



National University of Mongolia  
NUCLEAR RESEARCH CENTER



# Systematical Analysis of (n,2n) Reaction Cross Sections for 14 – 15 MeV Neutrons

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# Introduction

- Fast neutron induced nuclear reaction cross section data are necessary for both **nuclear energy technology** and the **understanding of fundamental nuclear physics problems**.
  - Radioactive nuclides produced in the reactor usually have short half-life.
  - So, direct measurement of their neutron cross sections is difficult. Therefore, model formulae are important to predict these cross sections theoretically.
- Biomedical applications such as production of radioisotopes and cancer therapy:  
 **$^{100}\text{Mo}(\text{n},2\text{n})^{99}\text{Mo}$**
- Accelerator driven transmutation of the long-lived radioactive nuclear wastes to short lived or stable isotopes
- Material irradiation experiments concerning research and development for fusion reactor technology.
- Study on the existence of Dineutron.

# Introduction

## The purpose of this work:

- In this work, in the framework of the statistical model we deduced some theoretical formulae for the  $(n,2n)$  cross section using the evaporation model, constant nuclear temperature approximation and Weizsäcker's formula for binding energy.
- Known experimental data of the  $(n,2n)$  cross sections at 14 - 15 MeV neutrons are analyzed with the help of the obtained formulae.

# Statistical Model formulae

In the framework of the statistical model based on the Bohr's assumption of a compound mechanism the cross section formula for  $(n,x)$  reaction is expressed as:

$$\sigma(n,x) = \sigma_c(n) \frac{2S_x + 1}{2S_n + 1} \frac{M_x}{M_n} e^{\frac{Q_{n,x} - V_x}{\Theta}} \left\{ \frac{1 - \frac{W_{n,x}}{\Theta} e^{-\frac{W_{n,x}}{\Theta}} - e^{-\frac{W_{n,x}}{\Theta}}}{1 - \frac{E_n}{\Theta} e^{-\frac{E_n}{\Theta}} - e^{-\frac{E_n}{\Theta}}} \right\}$$

G.Khuukhenkhuu *et al.* *J NUCL SCI TECHNOL*, Supp. 2, Vol. 1, 2001, pp. 782-784.

$\approx 1$

Following formula, which is similar to Cuzzocrea's *et al.* and Ericson's formulas, is obtained:

$$\sigma(n,2n) = \sigma_c(n) \frac{2S_{2n} + 1}{2S_n + 1} \frac{M_{2n}}{M_n} e^{\frac{Q_{n,2n} - V_{2n}}{\Theta}}$$

Here:  $\sigma_c(n)$  is the compound nucleus formation cross section;

$S_n$  and  $S_{2n}$  are the spin of the incident neutron and emitted  $2n$  respectively;

$M_n$  and  $M_{2n}$  are the masses of the neutron and  $2n$  respectively;

$Q_{n,2n}$  is the reaction energy;

$V_{2n}$  is the Coulomb potential for  $2n$ ;

$\Theta$  is the thermodynamic temperature;

$$\sigma_c(n) = \pi(R + \tilde{\lambda}_n)^2$$

Here:  $R$  is the radius of the target nucleus;

$\tilde{\lambda}_n$  is the wavelength of the incident neutron divided by  $2\pi$ .

# Statistical Model formulae

The Coulomb potential for neutrons  $V_{2n} = 0$ . So, taking into account the spin and mass of neutrons from the formula (2) we get:

$$\sigma(n,2n) = 4\pi(R + \tilde{\lambda}_n)^2 e^{\frac{Q_{n,2n}}{\Theta}}$$

Using the Weizsäcker's formula for binding energy we can obtain following expressions for the target and residual nuclei:

$$E_i = \alpha A - \beta A^{2/3} - \gamma \frac{Z^2}{A^{1/3}} - \xi \frac{(N-Z)^2}{A} \pm \frac{\delta_i}{A^{3/4}}$$

Reaction energy:

$$Q_{n2n} = E_f - E_i$$

$$E_f = \alpha(A-1) - \beta(A-1)^{2/3} - \gamma \frac{Z^2}{(A-1)^{1/3}} - \xi \frac{(N-1-Z)^2}{A-1} \pm \frac{\delta_f}{(A-1)^{3/4}}$$

(n,2n) cross section formula:

$$\sigma(n,2n) = 4\pi(R + \tilde{\lambda}_n)^2 \exp \left\{ \frac{-\alpha - \beta \left( (A-1)^{2/3} - A^{2/3} \right) - \gamma \left( \frac{Z^2}{(A-1)^{1/3}} - \frac{Z^2}{A^{1/3}} \right) - \xi \left( 1 - \frac{4Z^2}{A(A-1)} \right) \pm \frac{\delta_f}{(A-1)^{3/4}} \mp \frac{\delta_i}{A^{3/4}}}{\Theta} \right\}.$$

# Statistical Model formulae

In the case of  $A \gg 1$  can be obtain the following formulae for systematical analysis of the  $(n,2n)$  cross sections:

$$\frac{\sigma(n,2n)}{\pi(R + \hat{\lambda}_n)^2} = C \exp(-K \frac{Z^2}{A^2})$$

Here:  $Z$  and  $A$  are proton and mass numbers of the target nuclei;

The parameters  $K$  and  $C$  are expressed as:

$$K = \frac{4\xi}{\Theta}$$

$$C = 4 \exp \left\{ \frac{-\alpha - \beta \left( (A-1)^{2/3} - A^{2/3} \right) - \gamma \left( \frac{Z^2}{(A-1)^{1/3}} - \frac{Z^2}{A^{1/3}} \right) \pm \frac{\delta_f}{(A-1)^{3/4}} \mp \frac{\delta_i}{A^{3/4}} + \xi}{\Theta} \right\}$$

# Systematical analysis of (n,2n) reaction cross sections

In this paper the library of neutron cross sections known as **EXFOR**, **IAEA**. We've analyzed **147** experimental (n,2n) cross section data at the neutron energy of 14 – 15 MeV from EXFOR.

<https://www-nds.iaea.org/exfor/>

Target nuclei	A	N	Z	E <sub>n</sub> (MeV)	σ(n,2n) (mb)	Δσ(n,2n) (mb)	Authors
Li	6	3	3	14.06	78.1	4.1	Mather <i>et al.</i> (1969)
Li	7	4	3	14.06	49.7	3.2	Mather <i>et al.</i> (1969)
Be	9	5	4	14.1	478	14	Takahashi <i>et al.</i> (1987)
B	11	6	5	14.06	19	4	Mather <i>et al.</i> (1969)
C	12	6	6	14.1	6	6	Ashby <i>et al.</i> (1958)
C	13	7	6	14.28	255	25	Frehaut <i>et al.</i> (1978)
N	14	7	7	14.64	7.28	0.29	Sakane <i>et al.</i> (2001)
F	19	10	9	14.69	50.4	2.7	Ikeda <i>et al.</i> (1988)
Na	23	12	11	14.87	41.7	0.9	Hanlin <i>et al.</i> (1992)
Al	27	14	13	14.09	7.8	0.5	Wallner <i>et al.</i> (2003)
P	31	16	15	14.64	13.2	0.71	Sakane <i>et al.</i> (2001)
Cl	35	18	17	14.57	10.28	0.8	Molla <i>et al.</i> (1997)
K	39	20	19	14.66	4.55	0.25	Filatenkov (2016)
Ca	40	20	20	14.69	8	2	Braun <i>et al.</i> (1968)
Ca	48	28	20	14.7	850	35	Anders <i>et al.</i> (1985)
Sc	45	24	21	14.8	320	24	J.Luo <i>et al.</i> (2013)
Ti	46	24	22	14.72	42	2.3	Ikeda <i>et al.</i> (1998)
V	50	27	23	14.3	258	39	Greenwood <i>et al.</i> (1992)
Cr	50	26	24	14.6	21.2	1.2	Ribansky <i>et al.</i> (1985)
Cr	52	28	24	14.47	351	14.49	Mannhart <i>et al.</i> (2007)
Mn	55	30	25	14.58	812.9	28.9	Hanlin <i>et al.</i> (1980)
Fe	54	28	26	14.64	9	1.8	Sakane <i>et al.</i> (2001)
Fe	56	30	26	14.82	545.4	27.3	Wallner <i>et al.</i> (2011)

Hg	196	116	80	14.68	2220	170	Kasugai <i>et al.</i> (2001)
Hg	198	118	80	14.68	2060	130	Kasugai <i>et al.</i> (2001)
Hg	204	124	80	14.68	2140	100	Kasugai <i>et al.</i> (2001)
Tl	203	122	81	14.7	1970	110	Kiraly <i>et al.</i> (2001)
Tl	205	124	81	14.76	1895	143	Frehaut <i>et al.</i> (1980)
Pb	204	122	82	14.5	2161	172.45	Filatenkov (2016)
Pb	206	124	82	14.76	2028	155	Frehaut <i>et al.</i> (1980)
Pb	207	125	82	14.76	1976	161	Frehaut <i>et al.</i> (1980)
Pb	208	126	82	14.1	2380	140	Simakov <i>et al.</i> (1992)
Bi	209	126	83	14.74	2293	371.47	Filatenkov (2016)
Th	232	142	90	14.7	1177	78.86	Chatani <i>et al.</i> (1991)
Th	238	146	92	14.62	58	58	Raics <i>et al.</i> (1990)



# Systematical analysis of (n,2n) reaction cross sections

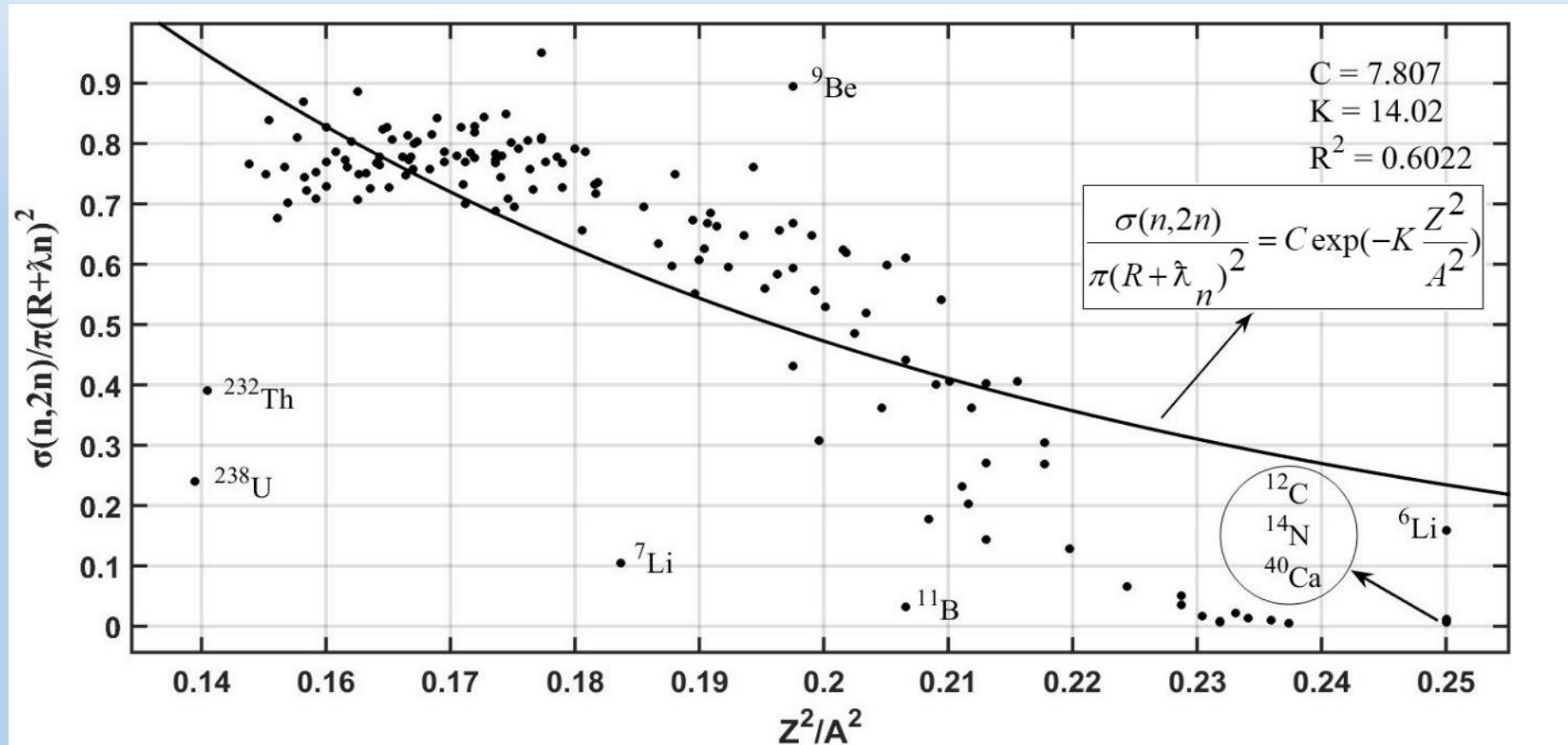


Figure 1. The dependence of the reduced (n,2n) cross sections on parameter  $Z^2/A^2$

# Discussions

A total cross section for fast neutrons can be approximated as following:

$$\sigma_n^{tot} = \sigma^{tot}(n, 2n) + \sigma^{tot}(n, \gamma) + \sigma^{tot}(n, \alpha) + \sigma^{tot}(n, p) + \sigma^{tot}(n, 3n) + \dots \approx \sigma^{tot}(n, 2n) \approx \pi(R + \tilde{\lambda}_n)^2.$$

$$^{197}\text{Au:} \quad \sigma_n^{tot} = \frac{\sigma^{tot}(n, 2n)}{2154mb} + \frac{\sigma^{tot}(n, \gamma)}{1.1mb} + \frac{\sigma^{tot}(n, \alpha)}{0.35mb} + \frac{\sigma^{tot}(n, p)}{2.2mb} + \frac{\sigma^{tot}(n, 3n)}{61mb} + \dots$$

$$^{127}\text{I:} \quad \sigma_n^{tot} = \frac{\sigma^{tot}(n, 2n)}{1655mb} + \frac{\sigma^{tot}(n, \gamma)}{1.12mb} + \frac{\sigma^{tot}(n, \alpha)}{1.2mb} + \frac{\sigma^{tot}(n, p)}{11.7mb} + \frac{\sigma^{tot}(n, 3n)}{38.5mb} + \dots$$

if we take into account the pre-equilibrium and direct mechanisms the total (n,2n) cross section can be obtained as follows:

$$\sigma_n^{tot} \approx \sigma^{tot}(n, 2n) \approx \pi(R + \tilde{\lambda}_n)^2 \approx \sigma^{comp}(n, 2n) + \sigma^{pre}(n, 2n) + \sigma^{dir}(n, 2n)$$

$$\sigma^{nonstat}(n, 2n) = \sigma^{pre}(n, 2n) + \sigma^{dir}(n, 2n)$$

$$\sigma^{nonstat}(n, 2n) = \sigma^{tot}(n, 2n) - \sigma^{comp}(n, 2n) = \pi(R + \tilde{\lambda}_n)^2 - C\pi(R + \tilde{\lambda}_n)^2 \exp(-K \frac{Z^2}{A^2}) = \pi(R + \tilde{\lambda}_n)^2 \left( 1 - C \exp(-K \frac{Z^2}{A^2}) \right).$$

The reduced (n,2n) cross section is expressed as:

$$\frac{\sigma^{nonstat}(n, 2n)}{\pi(R + \tilde{\lambda}_n)^2} = \left( 1 - C \exp(-K \frac{Z^2}{A^2}) \right).$$

# Discussions

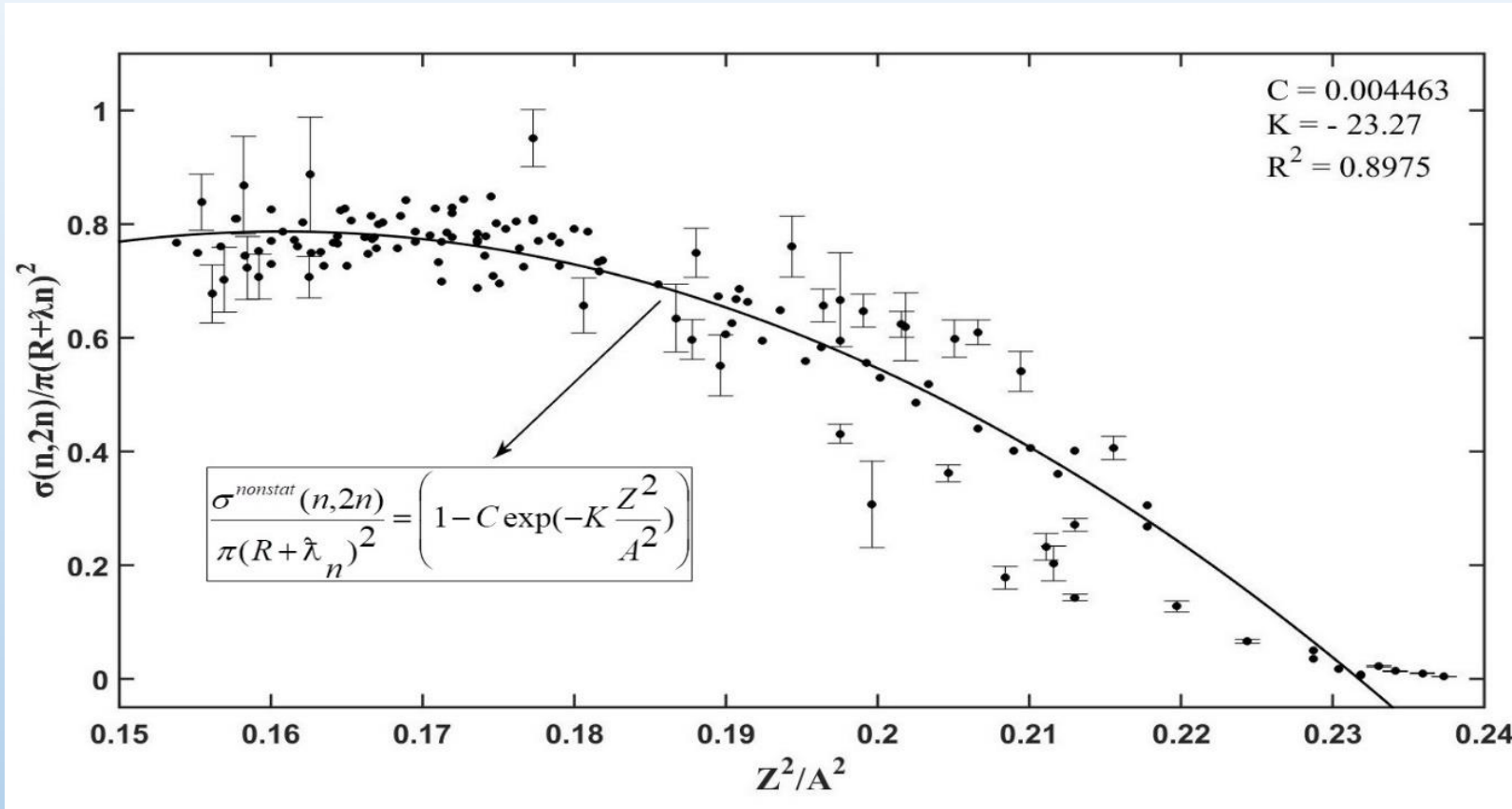


Figure 2. The dependence of the reduced (n,2n) cross sections on parameter  $Z^2/A^2$

The very heavy nuclei such as  $^{238}\text{U}$ ,  $^{232}\text{Th}$  and very light nuclei  $^6\text{Li}$ ,  $^7\text{Li}$ ,  $^9\text{Be}$ ,  $^{11}\text{B}$ ,  $^{12}\text{C}$ ,  $^{14}\text{N}$  are excluded from the consideration. Also, double magic nucleus  $^{40}\text{Ca}$  is not considered.

# Conclusions

1. In the framework of the statistical model a theoretical formula for the  $(n,2n)$  reaction cross section was deduced. In addition, a non-statistical share of the total neutron cross section was obtained.
2. Known experimental data of the  $(n,2n)$  cross sections for 14 – 15 MeV neutrons were analyzed using the obtained formulae. It was shown that the non-statistical share of the total cross section is in agreement with experimental data.

**Thank you for your  
attention**