# T-INVARIANCE AND NUCLEAR FISSION

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## 1.INTRODUCTION

The aim of this article is to demonstrate that **the using of the T-invariance condition** for manyparticles multistep nuclear reaction $a \to b$, in which among $n_a (n_a \geq 2)$ and $n_b (n_b \geq 2)$ particles of initial $a$ and final $b$ channels there are the different elementary particles and atomic nuclei, **allows to distinguish the possible mechanisms of the investigated reaction and to throw the mechanisms violating the T-invariance of the analyzed quantum system**. This statement will be used for the analysis of mechanisms of the appearance of anisotropies with different P- and T-parities in differential cross sections of nuclear reactions of the binary and ternary fission of oriented nuclei-targets by the cold polarized neutrons.

## 2. THE T-INVARIANCE CONDITION FOR QUANTUM MECHANICAL SYSTEMS

The T-invariance of the quantum system with the time independent Hamiltonian $\mathbf{H}(\xi)$, where $\xi$ is the total set of spatial, spin and analogous coordinates of this system, consists [E. P. Wigner, Gottingen Nachrichten, **51**, 546 (1932); M. L. Goldberger, K. M. Watson, *Collision theory* (Wiley J. & Son Inc., 1964)] in that for any possible states of the investigated system with the wave functions $\Psi(\xi,t)$ being the solutions of Schrödinger equation:

$$ih\frac{\partial}{\partial t}\Psi(\xi,t)=\mathbf{H}\Psi(\xi,t), \tag{1}$$

there are time-reversed states with the wave functions $\bar{\Psi}(\xi,t)$ being the solutions of the same Schrödinger equation (1). The wave functions $\bar{\Psi}(\xi,t)$ are connected [E. P. Wigner, Gottingen Nachrichten, **51**, 546 (1932); M. L. Goldberger, K. M. Watson, *Collision theory* (Wiley J. & Son Inc., 1964)] with the initial wave functions $\Psi(\xi,t)$ as

$$\bar{\Psi}(\xi,t) = \mathbf{T}\Psi(\xi,-t), \tag{2}$$

with the operator of time reversion **T**:

$$\mathbf{T} = \mathbf{OK}, \tag{3}$$

where **K** is the operator of the complex conjugation, **O** is the unitary operator satisfactory the condition for complex **H**:

$$\mathbf{OH}^{*}\mathbf{O}^{+} = \mathbf{H}. \tag{4}$$

For the wave functions $\Psi(\xi)$ of stationary states satisfactory the stationary Schrödinger equation:

$$\mathbf{H}\Psi(\xi) = E\Psi(\xi) \tag{5}$$

there are the time-reversed wave functions $\bar{\Psi}(\xi)$ being solutions of the same Schrödinger equation (5) and connected with $\Psi(\xi)$ by condition (2).

Using the formulae (2, 3), the matrix element $\langle \Psi_2(\xi) | \mathbf{Q} | \Psi_1(\xi) \rangle$ for arbitrary operator $\mathbf{Q}$ can be represented as

$$\langle \Psi_2(\xi) | \mathbf{Q} | \Psi_1(\xi) \rangle = \langle \bar{\Psi}_1(\xi) | \bar{\mathbf{Q}} | \bar{\Psi}_2(\xi) \rangle, \tag{6}$$

where $\bar{\mathbf{Q}}$ is the time-reversed operator $\mathbf{Q}$:

$$\bar{\mathbf{Q}} = \mathbf{T}\mathbf{Q}^{+}\mathbf{T}^{-1}. \tag{7}$$

In this case the time-reversed Hermitian operators of coordinate $\mathbf{r}$, moment $\mathbf{p}$, orbital moment $\mathbf{L}$ and spin $\mathbf{s}$ of the particle are expressed as

$$\bar{\mathbf{r}} = \mathbf{r};\ \bar{\mathbf{p}} = -\,\mathbf{p};\ \bar{\mathbf{L}} = -\,\mathbf{L};\ \bar{\mathbf{s}} = -\,\mathbf{s}. \tag{8}$$

The operator $\mathbf{Q}$ is invariant to the time-reversion if

$$\bar{\mathbf{Q}} = \mathbf{Q}. \tag{9}$$

## 3. THE CONDITIONS OF T-INVARINCE FOR AMPLITUDES OF MANYPARTICLES MULTISTEPS NUCLEAR REACTIONS

Let us consider the Hamiltonian $\mathbf{H}$ for the manyparticle multistep nuclear reaction $a \to b$ as

$$\mathbf{H} = \mathbf{H}_a + \mathbf{V}_a = \mathbf{H}_b + \mathbf{V}_b, \tag{10}$$

where $\mathbf{H}_a$, $\mathbf{H}_b$ and $\mathbf{V}_a$, $\mathbf{V}_b$ are nonperturbated parts of $\mathbf{H}$ and interaction potentials of particles in initial $a$ and final $b$ channels of the investigated reaction. Then the nonperturbated wave functions of initial $a$ and final $b$ channels are the solutions of Schrödinger equations:

$$\left(\mathbf{H}_a - E_a\right)\Phi_a = 0; \quad \left(\mathbf{H}_b - E_b\right)\Phi_b = 0, \tag{11}$$

where energies $E_a$ and $E_b$ lie on the mass surface of investigated reaction: $E_a = E_b = E$, at that $E$ is the reaction total energy included in equation (5).

The amplitude of reaction $a \to b$ can be expressed [M. L. Goldberger, K. M. Watson, *Collision theory* (Wiley J. & Son Inc., 1964); J. R. Taylor, *Scattering Theory: the Quantum Theory of Nonrelativistic Scattering* (New York, 1972)] by the matrix element $\mathcal{T}_{b,a}$:

$$\mathcal{T}_{b,a} = \langle \Phi_b | \mathcal{T} | \Phi_a \rangle, \tag{12}$$

where the reaction operator $\mathcal{T}$ has the form:

$$\mathcal{T} = \mathbf{V}_a + \mathbf{V}_b \left( E - \mathbf{H} + i\eta \right)^{-1} \mathbf{V}_a, \tag{13}$$

which demonstrates the pronounced dependence of this operator in matrix element (12) from the structures of the wave functions of initial *a* and final *b* channels.

Analogously the amplitudes of the inverse reaction $b \to a$ can be expressed by the matrix element $\tilde{\mathcal{T}}_{a,b}$ of the **inverse reaction operator** $\tilde{\mathcal{T}}$ by the formula:

$$\tilde{\mathcal{T}}_{a,b} = \langle \Phi_a | \tilde{\mathcal{T}} | \Phi_b \rangle, \tag{14}$$

where the operator $\tilde{\mathcal{T}}$ is determinated as

$$\tilde{\mathcal{T}} = \mathbf{V}_b + \mathbf{V}_a \left( E - \mathbf{H} + i\eta \right)^{-1} \mathbf{V}_b \tag{15}$$

and **is different from the operator $\mathcal{T}$ (12) by not only the transposition of potentials $\mathbf{V}_a$ and $\mathbf{V}_b$ of initial and final channels but and the transposition of potentials of intermediate stages evolution of inverse reaction in the comparison with analogical stages of initial reaction**.

Using the formulae (6, 2-4) the T-invariance condition for the matrix elements $\mathcal{T}_{b,a}$ for the manyparticle multistep nuclear reaction $a \to b$ can be represented [M. L. Goldberger, K. M. Watson, *Collision theory* (Wiley J. & Son Inc., 1964); J. R. Taylor, *Scattering Theory: the Quantum Theory of Nonrelativistic Scattering* (New York, 1972)] by the formula:

$$\mathcal{T}_{b,a} = \left\langle \Phi_b \middle| \mathcal{T} \middle| \Phi_a \right\rangle = \left\langle \Phi_{\bar{a}} \middle| \tilde{\mathcal{T}} \middle| \Phi_{\bar{b}} \right\rangle = \tilde{\mathcal{T}}_{\bar{a},\bar{b}}, \tag{16}$$

where $\Phi_{\bar{a}}$ and $\Phi_{\bar{b}}$ are time-reversed wave functions $\Phi_a$ and $\Phi_b$. In formula (16) the matrix element $\mathcal{T}_{b,a}$ of initial reaction $a \to b$ coincides with the matrix element $\tilde{\mathcal{T}}_{\bar{a},\bar{b}}$ of time-reversed reaction $\bar{b} \to \bar{a}$.

In the common case the matrix element $\mathcal{T}_{b,a}$ can be represented by the sum of matrix elements $\mathcal{T}^{i}_{b,a}$, which are connected with one of $k$ realized mechanisms of this reaction ($i = 1,\ldots, k$):

$$\mathcal{T}_{b,a} = \sum_{i=1}^{k} \mathcal{T}^{i}_{b,a}\,. \tag{17}$$

Then the differential cross section of the investigated reaction $d\sigma_{b,a}$ has the form:

$$d\sigma_{b,a} \sim \left|\sum_{i=1}^{k} \mathcal{T}^{i}_{b,a}\right|^2 \sim \left\{\sum_{i=1}^{k} \left|\mathcal{T}^{i}_{b,a}\right|^2 + \sum_{i\neq j=1}^{k} \left[\mathcal{T}^{i}_{b,a}\left(\mathcal{T}^{j}_{b,a}\right)^* + \left(\mathcal{T}^{i}_{b,a}\right)^* \mathcal{T}^{j}_{b,a}\right]\right\}. \tag{18}$$

The each mechanism $i$ corresponds one of two groups of matrix elements $\mathcal{T}^{i}_{b,a}$: T-even matrix elements $\mathcal{T}^{i_{ev}}_{b,a}$ for mechanisms $i=i_{ev}$ and T-odd matrix elements $\mathcal{T}^{i_{odd}}_{b,a}$ for mechanisms $i=i_{odd}$ having the quality:

$$\mathcal{T}^{i_{ev}}_{b,a} = \mathcal{T}^{i_{ev}}_{\bar{b},\bar{a}}, \quad \mathcal{T}^{i_{odd}}_{b,a} = -\mathcal{T}^{i_{odd}}_{\bar{b},\bar{a}}. \tag{19}$$

Using the T-invariance conditions (16) the following relations can be obtained for matrix elements of the initial $a \to b$ and inverse $b \to a$ reactions:

$$\mathcal{T}^{i_{ev}}_{b,a} = \mathcal{T}^{i_{ev}}_{a,b}, \quad \mathcal{T}^{i_{odd}}_{b,a} = -\mathcal{T}^{i_{odd}}_{a,b}. \tag{20}$$

Then the T-even $d\sigma^{ev}_{b,a}$ and T-odd $d\sigma^{odd}_{b,a}$ asymmetries in the differential cross section (18), defined as

$$d\sigma^{ev}_{b,a} = d\sigma^{ev}_{\bar{b},\bar{a}}; \; d\sigma^{odd}_{b,a} = -d\sigma^{odd}_{\bar{b},\bar{a}} \tag{21}$$

and with the usage of formula (18) can be brought to forms:

$$d\sigma_{b,a}^{ev} \sim \sum_{i}^{k}\left[\left|\mathcal{T}_{b,a}^{i_{ev}}\right|^2 + \left|\mathcal{T}_{b,a}^{i_{odd}}\right|^2\right] + \sum_{i\neq j}^{k}\left[\mathcal{T}_{b,a}^{i_{ev}}\left(\mathcal{T}_{b,a}^{j_{ev}}\right)^* + \left(\mathcal{T}_{b,a}^{i_{odd}}\right)^*\mathcal{T}_{b,a}^{j_{odd}}\right];$$
$$d\sigma_{b,a}^{odd} \sim \sum_{i\neq j}^{k}\left[\mathcal{T}_{b,a}^{i_{ev}}\left(\mathcal{T}_{b,a}^{j_{odd}}\right)^* + \left(\mathcal{T}_{b,a}^{i_{ev}}\right)^*\mathcal{T}_{b,a}^{j_{odd}}\right] \tag{22}$$

The cross section $d\sigma_{b,a}$ of the investigated reaction $a \to b$ contains the different asymmetries $d\sigma_{b,a}^{m}$ with coefficients $D_{b,a}^{m}$ defined [G. A. Petrov, et al., Nucl. Phys., **A 502**, 297 (1989); P. Jesinger et al., Nucl. Instrum. Methods, **A 440**, 618 (2000)] as

$$D_{b,a}^{m} = \frac{\left(d\sigma_{b,a}^{m}\right)^{+} - \left(d\sigma_{b,a}^{m}\right)^{-}}{\sum_{m}\left(d\sigma_{b,a}^{m}\right)^{+} + \left(d\sigma_{b,a}^{m}\right)^{-}}, \tag{23}$$

where the signs +/− correspond signs of orientations of certain vector parameters connected with investigated asymmetries. Using the T-invariance condition (16) it can been built the analogous condition for these coefficients:

$$D_{b,a}^{m} = D_{\bar{b},\bar{a}}^{m} \qquad (24)$$

Let us use the representation of article [F. Arash, J. Moravcsik, G.R. Goldstein, Phys. Rev. Let., **54**, 2649 (1985)], in which it was shown that it is impossible the vanishing of any observable in differential cross sections of every elastic and inelastic binary nuclear reactions with the conservation of T-invariance, and results of article [S.G. Kadmensky, P.V. Kostryukov, accepted for publication in Bull. RAS: Phys., (2016)], in which the T-invariance conditions have been generalized for manyparticle multistep statistical nuclear reaction $a \to b$ with $n_a (n_a \geq 2)$ and $n_b (n_b \geq 2)$ particles of initial $a$ and final $b$ channels. Then using the T-invariance condition (24) **it can been shown that the coefficients $D_{b,a}^{m}$ and $D_{\bar{a},\bar{b}}^{m}$ for similar asymmetries ($m$) in differential cross sections of initial $a \to b$ and time-reversed $\bar{b} \to \bar{a}$ nuclear reactions**, the appearances of which are caused clearly by the uniform mechanism, **can be represented by the unified scalar**

**(pseudoscalar) function $D^m$, depending from wave vectors and spins of particles of initial and final channels of investigated reactions and changing it's values correspondingly to formula (8) for the transition from initial reaction to time-reversed nuclear reaction**:

$$D_{b,a}^m = D^m\left(\mathbf{k}_b, \mathbf{s}_b; \mathbf{k}_a, \mathbf{s}_a\right);\; D_{\bar{a},\bar{b}}^m = D^m\left(-\mathbf{k}_a, -\mathbf{s}_a; -\mathbf{k}_b, -\mathbf{s}_b\right). \tag{25}$$

Then the T-invariance condition (24) brings to condition for the coefficients $D^m$:

$$D^m\left(\mathbf{k}_b, \mathbf{s}_b; \mathbf{k}_a, \mathbf{s}_a\right) = D^m\left(-\mathbf{k}_a, -\mathbf{s}_a; -\mathbf{k}_b, -\mathbf{s}_b\right). \tag{26}$$

Among the coefficients $D_{b,a}^m$ it can be marked T-even $D_{b,a}^{m_{ev}}$ and T-odd $D_{b,a}^{m_{odd}}$ coefficients:

$$D_{b,a}^{m_{ev}} = D_{\bar{b},\bar{a}}^{m_{ev}};\;\; D_{b,a}^{m_{odd}} = -D_{\bar{b},\bar{a}}^{m_{odd}}, \tag{27}$$

for which T-invariance conditions (26) are represented by the forms:

$$\begin{aligned} D^{m_{ev}}\left(\mathbf{k}_b, \mathbf{s}_b; \mathbf{k}_a, \mathbf{s}_a\right) &= D^{m_{ev}}\left(\mathbf{k}_a, \mathbf{s}_a; \mathbf{k}_b, \mathbf{s}_b\right), \\ D^{m_{odd}}\left(\mathbf{k}_b, \mathbf{s}_b; \mathbf{k}_a, \mathbf{s}_a\right) &= -D^{m_{odd}}\left(\mathbf{k}_a, \mathbf{s}_a; \mathbf{k}_b, \mathbf{s}_b\right). \end{aligned} \tag{28}$$

**The dependence of the functions $D^{m_{ev}}$ and $D^{m_{odd}}$ from wave vectors and spins of particles can be found (experimentally or theoretically) for initial reaction $a \to b$ and with the usage of the T-invariance condition (28) is used for testing of mechanisms of the appearance of anisotropies with different P- and T-parities in differential cross sections of different nuclear reactions satisfactory the T-invariance of nuclear system.**

# 4. THE MECHANISMS OF FORMATION OF DIFFERENT ASYMMETRIES IN DIFFERENTIAL CROSS SECTIONS OF BINARY AND TERNARY FISSION OF ORIENTED TARGET-NUCLEI BY POLARIZED NEUTRONS.

In differential cross sections of binary and ternary fission of non-oriented target-nuclei by cold polarized neutron the asymmetries coefficient $D^m\left(\mathbf{k}_b,\mathbf{s}_b;\mathbf{k}_a,\mathbf{s}_a\right)$ (25) of types $\left(\mathbf{k}_n,\mathbf{k}_{LF}\right)$, $\left(\mathbf{s}_n,\mathbf{k}_{LF}\right)$, $\left(\mathbf{s}_n,\left[\mathbf{k}_n,\mathbf{k}_{LF}\right]\right)$, $\left(\mathbf{k}_3,\mathbf{k}_{LF}\right)$, $\left(\mathbf{s}_n,\mathbf{k}_3\right)$, $\left(\mathbf{s}_n,\left[\mathbf{k}_3,\mathbf{k}_{LF}\right]\right)$, where $\mathbf{k}_n$ , $\mathbf{k}_{LF}$ $\mathbf{k}_3$ are wave vectors of going in neutron, light fission fragment and third particle; $\mathbf{s}_n$ is the vector of the neutron spin, were discovered [G.V. Danilyan et al., Lett. JETP, **26**, 197 (1977); V.N. Andreev et al., Lett. JETP, **28**, 58 (1978); A.K. Petukhov et al, Lett. JETP,

**30**, 324 (1979); V.A. Vesna et al, Lett. JETP, **31**, 704 (1980); G. A. Petrov, et al., Nucl. Phys., **A 502**, 297 (1989); P. Jesinger et al., Nucl. Instrum. Methods, **A 440**, 618 (2000); P. Jesinger, A. Kötzle, F. Göennenwien, Phys. Atom. Nucl., **65**, 630 (2002); F. Göennenwien et al., Phys. Lett., **B 652**, 13 (2007); G.V. Danilyan et al., in *Proceeding of the ISINN-15*, JIRN, Dubna, 111 (2008); G.V. Danilyan et al., Phys. Atom. Nucl., **73**, 1116 (2010)]. In the same time in theoretical articles [O.P. Sushkov, V.V. Flambaum, UPN, **136**, 3 (1982); V.E. Bunakov, V.P. Gudkov, Nucl. Phys, **A 401**, 23 (1983); S.G. Kadmensky, Phys. Atom. Nucl., **65**, 1390 (2002); V.E. Bunakov, F. Göennenwien, Phys. Atom. Nucl., **65**, 2036 (2002); V.E. Bunakov, S.G. Kadmensky, Phys. Atom. Nucl., **66**, 1846 (2003); S.G. Kadmensky, Phys. Atom. Nucl., **67**, 170 (2004); S.G. Kadmensky, V.E. Bunakov, S.S. Kadmensky, Phys. Atom. Nucl., **74**, 555 (2010); S.G. Kadmensky, V.E. Bunakov, L.V. Titova, Phys. Atom. Nucl., **78**, 708 (2015)] the different mechanisms of appearance of these asymmetries were proposed. P-even T-even asymmetry $\left(\mathbf{k}_n, \mathbf{k}_{LF}\right)$, P-even T-odd asymmetry $\left(\mathbf{s}_n,\left[\mathbf{k}_n,\mathbf{k}_{LF}\right]\right)$ and P-odd T-even asymmetries $\left(\mathbf{s}_n,\mathbf{k}_{LF}\right)$, $\left(\mathbf{s}_n,\mathbf{k}_3\right)$ appear in differential cross sections of analyzed reactions by taking into account the mechanisms connected with the interference of fission amplitudes of s- and p-neutron resonances of compound nuclei. P-even T-even asymmetry $\left(\mathbf{k}_3,\mathbf{k}_{LF}\right)$ are connected with the mechanisms, based on the interference of the same and different s-neutron resonances. Used mechanisms of the appearance of all considered above asymmetries and too analyzed theoretically asymmetries $\left(\mathbf{I},\mathbf{k}_{LF}\right)$, $\left(\mathbf{I},\left[\mathbf{k}_n,\mathbf{k}_{LF}\right]\right)$, $\left(\mathbf{I},\mathbf{k}_3\right)$, $\left(\mathbf{I},\left[\mathbf{k}_n,\mathbf{k}_3\right]\right)$, where **I** is the polarizability vector of target-nucleus, satisfactory the T-invariance conditions (28).

**In the same time asymmetry $\left(\mathbf{s}_n,\left[\mathbf{k}_3,\mathbf{k}_{LF}\right]\right)$ and theoretically investigated asymmetry $\left(\mathbf{I},\left[\mathbf{k}_3,\mathbf{k}_{LF}\right]\right)$ caused by the influence of Coriolis interaction on the angular distributions of third particle doesn't satisfactory the T-invariance condition (28) for the mechanism of simultaneous flights from fissile compound nucleus third**

**particle and fission fragments. Only for the mechanism of sequential flights from fissile compound nucleus third particle and fission fragments, which is connected with the non-evaporation mechanism** [O. Tanimura, T. Fliessbach, Z. Phys., **A 328**, 475 (1987); S.G. Kadmensky, L.V. Titova, A.O. Bulychev, Phys. Atom. Nucl., **78**, 672 (2015)] **of third particle flight caused by the nonadiabatical motion of fissile nucleus near it's scission point, satisfactory of the T-invariance condition (28).**

It is very interesting to analyze the mechanisms of appearance of new asymmetries $\left([\mathbf{I},\mathbf{s}_n],\mathbf{k}_{LF}\right)$, $\left([\mathbf{I},\mathbf{s}_n],\mathbf{k}_3\right)$, $\left([\mathbf{I},\mathbf{s}_n],[\mathbf{k}_n,\mathbf{k}_{LF}]\right)$, $\left([\mathbf{I},\mathbf{s}_n],[\mathbf{k}_n,\mathbf{k}_3]\right)$, $\left([\mathbf{I},\mathbf{s}_n],[\mathbf{k}_3,\mathbf{k}_{LF}]\right)$ for binary and ternary fission of oriented nucleus-target by cold polarized neutrons. **These asymmetries satisfy the T-invariance conditions (28) only for mechanisms taking into account the interference neutrons resonances states with different values of it's spins and corresponding to sequential flights of third particles and fission fragments for ternary fission.**

# Thank you for attention