Investigation of a neutron diffraction at surface acoustic waves

G.V. Kulin, A.I. Frank, S.V. Goryunov, N.V. Rebrova

V.A. Bushuev, Yu.N. Khaydukov, D.V. Roschupkin

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I.M. Frank JINR Comm. R4-8851 (1975) - possibility of inelastic process of neutron diffraction on surface (Rayleigh) waves in connection with the so-called UCN storage anomaly.

UCN scattering for the case of waves at the surface of a fluid:


First direct experiment aimed at observing neutron diffraction on travelling surface acoustic wave (SAW) excited on the surface (quartz plate)
Diffraction of neutrons on travelling SAW

\[ \Psi_0(x, z, t) = \exp \left( i k_{0x} x + i k_{0z} z - i \omega_0 t \right) \]

Due to diffraction on SAW:

\[ \Psi_R(x, z, t) = \sum_{n=-\infty}^{\infty} r_n \exp(i k_{nx} x - i k_{nz} z - i \omega_n t) \]

\[ \Psi_T(x, z, t) = \sum_{n=-\infty}^{\infty} t_n \exp(i k_{nx} x + i q_{nz} z - i \omega_n t) \]

At the interface \( z_s \) (boundary conditions):

\[ \Psi_0(x, z_s, t) = \sum_{m=-\infty}^{\infty} J_m(k_{0z} A) \exp(i k_{mx} x - i \omega_m t) \]

\[ k_{m_x} = k_{0x} + sQm \]

\[ \omega_m = \omega_0 + m\Omega \]

\[ \Psi_T(x, z_s, t) = \sum_{n} \sum_{m'} t_n J_{m'}(q_{nz} A) \exp[i(q_{nx} + m'Q)x - i(\omega_n + m'\Omega)t]. \]

\[ \Psi_R(x, z_s, t) = \sum_{n} \sum_{m'} r_n J_{m'}(-k_{nz} A) \exp[i(k_{nx} + sm'Q)x - i(\omega_n + m'\Omega)t] \]

Neutron wave number in a matter

\[ q_{nx} = k_{nx} \]

\[ q_{nz} = \sqrt{k_{nz}^2 - k_b^2} \]

\[ k_b = \sqrt{2MU_0} / \hbar \]

critical wave number
Diffraction of neutrons on travelling SAW

\[ \Psi_0(x, z, t) = \exp\left(ik_{0x}x + ik_{0z}z - i\omega_0 t\right) \]

Due to diffraction on SAW:

\[ \Psi_R(x, z, t) = \sum_{n=-\infty}^{\infty} r_n \exp(ik_{nx}x - ik_{nz}z - i\omega_n t) \]

\[ \Psi_T(x, z, t) = \sum_{n=-\infty}^{\infty} t_n \exp(ik_{nx}x + iq_{nz}z - i\omega_n t) \]

Continuity equations at the interface \( z_s \) (boundary conditions):

\[ J_m(k_{0z}A) + \sum_n r_n J_{m-n}(-k_{nz}A) = \sum_n t_n J_{m-n}(q_{nz}A) \]

\[ k_{0z}J_m(k_{0z}A) - \sum_n r_n k_{nz}J_{m-n}(-k_{nz}A) = \sum_n t_n q_{nz}J_{m-n}(q_{nz}A) \]

At experimental conditions arguments of Bessel functions \( J_m \) are small \((n, m = 0, \pm 1)\)

\[ R_m = \left| \frac{k_{nz}}{k_{0z}} \right|^2 \]

\[ R_0 = \left| \frac{k_{0z} - q_{0z}}{k_{0z} + q_{0z}} \right|^2, \quad R_{\pm 1} = k_{0z}k_{\pm 1z}A^2 \left| \frac{k_{0z} - q_{0z}}{k_{\pm 1z} + q_{\pm 1z}} \right|^2 \]

\[ k_{\pm 1z} = \sqrt{k_{0z}^2 \pm 2Q(k_V - sk_{0x})} \]

\[ q_{\pm 1z} = \sqrt{k_{\pm 1z}^2 - k_b^2} \]

\[ k_V = MV/\hbar \]
Diffraction of neutrons on travelling SAW

Intensities of diffraction orders:

\[ R_m = \left( \frac{k_{mx}}{k_{0x}} \right)^2 r_m^2 \]

\[ R_0 = \left( \frac{k_{0z} - q_{0z}}{k_{0z} + q_{0z}} \right)^2, \quad R_{\pm 1} = k_{0z} k_{\pm 1z} A^2 \left( \frac{k_{0z} - q_{0z}}{k_{\pm 1z} + q_{\pm 1z}} \right)^2 \]

\[ q_{nz} = \sqrt{k_{nz}^2 - k_b^2} \quad \text{neutron wave number in a matter} \]

\[ k_b = \sqrt{2MU_0 / \hbar} \quad \text{critical wave number} \]

Validity of the potential dispersion law is supposed

Diffraction angles:

\[ k_{mx} = k_{0x} + smQ \quad \omega_m = \omega_0 + m\Omega \]

\[ \Omega = 2\pi V / \Lambda \quad Q = 2\pi / \Lambda = \Omega / V \]

\[ \hbar \omega_m = \hbar^2 k_m^2 / 2M \]

\[ k_V = MV / \hbar \]

\[ \sin^2 \theta_m = \sin^2 \theta_0 + \frac{2mQ}{k_0} \left( \frac{k_V - sk_{0x}}{k_0} \right) - \frac{m^2 Q^2}{k_0^2} \]
• SAW arise due to periodical oscillation of the near-surface layer of a matter that moves with alternative velocity and acceleration

• The depth of penetration of the Rayleigh wave into the matter is of the order of the SAW wavelength

For a typical SAW with a frequency of tens MHz this acceleration reaches a value of $10^7g$!!!

Is the commonly accepted theory of UCN interaction with matter is valid in this case?
The theory of dispersion - the theory of multiple scattering of waves

Dispersion law of the neutron wave in accelerating matter — no any theoretical predictions at this moment

What is the region of validity of the potential dispersion law?

The hypothesis: Potential dispersion law is valid if phase distortion due to accelerating appeared at the interatomic distance is much less than the phase shift $kb$ due to scattering at the nuclei (A. I. Frank, JETP Lett., 100, p. 613, 2014).

$$\Delta \phi_w = \frac{mwx^2}{2E} \ll \delta \cong kb$$

for $x \approx$ interatomic distance $d$

$$w \ll \frac{4Eb}{md^2} = w_{\text{crit}}$$
Are there any experimental results?

<table>
<thead>
<tr>
<th>Type of experiment</th>
<th>E, eV</th>
<th>Acceleration, m/sec²</th>
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<td>2.8×10⁻⁴ 6.7×10⁻⁴</td>
<td>6.3×10⁷  4.8×10⁹</td>
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There are no contradictions with hypothesis of critical acceleration
NREX+ Reflectometer (FRM II, Garching, Germany)

Neutron wave length: 4.3 Å
Wavelength resolution: 1-2%
Angular divergence: 0.7 mrad

Sample: Single crystal Lithium Niobate (LiNbO$_3$)

Size of Surface Wave Region: 0.5 × 6 cm$^2$
SAW velocity: 3490 m/sec > $V_n = 920$ m/sec (4.3 Å)
SAW frequency: 69 MHz
SAW amplitude: ~ 2 nm
SAW wavelength: ~ 50 mkm

$\Delta E \approx 290$ neV (!)
SAW travelling direction

SAW ($s = -1$)

Standing SAW

SAW ($s = +1$)

No SAW

Standing SAW

V

V

A

V=0

Dubna, June 10, 2019
Kinematics is in good agreement with the calculations.

The reflection coefficients of the corresponding diffraction orders are consistent with the SAW amplitude $\sim 2$ nm.

Diffraction pattern in case of standing SAW looks if it is formed on two travelling waves independently.
Are there any experimental results?

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There are no contradictions with hypothesis of critical acceleration

Two experimental ways with SAW:

• Reflectometric experiment in a wide band of wavelengths
Accepted proposal to D17 reflectometer (@ILL, Grenoble France) – Autumn 2019

TOF mode measurements at number of fixed GSE for wide range of wavelength
Intensities of +/-1 diffraction orders: \[ R_{\pm 1} = k_{0z} k_{\pm 1z} a^2 \frac{k_{0z} - q_{0z}}{k_{0z} + q_{1z}} \]

\[ q_{nz} = \sqrt{k_{nz}^2 - k_b^2} \] neutron wave number in a matter, \[ k_b = \sqrt{2MU_0 / \hbar} \] critical wave number

No predictions for possible deviation term

\[ q_{nz}^2 = k_{nz}^2 - (1 + \delta) k_b^2 \]

\[ q_{nz}^2 = k_{nz}^2 - k_b^2 - \delta \varepsilon(k_0^2), \varepsilon(k_0^2) \sim 1/k_0^2 \]
Are there any experimental results?

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There are no contradictions with hypothesis of critical acceleration

Two experimental ways with SAW:

- Reflectometric experiment in a wide band of wavelengths
- Use neutrons with an energy at which acceleration of the matter is larger the critical one
Experiment on UCN upscattering at travelling SAW

Single crystal Lithium Niobate (LiNbO₃):

SAW frequency: 34 MHz
SAW amplitude: ~ 2 nm

\[ A\Omega^2 \approx 10^8 \text{ m/sec}^2 \text{ (SAW)}, \]

\[ \nu_{\text{crit}} = 4 \times 10^5 \text{ m/sec}^2 - \text{UCN} (E = 10^{-7} \text{ eV}) \]

In details: S.V. Goryunov — next talk
Summary

- SAWs arise due to periodical oscillation of the near-surface layer of matter that moves with alternative velocity and acceleration.

- Measurements with SAW for wide range of neutron wavelength can be suitable for the test of the concept of effective potential in case of giant acceleration.

- Existing experimental results are in agreement with potential dispersion law but there are no contradictions with hypothesis of critical acceleration.

- It is planned to perform TOF mode measurements at fixed GSE for wide range of wavelength.

- Experiment on UCN upscattering at SAW was started.
Thank you for your attention!