Angular anisotropy of fragments from neutron induced fission of nuclei at energies up to 200 MeV: Data and theoretical interpretation

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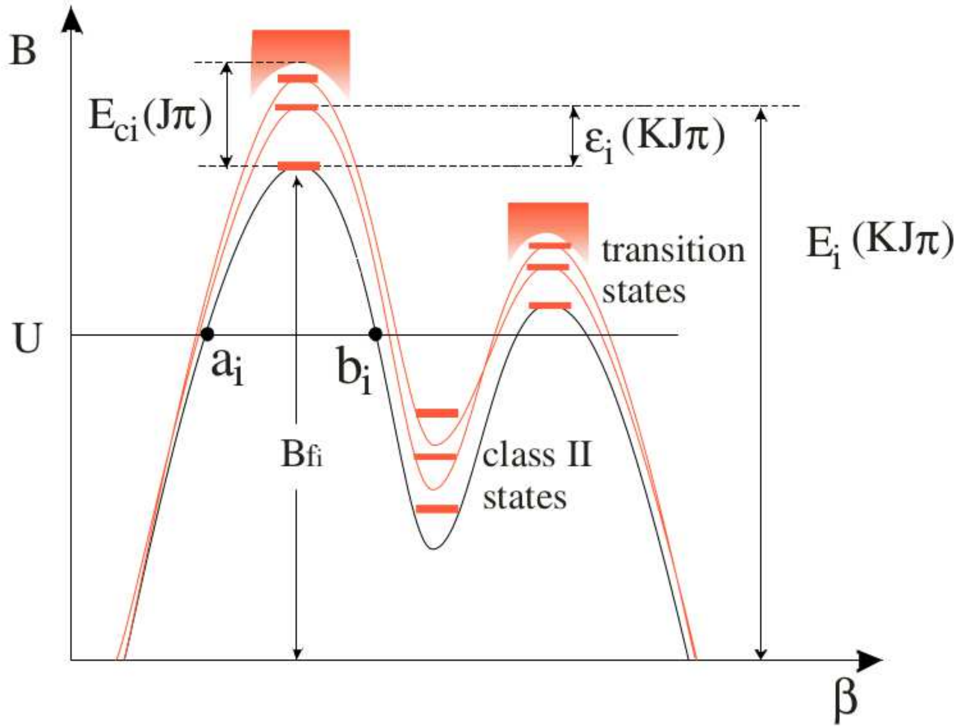
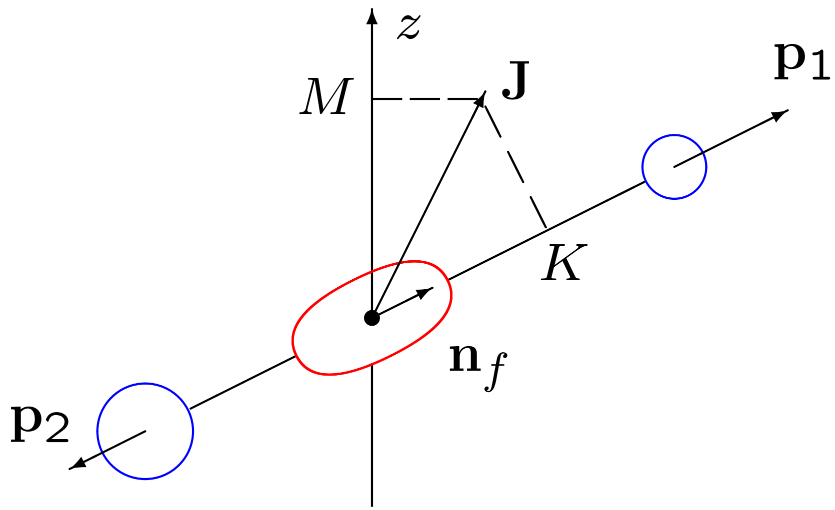
A. Bohr, 1955:

Angular distributions are due to transition states on the fission barrier





R.Capote et al. RIPL

Nuclear Data Sheets

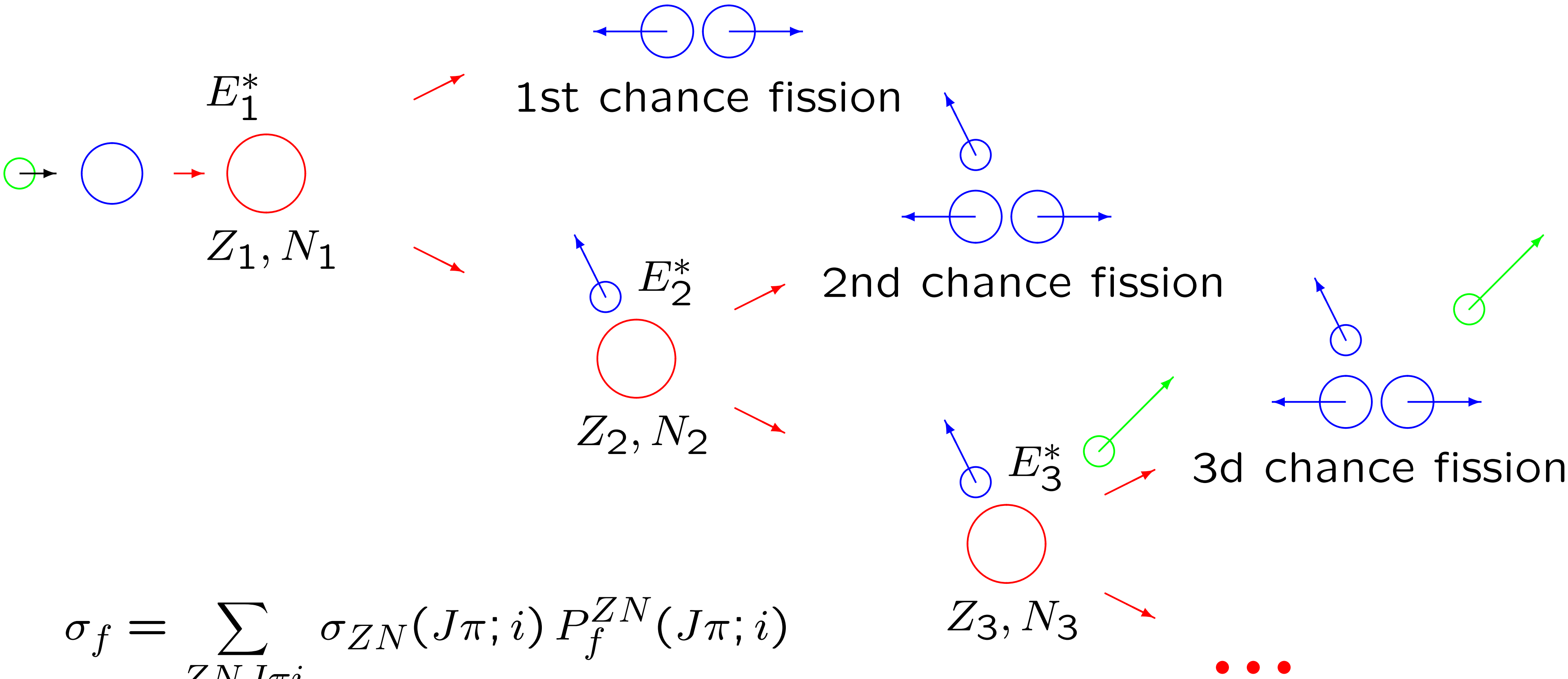
110 3107 (2009)

$$\Psi_{JMK} \sim D^J_{MK}(\mathbf{n}_f)$$

$$\frac{dw^J_{MK}(\mathbf{n}_f)}{d\Omega} = \frac{2J+1}{4\pi}\left|D^J_{MK}(\mathbf{n}_f)\right|^2$$

$$\frac{dw^{J\pi}(\mathbf{n}_f)}{d\Omega} = \sum_M \eta^{J\pi}(M)\sum_K \rho^{J\pi}(K)\,\frac{dw^J_{MK}(\mathbf{n}_f)}{d\Omega}$$

Reaction (n, f) at intermediate (up to 200 MeV) energies:

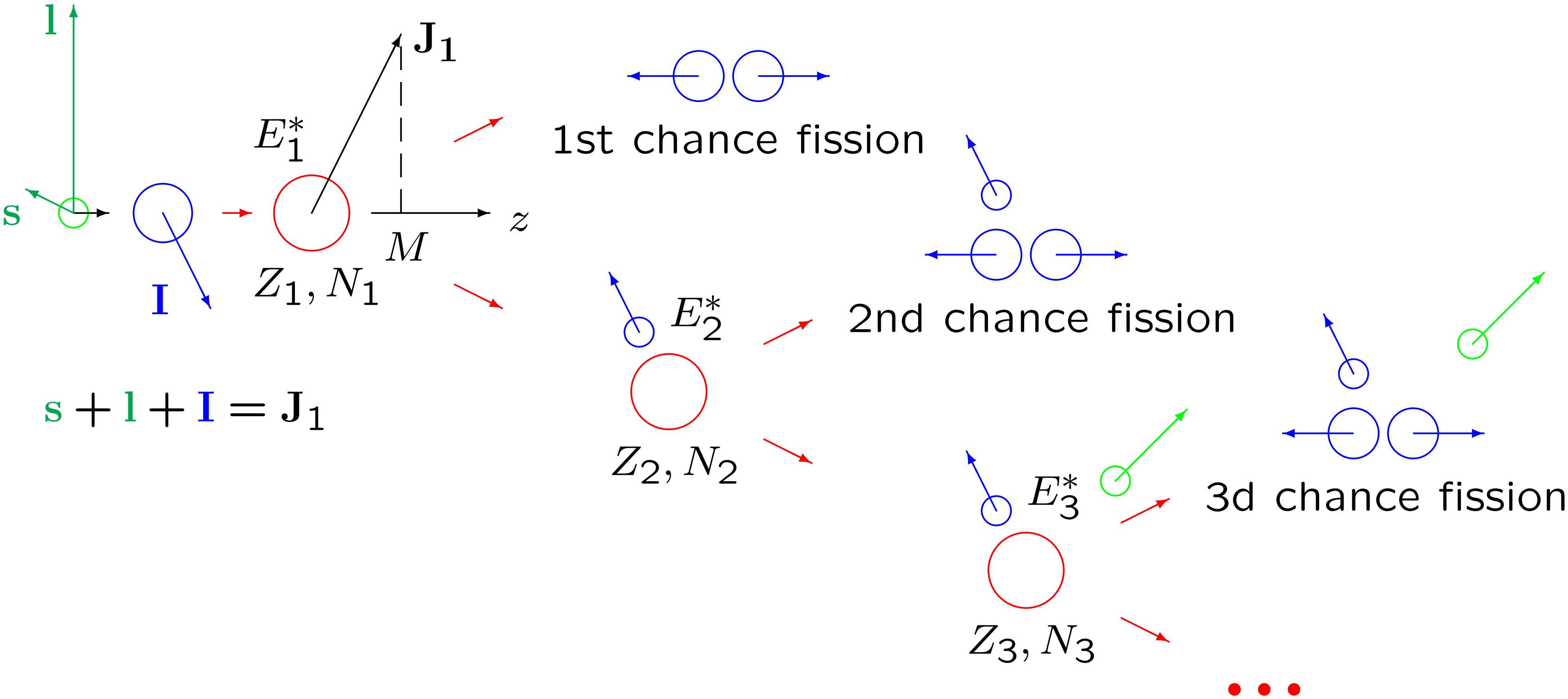


$$\sigma_f = \sum_{ZNJ\pi i} \sigma_{ZN}(J\pi; i)\, P_f^{ZN}(J\pi; i)$$

R.Capote et al. RIPL — Reference Input Parameter Library for Calculation of Nuclear Recations and Nuclear Data Evaluations. *Nuclear Data Sheets* **110** 3107 (2009):

released in January 2009, and is available on the Web through *http://www-nds.iaea.org/RIPL-3/*. This work and the resulting database are extremely important to theoreticians involved in the development and use of nuclear reaction modelling (ALICE, EMPIRE, GNASH, UNF, TALYS) both for theoretical research and nuclear data evaluations.

Nuclear spin orientation (alignment):

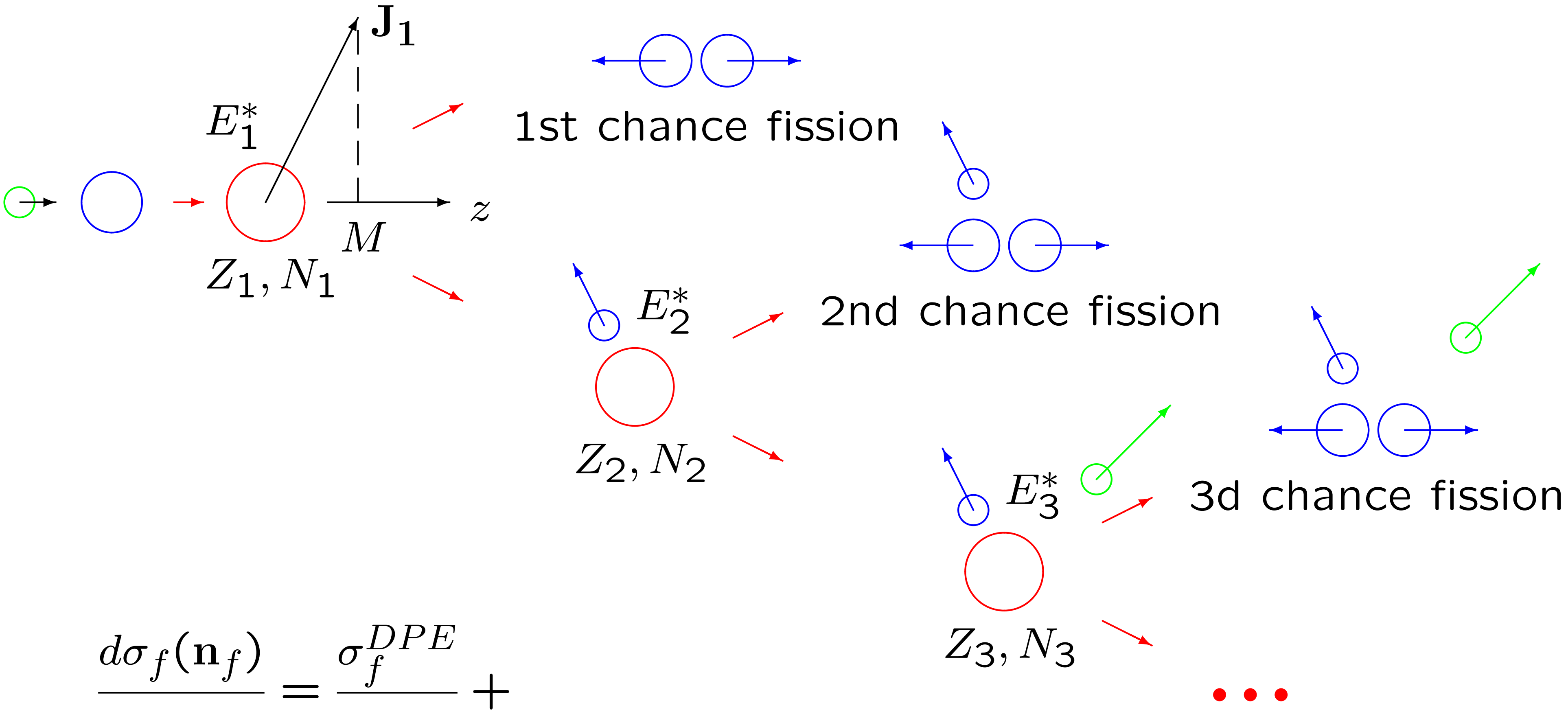


$$\mathbf{s} + \mathbf{l} + \mathbf{I} = \mathbf{J}_1$$

$$\sigma_f = \sum_{ZNJ\pi i} \sigma_{ZN}(J\pi; i)\, P_f^{ZN}(J\pi; i), \quad \sigma_{ZN}(J\pi; i) = \sum_M \sigma_{ZN}(J\pi M; i),$$

$$\frac{d\sigma_f(\mathbf{n}_f)}{d\Omega} = \sum_{ZNJ\pi i} \sum_M \sigma_{ZN}(J\pi M; i) P_f^{ZN}(J\pi; i) \sum_K \rho_{ZN}^{J\pi i}(K)\, \frac{2J+1}{4\pi} \left| D_{MK}^J(\mathbf{n}_f) \right|^2$$

Direct and Pre-Equilibrium processes: $\sigma_f = \sigma_f^{DPE} + \sigma_f^{C}$



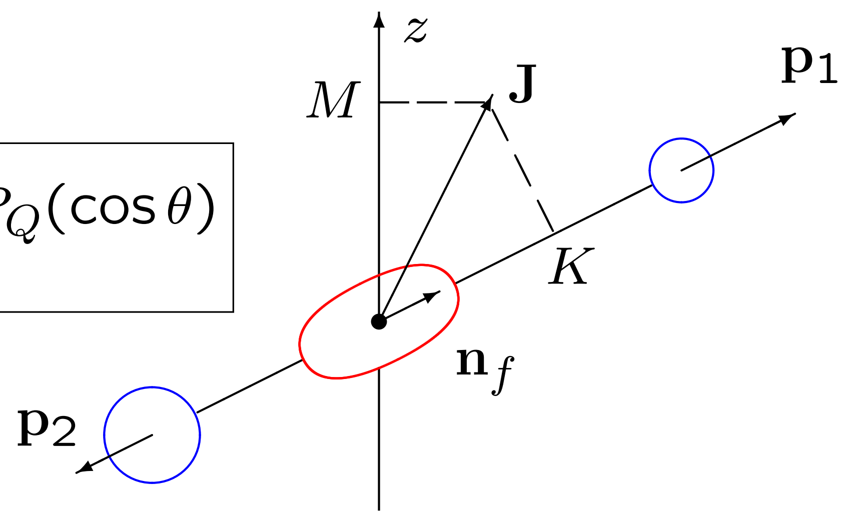
$$\frac{d\sigma_f(\mathbf{n}_f)}{d\Omega} = \frac{\sigma_f^{DPE}}{4\pi} +$$

$$\sum_{ZNJ\pi i}\sum_{M} \sigma^{C}_{ZN}(J\pi M; i)\, P_f^{ZN}(J\pi; i) \sum_{K} \rho^{J\pi i}_{ZN}(K)\, \frac{2J+1}{4\pi} \left| D^{J}_{MK}(\mathbf{n}_f) \right|^2$$

I.V.Ryzhov et al. Influence of multichance fission on fragment angular anisotropy in the ^{232}Th(n,f) and ^{238}U(n,f) reactions at intermediate energies. *Nucl. Phys. A* **760** 19 (2005): $E_n = 2 - 100$ MeV

Advanced method to account for nuclear alignment:

$$\boxed{(2J+1)\left|D^J_{MK}(\mathbf{n}_f)\right|^2 = \sum_Q (2Q+1) C^{JM}_{JMQ0} C^{JK}_{JKQ0} P_Q(\cos\theta)}$$



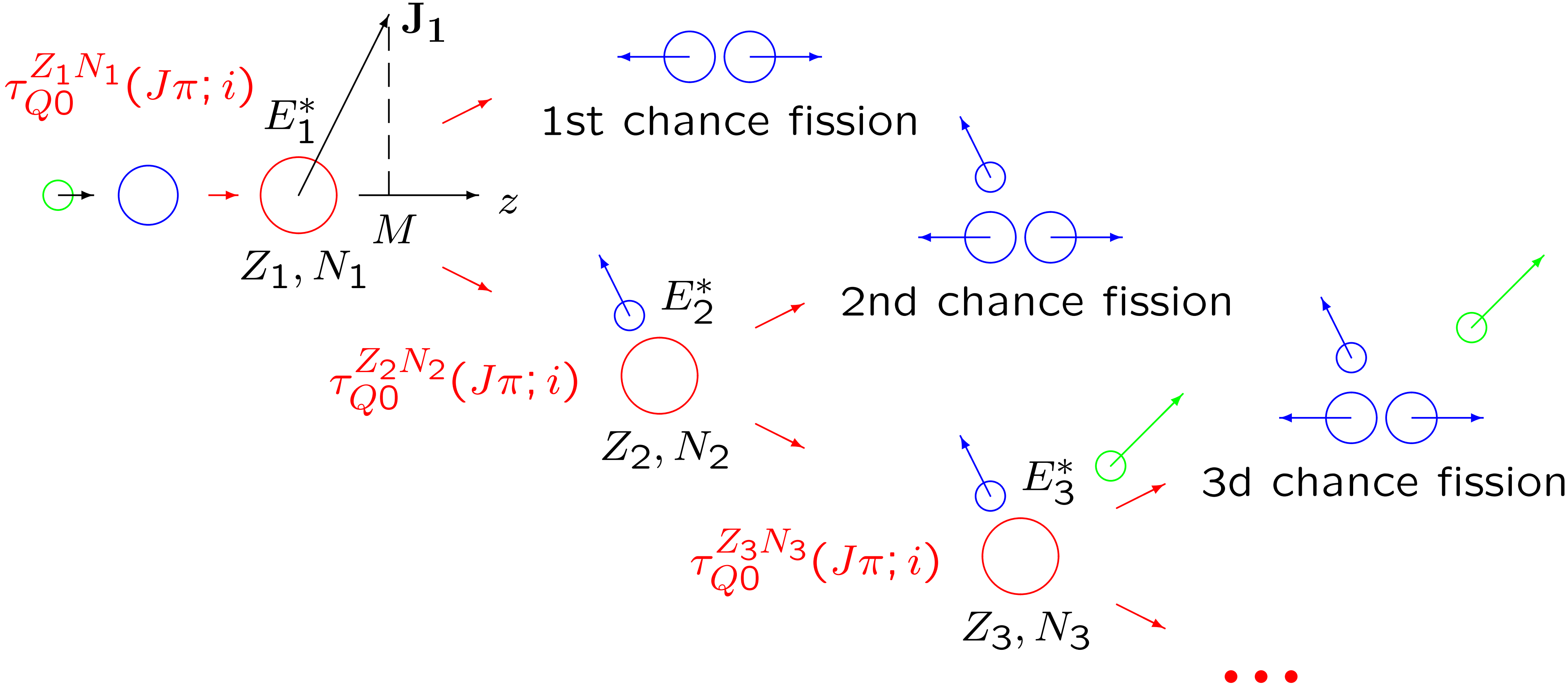
$$\frac{dw^J(\mathbf{n}_f)}{d\Omega} = \sum_M \eta^J(M) \sum_K \rho^J(K) \frac{2J+1}{4\pi} |D^J_{MK}(\mathbf{n}_f)|^2 \equiv$$

$$\frac{1}{4\pi} \sum_{Q=0,2,4,\ldots} (2Q+1) \underbrace{\left(\sum_M C^{JM}_{JMQ0}\, \eta^J(M)\right)}_{\substack{\parallel\!\!\parallel \\ \tau_{Q0}(J)}} \underbrace{\left(\sum_K C^{JK}_{JKQ0}\, \rho^J(K)\right)}_{\substack{\parallel\!\!\parallel \\ \beta_Q(J)}} P_Q(\cos\theta)$$

$\tau_{Q0}(J)$ — spin-tensor of orientation, $\beta_Q(J)$ — parameter of anisotropy

$$\frac{d\sigma_f(\theta)}{d\Omega} = \frac{\sigma_f}{4\pi} + \frac{1}{4\pi} \sum_{Q=2,4,\ldots} \sigma^C_{fQ} P_Q(\cos\theta)$$

Advanced method to account for nuclear alignment:



$$\frac{d\sigma_f(\theta)}{d\Omega} = \frac{\sigma_f}{4\pi} + \frac{1}{4\pi} \sum_{Q=2,4,\dots} \sigma_{fQ}^{C}\, P_Q(\cos\theta)$$

$$\sigma_{fQ}^{C} \sim \sum_{ZNJ\pi i} \cdots \underbrace{\tau_{Q0}^{ZN}(J\pi;i)}_{\displaystyle\downarrow\ \text{calculated with the use of TALYS}}\ \beta_Q^{ZN}(J\pi;i) \dots, \qquad \beta_Q^{ZN}(J\pi;i) = \sum_K C_{JKQ0}^{JK}\, \rho_{ZN}^{J\pi i}(K)$$

TALYS-1.9

New Edition December 21, 2017

A nuclear reaction program

Talys is a computer code system for the analysis and prediction of nuclear reactions.

The basic objective is the simulation of nuclear reactions that involve neutrons, photons, protons, deuterons, tritons, 3He- and alpha-particles, in the 1 keV – 200 MeV energy range and for target nuclides of mass 12 and heavier.

Free use, open software, always under development: from TALYS-1.0 — December 2007 to TALYS-1.9 — December 2017.

User Manual

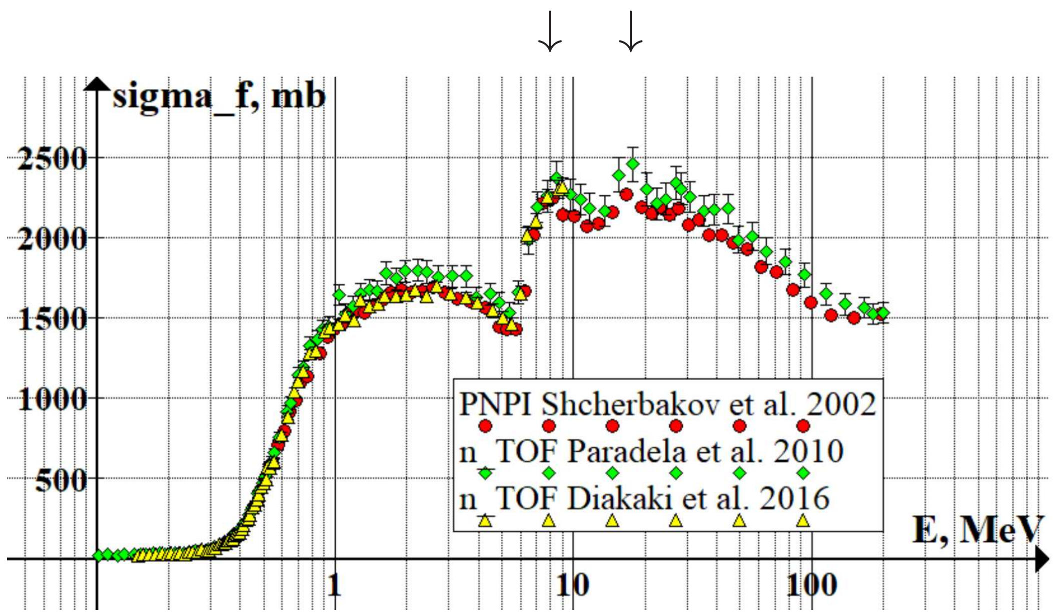
More than 300 subroutines, more than 100 000 lines (commands), more than 500 pages in the Manual.

Arjan Koning
Stephane Hilaire
Stephane Goriely

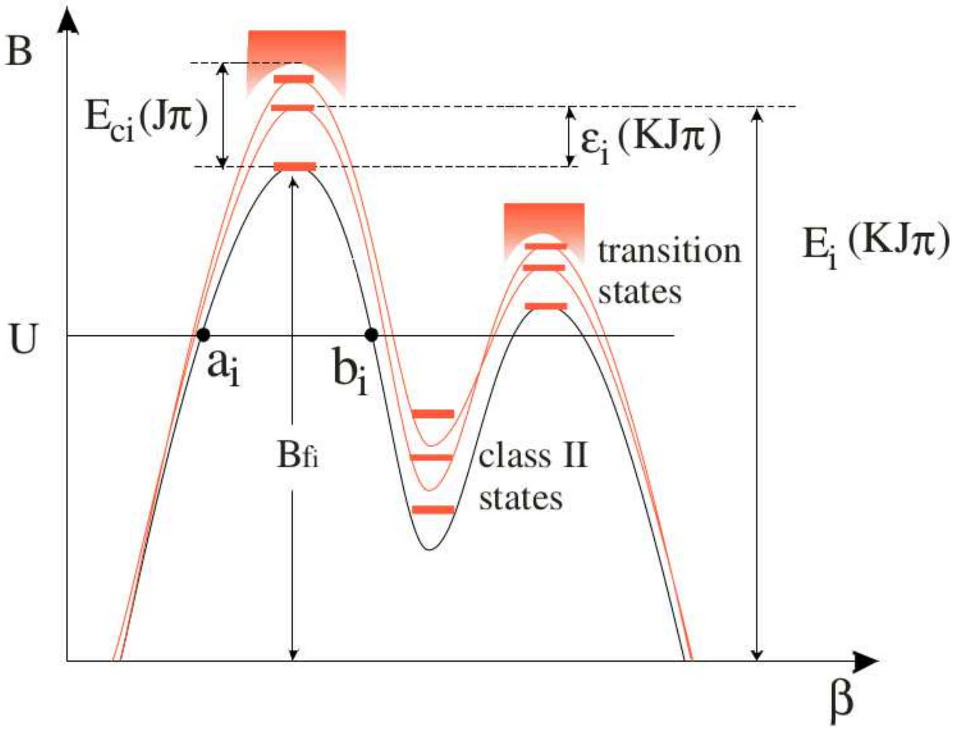
Completely integrated optical model and coupled-channels calculations by the ECIS-06 code.

Fission cross section for $n + {}^{237}\mathrm{Np}$ at 0.1 – 200 MeV:

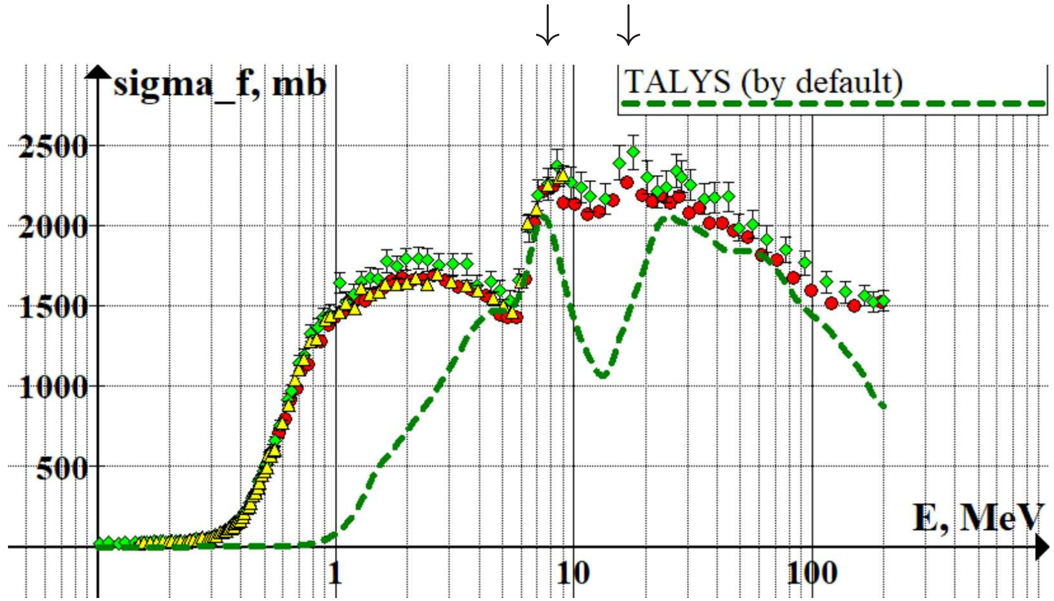
(n, f) $(n, n'f)$ $(n, 2nf)$ $\ldots$



Fission cross section for $n + {}^{237}$Np at $0.1 - 200$ MeV:



(n, f) $(n, n'f)$ $(n, 2nf)$ ...



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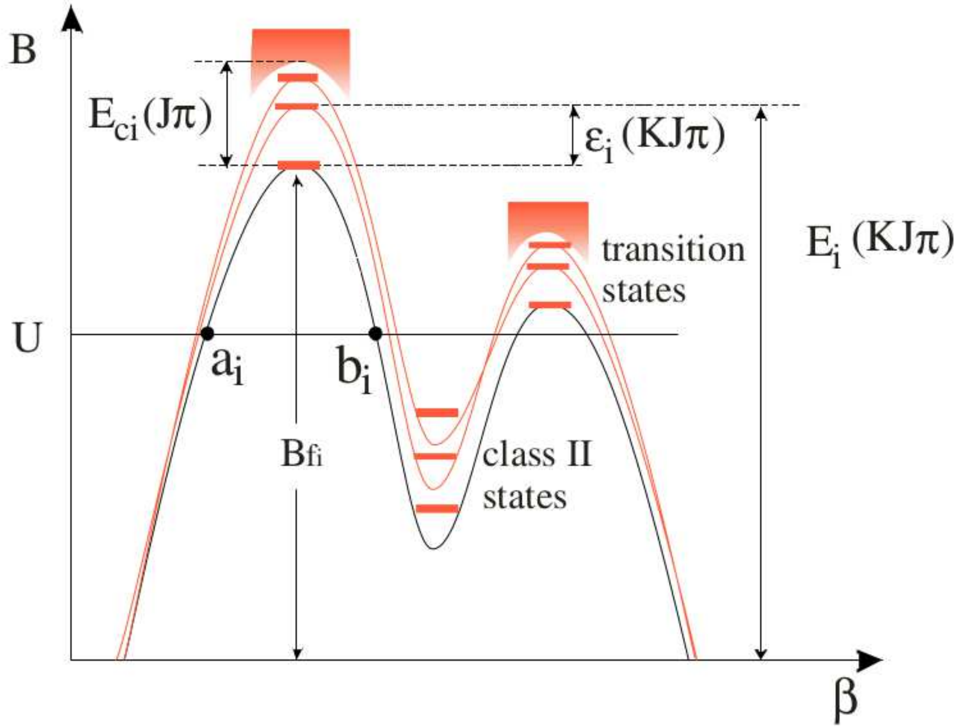
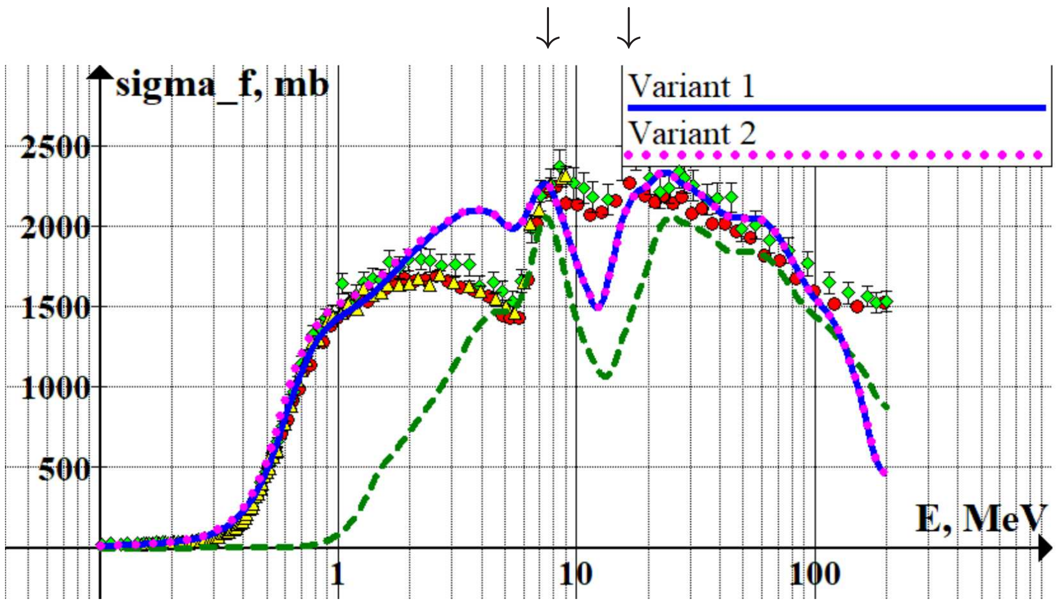
TALYS — parameters by default (RIPL):

Isotope	B_1	$\hbar\omega_1$	B_2	$\hbar\omega_2$	Trans. states
^{238}Np	6.50	0.6	5.75	0.4	yes
^{237}Np	6.00	1.0	5.40	0.5	yes
^{236}Np	5.90	0.6	5.40	0.4	yes

Fission cross section for $n + {}^{237}$Np at $0.1 - 200$ MeV:



(n,f) $(n,n'f)$ $(n,2nf)$ ...



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Adjusted parameters:

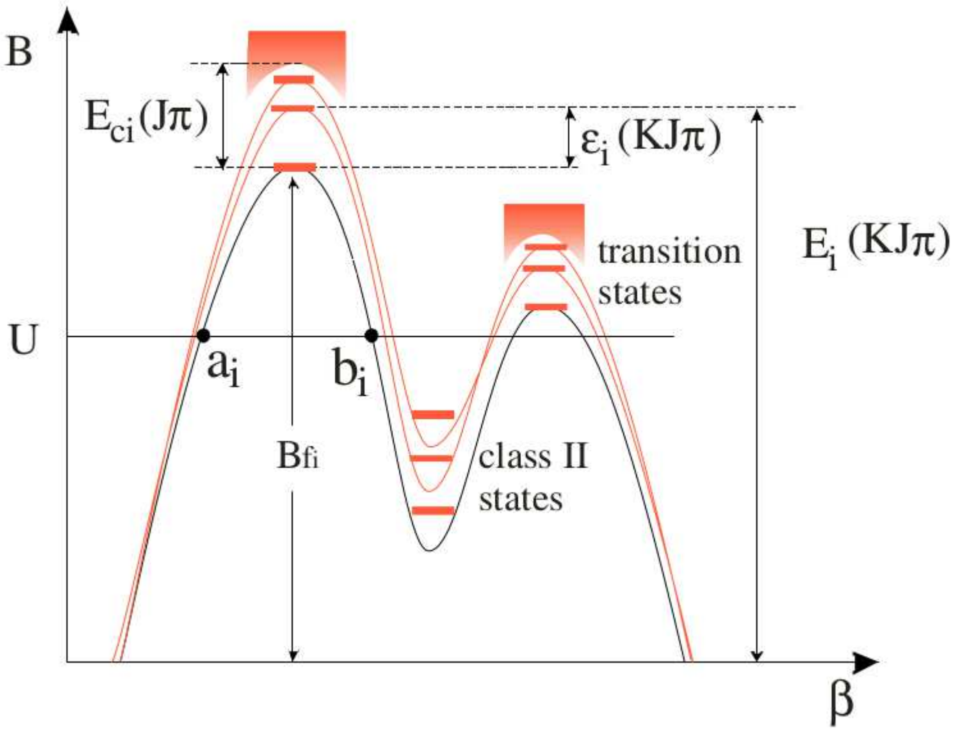
Isotope	B_1	$\hbar\omega_1$	B_2	$\hbar\omega_2$	Trans. states
^{238}Np	6.05	0.4	5.35	0.4	v. 1 or v. 2
^{237}Np	5.4	1.0	5.2	0.5	no
^{236}Np	5.1	0.6	5.0	0.4	no

Differential fission cross section $\rightarrow$ angular distribution for $n+{}^{237}$Np:

$$\frac{d\sigma_f(\theta)}{d\Omega} = \frac{\sigma_f}{4\pi} + \frac{1}{4\pi}\sum_{Q=2,4,\ldots}\sigma^C_{fQ}\,P_Q(\cos\theta) \quad\rightarrow\quad W(\theta) = \frac{1}{\sigma_f}\frac{d\sigma_f(\theta)}{d\Omega}$$

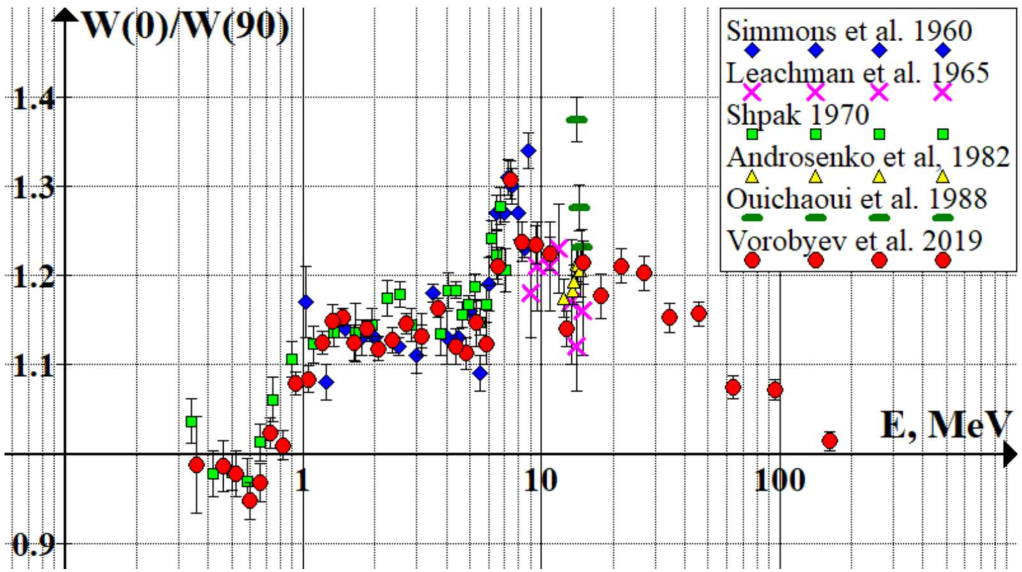
$$\sigma^C_{fQ} \sim \sum_{ZNJ\pi i}\ldots\underbrace{\tau^{ZN}_{Q0}(J\pi;i)}\ \beta^{ZN}_Q(J\pi;i)\ldots, \qquad \beta^{ZN}_Q(J\pi;i) = \sum_K C^{JK}_{JKQ0}\,\rho^{J\pi i}_{ZN}(K)$$

$\downarrow$
calculated with the use of TALYS



(n,f) $(n,n'f)$ $(n,2nf)$ $\ldots$

$\downarrow$ $\downarrow$

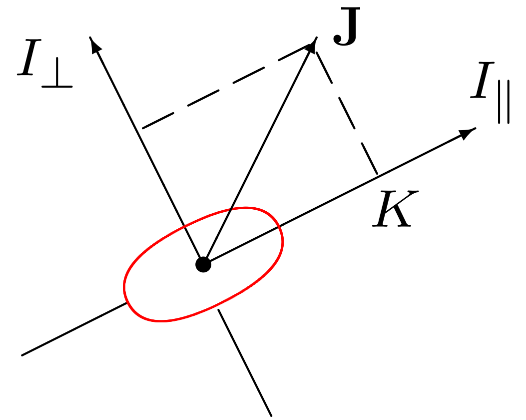


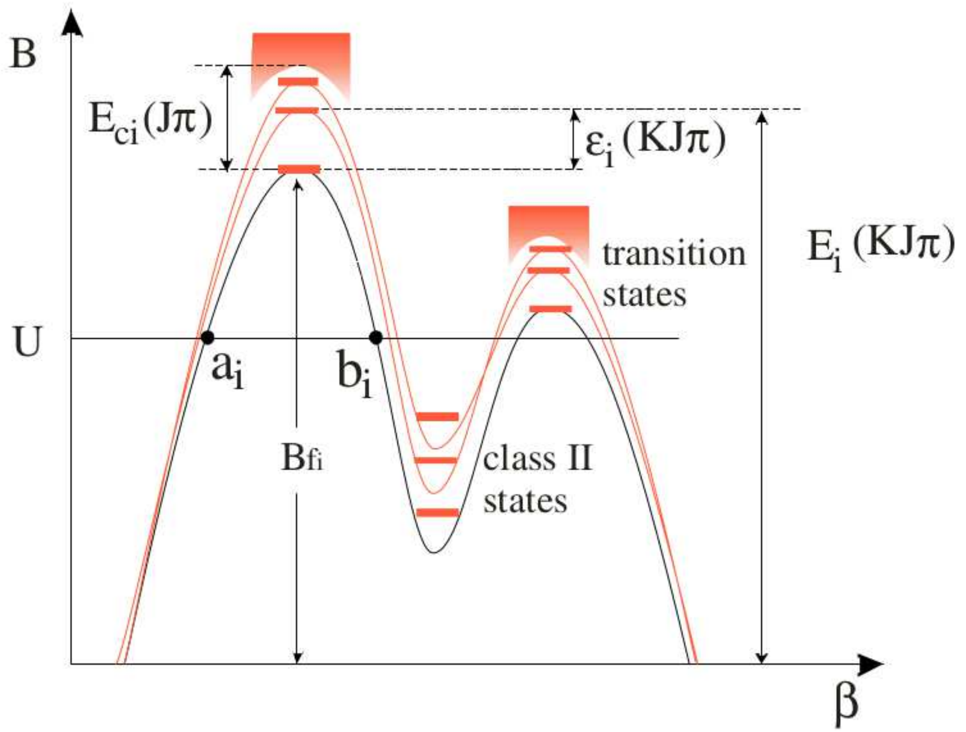
Statistical Model («high» energies):

$$\varepsilon(KJ\pi) = \frac{\hbar^2 J^2}{2I_\perp} + \frac{\hbar^2 K^2}{2I_{\text{eff}}}, \qquad \frac{1}{I_{\text{eff}}} = \frac{1}{I_\parallel} - \frac{1}{I_\perp}$$

$$E^* = E_{ex} - B_f > \Delta + U_{\text{up}} \quad \Rightarrow \quad \rho^{J\pi i}_{ZN}(K) \sim e^{-\frac{K^2}{2K_0^2}}$$

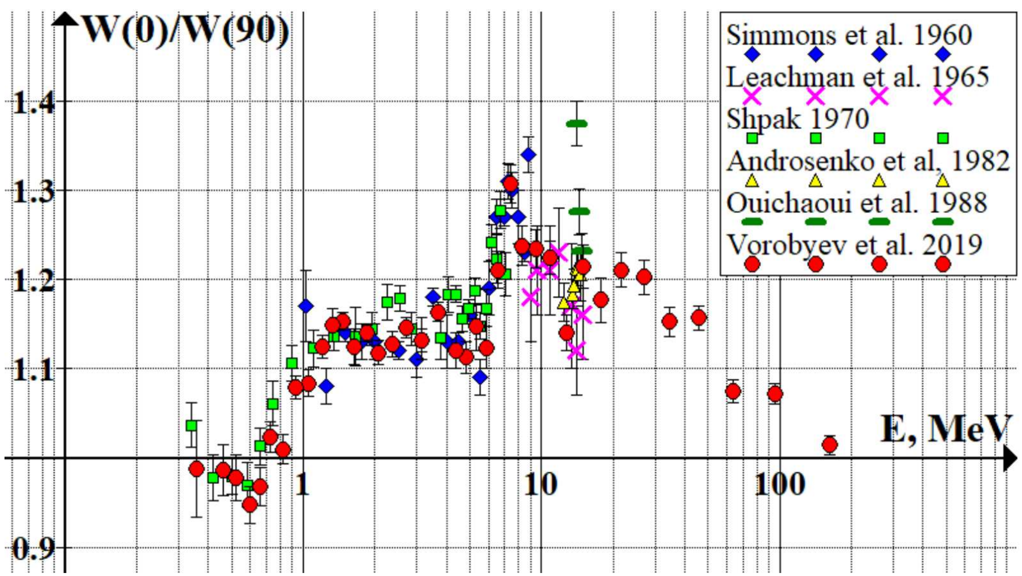
$$K_0^2 = \frac{I_{\text{eff}} T}{\hbar^2}, \quad T = \sqrt{\frac{U}{a(U)}}, \quad U = E^* - \Delta$$





$(n, f) \qquad (n, n'f) \qquad (n, 2nf) \qquad \ldots$

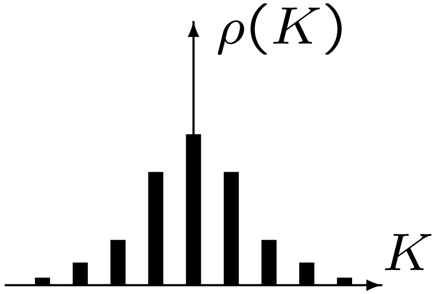
$\downarrow \quad \downarrow$



«High» energies:

$$E^* = E_{ex} - B_f > \Delta + U_{\text{up}}$$

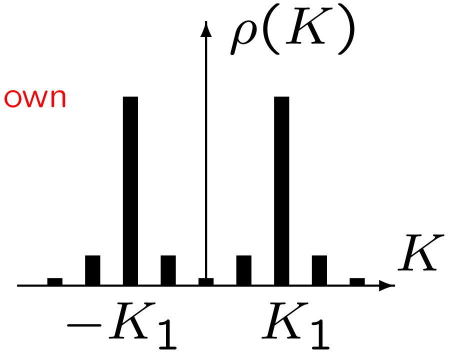
$$\rho_{ZN}^{J\pi i}(K) \sim e^{-\frac{K^2}{2K_0^2}}, \quad K_0^2 = \frac{I_{\text{eff}} T}{\hbar^2}$$



«Low» energies:

$$E^* = E_{ex} - B_f < \Delta + U_{\text{down}}$$

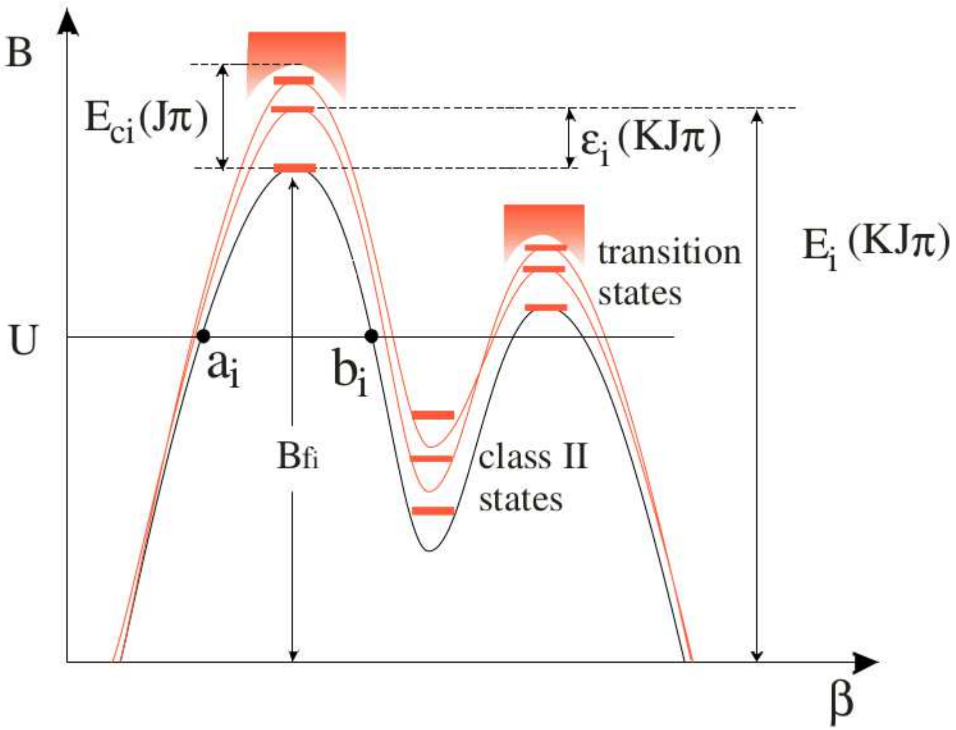
$$\rho_{ZN}^{J\pi i}(K) \sim e^{-\alpha(|K|-K_1)^2}$$



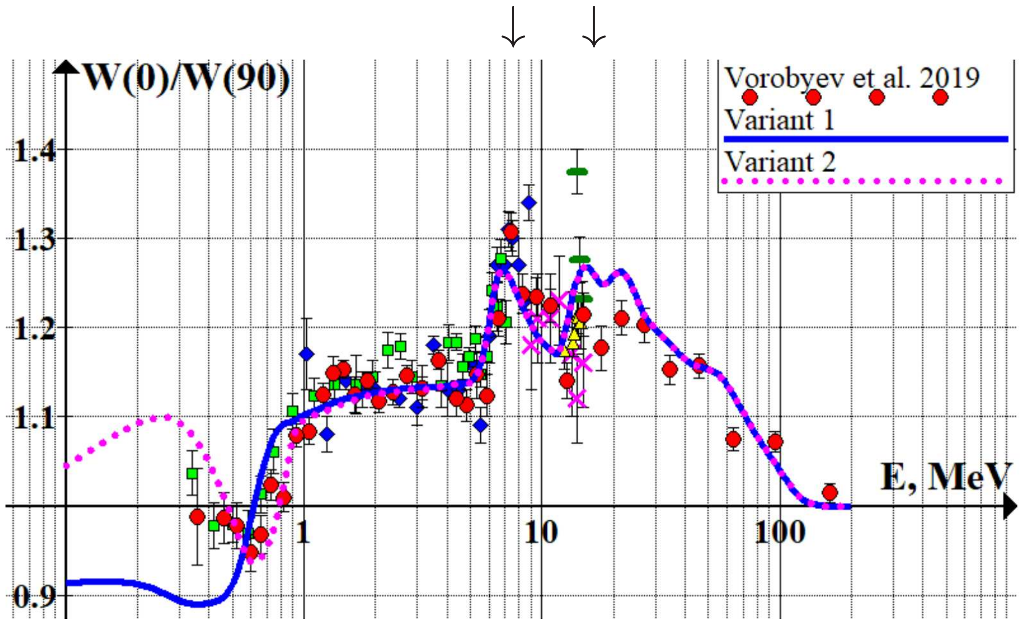
Additional parameters:

$$U_{\text{up}} = 0.4 \text{ MeV}, \quad U_{\text{down}} = -0.1 \text{ MeV}, \quad \frac{\hbar^2}{I_{\text{eff}}} = 0.017 \text{ MeV}, \quad \alpha = 0.15$$

$$K_1(^{238}\text{Np}) = \begin{cases} 0, & \text{Variant 1}, \\ 4, & \text{Variant 2}, \end{cases} \qquad K_1 = \begin{cases} 0.5, & ^{237}\text{Np}, \\ 1.5, & \text{all other isitopes.} \end{cases}$$

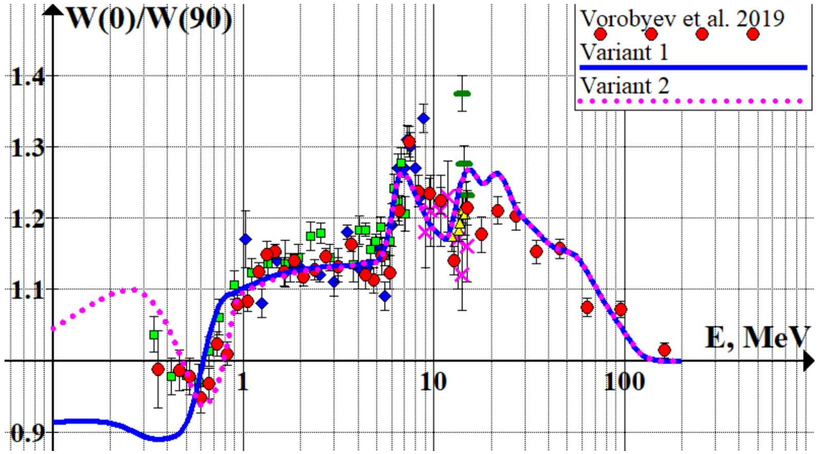


(n,f) $(n,n'f)$ $(n,2nf)$ $\ldots$



Multichance fission for $n + {}^{237}$Np at $E_n = 80$ MeV:

$$\sigma_f = \underbrace{\sigma_f^{DPE}}_{\sim 80\%} + \underbrace{\sigma_f^{C}}_{\sim 20\%}, \qquad a = \frac{W(0^\circ)}{W(90^\circ)} - 1 = 0.078$$



Main decay chaines of compound nucleus:

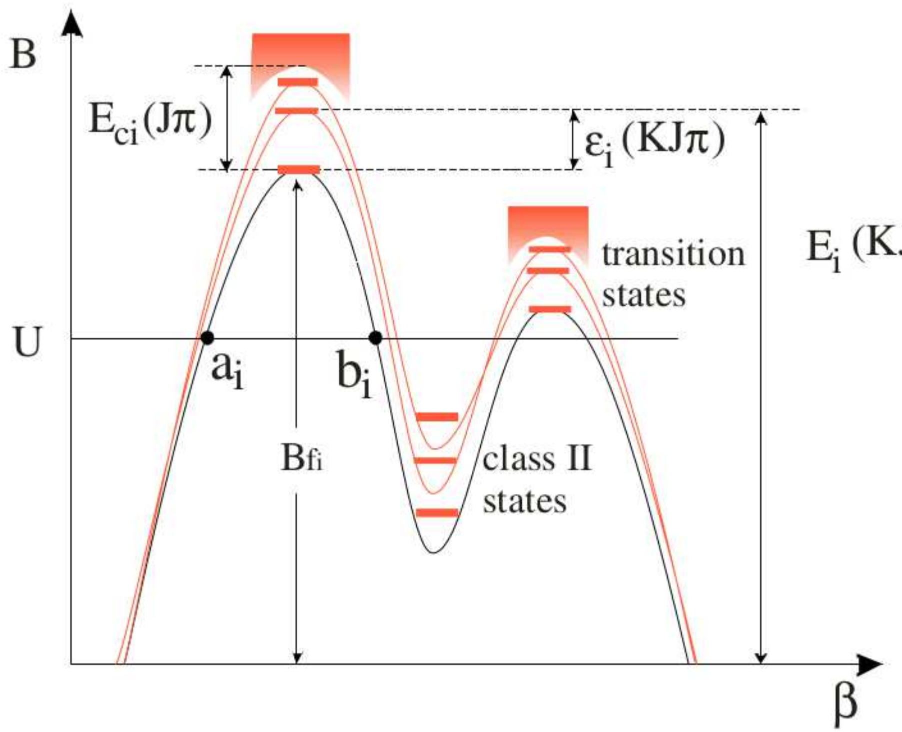
$$\begin{array}{ccccccccccccccccccc}
{}^{238}\mathrm{Np} & \rightarrow & {}^{237}\mathrm{Np} & \rightarrow & {}^{236}\mathrm{Np} & \rightarrow & {}^{235}\mathrm{Np} & \rightarrow & {}^{234}\mathrm{Np} & \rightarrow & {}^{233}\mathrm{Np} & \rightarrow & {}^{232}\mathrm{Np} & \rightarrow & {}^{231}\mathrm{Np} & \rightarrow & {}^{230}\mathrm{Np} & \rightarrow & \dots \\
 & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & \\
 & & {}^{237}\mathrm{U} & \rightarrow & {}^{236}\mathrm{U} & \rightarrow & {}^{235}\mathrm{U} & \rightarrow & {}^{234}\mathrm{U} & \rightarrow & {}^{233}\mathrm{U} & \rightarrow & {}^{232}\mathrm{U} & \rightarrow & {}^{231}\mathrm{U} & \rightarrow & {}^{230}\mathrm{U} & \rightarrow & \dots \\
 & & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & \\
 & & & & {}^{236}\mathrm{Pa} & \rightarrow & {}^{235}\mathrm{Pa} & \rightarrow & {}^{234}\mathrm{Pa} & \rightarrow & {}^{233}\mathrm{Pa} & \rightarrow & {}^{232}\mathrm{Pa} & \rightarrow & {}^{231}\mathrm{Pa} & \rightarrow & {}^{230}\mathrm{Pa} & \rightarrow & \dots \\
 & & & & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & \\
 & & & & & & {}^{235}\mathrm{Th} & \rightarrow & {}^{234}\mathrm{Th} & \rightarrow & {}^{233}\mathrm{Th} & \rightarrow & {}^{232}\mathrm{Th} & \rightarrow & {}^{231}\mathrm{Th} & \rightarrow & {}^{230}\mathrm{Th} & \rightarrow & \dots \\
 & & & & & & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & \\
 & & & & & & & & \dots & \rightarrow & \dots & \rightarrow & \dots & \rightarrow & \dots & \rightarrow & \dots & \rightarrow & \dots
\end{array}$$

8 isotopes, ^{238}Np, ^{236}U, ^{235}U, ^{233}U, ^{232}U, ^{231}U, ^{230}U, ^{234}Pa, give ~ 80 % to σ_f^C and a,

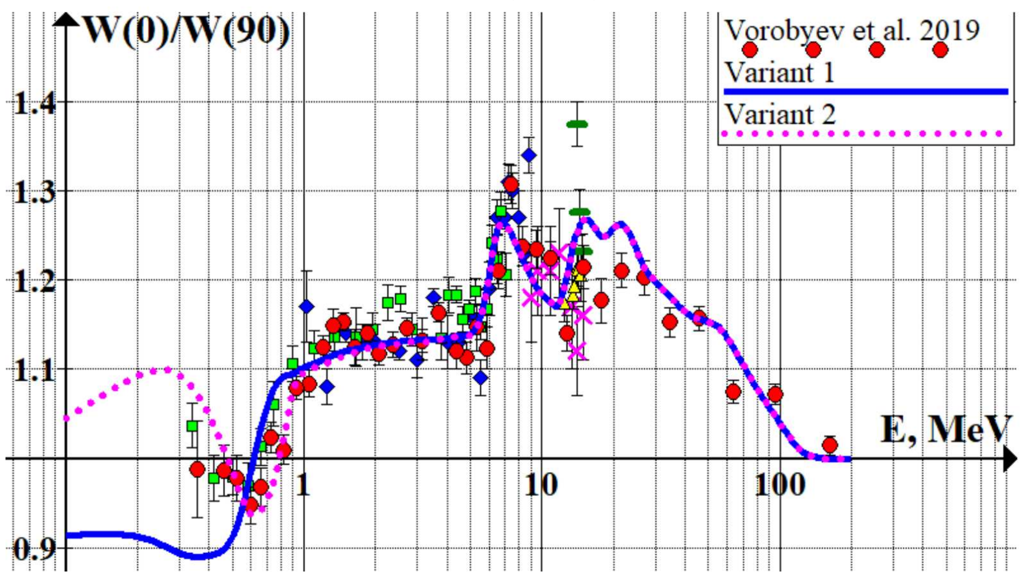
1 isotope, ^{230}U ($T_{1/2} = 20.23$ d), gives ~ 30 % to σ_f^C and a!

The used effective moment of inertia I_{eff} is the average of the moments of involved isotopes. Really I_{eff} depends at least on Z, N, E^*.

Problems with description of fragment angular anisotropy at low energies:



(n,f) $(n,n'f)$ $(n,2nf)$ $\ldots$



$$\sigma_f = \sum_{J\pi} \sigma(J\pi) P_f(J\pi), \qquad \frac{d\sigma_f(\mathbf{n}_f)}{d\Omega} = \sum_{J\pi}\sum_{M} \sigma(J\pi M) P_f(J\pi) \sum_{K} \rho^{J\pi}(K) \frac{2J+1}{4\pi} \left| D^J_{MK}(\mathbf{n}_f)\right|^2$$

$$P_f(J\pi) \sim \sum_i T_f^{J\pi}(E_{ex} - B_f - \varepsilon_i(KJ\pi)) + \int_{E_c(J\pi)}^{E_{ex}} \rho(\varepsilon, J, \pi)\, T_f^{J\pi}(E_{ex} - B_f - \varepsilon)\, d\varepsilon,$$

But really:

$$\frac{d\sigma_f(\mathbf{n}_f)}{d\Omega} = \sum_{J\pi}\sum_{M} \sigma(J\pi M) \sum_{K} P_f(J\pi K) \frac{2J+1}{4\pi} \left| D^J_{MK}(\mathbf{n}_f)\right|^2, \qquad P_f(J\pi K) \neq P_f(J\pi)\, \rho^{J\pi}(K)\,!$$

Summary

1. «TALYS-based» method for calculation of fission fragment angular distribution for neutron-induced reaction is presented.

2. High degree of relevancy of the method is shown: the gross structure of energy dependence of fragment angular anisotropy is described for the reaction $^{237}Np(n,f)$ with the use of a minimal set of additional parameters.

3. At intermediate energies the fragment angular anisotropy is very sensitive to pre-equilibrium processes and multichance contributions. In the planned detail analysis, we expect to clarify pre-equilibrium contributions to the reaction cross sections and to test possible dependence of I_{eff} on N, Z and excitation energy.

4. At low energies more consistent methods are needed to describe the fragment angular anisotropy and to obtain new information on fission transition states.