THE ANGULAR AND SPIN DISTRIBUTIONS OF THE BINARY AND TERNARY LOW-ENERGY NUCLEAR FISSION PRODUCTS AND THE TRANSVERSED WRIGGLING- AND BENDING-VIBRATIONS OF THE COMPOUND FISSILE NUCLEI

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1. INTRODUCTION

The aims of this presentation are the description for the low-energy nuclear fission of:

1. the spin and angular distributions of the fission fragments;
2. the characteristics of products of these fragments decays.

The fission fragments angular distribution $T^J_{MK}(\Omega)$ in the laboratory coordinate system (LCS) for the low-energy nuclear fission from the transition fission state (TFS) [Bohr, and B. Mottelson, *Nuclear Structure* (NY, Amsterdam, 1969, 1975) V. 1, 2] *JMK* of compound fissile nucleus (CFN) can be represented [3-6] through the analogous distribution $T(\Omega')$ in the internal coordinate system (ICS):

$$T^J_{MK}(\Omega)=\frac{2J+1}{16\pi^2}\int d\omega\left[\left|D^J_{MK}(\omega)\right|^2+\left|D^J_{M-K}(\omega)\right|^2\right]T(\Omega'), \qquad (1)$$

where the solid angles Ω, Ω' are defined by the fragment emission angles φ, θ and φ', θ' in LCS and ICS, correspondently, $D^J_{MK}(\omega)$ is the generalized spherical function depending from Euler angles $\omega=(\alpha,\beta,\gamma)$, which define the orientation of the axially symmetrical fissile nucleus axes relatively to the axes LCS.

As a rule, the distribution $T(\Omega')$ is constructed with the usage of O. Bohr's hypothesis [Bohr, and B. Mottelson, *Nuclear Structure* (NY, Amsterdam, 1969, 1975) V. 1, 2]: $T(\Omega')=T_0(\Omega')$, where the fission fragments distribution $T_0(\Omega')$ has a δ-function character [S.G. Kadmensky, Phys. At. Nucl. **65**, P. 1390 (2002); S.G. Kadmensky, L.V. Rodionova, Phys. At. Nucl. **66**, 1219 (2003); S.G. Kadmensky, L.V. Rodionova, Bull. Russ. Sci. Phys. **69**, 751 (2004)]:

$$T_0(\Omega') = \frac{1}{2\pi}\left[\delta(\xi' - 1)\right], \tag{2}$$

where $\xi' = \cos\theta'$. Then the distribution $T^J_{MK}(\Omega)$ (1) is determined by the distribution $\left(T^J_{MK}\right)_0(\Omega)$:

$$\left(T^J_{MK}\right)_0(\Omega) = \frac{2J+1}{8\pi^2}\left[\left|D^J_{MK}(\omega)\right|^2 + \left|D^J_{M-K}(\omega)\right|^2\right]_{\alpha=\varphi,\beta=\theta,\gamma=0}. \tag{3}$$

coinciding with the probability of the orientations of ICS axes in LCS defined by Euler angles $\alpha = \varphi$, $\beta = \theta$, $\gamma = 0$.

The angular distribution (2) differs from zero for a fixed value of the angle $\theta' = 0$, when $\Delta\theta' = 0$. Then from the quantum-mechanical uncertainty relation [S.G. Kadmensky, Phys. Atom. Nucl. **65**, 1390 (2002);. **66**, 1785 (2002); V.E. Bunakov, S.G. Kadmensky, Bull. Bull. Russ. Acad. Sci. Phys. **71**, 346 (2007)] for ΔL and $\Delta\theta'$:

$$\left(\Delta L\right)^2\left(\Delta\theta'\right)^2 \geq \hbar^2/4,$$

it follows that the distribution (2) corresponds to the case of complete uncertainty $\Delta L = \infty$ in the values of the orbital angular moment L. Therefore for real experimental situations it is necessary instead of distribution (2) to use [S.G. Kadmensky, Phys. At. Nucl. **65**, P. 1390 (2002); S.G. Kadmensky, L.V. Rodionova, Phys. At. Nucl. **66**, 1219 (2003); S.G. Kadmensky, L.V. Rodionova, Bull. Russ. Sci. Phys. **69**, 751 (2004)] the physically close distribution $T(\Omega')$, which differs from zero in a small vicinity of the angle $\theta' = 0$ at $\Delta\theta' \ll 1$:

$$T(\Omega') = \left|A(\Omega')\right|^2. \tag{4}$$

The amplitude $A(\Omega')$ of the named above distribution is represented as

$$A(\Omega') = \sum_L \psi_L Y_{L0}(\Omega'), \tag{5}$$

where the wave function ψ_L defines the L-distribution $W(L) = |\psi(L)|^2$ of fission fragments in the vicinity of the scission point of the CFN. Because of the approximate validity of formula (2) the characteristic orbital moments L in the sum (5) will have large values in comparison with the value of the compound fissile nucleus spin J. Then from the law of total spin conservation $\mathbf{J} = (\mathbf{F} + \mathbf{L})$, where $\mathbf{F} = (\mathbf{J}_1 + \mathbf{J}_2)$ is the summary fission fragments spin, it follows $\mathbf{F} \approx -\mathbf{L}$ and for $L \gg J$ the summary spin F must have the large values $F \gg J$.

The experimental multiplicities, energy and angular distributions of neutrons and γ-quanta, evaporated from thermalized fragments of low-energy fission of actinide nuclei, isomeric ratios of yields of final fragments, as well as characteristics of delayed neutrons emitted during the β-decay of named above final fragments are consistent with the fact [Hoffman, Phys. Rev. B **133**, 714 (1964); J. O. Rasmussen, W. Norenberg, H. J. Mang, Nucl. Phys. A **136**, 456 (1969); J. B. Wilhelmy, *et al.*, Phys. Rev. C **5**, 2041 (1972); M. Zielinska-Pfabe, K. Diethrich Phys. Let. B 49, 123, (1974); T. M. Sheidman *et al*, Phys. Rev. C **65**, 064302 (2002); A. Gavron, Phys. Rev. C **13**, 2562 (1976); L. G. Moretto, G. F. Peaslee, G. J. Wozniak, Nucl. Phys. A **502**, 453 (1989); S. Musicu et al, Phys. Rev. C **60**, 034613 (1999); O. T. Grudzevich *et. al*, Phys. At. Nucl. **64**, P. 1643 (2001)] the appearance of large values of fission fragments spins $\mathbf{J}_1$, $\mathbf{J}_2$, which are oriented perpendicularly to the direction of CFN symmetry axis.

The question arises about mechanisms of the large values of the relative orbital moments L and spins J_1, J_2 fission fragments appearance. Attempts of the explanation of these facts through the Coulomb interaction of strongly deformed primary fission fragments are unsatisfactory, since this interaction can change [20, 21] the average values of fission fragments spins and orbital fragments only on small quantities $\Delta\bar{L}$, $\Delta\bar{J}_1$, $\Delta\bar{J}_2 \leq 2$. The answer to this question can be obtained within the framework of the quantum theory of fission [Bohr, and B. Mottelson, *Nuclear Structure* (NY, Amsterdam, 1969, 1975) V. 1, 2; J.R. Nix, W.J. Swiatecki, Nucl. Phys. A **71**, 1(1965)] in the development the representations of papers [S. G. Kadmensky, D. E. Lyubashevsky, L. V. Titova, Bull. Russ. Acad. Sci.: Phys., **75**, P. 989 (2011); **79**, P. 975 (2015); **81**, P. 792 (2017); S. G. Kadmensky, D. E. Lyubashevsky, V.E. Bunakov, Phys. At. Nucl., **77**, P. 198 (2015)], allowing simultaneously to take into account effects of transverse wriggling- and bending-vibrations of the composite fissile nucleus with the defining role of wriggling oscillations.

2. CHARACTERISTICS OF THE LOW-ENERGY BINARY AND TERNARY NUCLEAR FISSION.

For the description of the binary and ternary low-energy fission it can be used the following facts:

1. the conservation of the direction of the symmetry axis of the fissile nucleus at all stages of its internal collective deformation motion [M. Brack, *et al.*, Rev. Mod. Phys. **44**, 320 (1972)] to scission point of this nucleus to the fission fragments;

2. the coldness [Bohr, and B. Mottelson, *Nuclear Structure* (NY, Amsterdam, 1969, 1975) V. 1, 2] of fissile nucleus at all fission stages after passage of the second fission barrier to it's scission point;
3. connected with named above coldness the conservation of the projection K of the spin J of fissile nucleus on the symmetry axis [Bohr, and B. Mottelson, *Nuclear Structure* (NY, Amsterdam, 1969, 1975) V. 1, 2; S.G. Kadmensky, Phys. At. Nucl. **65**, 1390 (2002); S.G. Kadmensky, L.V. Rodionova, Phys. At. Nucl. **66**, 1219 (2003); S.G. Kadmensky, L.V. Rodionova, Bull. Russ. Sci. Phys. **69**, 751 (2004); S.G. Kadmensky, Phys. At. Nucl. **68**, 2030 (2005); S. G. Kadmensky, L. V. Titova, Phys. At. Nucl., **72**, 1738 (2009)];
4. connected with named above coldness the necessary of taking into account [S. G. Kadmensky, D. E. Lyubashevsky, L. V. Titova, Bull. Russ. Acad. Sci.: Phys., **79**, P. 975 (2015); S. G. Kadmensky, D. E. Lyubashevsky, V.E. Bunakov, Phys. At. Nucl., **77**, P. 198 (2015)] only zero wriggling- and zero bending-vibrations for the formation of angular and spin distributions of the fission fragments;

3. CHARACTERISTICS OF THE ZERO BENDING- AND WRIGGLING-VIBRATIONS OF FISSILE NUCLEI FOR THE LOW-ENERGY NUCLEAR FISSION

A modern understanding of the nature of the appearance of large values of the relative orbital moments and spins of fission fragments is based [S. G. Kadmensky, D. E. Lyubashevsky, L. V. Titova, Bull. Russ. Acad. Sci.: Phys., **75**, 989 (2011); **79**, 975 (2015); S. G. Kadmensky, D. E. Lyubashevsky, V.E. Bunakov, Phys. At. Nucl., **77**, 198 (2015)] on account of the two types of collective transverse vibrations of the fissile nucleus introduced in paper [J.R. Nix, W.J. Swiatecki, Nucl. Phys. A **71**, 1(1965)] the first time in the vicinity of its scission point.

The first type includes bending-vibrations associated with rotations of the symmetry axes of two strongly deformed fission fragments that touch their vertices in the neck region of a strongly deformed fissile nucleus and pass after the scission of named above neck into primary fission fragments. These rotations occur in opposite directions around parallel axes perpendicular to the axis of symmetry of the fissile nucleus. Because of the law of conservation of the total spin of the fissile nucleus, the fission fragments due to the bending-vibrations of the spins emitted from the fissile nucleus satisfy the condition $\mathbf{J}_{b1} = -\mathbf{J}_{b2}$.

The second type of transverse vibrations corresponds to the wriggling-vibrations of the fissile nucleus, which are associated with rotations of the symmetry axes of the two fission prefragments in one direction around parallel axes perpendicular to the axis of symmetry of the composite fissile nucleus, which leads to the appearance of equally directed and large in magnitude compared with spin J of the said nucleus with spins of the emitted fission fragments J_{w1} and J_{w2}. In order to compensate for the nonzero total spin of these fragments $\mathbf{J}_w = \left(\mathbf{J}_{w1} + \mathbf{J}_{w2}\right)$, the entire fissile nucleus rotates about an axis parallel to the axis of rotation of the fission fragments in the opposite direction, which leads to the appearance of relative orbital momenta of the fission fragments L, which due to the law of conservation of the total spin of the fissile nucleus take values $\mathbf{L} = -\mathbf{J}_w$.

Both types of transverse vibrations contribute to the values of the spins $\mathbf{J}_1$ and $\mathbf{J}_2$ of the emitted fission fragments, the average values of which turn out to be appreciably larger than the spin J of the compound fissile nucleus. In this case it is necessary to take into account that most of the spin J in fission passes [J. B. Wilhelmy, *et al*., Phys. Rev. C **5**, 2041 (1972)] not to spins J_1 and J_2 of the fission fragments, but to their relative orbital the moment L. At the same time, only wriggling-vibrations actually define the distribution of the moments L of the fission fragments, since, as will be shown below, the average values L for wriggling-vibrations significantly exceed the values of J.

Since for a low-energy fission the compound fissile nucleus and the fission fragments emitted from it in the vicinity scission point of the indicated nucleus should be located only in cold non nonthermalized states [Bohr, and B. Mottelson, *Nuclear Structure* (NY, Amsterdam, 1969, 1975) V. 1, 2; S.G. Kadmensky, Phys. At. Nucl. **65**, P. 1390 (2002); S.G. Kadmensky, L.V. Rodionova, Phys. At. Nucl. **66**, 1219 (2003); S.G. Kadmensky, L.V. Rodionova, Bull. Russ. Sci. Phys. **69**, 751 (2004); S.G. Kadmensky, Phys. At. Nucl. **68**, P. 2030 (2005); S. G. Kadmensky, L. V. Titova, Phys. At. Nucl., **72**, P. 1738 (2009)], when constructing their spin distributions, it is necessary to take into account [S. G. Kadmensky, D. E. Lyubashevsky, L. V. Titova, Bull. Russ. Acad. Sci.: Phys., **75**, 989 (2011); **79**, 975 (2015); S. G. Kadmensky, D. E. Lyubashevsky, V.E. Bunakov, Phys. At. Nucl., **77**, 198 (2015)] only the zero transverse bending- and wriggling-vibrations of the fissile nucleus.

The wave functions of zero wriggling and bending vibrations in the momentum representation $\Psi_0(J_{w_x})$, $\Psi_0(J_{w_y})$ and $\Psi_0(J_{b_x})$, $\Psi_0(J_{b_y})$ depend from the momentum for wriggling- and bending-vibrations J_{w_x}, J_{w_y} and J_{b_x}, J_{b_y}. These moments are related with projections of the spins $\mathbf{J}_1$ and $\mathbf{J}_2$ fission fragments on the axis X, Y perpendicular to the axis of symmetry of the fissile nucleus:

$$J_{w_x} = J_{1x} + J_{2x},\ J_{w_y} = J_{1y} + J_{2y};\ J_{b_x} = J_{1x} - J_{2x},\ J_{b_y} = J_{1y} - J_{2y};\ J_1^2 = J_{1x}^2 + J_{1y}^2,\ J_2^2 = J_{2x}^2 + J_{2y}^2. \tag{6}$$

As a result $\Psi_0(J_{w_x})$ and $\Psi_0(J_{b_x})$ they are represented in the form [30]:

$$\Psi_0(J_{w_x}) = (\pi C_w)^{-1/4} \exp\left(-\frac{J_{w_x}^2}{4C_w}\right); \qquad \Psi_0(J_{b_x}) = (\pi C_b)^{-1/4} \exp\left(-\frac{J_{b_x}^2}{4C_b}\right), \tag{7}$$

$C_w = M_w \hbar \omega_w$, $C_b = M_b \hbar \omega_b$, frequencies ω_w and ω_b of both wriggling- and bending-vibrations are determined by the classical formulas $\omega_w = \sqrt{K_w / M_w}$ and $\omega_b = \sqrt{K_b / M_b}$, where K is the stiffness parameter, and M is the mass parameter. Expressing the distribution function $W(\mathbf{J}_1, \mathbf{J}_2)$ of the fission fragments along the spins $\mathbf{J}_1$ and $\mathbf{J}_2$ through the products of squares of the modules of the wave functions of zero bending and wriggling vibrations (7):

$$W(\mathbf{J}_1, \mathbf{J}_2) = \left|\Psi_0(J_{w_x})\right|^2 \left|\Psi_0(J_{w_y})\right|^2 \left|\Psi_0(J_{b_x})\right|^2 \left|\Psi_0(J_{b_y})\right|^2 , \tag{8}$$

One can obtain [J.R. Nix, W.J. Swiatecki, Nucl. Phys. A **71**, 1(1965)] the explicit form of the distribution (8):

$$W(\mathbf{J}_1, \mathbf{J}_2) = \frac{4J_1J_2}{\pi C_b C_w} \exp\left[-\frac{1}{2}\left(\frac{1}{C_b} + \frac{1}{C_w}\right)\left(J_1^2 + J_2^2\right) + \left(\frac{1}{C_b} - \frac{1}{C_w}\right) J_1 J_2 \cos\phi\right], \tag{9}$$

where $\phi(0 \le \phi \le 2\pi)$ is the angle between the two-dimensional spin vectors of the fragments $\mathbf{J}_1$ and $\mathbf{J}_2$ lying in the plane *xy*. By integrating in (9) with respect to the variables J_2 and ϕ, one can obtain [J.R. Nix, W.J. Swiatecki, Nucl. Phys. A **71**, 1(1965)] the distribution of the spins of one of the fission prefragments:

$$W(J_1) = \frac{4J_1}{C_b + C_w} \exp\left[-\frac{2J_1^2}{C_b + C_w}\right]. \tag{10}$$

With usage of the formula (10), we can calculate the average spin of one of the fission fragments:

$$\bar{J}_1 = \int_0^\infty J_1 W(J_1) dJ_1 = \frac{1}{2}\sqrt{\frac{\pi}{2}}\left(C_b + C_w\right)^{1/2}. \tag{11}$$

From the estimates of [J.R. Nix, W.J. Swiatecki, Nucl. Phys. A **71**, 1(1965)] for a fissioning nucleus for values of deformation parameters of fission prefrages ≈ 0.2, we can obtain: : M_w = 1.6·10^6 MeV·Fm^2·s^2; M_b = 2.0·10^6 MeV·Fm^2·s^2; K_w = 295 MeV·rad^{-2}; K_b = 52 MeV·rad^{-2}; $\hbar\omega_w$ = 2.3 MeV; $\hbar\omega_b$ = 0.9 MeV; $C_w = 132$ and $C_b = 57$, from which it follows that the stiffness parameters, the quantum energies and the coefficients for the wriggling-vibrations turn out to be noticeably larger than the analogous values for the bending-vibrations . Then the quantity $\left(C_b + C_w\right)/2$ determining the character of the distribution (10) turns out to be 95, which leads to the average value (11) of the spin of the fission fragment $\bar{J}_1 \approx 8.6$. At the same time, if the value $C_b = 57$ is neglected in comparison with the value $C_w = 132$ in the formula $\left(C_b + C_w\right)/2$, the average value of the spin of the fission fragment turns out to be equal to 6, which differs by a factor of 1.5 from the spin value obtained above, while taking into account the wriggling- and bending-vibrations.

Hence, the conclusion is drawn that the wriggling-vibrations play a predominant role in comparison with bending-vibrations in the formation of the distribution of the spins of fission fragments.

The approach developed above to the description of the spin distribution of fission fragments based on the concept of the coldness of the fissioning nucleus at the point of discontinuity and taking into account the zero transverse vibrations of the fissioning nucleus is fundamentally different from the approach of [K. Skarsvag, *and* K. Bergheim, Nucl. Phys. **45**, 721 (1963), J. O. Rasmussen, W. Norenberg, H. J. Mang, Nucl. Phys. A **136**, 456 (1969); J. B. Wilhelmy, *et al*., Phys. Rev. C **5**, 2041 (1972); M. Zielinska-Pfabe, K. Diethrich Phys. Let. B 49, 123, (1974); T. M. Sheidman *et al*, Phys. Rev. C **65**, 064302 (2002)], in which the assumption of appreciable thermalization of fission fragments near scission point of the fissile nucleus, when the temperature T of the fission fragments exceeds 1 MeV. In this case, due to the significantly lower energy of the $\hbar\omega_b$ quantum of bending-vibrations compared to the analogous energy $\hbar\omega_w$ of the quantum of wriggling-vibrations (for example, for the nucleus), the main role in the temperature distribution of the fission fragments in terms of the number n_b and n_w quanta of bending- and wriggling- vibrations is played by the bending-vibrations. But, since the fissile nucleus remains in the cold state near it’s scission point to fragments of fission, the representation of papers are not realized, and the formation of spin distributions of fission fragments is determined by zero wriggling- and bending- vibrations of the fissile nucleus with the predominant role of wriggling-vibrations.

Now consider the distribution $W(L)$ of the relative orbital moments of the fission fragments. To do this, transform the spin distribution of the fission fragments (8) to the form

$$W\left(\mathbf{L},\mathbf{J}'\right)=\frac{1}{\pi^2 C_w C_b}\exp\left[-\frac{\mathbf{L}^2}{2C_w}-\frac{\mathbf{J}'^2}{2C_b}\right], \tag{12}$$

where the definitions of the relative orbital angular momentum $\mathbf{L}$ and the relative spin $\mathbf{J}'$ of the fission fragments through the spins of the first and second fission fragments are introduced $\mathbf{J}_1$ and $\mathbf{J}_2$:

$$\mathbf{L}=-(\mathbf{J}_1+\mathbf{J}_2),\ \ \mathbf{J}'=\left(\mathbf{J}_1-\mathbf{J}_2\right)/2; \tag{13}$$

$$\mathbf{J}_1=-\mathbf{L}/2+\mathbf{J}',\ \ \mathbf{J}_2=-\mathbf{L}/2-\mathbf{J}', \tag{14}$$

and the Jacobian of the change in the transition from the phase volume element $d\mathbf{J}_1 d\mathbf{J}_2$ to the element $d\mathbf{L}d\mathbf{J}'$ is equal to 1. Taking into account that the elements of the phase volume $d\mathbf{L}$, $d\mathbf{J}'$ taking into account the two-dimensionality of the vectors $\mathbf{L},\mathbf{J}'$ are represented in the cylindrical coordinate system as

$$d\mathbf{L}=LdLd\varphi_{\mathbf{L}}\ ,\ d\mathbf{J}'=J'dJ'd\varphi_{\mathbf{J}'} \tag{15}$$

and integrating the distribution (12) with respect to $dJ', d\varphi_{\mathbf{J}'}, d\varphi_{\mathbf{L}}$ can obtain the distribution normalized by integration with respect to unity:

$$W(L)=\frac{L}{C_w}\exp\left[-\frac{L^2}{2C_w}\right] \tag{16}$$

As expected, the obtained distribution $W(L)$ is determined only by a constant C_w for wriggling vibrations. Then the average value $\bar{L}$ of the relative orbital angular momentum L of the fission fragments, defined as

$$\bar{L}=\int_0^\infty L\left|\psi(L)\right|^2 dL=\frac{1}{C_w}\int_0^\infty L^2\exp\left(-\frac{L^2}{2C_w}\right)dL=\sqrt{\frac{\pi}{2}}(C_w)^{1/2}, \tag{17}$$

using the value found above $C_w = 132$ for the nucleus $^{236}\mathrm{U}$, it turns out to be equal and does not differ much from the obtained with simultaneous allowance for the wriggling- and bending-vibrations of the expectation value $2\bar{J}_1$ of the doubled spin of one fission fragment equal to 17.2.

4. THE DESCRIPTION OF SPIN DISTRIBUTIONS OF FISSION FRAGMENTS

During the spacing, initially the cold fragments of fission in a fairly short time (of the order of 10^{-21}s) go over to equilibrium thermalized states, for the description of which, in a number of papers [T. Ericson, *and* V. Strutinsky Nucl. Phys. **8**, 284 (1958); V. M. Strutinsky, JETP. **37**, 613 (1960)], the Gibbs distributions $\rho_i(E_i^*, J_i)$ in excitation energy E_i^* and J_i spin for the *i*-th fission fragment (*i* = 1, 2):

$$\rho_i(E_i^*, J_i) = \rho_i(E_i^*)\rho_i(J_i), \tag{18}$$

where the energy $\rho_i(E_i^*)$ and spin $\rho_i(J_i)$ distributions of the *i*-th fission fragment with temperature T_i and moment of inertia $\Im_i$ have the form:

$$\rho_i(E_i^*) \sim \exp(-E_i^* / kT_i), \tag{19}$$

$$\rho_i(J_i) \sim (2J_i + 1)\exp[-\hbar^2 J_i(J_i + 1) / \Im_i kT_i]. \tag{20}$$

In later papers [J. O. Rasmussen, W. Norenberg, H. J. Mang, Nucl. Phys. A **136**, 456 (1969). J. B. Wilhelmy, *et al.*, Phys. Rev. C **5**, 2041 (1972); L. G. Moretto, G. F. Peaslee, G. J. Wozniak, Nucl. Phys. A **502**, 453 (1989)], a representation was used according to which the statistical equilibrium in thermalized fission fragments only arises from their excitation energies, and the spin distribution $\rho_i(J_i)$ of fission fragments is nonequilibrium, since it forms at the scission point of the cold nucleus and is not associated with the temperature of the fission fragments after their thermalization. In this case $\rho_i(J_i)$ it is presented in the "standard" form :

$$\rho_i(J_i) \sim (2J_i+1)\exp[-J_i(J_i+1)/B^2], \tag{21}$$

where the value B^2 can differ markedly from the value $\Im_i kT/\hbar^2$ in formula (20). Based on the statistical model of nuclear reactions [H. Hauser, *and* H. Feshbach, Phys. Rev. **87**, 366 (1952); P. A. Moldauer, Phys. Rev. B **135**, 6421 (1964)] and the cascade-evaporation model [E. S. Troubetzkoy, Phys. Rev. **122**, 212 (1961)] using energy (3) and spin (5) distributions of thermalized fragments of spontaneous and low-energy stimulated fission of actinide nuclei, multiplicities, energy and angular distributions instant neutrons [24, 44] and gamma quanta [J. B. Wilhelmy, *et al.*, Phys. Rev. C **5**, 2041 (1972); L. G. Moretto, G. F. Peaslee, G. J. Wozniak, Nucl. Phys. A **502**, 453 (1989)], evaporated from thermalized fragments, relative yields of the main and isomeric states of the final fission fragments [J. B. Wilhelmy, *et al.*, Phys. Rev. C **5**, 2041 (1972); L. G. Moretto, G. F. Peaslee, G. J. Wozniak, Nucl. Phys. A **502**, 453 (1989)], as well as the characteristics of the delayed neutrons emitted during the β-decay of these fragments. The parameter B^2 in formula (5) was assumed to be 80-120, which leads to the average values of the spins of the fission fragments $\bar{J}$ in the range 7-9, close to their experimental values.

The spin distribution of the fission fragments (21) coincides in form with the analogous introduced above distribution (10) with the choice of the constant B^2 as

$$B^2 = \frac{C_b + C_w}{2}. \qquad (22)$$

1. The values of $\left(C_b + C_w\right)/2 \approx 95$ and $\bar{J} \approx 8.6$ from (10) obtained above using the results of [J.R. Nix, W.J. Swiatecki, Nucl. Phys. A **71**, 1(1965)] are clearly correlated with the range of values (80-120) for the value B^2 and (7-9) for the average spins of the fission fragments $\bar{J}$ calculated in [A. Bohr Proc. Intern. Conf. on the Peaceful Uses of Atomic Energy vol. 2, United Nations, New York. N. Y., 1956., S.G. Kadmensky, V. P. Markushev, V.I. Furman, Sov. J. Nucl. Phys. **35**, 166 (1982)]. Since neglecting the value C_b leads to a change in the value B^2 from 95 to a value of 66, which changes the average values of the spins of the fragments from 9 to 6, i.e. no more than 30%, then we can conclude that the prevailing role of wriggling oscillations in the formation of spin distributions of fission fragments in comparison with bending oscillations.

5. THE ANGLE DISTRIBUTIONS OF FRAGMENTS OF THE LOW-ENERGY PHOTOFISSION

The complete angular distribution $W(\Omega)$ of the fragments of the photofission reaction of an even-even target nucleus using the formalism of [1] can be represented as

$$W(\Omega) \equiv \frac{d\sigma_{\gamma f}(\theta)}{d\Omega} = \sum_J \sum_M \sigma(E_\gamma, JM) \sum_{K=0}^{J} \frac{\Gamma_f(JK)}{\Gamma(J)} T^J_{MK}(\Omega) . \qquad (22)$$

2. where is the cross section for the formation of a compound nucleus with a total spin J ($J=1$ for electric dipole and $J=2$ for electric quadrupole photons) and its projection on the direction of the incident beam of photons with energy E_γ, $\Gamma_f(JK)$ is the dividing width of the transition fission state of the compound nucleus, $\Gamma(J)$ is the total width of the decay from the state of the composite fissile nucleus in the first well of the deformation potential. Using the formulas (1, 4-5) and the actual wave function ψ_L associated with the distribution $W(L)$ (16) of fission fragments by relative orbital angular momentum for the wriggling-vibrations an expression $\psi_L=\sqrt{W(L)}$, we can obtain [S.G Kadmensky, L. V. Titova, D.E. Lyubashevsky, Bull. Bull. Russ. Acad. Sci. Phys. **81**, 792 (2017)] for the anisotropy of the angular distribution of the nuclear fission fragments, where θ the angle is measured from the direction of the incident photons:

$$\frac{d\sigma_{\gamma f}(\theta)\big/d\Omega}{d\sigma_{\gamma f}(90^\circ)\big/d\Omega}=a+b\sin^2\theta+c\sin^2(2\theta). \tag{23}$$

The coefficients a, b, c in formula (23) are defined by the expressions:

$$a=\left(\alpha_{10}+\alpha_{11}G(11)+\alpha_{20}G(20)\right)/d,\quad b=\left(\beta_{10}+\beta_{11}G(11)+\beta_{20}G(20)\right)/d,\quad c=\gamma_{20}G(20)/d, \tag{24}$$

where

$$d=(\alpha_{10}+\beta_{10})+(\alpha_{11}+\beta_{11})G(11)+(\alpha_{20}+\beta_{20})G(20),$$

$$\alpha_{JK}=2\pi\left(B_{JK0}+B_{JK2}+B_{JK4}\right);\ \beta_{JK}=2\pi\left(-\frac{3}{2}B_{JK2}-\frac{5}{8}B_{JK4}\right);\ \gamma_{JK}=2\pi\left(-\frac{35}{32}B_{JK4}\right), \tag{25}$$

$$B_{JKMF} = \frac{2J+1}{4\pi} \sum_{LL'} \psi_L \psi_{L'} \sqrt{(2L+1)(2L'+1)} \sum_{jF} (-1)^{j+J+F} \frac{\sqrt{2F+1}}{\sqrt{2J+1}} C^{jK}_{JLK0} C^{jK}_{JL'K0} C^{F0}_{LL'00} C^{JM}_{JFM0} \begin{Bmatrix} L & j & J \\ J & F & L' \end{Bmatrix}, \qquad (26)$$

and at $F = 0$ a value $B_{JKM0} = 1/4\pi$.

In Fig. 1, the theoretical values of the asymmetry coefficients *a*, most sensitive to the choice of the wriggling-vibrations parameter C_w and calculated from the formula (24), were compared with the corresponding experimental values [N.S. Rabotnov , Sov. J. Nucl. Phys. **11**., 285 (1970); P.P. Ganich, Sov. J. Nucl. Phys. **52**, 23 (1990); L.J. Lindgren et al., Sov. J. Nucl. Phys. **32**, 173. (1980)] for the target nucleus ^{234}U as a function of the γ-quanta energy E_γ.

From Fig. 1 that the optimum values of the parameter C_w lie in the range $C_w = 130 \pm 40$ and are in good agreement with the estimate of the wriggling-vibrations parameter $C_w = 132$ for ^{234}U obtained in [J.R. Nix, W.J. Swiatecki, Nucl. Phys. A **71**, 1(1965)]. Similar values of the parameters C_w prove to be optimal when comparing the theoretical and experimental values of the asymmetry coefficients *a* for the photofission of the ^{236}U and ^{238}U nuclei.

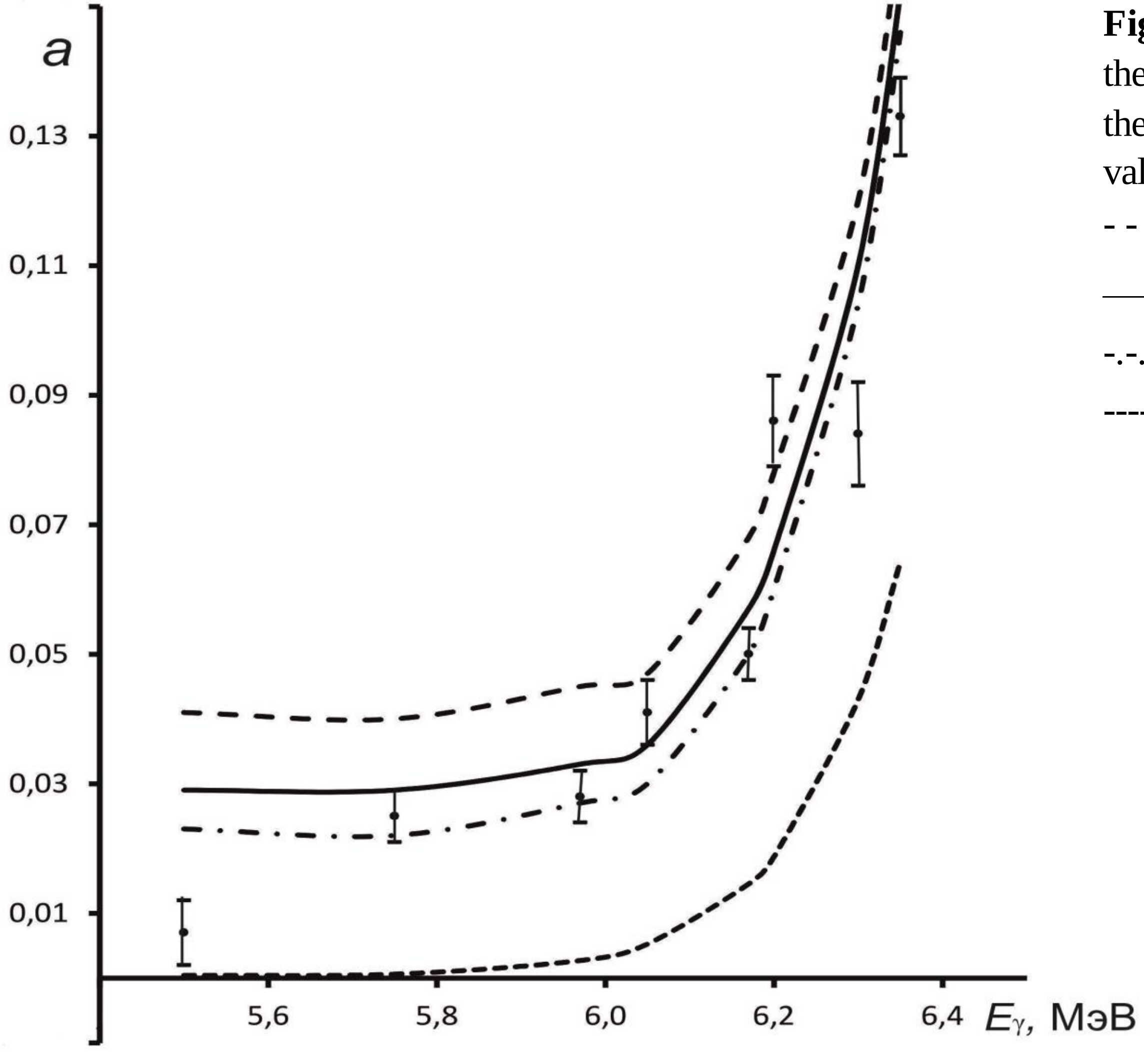

Fig. 1. Approximation of the energy dependence of the angular distribution coefficients of fragments of the subthreshold photofission ^{234}U for different values of the wriggling-vibrations parameter.

- - - - - - $C_w = 70$;

———— $C_w = 110$;

-.-.-.-.-.- $C_w = 140$;

----------$C_w \to \infty$ (O. Bohr limit).

6. CONCLUSION.

1. The successful description of the angular distributions of fragments of spontaneous and stimulated low-energy nuclear fission, as well as the characteristics of the instant neutrons and gamma quanta, evaporated from fission fragments after their thermalization, is based on the use of three basic concepts.
2. The first of these representations is related to the **coldness** of the fissioning nucleus at the point of its discontinuity. The second is based on taking into account the transverse **zero bending and wriggling** vibrations of this nucleus [J.R. Nix, W.J. Swiatecki, Nucl. Phys. A **71**, 1(1965)], which lead to the appearance of large values of the spins and relative orbital angular momenta of the fission fragments oriented perpendicular to the axis of symmetry of the fissioning nucleus at the moment of its rupture.
3. And, finally, the third representation is based on the use of a cascade-evaporation model, taking into account the **nonequilibrium character** of the spin distributions and the relative orbital moments of the fission fragments due to the bending- and wriggling-vibrations considered above with prevailing wriggling vibrations.