Group delay time and neutron optics

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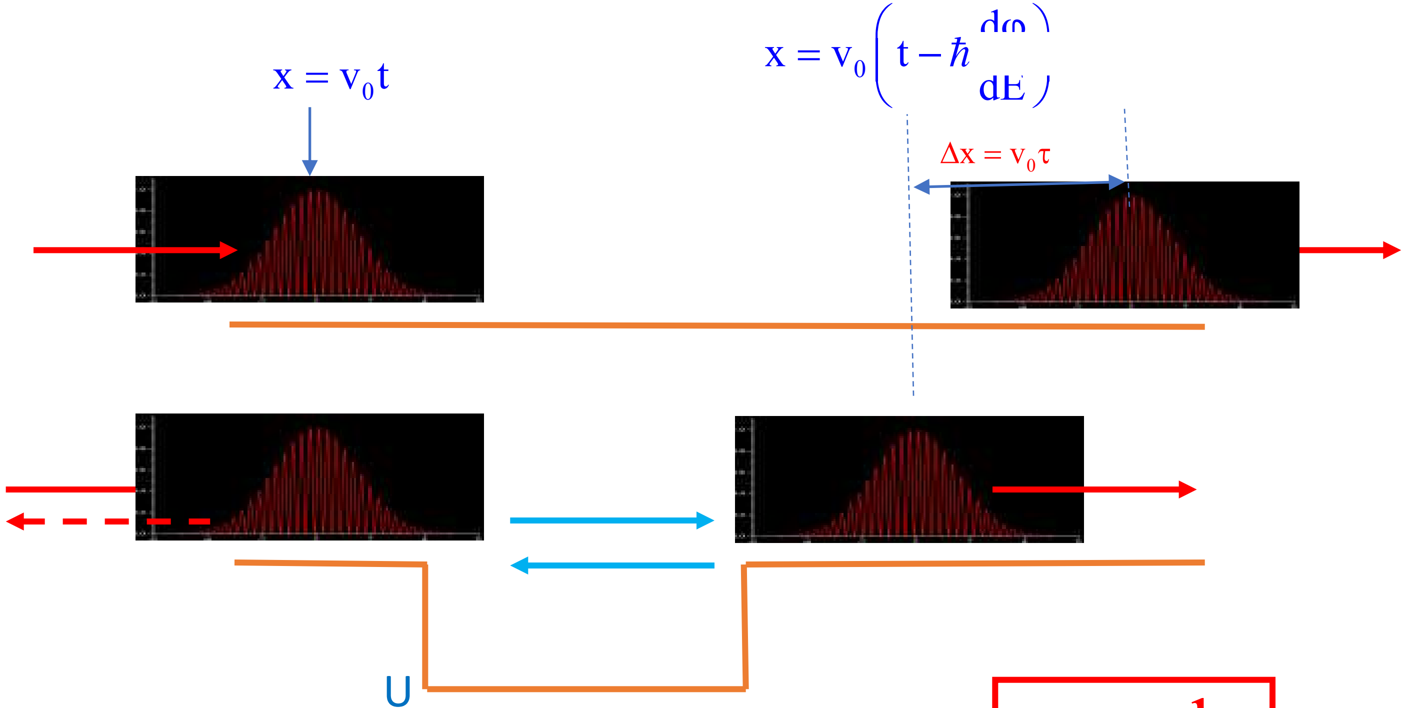
Group Delay Time in neutron optics

1. *Introduction*
2. *Larmor clock as theoretical and experimental approach to the problem of interaction time of neutron with quantum object*
3. *Goos-Hänchen shift at neutron reflection from a matter*
4. *Some aspects of the refraction theory*

Introduction

Group delay time



$$\tau = \hbar \frac{d\omega}{dE}$$

Bohm, D. ,1951, Quantum Theory (Prentice-Hall. N.-Y), pp 257-261
Wigner, E.P., 1955, Phys,Rev. 98, 145

Foundations of neutron optics

Dispersion law

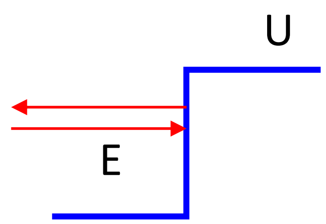
$$k^2 = k_0^2 - 4\pi\rho b \qquad b = b' - ib'' \qquad b'' << b' \qquad \frac{b''}{b'} \approx 10^{-4} - 10^{-5}$$

$$U = \frac{2\pi\hbar^2}{m}\rho b$$

b = const (?)

Refraction index

$$n = \frac{k}{k_0} \qquad n^2 = 1 - \frac{4\pi\rho}{k_0^2} b$$



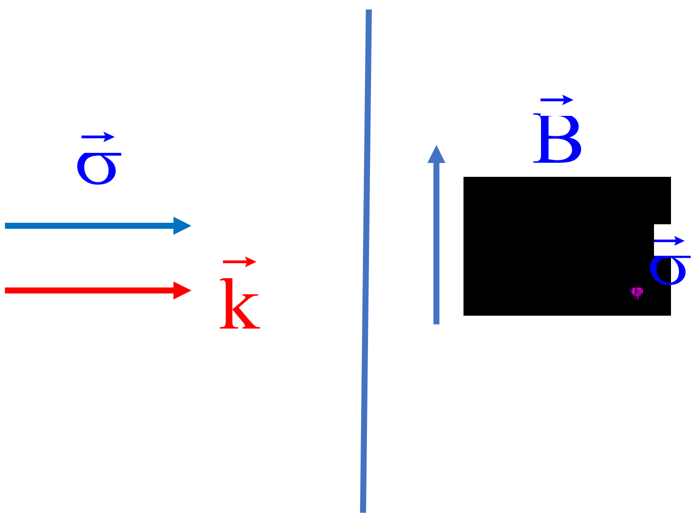
$$k_{0\perp} \le k_b = (4\pi\rho b)^{1/2} \longrightarrow$$ ***Total reflection***

$$k_\perp^2 = k_{0\perp}^2 - 4\pi\rho b \longrightarrow$$

Optical properties of an object do not depend on the longitudinal component of k

Group delay time
and Larmor clock

Spin precession in the stationary field



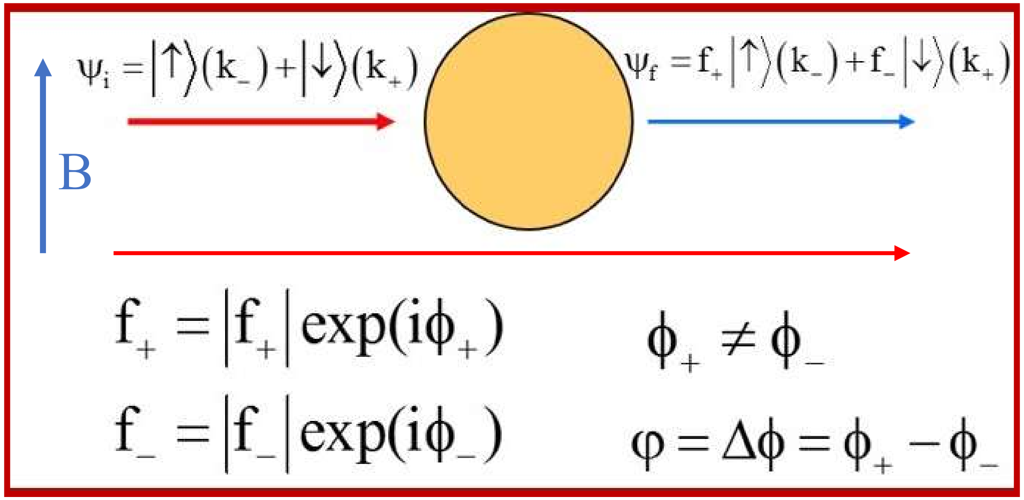
$$\Psi(x,t) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{ik_+x} \\ e^{ik_-x} \end{pmatrix} e^{-\omega t} \qquad k_\pm = k_0\left(1 \mp \frac{\mu B}{E}\right)^{1/2}$$

$$\psi(x) = \begin{pmatrix} e^{i\varphi/2} & 0 \\ 0 & e^{-i\varphi/2} \end{pmatrix}\psi_0, \qquad \psi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{i(kx-\omega t)}$$

$\hat{R}$ Rotation operator

$$\varphi = \left(k_- - k_+\right)x \qquad \varphi = \omega_L \frac{x}{v} \qquad \omega_L = \frac{2\mu B}{\hbar}$$

Larmor clock and group delay time



A.I.Baz,
Soviet. J. Nucl. Phys. 4 (1967) 182

In the presence of magnetic field, B interaction results in extra spin precession

Delay time due to interaction can be defined as

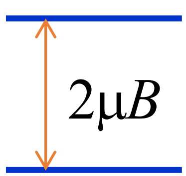
$$\tau = \frac{\Delta\phi}{\omega_L} \qquad \Delta\varphi = \omega_L \tau \qquad \omega_L = \frac{2\mu B}{\hbar}$$

$$\tau = \hbar \frac{\Delta\phi}{2\mu B} \qquad \hbar \frac{\Delta\phi}{\Delta E}$$

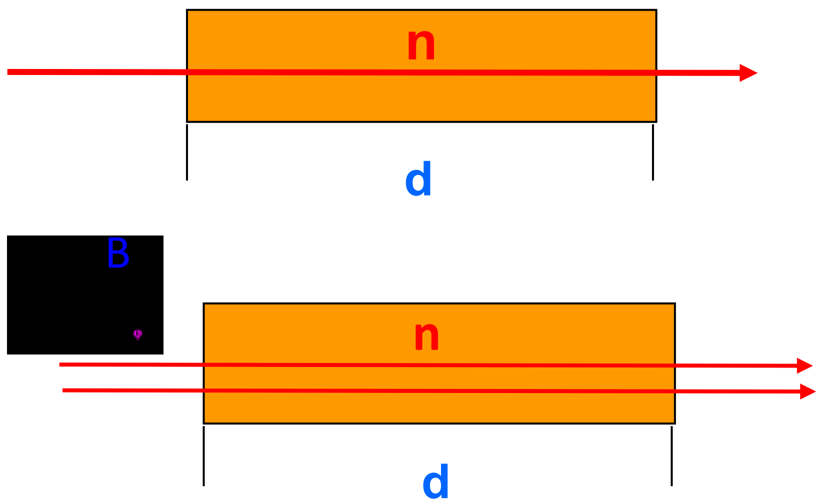
In a limit $B, \Delta E \to 0$

$$\tau = \hbar \frac{d\phi}{dE}$$

Group delay time



Extra spin precession at refraction



$$\phi = k(n-1)d$$

$$\phi_+ = k_+\left[n(k_+)-1\right]d$$

$$\phi_- = k_-\left[n(k_-)-1\right]d$$

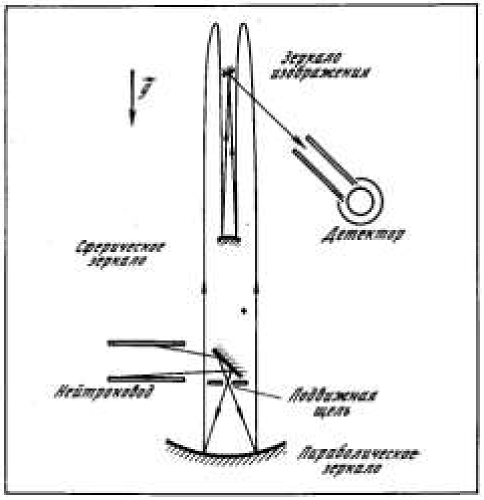
Extra spin precession

$$\boxed{\Delta\varphi = \omega_L\left(\frac{n-1}{n}\right)\frac{d}{v}}$$

$$\Delta\varphi = \omega_L\left(\frac{d}{v}-\frac{d}{nv}\right)$$

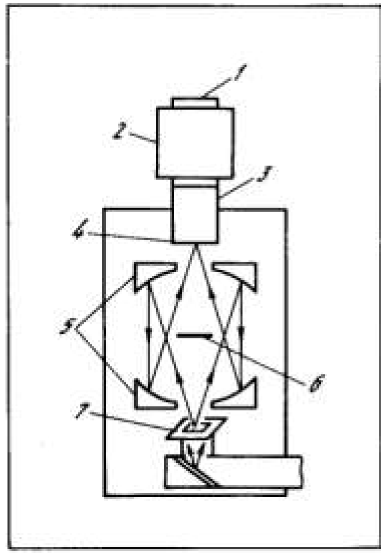
А.И.Франк. Вопр. Ат.науки и техники, Серия: Общая и ядерная физика, (36), с.69, 1986.
V.G.Baryshevskii, S.V.Cherepitsa, A.I.Frank.Phys.Lett.A, 1991, V.153, 299.
A.I.Frank Soviet Physics Uspekhi **34** (11) 980–987 (1991)

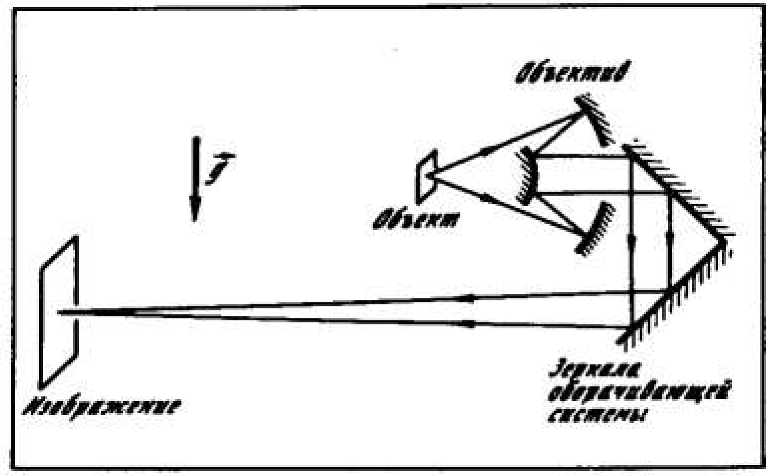
Neutron Microscopy using ultra-cold neutrons



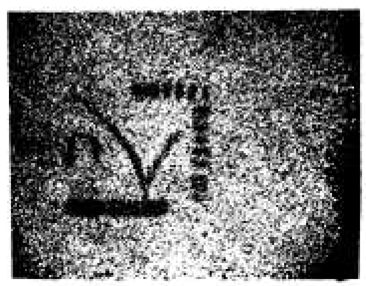
Resolution ≈ 10 μ

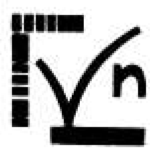
A.Steyerl et al, Neutron Microscopy Rev.Phys.Appl, **23**, *171, (1988)*

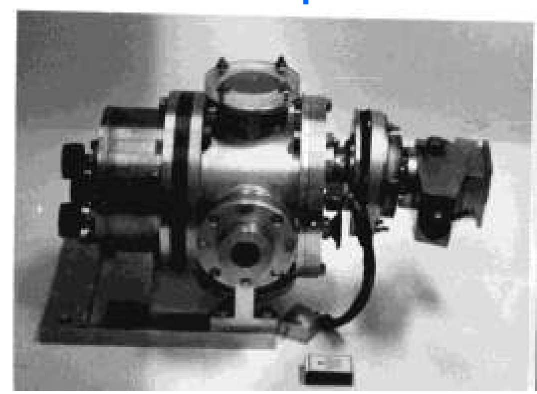




Resolution ≈ 15 μ

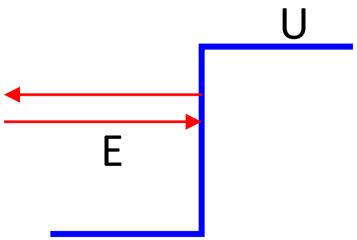






A.I.Frank *Ultra-Cold neutron microscopy*. Atomic energy, **66**, p.106 (1987)
A.I.Frank. *Optics of very slow neutrons and neutron microscopy*. Nucl.Instr.Meth. A, **284,** (1989)

Pseudo-Larmor spin precession at reflection from potential step



$$\psi(x) = Re^{-ikx}; \quad R = e^{i\varphi} \qquad \cos\varphi = \frac{2E - U}{U}$$

In magnetic field $\quad k_0^2 \to k_\pm^2, \qquad E \to E \mp$

$$\cos\varphi_\pm = \frac{2E \mp \mu \quad -}{U}$$

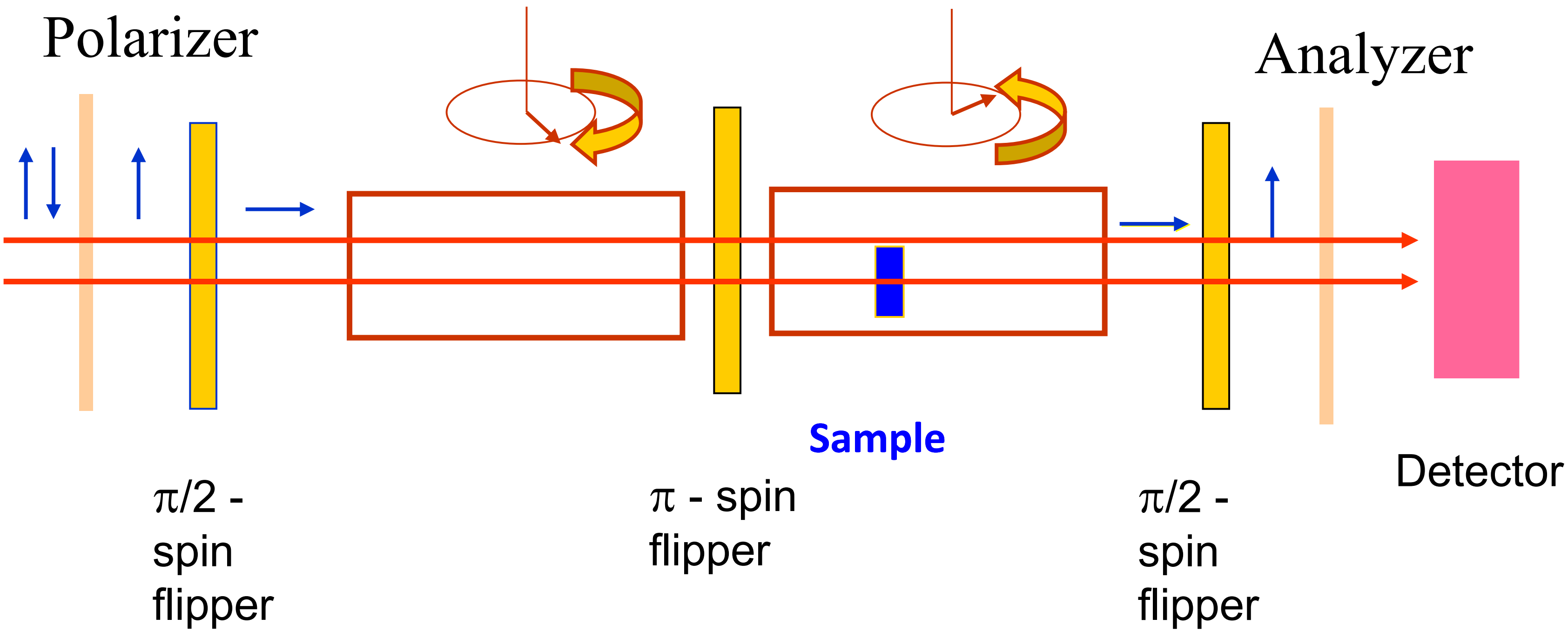
Spin rotation at reflection

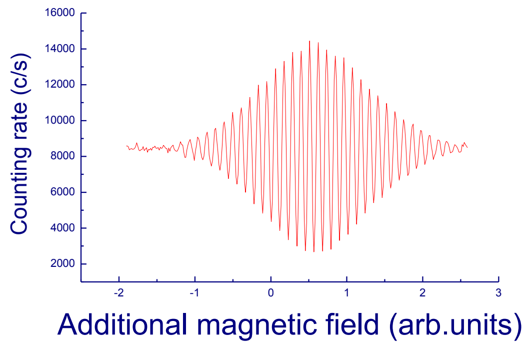
$$\Delta\varphi = \varphi_+ - \varphi_- = \frac{2\mu B}{\sqrt{E(U - E)}}$$

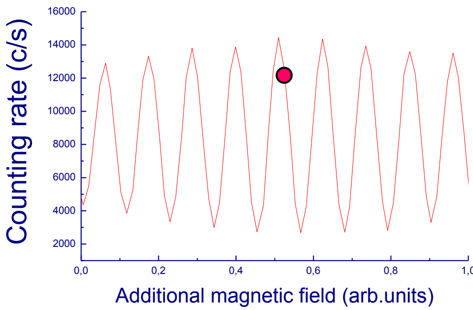
$$\tau = \frac{\hbar}{\sqrt{E_\perp (U - E_\perp)}}$$

N. K. Pleshanov, Physica B **198**, 70 (1994)

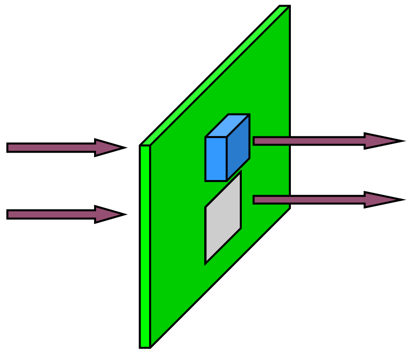
Spin – echo machine and detection of the extra-precession angle

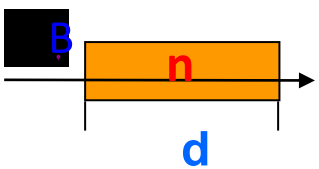




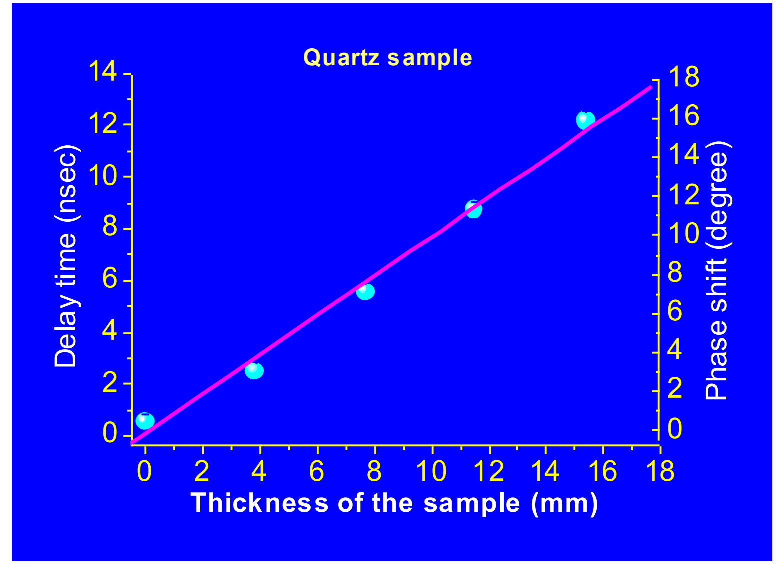


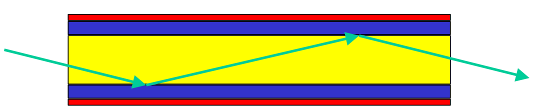
Time delay measure: refraction and Bragg reflection





$$\Delta\Phi = \omega_L \left(\frac{1-n}{n}\right)\frac{d}{v}$$

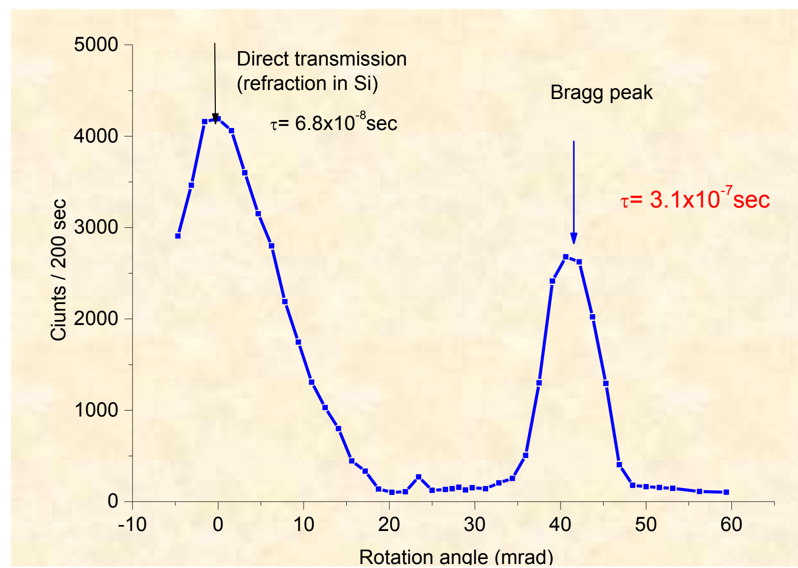




Si wafer (two-side polished)

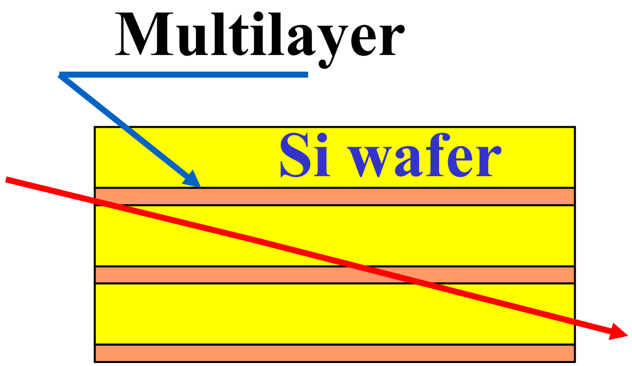
Multilayers
NiV(7) (130A+Ti 70A)x30

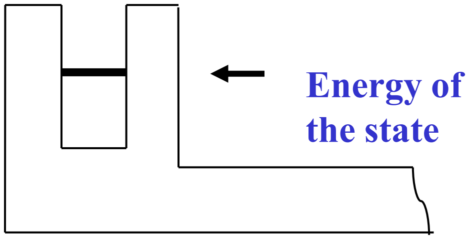
Gd absorber 700-100A



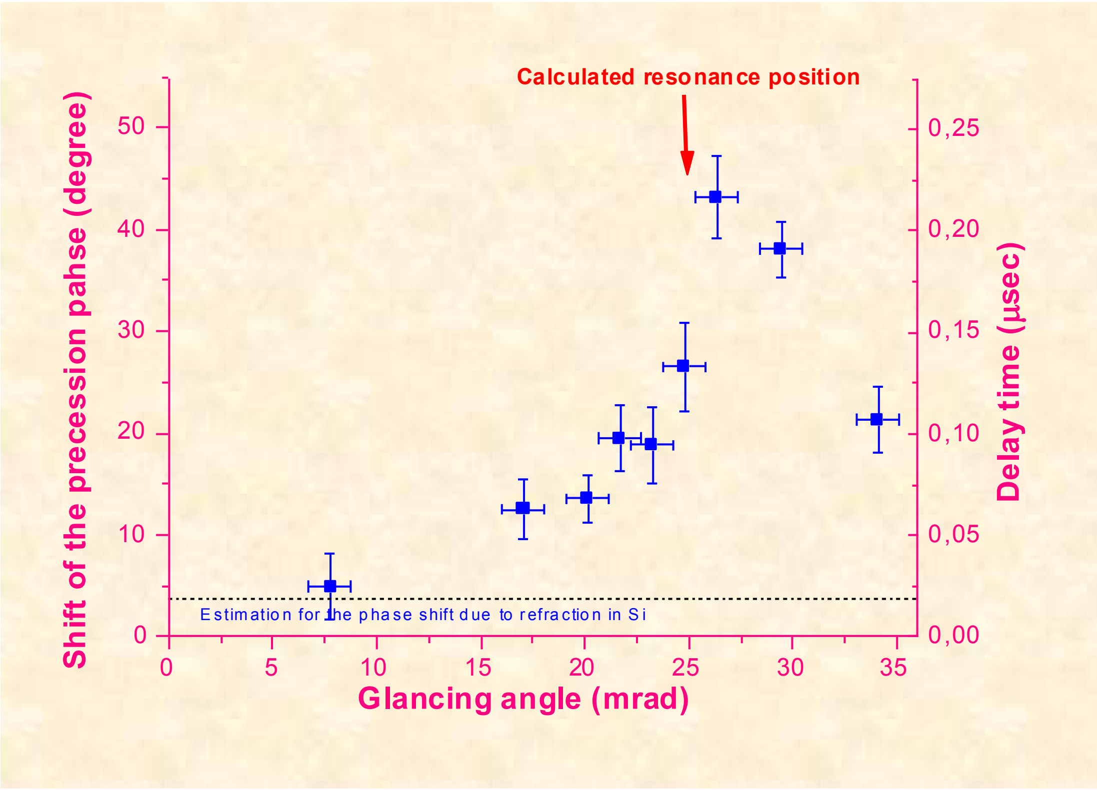
A.I. Frank, I.V. Bondarenko, A.V. Kozlov, P. Høghøj and G. Ehlers. Physica B: Condensed Matter 297, 307 (2001)

Tunneling time in a resonance of quasi-bound state





$$U = \frac{2\pi\hbar}{m}\rho b$$



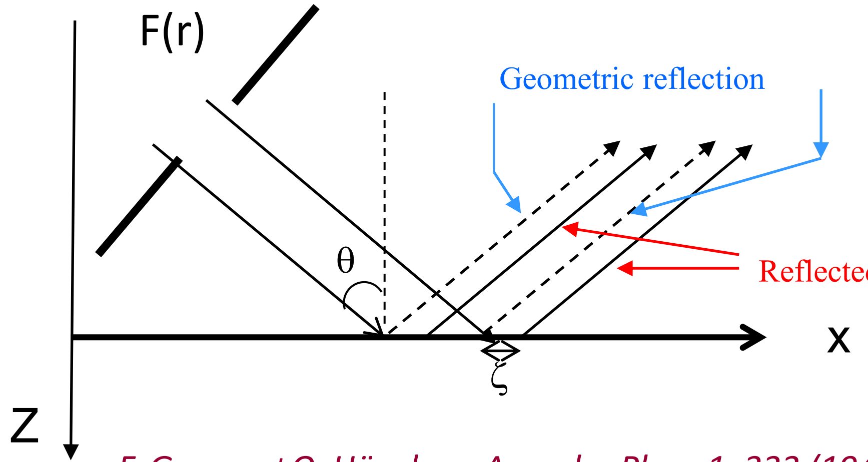
A.I.Frank, I.V. Bondarenko, V.V.Vasil'ev, I.Anderson, G.Ehlers, P. Høghøj. JETP Letters, 75, 705 (2002)
A.I.Frank, I.V.Bondarenko, A.V.Kozlov, G.Ehlers and P. Høghøj. In: Neutron Spin Echo Spectroscopy, Springer, pp 164-175.

1. Extra (or pseudo-Larmor) spin precession in neutron optics must be interpreted as a manifestation of the delay time.

2. Larmor clock is a good tool the calculation and measurement of the interaction time of neutron with various objects.

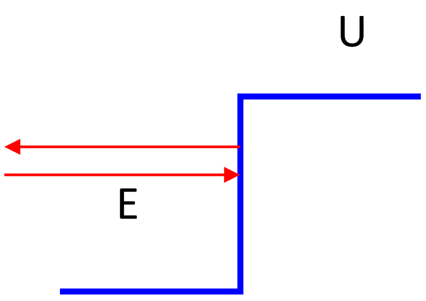
Goos – Hänchen shift at neutron reflection from a matter and group delay time

G.-H. shift. Formulation of the problem



F. Goos und O. Hänchen, Ann. der Phys. 1, 333 (1947).
F. Goos und H. Lindberg-Hanchen, Ann. der Phys. 5, 251 (1949)

$$\psi_{in}(r) = A_0(r)\exp(ik_0 r)$$



At z = 0

$$\Psi_{in}(x) \Leftrightarrow \Psi_R(x)$$

K. Artmann, Ann. der Phys. 2, 87 (1948)
L.M. Brechovskikh, Usp. Fiz. Nauk 50, 539 (1953)
H. Hora, Optik 17, 409 (1960)
J. L. Carter and H. Hora. J. Opt. Soc. Am, 61, 1640, (1971)

G.-H shift. Simplified solution

Fourier transform of the incoming and reflected waves

$$A_{in}(x) = \int_{-\infty}^{\infty} A_{in}(k_x) exp(ik_x x) dk_x$$

$$A_R(x) = \int_{-\infty}^{\infty} A_{in}(k_x) R(k_x) \exp(ik_x x) dk_x$$

G.-H shift. Simplified solution

Fourier transform of the incoming and reflected waves

$$A_{in}(x) = \int_{-\infty}^{\infty} A_{in}(k_x) exp(ik_x x) dk_x$$

$$A_R(x) = \int_{-\infty}^{\infty} A_{in}(k_x) R(k_x) \exp(ik_x x) dk_x$$

$$R(k_x) = |R(k_x)| \exp[i\varphi(k_x)]$$

Approximation

$$\varphi(k_x) \cong \varphi_0(k_x)_{x0} + \varphi'(k_x)_{x0}(k_x - k_{x0})$$

G.-H shift. Simplified solution

Fourier transform of the incoming and reflected waves

$$A_{in}(x) = \int_{-\infty}^{\infty} A_{in}(k_x) exp(ik_x x) dk_x$$

$$A_R(x) = \int_{-\infty}^{\infty} A_{in}(k_x) R(k_x) \exp(ik_x x) dk_x$$

$$R(k_x) = |R(k_x)| \exp[i\varphi(k_x)]$$

Approximation

$$\varphi(k_x) \cong \varphi_0(k_x)_{x0} + \varphi'(k_x)_{x0}(k_x - k_{x0})$$

Solution

$$A_R(x) \cong |R(k_{x0})| \int_{-\infty}^{\infty} A_{in}(k_x) exp[ik_x(x - \varsigma] dx.$$

$$\zeta = -(d\varphi/dk_x)_{x_0}$$

G.-H shift of the matter waves and its relation with the group delay time

$$\varsigma = -\left(\frac{d\varphi}{dk_x}\right)_{x_0} = -\left(\frac{d\varphi}{dk_z^2}\right)\frac{dk_z^2}{dk_x}$$

$$k_z^2 = k_0{}^2 - k_x^2$$

$$k_z^2 = \frac{2m}{\hbar^2}E_z$$

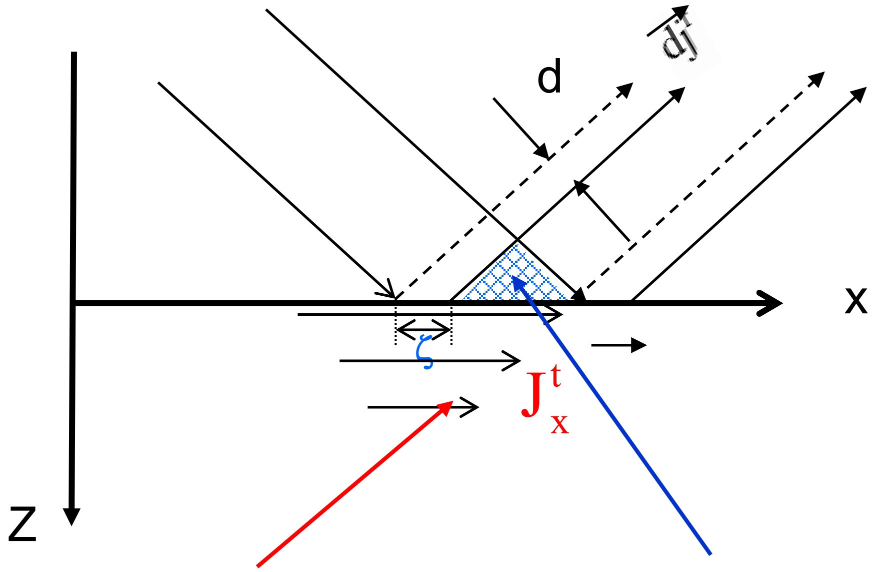
$$\varsigma = \hbar\left(\frac{d\varphi}{dE_z}\right)v_x = \tau_g v_x$$

$$\tau_g = \hbar\left(\frac{d\varphi}{dE_z}\right)$$

Group delay time

For the total reflection $\tau \approx 5 \div 7$ ns

G.-H shift and flux balance



R. H. Renard. J. Opt. Soc. Am. 54, 1190 (1964)
with correction of
K. Yasimoto, Y.Oishi. J.Apll. Phys. 54, 2170 (1983)
V. G. Fedoseyev J. Opt. Soc. Am. A 3, 826, (1986)

$$J_{extra} = d \cdot j^r = d \cdot j^{in}$$

$$J_x^t = \frac{2\hbar}{m} \frac{1}{\sqrt{k_b^2 - k_z^2}} \frac{k_z^2}{k_b^2}$$

$$J_x^{ir} = -(V_x / k_z)\sin\varphi$$

$$J_{extra} = J_x^t + J_x^{ir} = \frac{2\hbar}{m\sqrt{k_b^2 - k_z^2}}$$

$$k_b = \sqrt{2mU} / \hbar$$

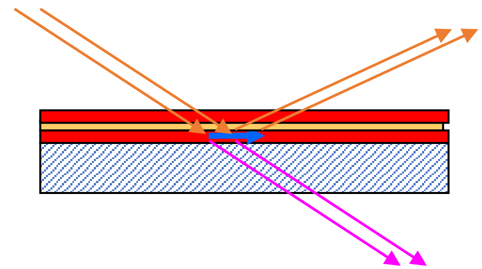
$$\varsigma = \frac{2k_x}{k_y\sqrt{k_b^2 - k_z^2}}$$

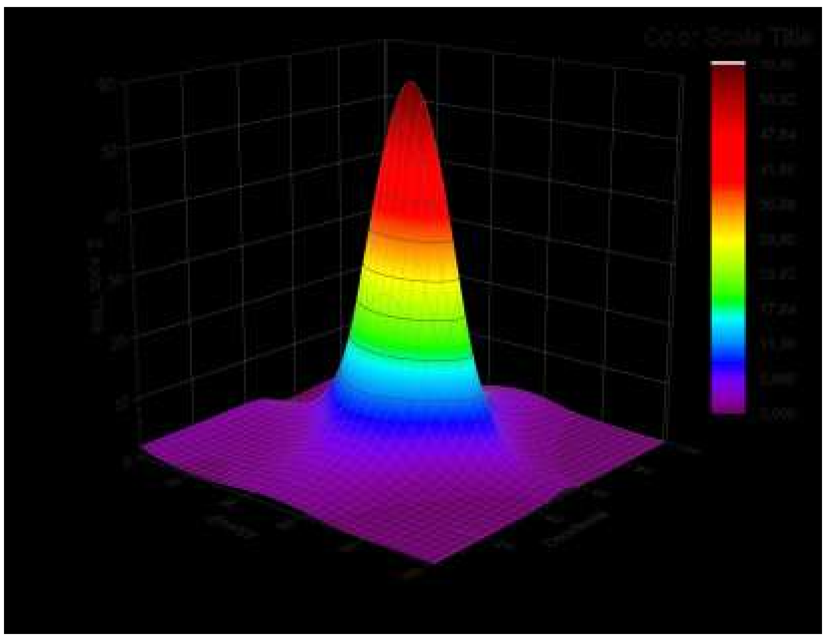
$$\zeta = -(d\varphi / dk_x)_{x_0}$$

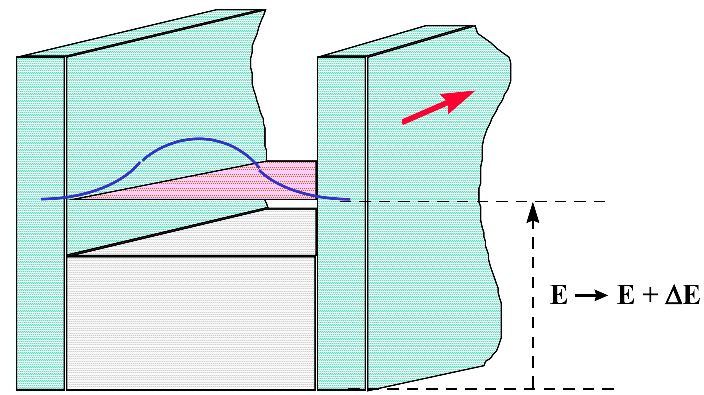
$$\varsigma = v_x \tau$$

Alternative definition of the interaction time and resonant amplification of the GDT

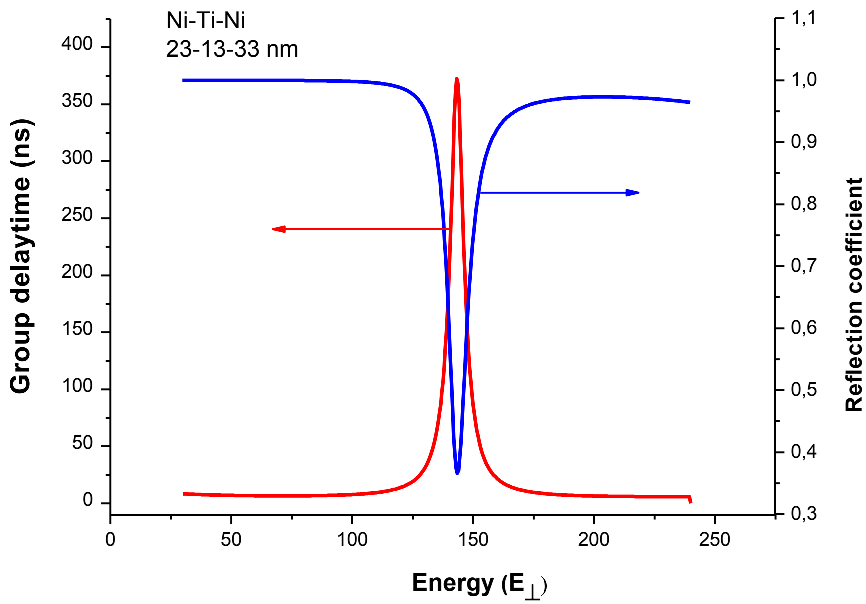
$$\tau = \frac{|\psi_{extra}|^2}{\dot{j}_{in}}$$

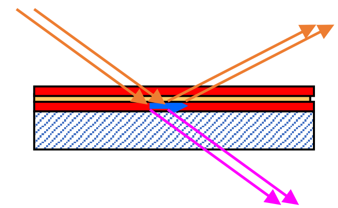


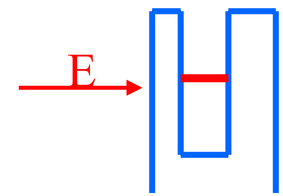


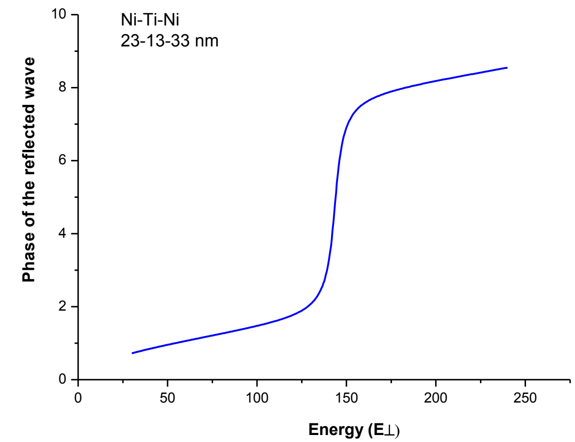


Giant group delay time









Group delay time in reflection and reflection coefficient for the asymmetric three-layered resonant structures NiMo-Ti-NiMo

Phase of the reflection wave

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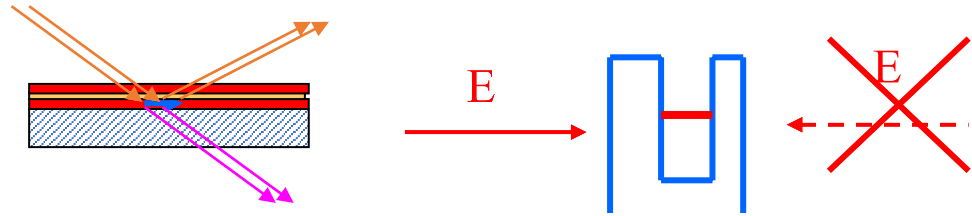
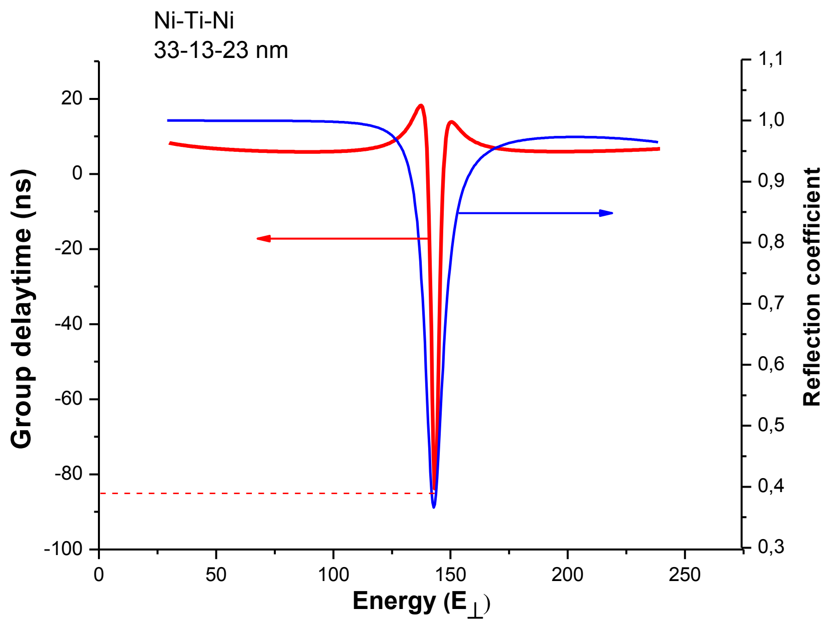
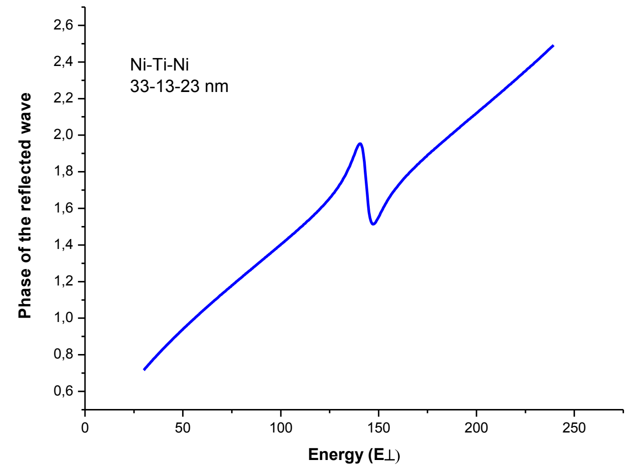
Negative group delay time

Asymmetric interference filter







Group delay time in reflection and reflection coefficient for the asymmetric three-layered resonant structures NiMo-Ti-NiMo

Phase of the reflected wave

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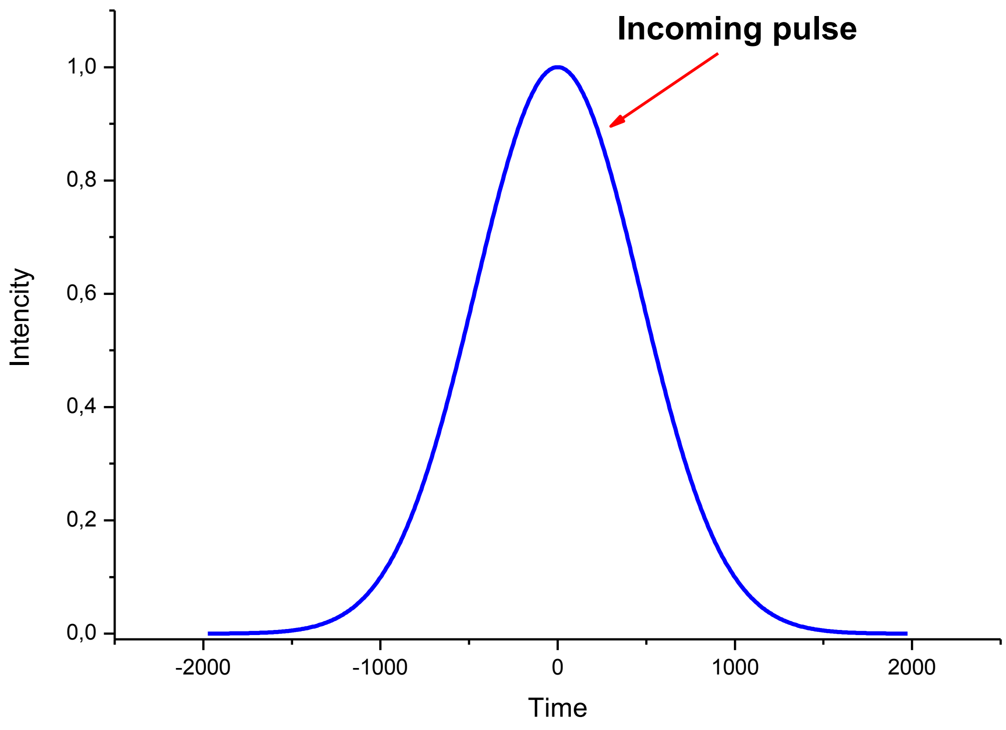
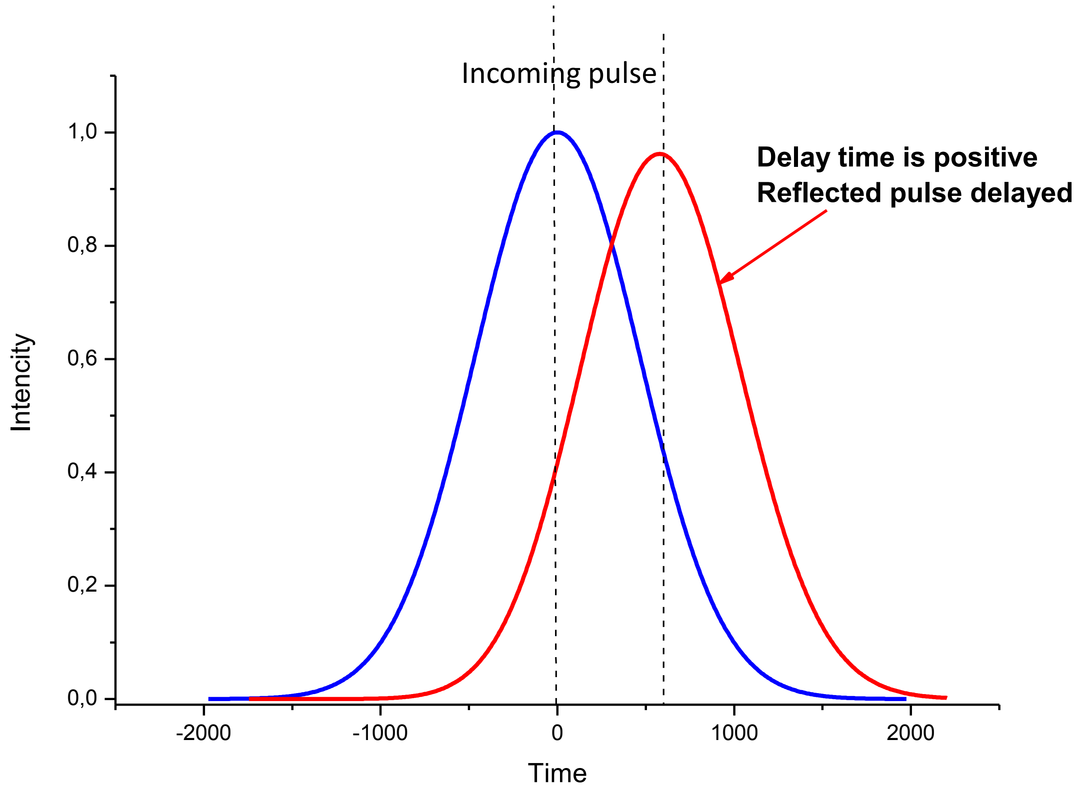
Positive and negative delay time. What is it?

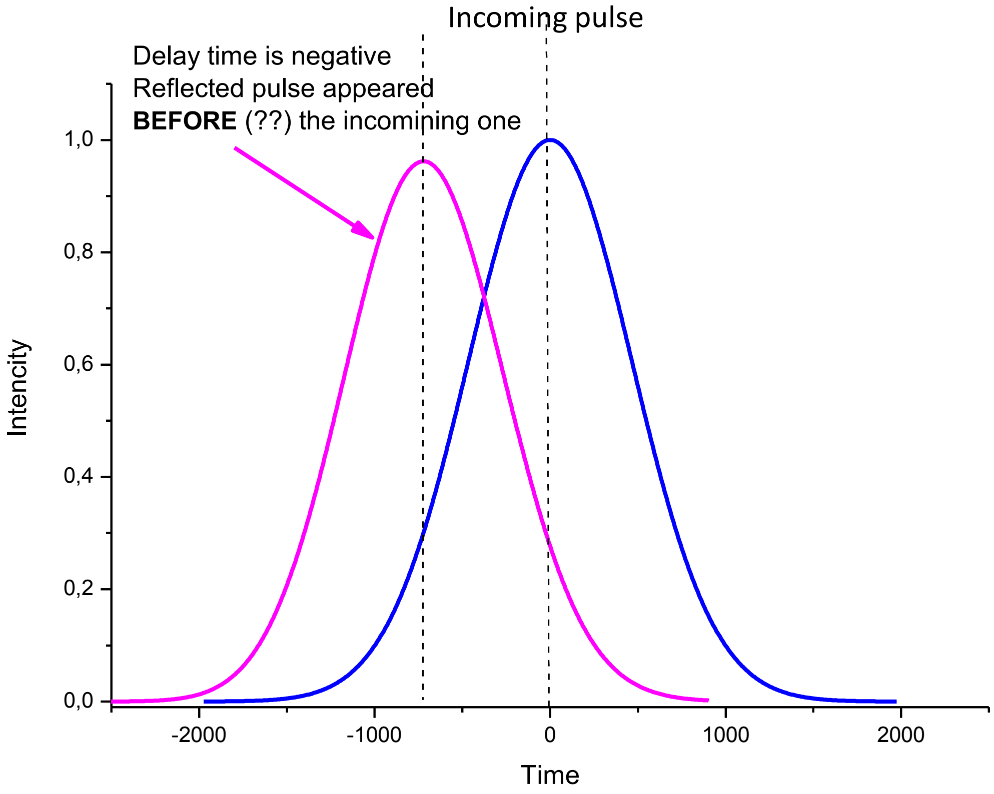
Incoming pulse



Positive and negative delay time. What is it?



Positive and negative delay time. What is it?



Group delay time at the reflection of wave packet

At z = 0

$$A_{in}(t) = A_0(t)exp(-i\omega t)$$

$$A_{in}(t) = \int_{-\infty}^{\infty} A_{in}(\omega)exp(-i\omega t)d\omega$$

$$A_R(t) = \int_{-\infty}^{\infty} A_{in}(\omega)R(\omega)\exp(-i\omega t)d\omega,$$

$$\varphi(\omega) \cong \varphi_0(\omega_0) + \varphi'(\omega_0)(\omega - \omega_0)$$

$$\tau = \left(\frac{d\varphi}{d\omega}\right)_{\omega 0} = \hbar\left(\frac{d\varphi}{dE}\right)_{E0}$$

That is very rough approximation

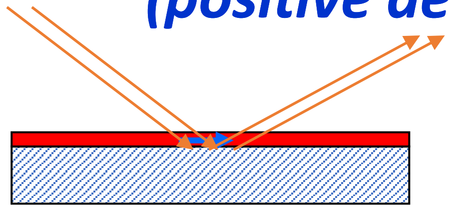
$$A_R(t) = \int_{-\infty}^{\infty} A_{in}(\omega)R(\omega)\exp(-i\omega t)d\omega,$$

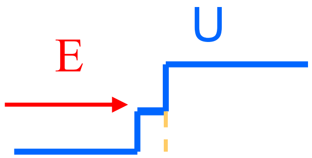
↑

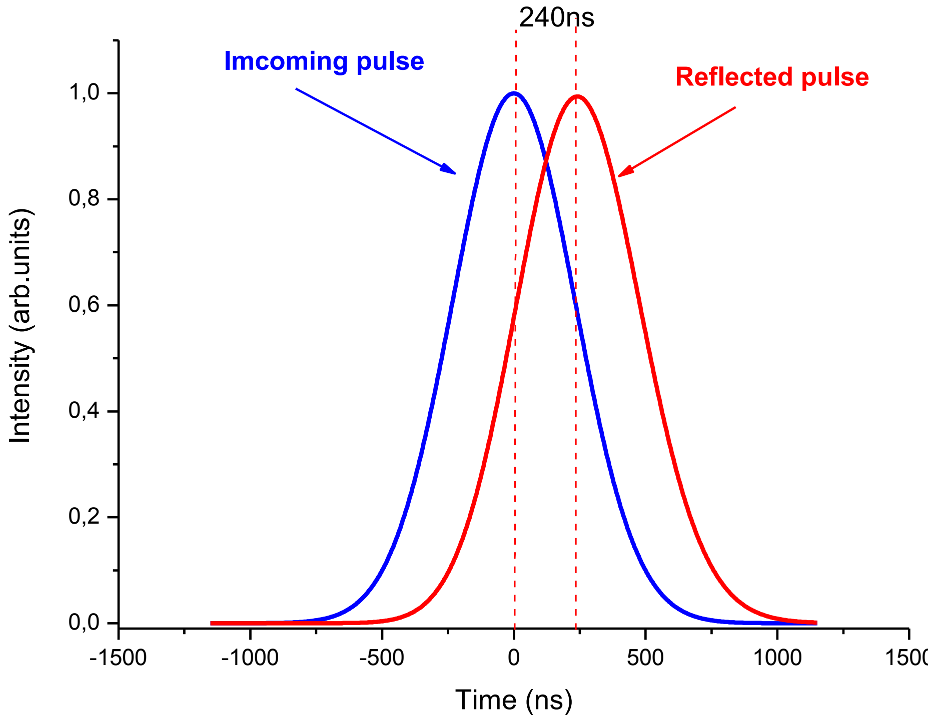
Must be calculated

where $A_{in}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_{in}(t)exp(i\omega t)dt$

Reflection of the short time pulse (positive delay time)



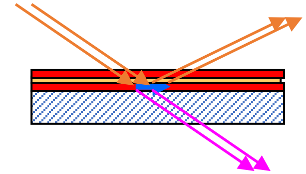


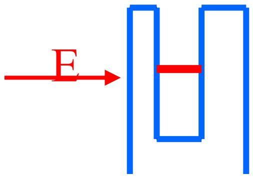


Energy 121 nev
Spectrum width (FWHM) 2 nev
Duration of the time pulse 330 ns
Ideal time shift (GDT) 277ns
Real time shift 240 ns

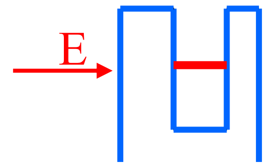
More details in the talk of German Kulin today later

Reflection of the short time pulse

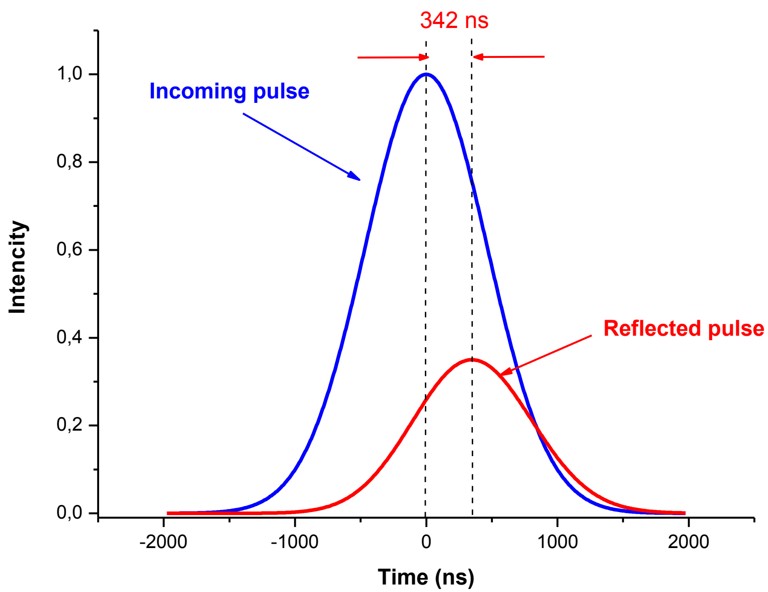




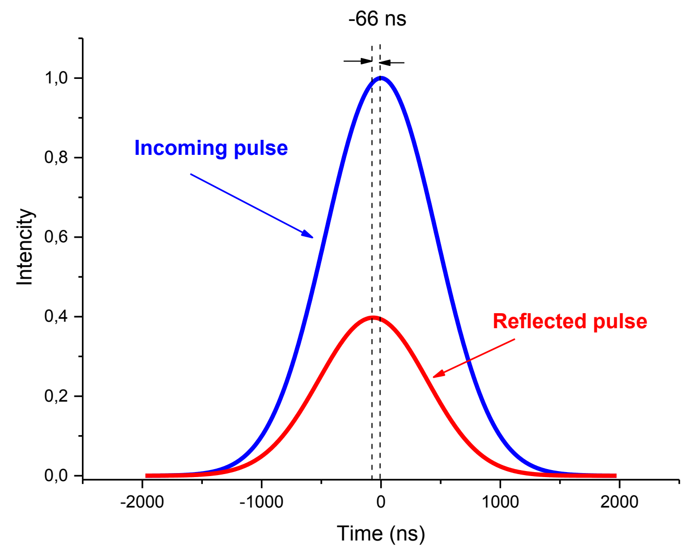
Positive delay time



Negative delay time



Energy 144nev
Spectrum width (FWHM) 1 nev
Duration of the time pulse 660 ns
Ideal time shift (GDT) 349ns
Real time shift 342ns



Energy 144nev
Spectrum width (FWHM) 1 nev
Duration of the time pulse 660 ns
Ideal time shift (GDT) -80ns
Real time shift -66 ns

1. *In the first approximation G-Ch. shift at the neutron reflection is proportional to the GDT*

2. *In the case of neutron reflecton from resonant multilayered structures (GDT) may reach very large positive and negative value but that is not contradict to the causality principle*

Group delay time and refraction of neutron waves

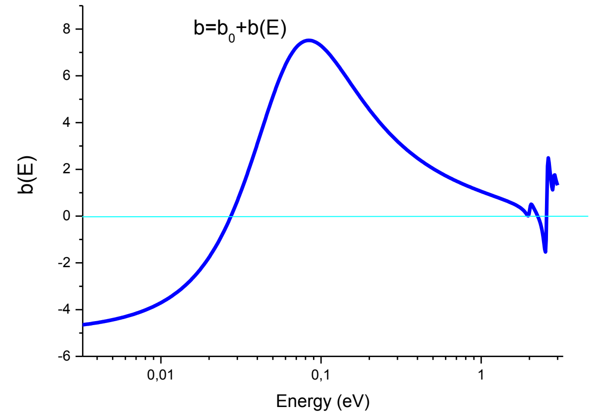
GLT , dispersion law and refraction

Dispersion law $k^2 = k_0^2 - 4\pi\rho b$ *is not exactly correct*

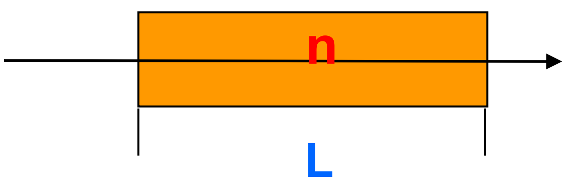
$$k^2 = k_0^2 - 4\pi\rho b(k_0)c(k_0),$$

M.Lax. 1952,1952; F. Sears, 1985

$$k = F(k_0^2)$$



Behavior of the resonant part of scattering amplitude b for the natural Gd



$$\Delta\Phi = kL \qquad \tau = \hbar \frac{d(\Delta\Phi)}{dE} \qquad v = \frac{L}{\tau}$$

$$v = \frac{\hbar}{2m}(F\ )$$

$$v \overset{?}{\neq} nv_0$$

A.I.Frank. Physics Uspekhy (in print)

GLT , dispersion law and refraction

$$\mathbf{v} \neq n\mathbf{v}_0 \qquad k = nk_0 \qquad m^*\mathbf{v} = nm\mathbf{v}_0$$

$$v = \frac{\hbar k}{m^*} = nv_0 \frac{m}{m^*} \qquad m^* = 2mkF' \qquad \frac{1}{m^*} = \frac{\partial^2 E}{\partial p^2}$$

For negative neutron effective mass at Bragg diffraction see Zeilinger A. Et al. *Phys. Rev.Lett.* **57,** 3089 (1986)

Generally speaking neutron inside a refractive matter is not a particle

but quasi particle

Putting $\mathbf{v} = n\mathbf{v}_0$ We arrive immediately to

$$\boxed{k^2 = k_0^2 + \chi^2} \qquad \boxed{m^* = m} \qquad k^2 = k_0^2 - 4\pi\rho b$$

Conclusion

1. ***Group delay time (GDT) play an important role in neutron optics.***

2. ***In the spin optics GDT may be related with the extra precession angle. Larmor clock may be used for the calculation and measure of GDT***

3. ***Seems that we understand the physical nature of the G.H. effect as a general wave phenomenon but for the neutron beam it was not yet directly observed.***

4. ***The concept of GDT was used for the derivation the equation for the neutron velocity in the refractive matter.***

Thank you for your attention