NEUTRON RESONANCES AND QUANTUM CHAOS

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After a long search, the main source of classical chaos was stated to be the instability of the system's trajectories in the phase space with respect to the small variations of the initial conditions. The trajectories of the chaotic system diverge with time according to the exponential law as $\exp(\Lambda t)$, where the rate of divergence is defined by the Lyapounov exponent Λ.

However, the uncertainty relation in quantum mechanics deprives the trajectory of the necessary precision. Therefore, the modern definition of quantum chaos as "dynamics of the quantum systems which are chaotic in the classical limit" is quite vague. It is also proposed to search in these quantum systems for "the quantum signatures of the classical chaos". During the almost 40 years of this search the only more or less generally recognized such signature turned out to be the system's level distribution law. For the quantum analogs of the chaotic systems the level spacing distribution turned out to be close to the Wigner law with its characteristic level repulsion, which was experimentally established for the neutron resonances of compound nuclei. It is also often claimed that the level distribution of the quantum analogs of the classical regular systems is governed by the Poisson law.

It can be easily shown [1] that the level distribution criterion is a rather weak "signature" of quantum chaoticity or regularity. Wigner's law describes the chaotic system only if one selects a sequence of levels with fixed values of quantum numbers, while the Poisson law just shows that we have a mixture of many independent sequences of energy levels with different quantum numbers (irrespective of the system's regularity or chaoticity).

To avoid all these ambiguities and difficulties we suggest (see e.g. [1–3]) to use for the definition of regular and chaotic systems both in classical and quantum mechanics the Liouville-Arnold theorem, which states that the system is regular if the number N of its degrees of freedom equals the number M of its first integrals of motion associated with the symmetries of the system. If the symmetry of the system is broken so that M becomes smaller than N, the system becomes chaotic. Unlike the trajectory, the concept of symmetry applies both in classical and quantum mechanics. The quantum analog of the first integral of motion is a "good" quantum number. It was shown [4] that while applying the Liouville-Arnold theorem to quantum systems one should not include in the M number purely quantal characteristics (e.g., parity), which disappear in the classical limit.

This approach shows that neutron resonances of compound nuclei are indeed nice examples of the quantum chaos signature since for the compound nucleus the number N of its degrees of freedom is always much larger than M. An example of the regular quantum system is given by the shell model of independent particles moving in a spherical potential field. The existence of the neutron strength-function maxima shows that some remnants of the regular shell model motion symmetries destroyed by the pairwise nuclear interaction still exist in the compound nucleus. It is shown that the single-particle resonance spreading width Γ_{spr} (to be exact, $\Gamma_{spr}/\hbar$) is a measure of quantum chaoticity, which transforms into the Lyapounov exponent in the classical limit $\hbar \to 0$.

References

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