AN INVERSE-PROBLEM SOLVING BY THE EXAMPLE OF $^{238}U(n,2\gamma)^{239}U$ REACTION ANALYSIS

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- The wave function of any excited level includes both quasi-particle and phonon components.
- Information about the nuclear-matter behavior can be received if only the strong-correlated nuclear-physical parameters (the nuclear level density **ρ** and partial widths **Γ** of reaction-products emission) are obtained **simultaneously** at the nuclear-reaction investigation.
- The strong-correlated **ρ** and **Γ** parameters enter into measured spectra as a product ρ×Γ, so their extraction from indirect experiment is a search for inverse-task solution.
- The Dubna empirical method allows simultaneous extraction of $\rho(E_{ex})$ and $\Gamma(E_\gamma)$ functions from approximation of the experimental $I_{\gamma\gamma}(E_1)$-intensities. This method was applied to analyze experimental γ-spectrum from the $^{238}U(n,2\gamma)^{239}U$ reaction, which has been measured using nearly 4π γ-ray calorimeter DANCE composed of a spherical array of 160 BaF_2 crystals.

J.L. Ullmann, T. Kawano, B. Baramsai, et al., Phys.Rev. C 96, 024627 (2017)

The basis of the Dubna empirical method

The $\rho(E_{ex})$ and $\Gamma(E_\gamma)$ functions are obtained **simultaneously** from the fitting of calculated $I_{\gamma\gamma}^{cal}(E_1)$-intensities to the experimental $I_{\gamma\gamma}^{exp}(E_1)$, when a system of nonlinear equations is Monte-Carlo solving. Each of equations connects $I_{\gamma\gamma}(E_1)$-intensities with Γ of γ-transitions between initial cascade level λ and a group of final levels *f* via all possible intermediate levels *i:*

$$I_{\gamma\gamma}(E_1)=\sum_{\lambda,f}\sum_{i}\frac{\Gamma_{\lambda i}}{\Gamma_\lambda}\frac{\Gamma_{if}}{\Gamma_i}=\sum_{\lambda,f}\sum_{j}\frac{\Gamma_{\lambda j}}{<\Gamma_{\lambda j}>M_{\lambda j}}n_j\frac{\Gamma_{jf}}{<\Gamma_{jf}>m_{jf}}$$

$$\chi^2=\sum_{n_j}\frac{(I_{\gamma\gamma}^{cal}(E_1)-I_{\gamma\gamma}^{\exp}(E_1))^2}{\sigma^2}$$

In a small interval ΔE_j there are n_j levels, to which $M_{\lambda j}=\rho\Delta E_j$ primary transitions go from initial level *λ*, and m_{jf} secondary transitions go to final level *f.* ($n_j=\rho\Delta E_j$ and $m_{jf}=\rho\Delta E_j$).

- The system solving is impossible without using model representations of the $\rho(E_{ex})=\phi(p_1,p_2, ...)$ and $\Gamma(E_1)=\psi(q_1, q_2, ...)$ functions.
- For given $\rho(E_{ex})=\phi(p_1, p_2, ...)$ and $\Gamma(E_1)=\psi(q_1, q_2, ...)$ model functions there is only one solution of this system of equations.
- Uncertainties of the obtained $\rho(E_{ex})$ and $\Gamma(E_1)$ functions is mainly determined by inexactness of their model representations, so there is a problem of a choice of their most realistic parametrizations.
- In the two-step cascade the γ-quanta sequence can be determined only for a part of energy-resolved cascades corresponded to available spectroscopic data. The decay-scheme application is impossible to separate carefully $I_{\gamma\gamma}(E_1)$-distribution from insufficiently detailed analyzed $I_{\gamma\gamma}(E_\gamma)$-spectrum of ^{239}U.

The nuclear-parameters' representations

Available in RIPL-file models of the nuclear parameters were modified for potential possibility of disturbance in their smooth energy dependences for exact experimental-spectrum description.

For **the level density** $\rho(E_{ex})$ function – a modern Strutinsky model (***in Proceedings of the International Congress on Nuclear Physics, Paris, France, 1958, p. 617***) and a balance between changes of entropy and energy of quasi-particles' states (***A.V. Ignatyuk, Report INDC-233(L), IAEA (Vienna, 1985)***).

$$\rho_l = \frac{(2J+1)\exp(-(J+1/2)^2/2\sigma^2)}{2\sqrt{(2\pi)}\sigma^3}\Omega_n(E_{ex}),$$

$$\Omega_n(E_{ex}) = \frac{g^n(E_{ex}-U_l)^{n-1}}{((n/2)!)^2(n-1)!}.$$

Ω_n is of n-quasi-particle states, a cut-off factor σ of spin J of excited state of compound-nucleus above the energy E_d and density $g=6a/\pi^2$ of single-particle states near Fermi-surface are taken from back-shifted Fermi-gas model, U_l is the energy of step (l-th Cooper pair breaking threshold).

Generally accepted coefficient C_{col} of enhancement of collective level density:

$$C_{col} = A_l\exp(\sqrt{(E_{ex}-U_l)/E_\nu} - (E_{ex}-U_l)/E_\mu) + \beta.$$

A_l - parameters of densities of vibrational levels above the breaking point of each l-th Cooper pair, E_μ - a change in the nuclear entropy, E_ν - a change of quasi-particles excitation energies, $\beta \geq 1$ (can differ from 1 for deformed nuclei).

For the smooth parts of the **dipole radiative strength functions**, $k(E_\gamma)=\Gamma/(A^{2/3}\cdot E_\gamma^3\cdot D_\lambda)$, – model of Kadmensky-Markushev-Furman (***Sov. J. Nucl. Phys. 37, 165 (1983)***) + 1-2 local peaks

$$k(E_\gamma) = w\ \frac{1}{3\pi^2\hbar^2c^2A^{2/3}}\frac{\sigma_G\Gamma_G^2(E_\gamma^2+\kappa\ 4\pi^2T_E^2)}{(E_\gamma^2-E_G^2)^2+E_G^2\Gamma_G^2}$$

T - varied thermodynamic parameters, w and κ - parameters of weight and of a change of derivatives of the strength function, E_G, Γ_G and σ_G - location of the center, width and cross section in maximum of giant dipole resonance.

Analysis of the data from $^{238}U(n,2\gamma)^{239}U$ reaction

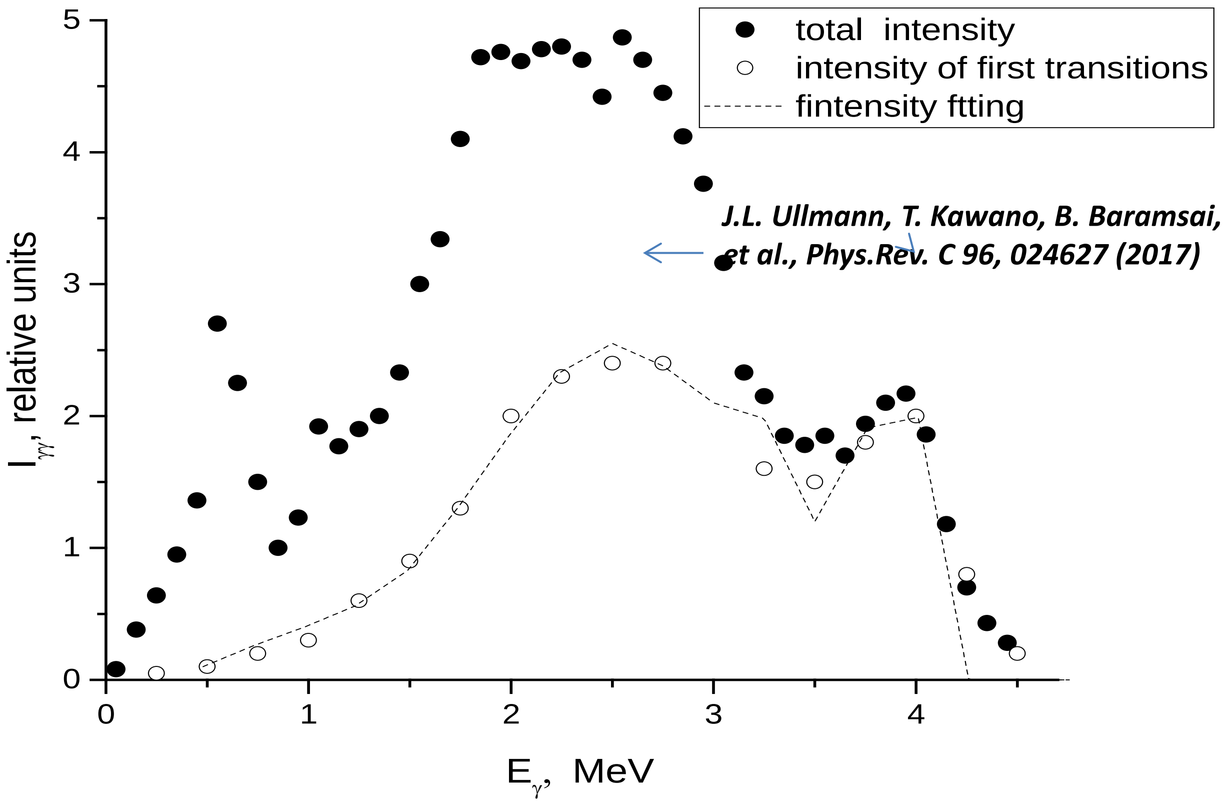

Fig.1. The dependence of intensities of the two-step γ-cascades (relative to their total area) on γ-quanta energies at the decay of compound-state in ^{239}U nucleus (36 eV): close points are the total experimental $I_{\gamma\gamma}(E_\gamma)$-intensities; open points are $I_{\gamma\gamma}(E_1)$-intensities; dashed line is the best fit of $I_{\gamma\gamma}(E_1)$-intensity.

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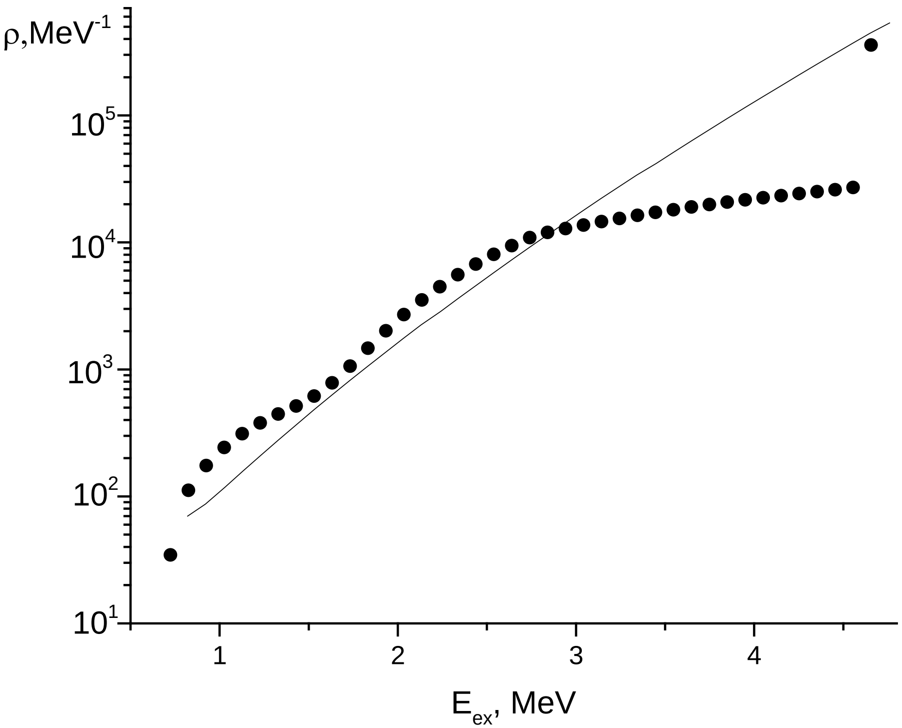

Fig.2. The expected level density of ^{239}U nucleus: close points is calculation (using the best fitted parameters from the $I_{\gamma\gamma}(E_1)$-fitting); line is calculation according to the back-shifted Fermi-gas model (***W. Dilg, W. Schantl, H. Vonach, and M. Uhl, Nucl. Phys. A 217, 269 (1973)***).

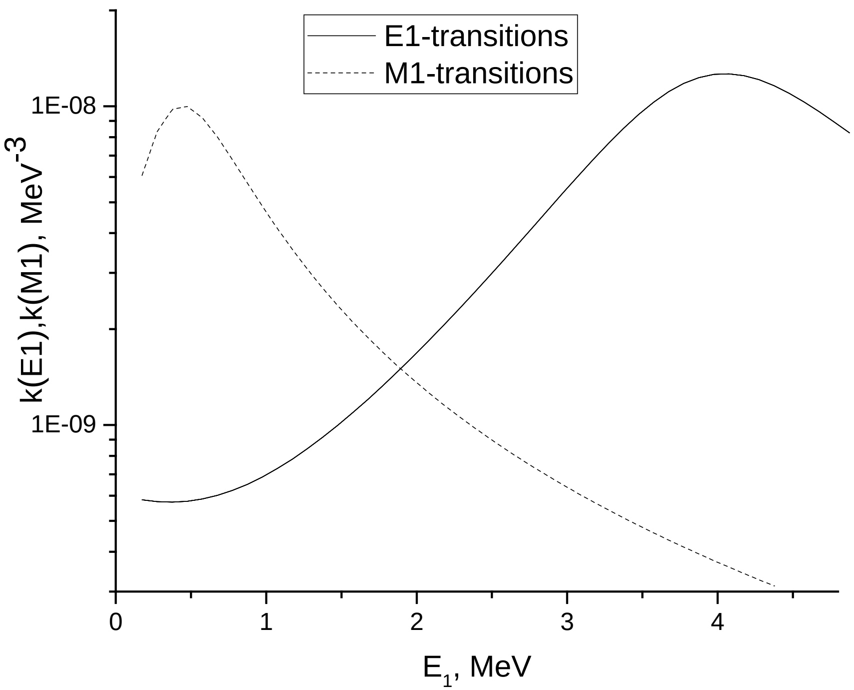

Fig.3. The radiative strength functions of ^{239}U nucleus (calculations for the best fit of the $I_{\gamma\gamma}(E_1)$-intensity): solid line is k(E1) of electrical γ-transitions; dashed line is k(M1) of magnet transitions.

Conclusion

- ✓ In spite of the absence of individual cascade peaks in measured γ-spectrum of ^{239}U nucleus, we have been successful in a description of its roughly separated $I_{\gamma\gamma}(E_1)$-spectrum.

- ✓ A successful approximation of $I_{\gamma\gamma}(E_1)$-spectrum allowed us to ascertain, at least, a behavior of obtained simultaneously $\rho(E_{ex})$ and $k(E_1)$ functions.

- ✓ High-transmission spectrometers of gammas are needed in order for the strong-correlated nuclear-physical parameters extracted from indirect experiment will come to more reliability.

- ✓ Possibility of testing of suitable model parametrizations of $\rho(E_{ex})$ and $k(E_1)$ functions allows to decrease a main uncertainty of their simultaneous extracting.