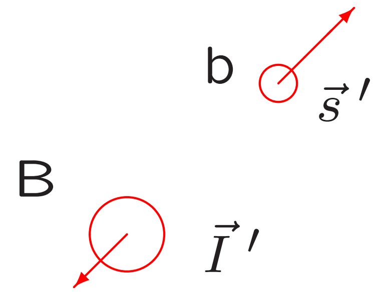
Angular distribution for a+A $\rightarrow$ C$\rightarrow$ b + B

before:



$\vec{l}, \quad \vec{j} = \vec{s} + \vec{l}, \quad \vec{J} = \vec{I} + \vec{j}$

after:



$\vec{l}\,', \quad \vec{j}\,' = \vec{s}\,' + \vec{l}\,', \quad \vec{J} = \vec{I}\,' + \vec{j}\,'$

TALYS also computes the compound nucleus formula for the angular distribution. It is given by

$$\frac{d\sigma^{comp}_{\alpha\alpha'}(\theta)}{d\Omega} = \sum_L C^{comp}_L P_L(\cos\Theta), \tag{4.180}$$

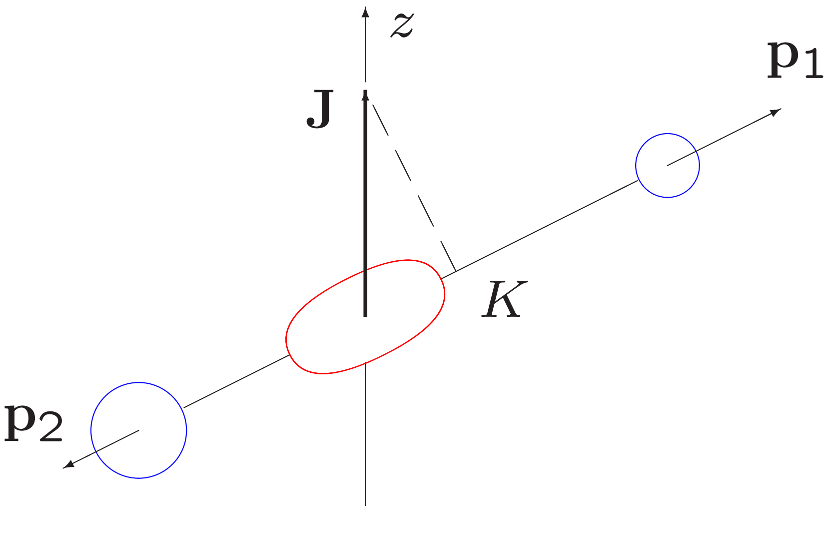
where P_L are Legendre polynomials. The Legendre coefficients C^{comp}_L are given by

$$\begin{aligned} C^{comp}_L &= D^{comp}\frac{\pi}{k^2}\sum_{J,\Pi}\frac{2J+1}{(2I+1)(2s+1)}\sum_{j=|J-I|}^{J+I}\sum_{l=|j-s|}^{j+s}\sum_{j'=|J-I'|}^{J+I'}\sum_{l'=|j'-s'|}^{j'+s'} \\ &\times \quad \delta_\pi(\alpha)\delta_\pi(\alpha')\frac{T^J_{\alpha lj}(E_a)\left\langle T^J_{\alpha' l' j'}(E_{a'})\right\rangle}{\sum_{\alpha'',l'',j''}\delta_\pi(\alpha'')\left\langle T^J_{\alpha'' l'' j''}(E_{a''})\right\rangle} W^J_{\alpha lj\alpha' l' j'} A^J_{IljI'l'j';L}, \end{aligned} \tag{4.181}$$

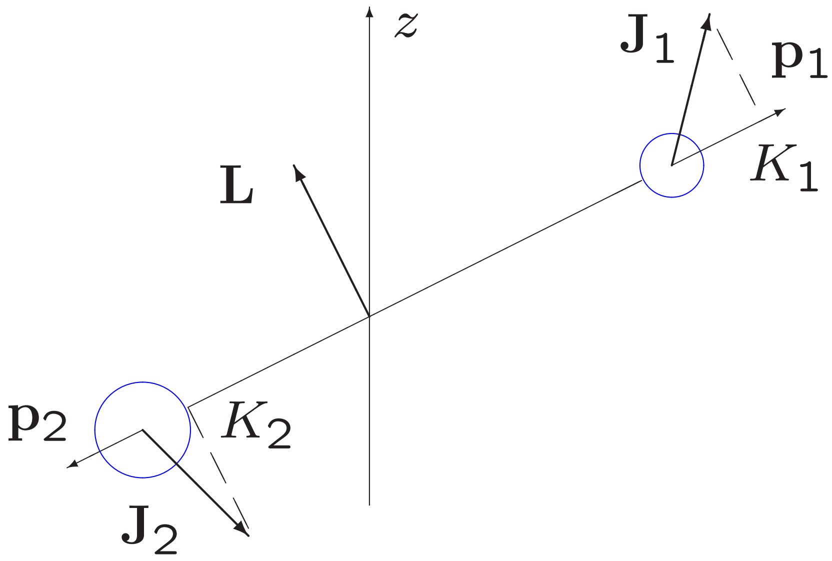
where the Blatt-Biedenharn factor A is given by

$$\begin{aligned} A^J_{IljI'l'j';L} &= \frac{(-1)^{I'-s'-I+s}}{4\pi}(2J+1)(2j+1)(2l+1)(2j'+1)(2l'+1) \\ &(ll00|L0)\,\mathcal{W}(JjJj;IL)\mathcal{W}(jjll;Ls)\,(l'l'00|L0)\,\mathcal{W}(Jj'Jj';I'L)\mathcal{W}(j'j'l'l';Ls'), \end{aligned} \tag{4.182}$$

where $(\quad|\quad)$ are Clebsch-Gordan coefficients and $\mathcal{W}$ are Racah coefficients.



A. Bohr, 1955

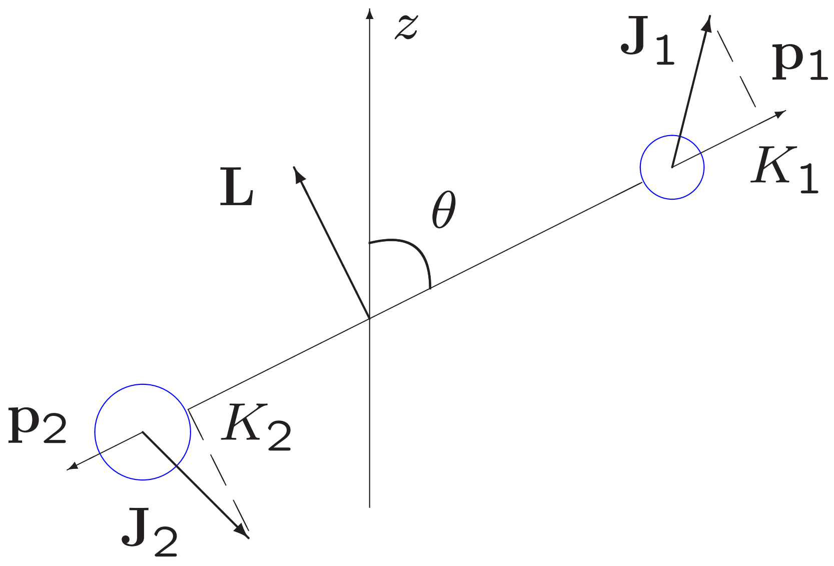


V.M. Strutinsky, 1956

$$
\begin{array}{ccccccccc}
\mathbf{J} & \rightarrow & \mathbf{J}_1 & + & \mathbf{J}_2 & + & \mathbf{L} \\
K & \rightarrow & K_1 & + & K_2 & &
\end{array}
$$

K_1 and K_2 — fragment's helicities

Helicity representation in fission channels (A.Barabanov and W.Furman, 1997):



$$\Psi_J \to \frac{e^{ik_\alpha r}}{r} \sum_M a_M(J) \sum_{LF} (-i)^{L+1} g^\alpha(LF) \sum_{\nu m} C^{JM}_{F\nu Lm}\, \chi^\alpha_{F\nu}\, i^L Y_{Lm}(\mathbf{n}_\alpha)$$

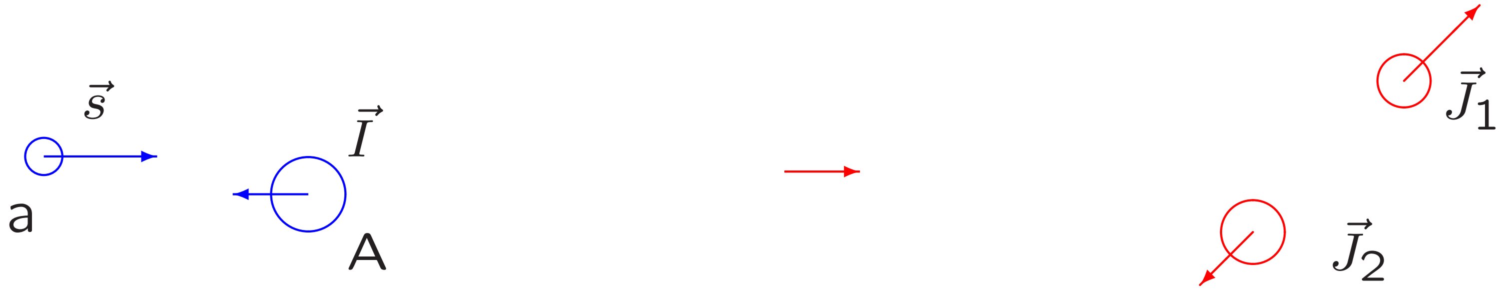
$$\chi^\alpha_{F\nu} = \sum_K D^F_{\nu K}(\mathbf{n}_\alpha)\chi^\alpha_{FK}$$

$$\downarrow$$

$$\frac{e^{ik_\alpha r}}{r} \sum_M a_M(J) \sum_{FK} g^\alpha(FK)\, \underbrace{\chi^\alpha_{FK} D^J_{MK}(\mathbf{n}_\alpha)}_{=\ \varphi^\alpha_{FKJM}}$$

$$g^\alpha(LF) = \sqrt{\frac{2L+1}{2J+1}}\, \sum_K C^{JK}_{FKL0}\, g^\alpha(FK), \qquad g^\alpha(FK) = \sum_L \sqrt{\frac{2L+1}{2J+1}}\, C^{JK}_{FKL0}\, g^\alpha(LF)$$

Angular distribution for fission

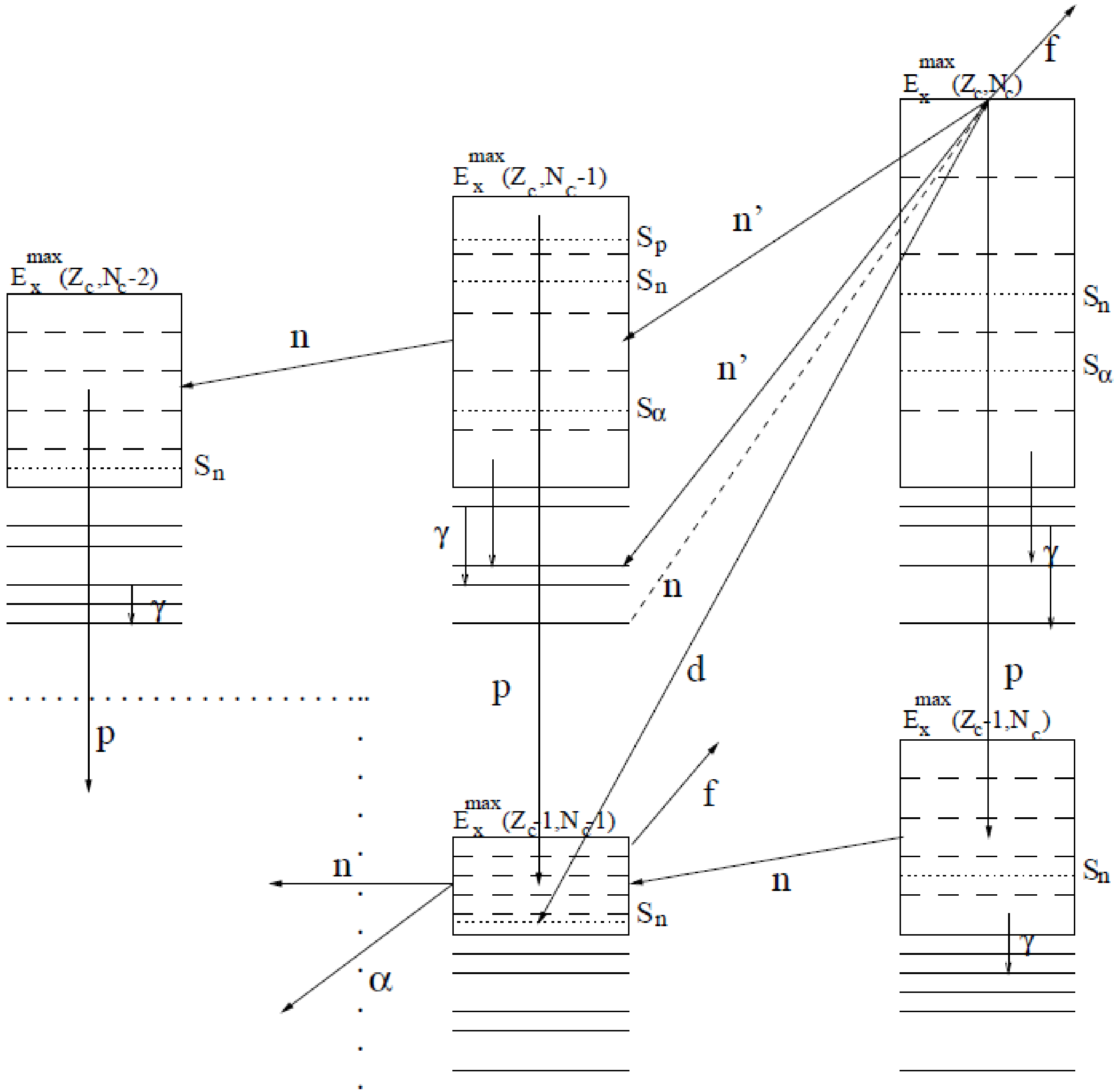


$$\vec{l}, \quad \vec{j} = \vec{s} + \vec{l}, \quad \vec{J} = \vec{I} + \vec{j} \qquad\qquad \vec{L}, \quad \vec{F} = \vec{J}_1 + \vec{J}_2, \quad \vec{J} = \vec{L} + \vec{F}$$

$$\frac{d\sigma_f}{d\Omega} = \sum_{J\pi} \sigma_f(J^\pi) \frac{dw(J^\pi, \mathbf{n})}{d\Omega}, \qquad \frac{dw(J^\pi, \mathbf{n})}{d\Omega} = \frac{1}{4\pi} \sum_Q (2Q+1)\, \tau_{Q0}(J^\pi)\, b_Q(J^\pi)\, P_Q(\cos\theta),$$

$$b_Q(J^\pi) = \sum_K C^{JK}_{JKQ0}\, \beta_K(J^\pi), \qquad \beta_K(J^\pi) = \frac{1}{\Gamma^f_{J\pi}} \sum_\alpha \sum_F \langle\, |g^\alpha(FKJ^\pi)|^2 \,\rangle$$

$$\tau_{Q0}(J^\pi) = \frac{\sum_{lj} T^{J\pi}_{lj} C^{l0}_{l0Q0}\, U(sjlQ, lj)\, U(IjJQ, Jj)}{\sum_{lj} T^{J\pi}_{lj}}$$



What should be added to TALYS:

Transition probabilities:

$w(i \to i')$, where

$i \equiv (Z_i, N_i, E_i^*, J_i, \pi_i, M_i)$

or spin-orientation transfer:

$\tau_{Q0}(J_i) \to \tau_{Q0}(J_{i'})$, where

$i \equiv (Z_i, N_i, E_i^*, J_i, \pi_i)$

Summary

1. TALYS seems to be an appropriate code for the simulation of heavy nuclei fission by neutrons with energy up to 200 MeV. But the current version of TALYS do not give the possibility to analyze angular distribution of fission fragments.

2. TALYS may be extended to take into account the spin-tensors of orientation of all nucler states involved into the reaction. The mechanism of orientation transfer is clear for the usual compound-nuclear mechanisms of particle emission.

3. Pre-equilibrium particle emission need special attention. Only the use of consistent quantum mechanical models for pre-equilibrium processes provides reliable evaluation for spin-tensors of orientation.