NEUTRON RESONANCES AND QUANTUM CHAOS

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Investigation of quantum chaos in nuclear physics is strongly hampered by the absence of even the definition of quantum chaos, not to mention the numerical criterion of the quantum chaoticity. These drawbacks are caused by the fact that the present concept of chaos is based on the Lyapunov trajectory instability. For the classical chaotic system, a small variation of initial conditions causes exponential divergence of the trajectories in the phase space. The distance between two initially close trajectories increases as

$$\exp \Lambda t$$

Λ - Lyapunov exponent.

But the trajectory is smeared by the uncertainty principle. Therefore:

“Incorrect term “quantum chaos” means quantum phenomena characteristic of the classically chaotic systems, quantum signatures of the classical chaos.” (M. Berry (1991)).

The modern definition of quantum chaos is quite vague - “dynamics of the quantum systems which are chaotic in the classical limit”.

Search for the “quantum signatures” of the classical chaos:

1. Take two classical systems - regular and chaotic.
2. Construct and solve the two corresponding Schrödinger equations.
3. Compare all the properties of the solutions in hope to find something distinguishing the quantum analogue of the classically chaotic system.

The only such signature found in 50 years of intensive search turned out to be the level distribution law.

For the quantum analogues of the classically chaotic systems - Wigner law

$$P(\varepsilon) = \frac{\pi\varepsilon}{2D^2}\exp\left(-\frac{\pi\varepsilon^2}{4D^2}\right)$$

$P(\varepsilon) \to 0$ for $\varepsilon \to 0$ -- level repulsion (D - average level spacing)

This immediately brings us to nuclear physics. Back in 1950-ies Wigner developed the random matrix approach to the statistical description of neutron resonances in compound nuclei and derived the above law for the distribution of those resonances.

Sometimes - statements made that for regular systems Poisson law

$$P(\varepsilon) = \frac{1}{D}\exp\left(-\frac{\varepsilon}{D}\right)$$

Since the majority of natural systems - intermediate between the regular and chaotic ("soft chaos"), attempts to invent the interpolation formulae between the two laws.

However, it can be easily shown (e. g. Bunakov. Phys. Atom. Nucl. 2016. V.79. P. 995) that:

1. Wigner repulsion applies to chaotic system only when we select energy levels with the same fixed quantum numbers. If we mix levels with even two different quantum numbers (say, spin) this repulsion (and Wigner law) disappears while the system is still chaotic.
2. Poisson law shows only that we have mixed several independent sequences of energy levels with different quantum numbers (irrespective of the system's regularity or chaoticity).

3. No general law for the energy distribution of the quantum regular systems.

Since the main feature of compound resonances is their complex structure, attempts were made to invent different measures of their wave functions' complexity as the measures of quantum chaoticity. Still the main drawback - lack of the quantum chaos definition. The definition "quantum signatures of the classical chaos" seems quite strange and wrong. Nobody considers the laws of relativistic mechanics as "relativistic signatures" of the classical mechanics laws.

Our idea is to stop clinging to Lyapunov trajectory instability as the main source of chaos in classical mechanics.

There is a well-known Liouville-Arnold theorem in classical mechanics which states that a regular system with N degrees of freedom should also have M=N independent global integrals of motion (conservation laws) connected with the symmetries of the system's Hamiltonian. I.e., the regular system possesses a sufficiently high symmetry to have the number M of integrals of motion equal to the number N of its degrees of freedom.

Contrary to trajectory, the notion of symmetry applies to all the fields of physics, including classical and quantum mechanics. As a consequence of this theorem one can define the quantum regular system as that whose Hamiltonian possesses sufficiently high symmetry to have the number M of conservation laws ("good quantum numbers") which is equal to the number N of its degrees of freedom. If one switches on the perturbation which destroys the systems symmetries so that M<N, the system becomes chaotic.

Thus, we suggest to define as chaotic the quantum system whose symmetry is so low that M<N. (see Phys. Lett. **A243** (1998) 288; Phys. At. Nucl. **62** (1999) 1; J. Phys. **A35** (2002) 1907; Phys. Atom. Nucl. **79** (2016) 995; Phys. Atom. Nucl. **79** (2016) 561).

We had also investigated the question which arises in application of the theorem to quantum systems: There are purely quantum integrals of motion (e. g. parity) which disappear in the classical limit. Should one include them in the number M while comparing them to the number N? We had shown (Bull. Russ. Acad. Sci. Phys. 2021. V.85. P.538) that in applying Liouville-Arnold theorem to quantum system one should count only the "classical" integrals of motion M_{cl}.

Let us apply this definition to nuclei.

Actually, each system of more than 2 interacting particles has less integrals of motion than its degrees of freedom. Therefore, all the nuclei heavier than deuteron are quantum chaotic systems. The neutron resonances of medium and heavy nuclei for whom Wigner stated his level distribution law have only 3 good quantum numbers (energy, spin and parity) and only one “classical” integral of motion (energy). The number of quasi-particles defining the structure of the low-lying neutron resonances is about 8–10. Therefore, these resonances are of course quite typical examples of the quantum chaotic systems.

Moreover, the theory of the neutron strength function describing these resonances allows to introduce the quantitative measure of the quantum chaoticity.

Indeed, bearing in mind the Hartree-Fock method which proved quite successful in constructing a self-consistent mean field in atomic physics, we can try to represent the Hamiltonian of a compound nuclear system as

$$H = H_0 + V$$

were the regular part H_0 describes the motion of noninteracting nucleons in their mean field, while V takes into account residual pair interactions, which cannot be included in the mean field. We can now seek the wave functions of the system (eigenfunctions ψ_i of the full Hamiltonian H) as a superposition of various (configurations) ϕ_k of the Hamiltonian H_0 describing noninteracting particles. Expand now the wave functions ψ_i of the total Hamiltonian H over the basis of the "regular" states ϕ_k :

$$\psi_i = \sum_k \langle \phi_k | \psi_i \rangle \phi_k = \sum_k c_i^k \phi_k$$

and consider the probability $P_s(E_i) = \left| c_i^s \right|^2$ to find the initial regular single-quasiparticle component ϕ_s in the eigenfunctions ψ_i .

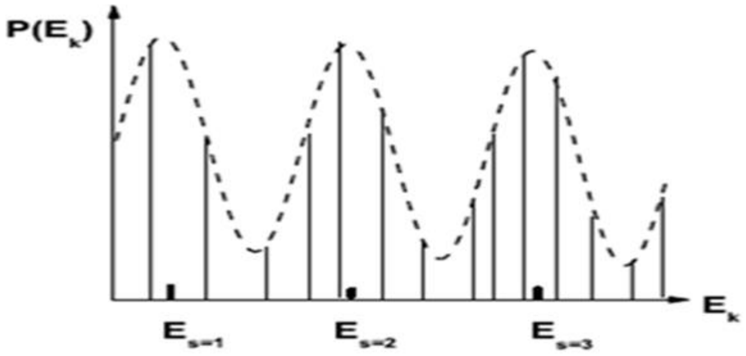
The neutron strength function theory shows that this probability is concentrated in the energy interval Γ_{spr} around the "initial" value ε_k. The strength function $S_k(E_i) = P_k(E_i)/D$ under rather general assumptions (see A. Bohr, B. Mottelson, Nuclear Structure. V.I) shows how the regular state is fragmented over the states of the chaotic system:

$$S_k(E_i) = \frac{\left|c_i^k\right|^2}{D} = \frac{1}{2\pi}\frac{\Gamma_{spr}^k}{(E_i - \varepsilon_k)^2 + \left(\Gamma_{spr}^k\right)^2/4}$$

When the localization interval Γ_{spr} is smaller than the average level distance D_0 of the regular system

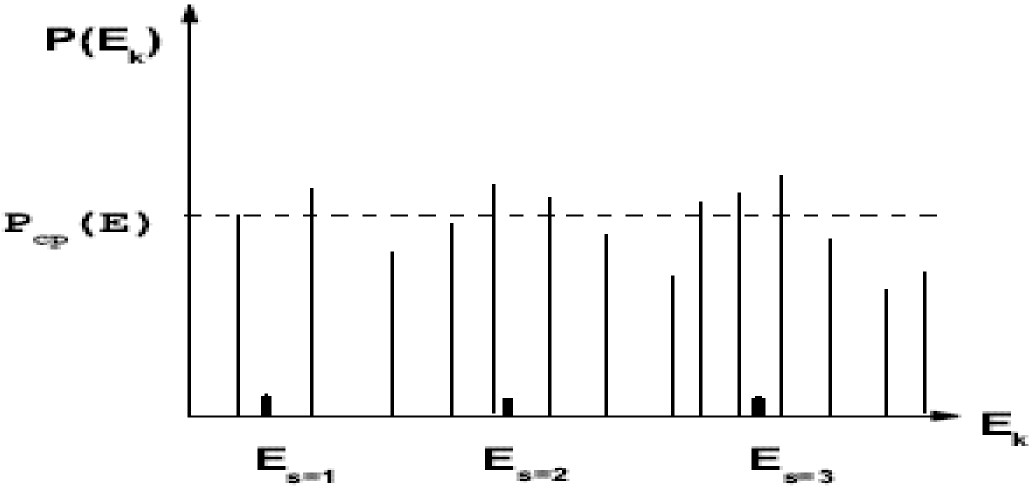
$$\Gamma_{spr} < D_0$$

the strength function peaks at ε_k show vivid traces of the initial regular states ϕ_k with quantum numbers k. Although the symmetry of the regular system is formally broken, its traces are quite visible - soft chaos.



When $\Gamma_{spr} \geq D_0$

the $S_k(E_i)$ maxima disappear. Disappearance of all the traces of the regular states broken symmetries. Hard chaos.



Thus, the dimensionless parameter

$$\kappa = \frac{<\Gamma_{spr}^{k}>}{D_0} = \frac{\Gamma_{spr}}{D_0}$$

is a natural measure of quantum chaoticity.

Position of the s- resonances in nuclei:

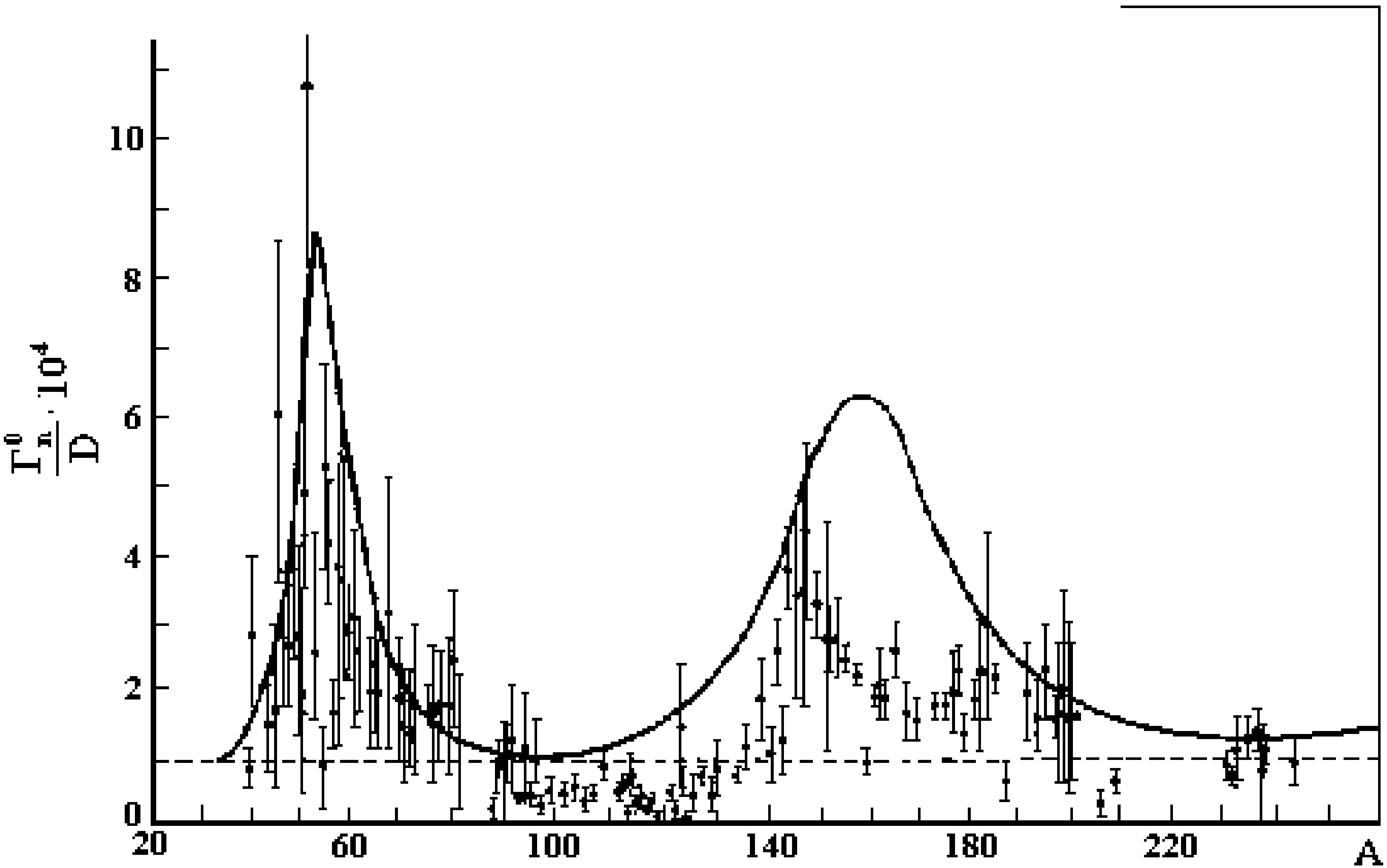
$$KR = (2n+1)\frac{\pi}{2}$$

$$K = \sqrt{\frac{2mE}{\hbar^2}} \qquad R = r_0 (A)^{1/3}$$

Thus, one can either fix the nucleus A (i.e., R) and see the dependence $S(E)$, or

fix E≈1 eV and look for the dependence $S(A)$

Neutron strength function for s-wave neutrons



In nuclei A=50 and A=160 single particle states 3s and 4s appear at neutron energy ≈1 eV. Nuclei with 140<A<200 are deformed - spherical symmetry broken and single-particle states with fixed angular momentum are fragmented.

Typical example of weak chaos: the symmetries of the spherical mean field $V(\vec{r}_i)$ are broken by the pairwise N-N interaction $v(\vec{r}_i - \vec{r}_j)$, but the traces of those symmetries are still clearly seen (quantum analog of classical KAM theorem).

We see that nuclear physics had to deal with quantum chaotic objects for more than half a century. Not only does the experience of nuclear physics enable us to clarify connections between symmetries and chaos, but it also proposes the effective methods to trace numerically the transition of the quantum system from regularity to hard chaos.

$\kappa < 1$ is the principal small parameter in nuclear physics, which makes the shell-model basis most adequate because of its rapid convergence and allows to use optical model as the main component of various reaction theories.

We have demonstrated (J. Phys. A**35** (2002) 1907; Phys. Atom. Nucl. **79** (2016) 995) that in the classical limit

$$(\Gamma_{spr} / \hbar) \rightarrow \Lambda$$

and

$$\kappa \rightarrow \frac{\Lambda T}{2\pi} = \frac{\chi}{2\pi}$$

Here T is the period of the classical orbit.

We had also applied our ideas to the only 2 "textbook" systems where the transition from regularity to chaos with the destruction of symmetry was accurately traced:

1. To the Henon-Heiles system with the Hamiltonian

$$H = \frac{1}{2}(p_x^2 + x^2 + p_y^2 + y^2) + \lambda(x^2 y - y^3 / 3) \equiv H_0 + \lambda V$$

Here the regular two-dimensional oscillator H_0 is perturbed by the non-linear potential V.

2. To the diamagnetic Kepler system - merely a classical spinless analog of the hydrogen atom in a homogeneous magnetic field. The symmetry of the regular Coulomb potential H_o is violated by the external homogeneous magnetic field.

We had calculated the energy spectra of the analogous quantum systems and the values:

$$\kappa = \frac{< \Gamma_{spr}^{k} >}{D_0} = \frac{\Gamma_{spr}}{D_0}$$

for the gradually increasing perturbations. It turned out that this purely quantum parameter reached the value unity at the same perturbation strength which marked the onset of hard chaos in the classical systems (see e. g. V.E. Bunakov, I.B. Ivanov, J. Phys. **A35**, 1907 (2002)).

THANKS FOR YOUR ATTENTION!