AB INITIO STUDY OF DECAY WIDTHS AND BRANCHING RATIOS OF NEUTRON RESONANCES OF LIGHT NUCLEI

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Ab initio calculations of light nuclei using NCSM

The formalism of No-Core Shell Model (NCSM) is following:

1) Basis components are

$$\psi_i = \begin{vmatrix} \psi_{n_1 l_1 j_1 m_1}(r_1) & ... & \psi_{n_A l_A j_A m_A}(r_1) \\ ... & ... & ... \\ \psi_{n_1 l_1 j_1 m_1}(r_A) & ... & \psi_{n_A l_A j_A m_A}(r_A) \end{vmatrix}. \qquad (1)$$

The basis limit is $\sum_{k=1}^{A} 2n_k + l_k \leq N_{\max}^{sum}$.

2) The A-nucleon Schrödinger equation is solved on the built basis

$$H\psi = E\psi, \psi = \sum_i c_i \psi_i, H = T + U$$

3) The solution of the equation is equal to the calculation of lowest eigenvalues of the following matrix

$$\begin{Vmatrix} \langle \psi_1 | H | \psi_1 \rangle & ... & \langle \psi_N | H | \psi_1 \rangle \\ ... & ... & ... \\ \langle \psi_1 | H | \psi_N \rangle & ... & \langle \psi_N | H | \psi_N \rangle \end{Vmatrix}$$

The dimension of the Slater determinants basis is up to 10^{10} on modern supercomputers.

4) Calculation method is iterative Lanczos algorithm for eigenvalues and eigenvectors usually.

5) The results are total binding energies and wave functions of the ground and lower exited states of light nuclei obtained in ab initio calculations.

DISADVANTAGES: Difficulties in calculations of the asymptotic characteristics of various nucleon and cluster channels (it means difficulties with decay widths and cross sections) due to extremely growth of SD basis.

Cluster Channel Orthogonal Functions Model (CCOFM)

CCOFM model could be used for ab initio calculations of various nuclear characteristics:

CCOFM

Ab initio nuclear spectra calculations in clustered and combined bases

Calculations of cluster and asymptotic properties of ground and lower exited states:
1) **spectroscopic factors**
2) **nucleon and cluster formfactors**
3) **partial decay widths of resonances**
4) **asymptotic normalizing coefficients of bound states**

Construction of cluster terms of CCOFM basis

The cluster-channel terms are built in the form of translationally-invariant A-nucleon wavefunctions

$$\Psi_A^i = \frac{1}{W} A\{\Psi_{A_1} \Psi_{A_2} \varphi_{nlm}(\rho)\}_{JM_J},$$

For ab initio calculations cluster-channel terms must be expressed in the form of Slater determinant superposition:

$$\Psi_A^i = \sum_{j=1}^{N_{size}} C_j^i \psi_j, N_{size} < N_{NCSM}^{max}$$

The algorithm for construction of the basis WF:

$\Phi_{000}^{A_i}(R_i)\Psi_{A_i}$ - NCSM solutions for individual clusters with zero vibrations along the coordinate of the center of mass.

Calculations of cluster WF with nonzero vibrations along the coordinate of the center of mass $\Phi_{NLM}^{A_i}(R_i)\Psi_{A_i}$ as SD superpositions using cluster coefficients method.
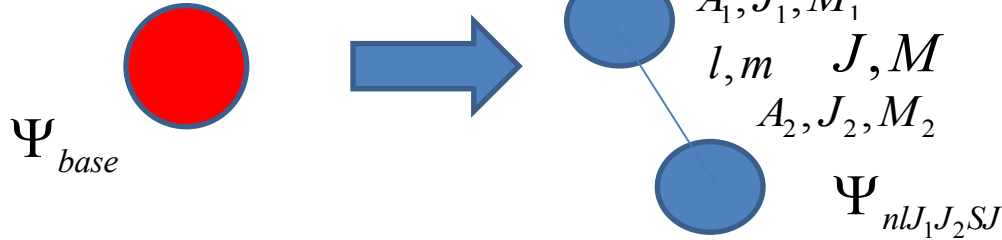
Obtaining cluster-channel terms by Talmi-Moshinsky transformation.

$$\Psi_A^i = \frac{1}{W} A\left\{ \sum_{N_1,L_1,M_1,N_2,L_2,M_2} \left\langle \begin{matrix} 000 \\ nlm \end{matrix} \middle| \begin{matrix} N_1,L_1,M_1 \\ N_2,L_2,M_2 \end{matrix} \right\rangle \Phi_{N_1,L_1,M_1}^{A_1}(R_1)\Psi_{A_1} \Phi_{N_2,L_2,M_2}^{A_2}(R_2)\Psi_{A_2} \right\}.$$

The algorithm of cluster properties calculation

1) The task is: J, M

Ψ_{base}

A_1, J_1, M_1
l, m J, M
A_2, J_2, M_2

$\Psi_{nlJ_1J_2SJ}$

2) Basis functions: $\Psi_A^{nlJ_1J_2SJ} = A\{\{\Psi_{A_1}^{J_1M_1}\Psi_{A_2}^{J_2M_2}\}_{SM_S}\varphi_{nlm}(\rho)\}_{JM_J}$,

3) Orthogonalization of basis WF for each channel

$$\Psi_A^i = A\{\{\Psi_{A_1}^{J_1M_1}\Psi_{A_2}^{J_2M_2}\}_{SM_S}\varphi_{ilm}(\rho)\}_{JM_J}, \quad \varphi_{ilm}(\rho) = \sum_n B_n^i\varphi_{nlm}(\rho)$$

4) The cluster formfactor of fragmentation of the A-nucleon state characterized by the wavefunction Ψ_{base} into this channel has the form

$$F_l(r) = \sum_i \frac{1}{\sqrt{\varepsilon_i}}\langle\Psi_A^i|\Psi_{base}\rangle\varphi_{il}(r),$$

5) Obtaining the cluster formfactor as a superposition of oscillator functions and spectroscopic factor, as its normalization.

$$F_l(r) = \sum_n A_n^l\varphi_{nl}(r), \quad A_n^l = \sum_{i,n'}(\varepsilon_i)^{-1/2}\langle\Psi_{base}|\Psi_A^{n'lJ_1J_2SJ}\rangle B_n^iB_{n'}^i$$

Calculation of asymptotic properties

1) Narrow neutron resonances calculations

In this case, in a sufficiently wide subbarrier region, nuclear attraction is negligible.

For all this region the relation $n_l(\mathrm{r}) \gg \mathrm{j}_l(\mathrm{r})$ takes place.

It means that regular function could be neglected.

To determine the position of the matching point R_{point} of the cluster formfactor and irregular wavefunction in this range, we use the condition of equality of the logarithmic derivatives:

$$\frac{F'_l(r)}{F_l(r)} = \frac{n'_l(r)}{n_l(r)}.$$

Therefore, the decay width of the neutron resonance is given by the expression:

$$\Gamma = \frac{h^2}{\mu k}\left(\frac{F_l(R_{point})}{n_l(R_{point})}\right)^2$$

2) Proton and cluster resonances calculations

For proton and cluster resonances Coulomb functions are used instead of Neiman and Bessel functions.

3) Calculations of asymptotic normalizing coefficients of bound states

$$ANC = \frac{rF_l(r)}{W_{-\eta,l+1/2}(2kr)}$$

Calculation results

For ab initio calculations of light nuclei properties, modern realistic NN-potentials Daejeon16 and JISP16 were used. These universal potentials describe the spectra of all light nuclei up to ^{16}O without additional phenomenological parameters.

For the oscillator basis, the distance region, where the solutions of the Schrödinger equation are described correctly, expands in proportion to $[N_{max}^{tot}]^{1/2}$. In this connection, a microscopic description of cluster channels at distances where asymptotic representation is correct requires an extremely large NCSM basis.

Therefore, the cutoff parameter for the ^{7}Li basis is 16, and the corresponding SD basis is $5.1 * 10^{8}$, in the case of the ^{5}He, the cutoff parameter is 17.

Total binding energies of ^{7}Li, ^{5}He and their subsystems

Calculations of ground states of ^{7}Li and ^{5}He were held for cutoff parameters N^{max} = 16 (17) for different values of oscillator parameter $\hbar\omega$. Subsystems ^{4}He, ^{3}H, ^{6}Li were calculated with cutoff parameter N^{max} = 4.

	Total binding energies of ^{7}Li, ^{5}He, ^{4}He, ^{3}H, ^{6}Li.						
	$\hbar\omega$ =12.5	$\hbar\omega$ =15	$\hbar\omega$ =17.5	$\hbar\omega$ =20	$\hbar\omega$ =22.5	$\hbar\omega$ =25	exp.
^{5}He	27.335	27.279	27.209	-----	------	--------	27.41
^{7}Li	------	------	------	39.110	38.909	38.620	39.245
^{4}He	26.778	27.784	28.102	28.158	28.126	28.051	28.296
%($0s1/2^4$)	0.891	0.9373	0.9694	0.9848	0.9861	0.97724	-----
^{3}H	------	------	------	7.976	7.802	7.598	8.482
%($0s1/2^3$)	------	------	------	0.9740	0.9628	0.9489	-----
^{6}Li	-------	------	-------	29.186	28.409	27.325	31.995

For given basis cutoff parameters, the ground states of ^{7}Li and ^{5}He and their subsystems ^{4}He, ^{3}H, ^{6}Li are described with good accuracy.
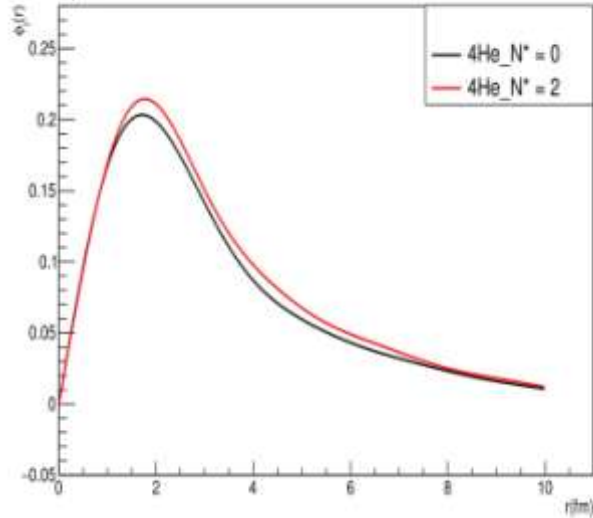
Decay width of 3/2⁻ resonance of ⁵He
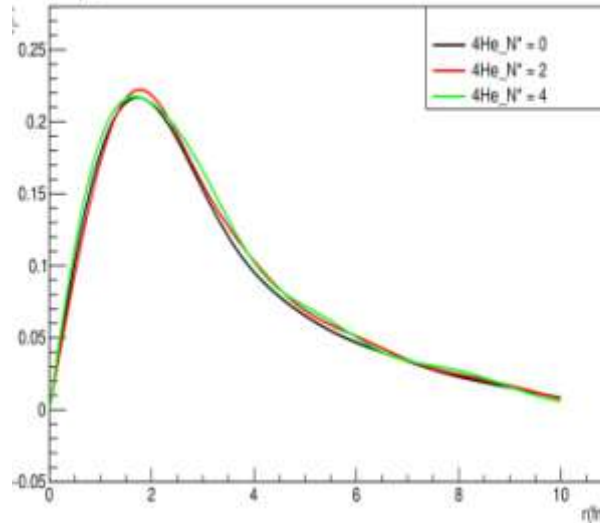
$\hbar\omega$ = 12.5, E_{th} = 27.335 $\hbar\omega$ = 15, E_{th} = 27.279 $\hbar\omega$ = 17.5, E_{th} = 27.209
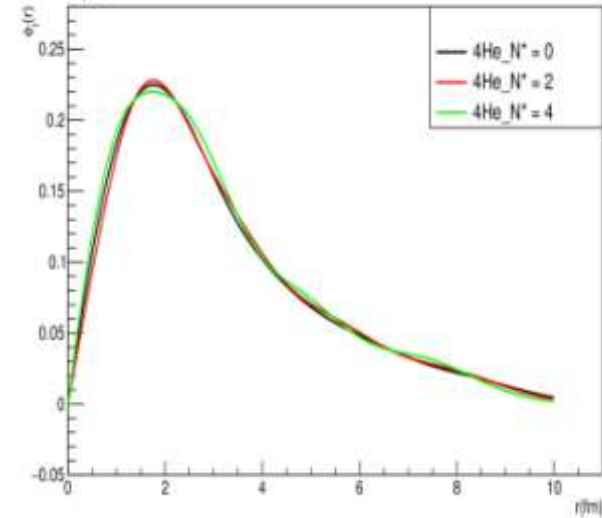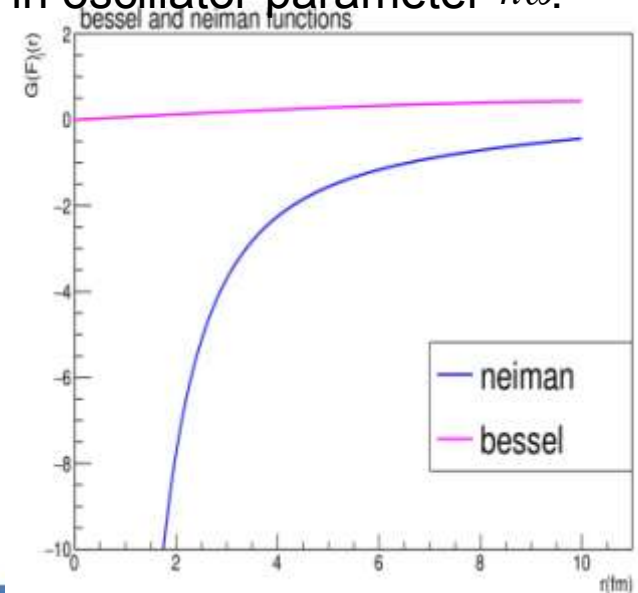

Function of neutron formfactor 3/2⁻ turns out to be stable with respect to changes in the accuracy of description of ⁴He and changes in oscillator parameter $\hbar\omega$.
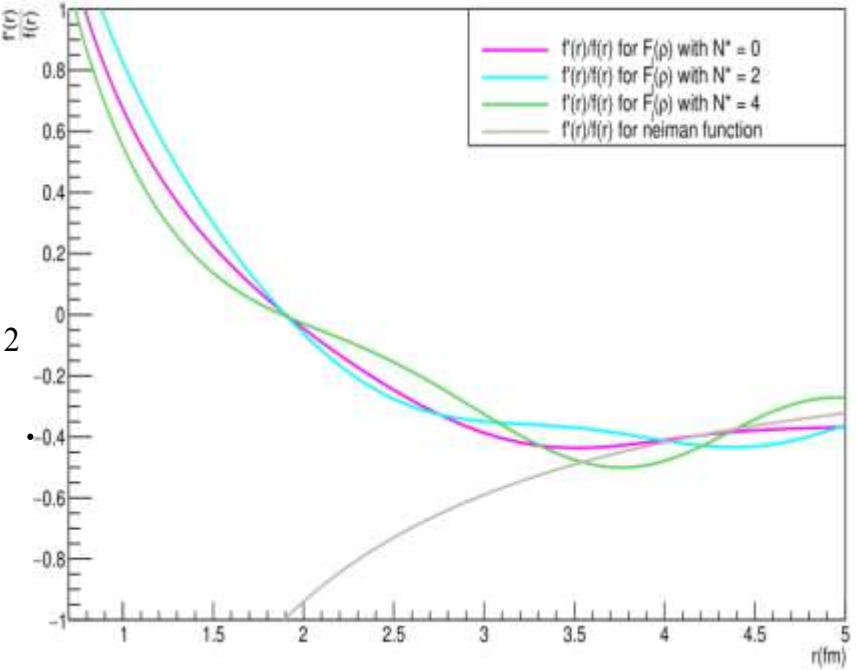
For the value of the energy of the given resonance (E_{exp} = 27.4 MeV, E_{res} = 0.890 MeV) in the all region of interest to us along the coordinate r $n_l(r) \gg j_l(r)$.

In this case $P_l(r) \approx (kr)^{-2}(n_l(r))^{-2}$.

For the matching point R_{point} of ab initio WF with a two-body solution, the following equation is used for ^{4}He with the cutoff parameter $N_{He} = 0,2,4$.

$$\frac{F'_l(r)}{F_l(r)} = \frac{n'_l(r)}{n_l(r)} \implies \Gamma_{3/2-} = \frac{h^2}{\mu k}\left(\frac{F_l(R_{point})}{n_l(R_{point})}\right)^2$$

Logarithmic derivatives of neutron formfactors and Neiman function, $\hbar\omega = 17.5$ MeV.

As follows from the ratio between the asymptotic solution and the neutron formfactor, the ab initio calculations accurately describe the asymptotic region.
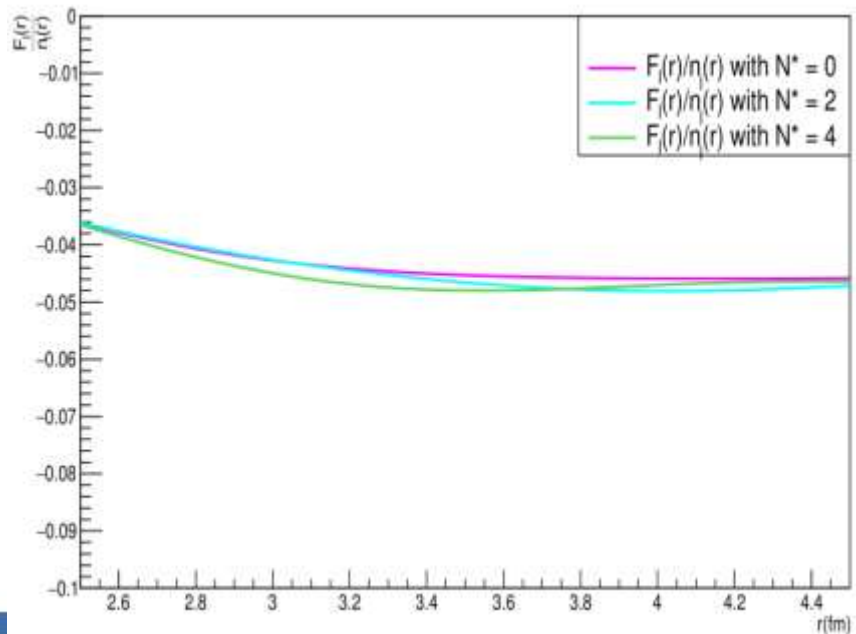
The results of the calculation of the width of 3/2⁻ resonance ⁵He

Calculations of the 3/2⁻ ⁵He state were carried out for a wide range of NCSM bases cutoff parameters for ⁵He (N_{max}) and ⁴He (N_{He}) and the oscillatory parameter $\hbar\omega$. The results are shown in the table. The experimental value of the resonance width is 600 keV.

	N_{max}=13 $\hbar\omega$ = 12.5	N_{max}=13 $\hbar\omega$ = 15	N_{max}=13 $\hbar\omega$ = 17.5	N_{max}=15 $\hbar\omega$ = 12.5	N_{max}=15 $\hbar\omega$ = 15	N_{max}=15 $\hbar\omega$ = 17.5	N_{max}=17 $\hbar\omega$ = 12.5	N_{max}=17 $\hbar\omega$ = 15	N_{max}=17 $\hbar\omega$ = 17.5
	Decay width of 3/2⁻ ⁵He								
N_{He} = 0	469 кэВ	548 кэВ	628 кэВ	443 кэВ	530 кэВ	597 кэВ	427 кэВ	578 кэВ	577 кэВ
N_{He} = 2	545 кэВ	588 кэВ	625 кэВ	545 кэВ	590 кэВ	615 кэВ	515 кэВ	587 кэВ	600 кэВ
N_{He} = 4	642 кэВ	678 кэВ	642 кэВ	563 кэВ	629 кэВ	630 кэВ	------	611 кэВ	620 кэВ

Theoretical calculations show good agreement with experiment
and stability with respect to changing free parameters of the basis.

Calculations of ground and exited states of ^{7}Li

Spectrum of lowest states of ^{7}Li					
	exp. (E)	T	Γ (exp.)	$\hbar\omega$ =20 (E) Daejeon16	$\hbar\omega$ =22.5 (E) JISP16
$3/2^-$	39.245	1/2	-----	39.110	37.929
$1/2^-$	38.788	1/2	73 fs	38.279	37.290
$7/2^-$	34.615	1/2	93 keV	34.410	32.625
$5/2^-$	32.565	1/2	880 keV	31.613	30.332
$5/2^-$	31.786	1/2	89 keV	30.816	29.491
$7/2^-$	29.575	1/2	400 keV	28.489	26.840
$3/2^-$	29.335	1/2	1200 keV	28.175	27.286
$3/2^-$	28.000	3/2	260 keV	27.248	25.383

The spectrum of calculated levels shows good agreement with the experiment. This means that these wavefunctions can be used to calculate the partial decay widths of resonances and the asymptotic normalization coefficients of the bound states.
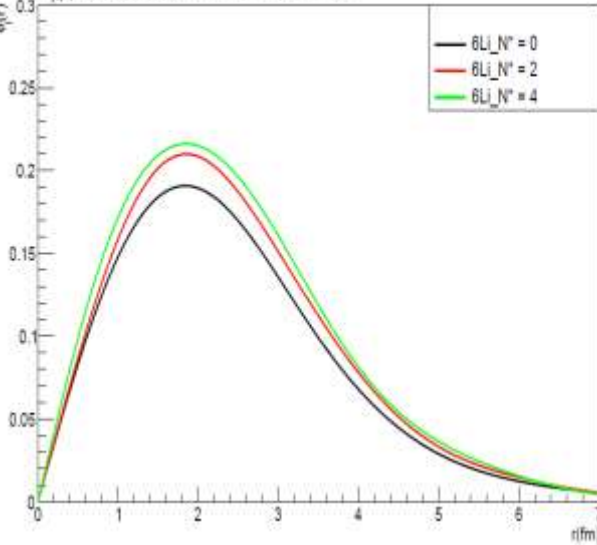
ANC calculations of $3/2^-$, $1/2^-$ states of ^{7}Li.
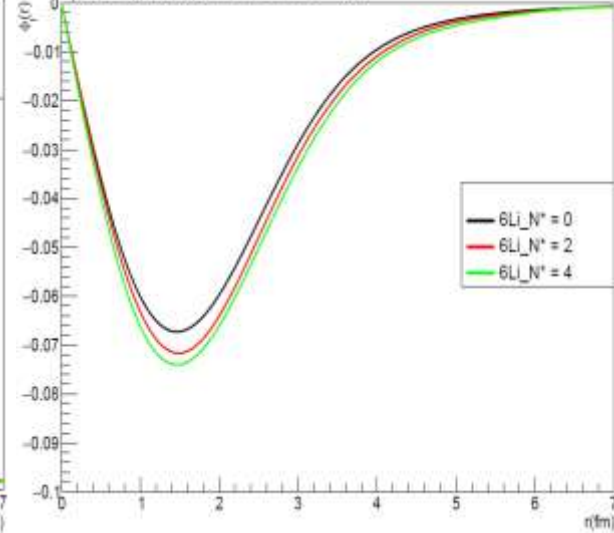
For channel ^{6}Li+n with l=1 s=1/2

For channel ^{6}Li+n with l=1 s=3/2

For channel ^{4}He+^{3}T
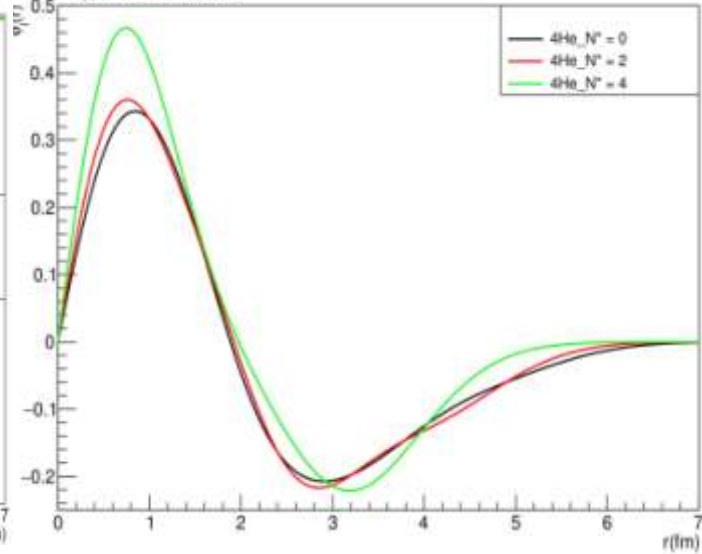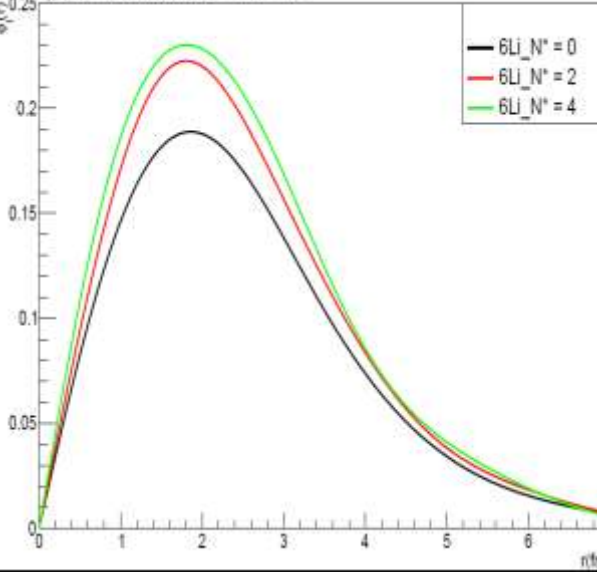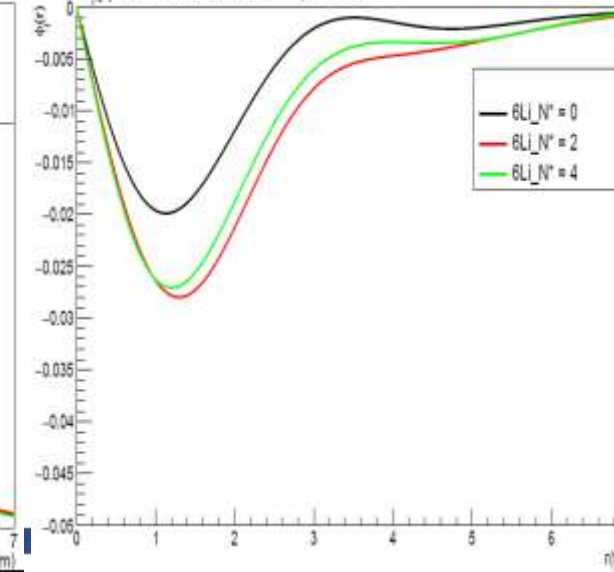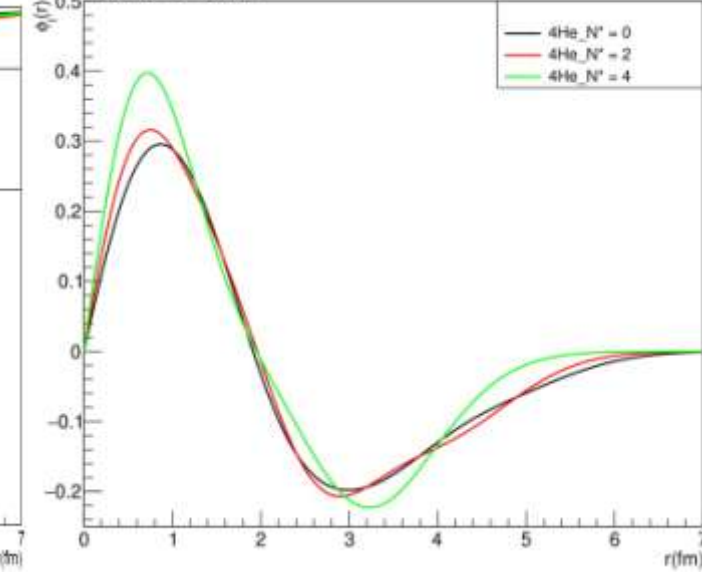
Results of calculations of ANC

	exp.	theor.	Daejeon16	JISP16
$3/2^-$: ^{4}He+^{3}T channel	3.57+-0.15	----------	3.44	2.84
$3/2^-$: ^{6}Li+n channel with l=1 j=1/2	ANC^2=1.22	1.652	-1.6168	-1.4114
$3/2^-$: ^{6}Li+n channel with l=1 j=3/2	ANC^2=1.66	1.890	1.317	1.1986

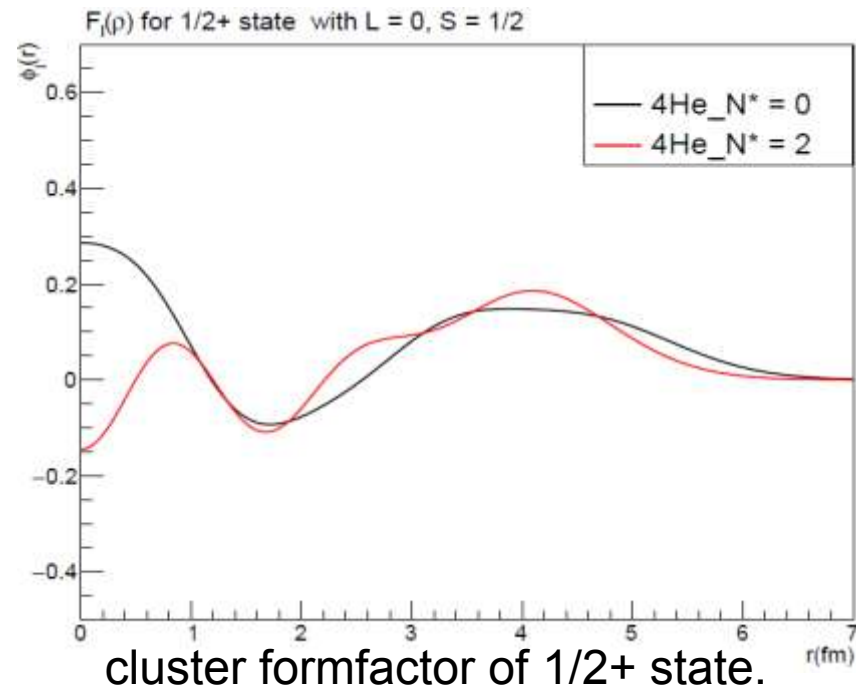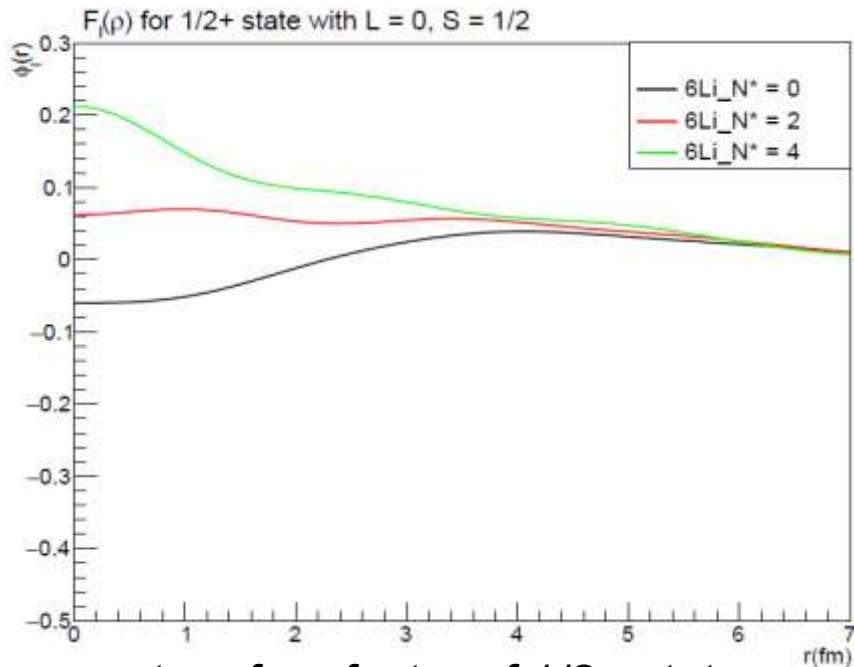|	exp.	theor.	Daejeon16	JISP16
$1/2^-$: channel ^{4}He+^{3}T	3.0+-0.15	----------	2.95	2.42
$1/2^-$: channel ^{6}Li+n with l=1 j=1/2	--------	-0.543	-0.531	-0.3629
$1/2^-$: channel ^{6}Li+n with l=1 j=3/2	---------	-2.543	1.979	1.637

Theoretical calculations show agreement with the application of realistic potentials based on different principles - Daejeon16 and JISP16, and in the case of the ^{4}He + ^{3}T channel show good agreement with the experiment.

ANC and partial decay widths of highly excited states ^{7}Li

	exp.	Daejeon16	JISP16
$7/2^-$: ^{4}He+^{3}T channel (Gamma)	92 (69) keV	65 keV	34.8 keV
$7/2^-$: ^{6}Li+n channel with l=3 s=1/2 (ANC)	---------	0.01356	0.0121
$5/2^-$: ^{4}He+^{3}T channel (Gamma)	880 (918) keV	564 keV	438 keV
$5/2^-$: ^{6}Li+n channel with l=1 s=3/2 (ANC)	----------	0.199	0.033
$5/2^-$ (2): ^{4}He+^{3}T channel (Gamma)	Total: 89 (80) keV	797 keV	9.7 keV
$5/2^-$ (2): ^{6}Li+n channel with l=1 s=3/2 (Gamma)	57 keV	53.15 keV	86.6 keV
$7/2^-$ (2): ^{6}Li+n channel with l=3 s=1/2 (Gamma)	------------	0.72 keV	0.353 keV
$7/2^-$ (2): ^{6}Li+n channel with l=3 s=3/2 (Gamma)		44.6 eV	13.7 eV
$3/2^-$ (2): ^{6}Li+n channel with l=1 s=1/2 (Gamma)	867 keV	223 keV	48.9 keV
$3/2^-$ (2): ^{6}Li+n channel with l=1 s=3/2 (Gamma)		1.88 MeV	1.88 MeV

Results of calculations for 1/2⁺ state of ⁷Li

Exp. data	J	E	E_n	$E_{\alpha+t}$	$\Gamma_n^0 (1eV)$	Γ_α
	$1/2^+$	32.803 MeV	-0.808 MeV	3.974 MeV	0.295 keV	3.152 MeV

neutron formfactor of 1/2+ state.

cluster formfactor of 1/2+ state.

Total binding energies of 3/2⁻, 1/2⁺ states in NCSM calculations		
N*	3/2⁻ state	1/2⁺ state
9	38.525	27.343
11	38.901	28.921
13	39.110	30.061

According to work of A. M. Mukhamedzhanov and R. E. Tribble (Phys. Rev. C Vol. 59 N. 6) the partial width of the subthreshold resonance at E>0 could be expressed throw ANC:

$$\Gamma_n^0(E) = \frac{\hbar}{\mu_{ab}} \kappa r_0 r_l(E) \frac{W_{-\eta_0, l+1/2}^2(2k_0 r_0)}{r_0} |C_{ab}|^2$$

	exp.	Daejeon16	JISP16
^{4}He+^{3}T channel (Γ_α)	3.152 MeV	7.4 MeV	6.96 MeV
^{6}Li+n channel with l=0 s=1/2 (ANC)	----------	0.595	0.544
^{6}Li+n channel with l=0 s=1/2 ($\Gamma_n^0(1eV)$)	0.295 keV	0.540 keV	0.529 keV

Conclusions

I. New ab initio multichannel scheme was developed for calculating the asymptotic characteristics of both bound and resonant states, namely the asymptotic normalization coefficients of bound states and partial decay widths of resonant states. This scheme allows to calculate the asymptotic characteristics of several decay channels of the same state of a light nucleus.

II. The asymptotic characteristics of the bound and resonant states of ^{5}He and ^{7}Li nuclei were calculated.

III. The theoretical calculation of lowest resonance state 3/2$^-$ of ^{5}He shows high-quality agreement with experimental data.

IV. For the ^{7}Li nucleus, calculations were performed for all experimentally identified states and for various decay channels. These results, in general, show rather good agreement with experimental data.

V. This approach provides possibilities to test universal realistic NN-potentials.

THANK YOU FOR YOUR ATTENTION!