



## **ISINN-28 Agenda**

**The angular and spin distributions of low-energy nuclear fission fragments and collective wriggling - and bending - vibrations of the fissile nucleus in the vicinity of its scission point**

**Lyubashevsky Dmitriy**

**Voronezh State University**

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# Relevance

Theoretical understanding of the:

- nature of experimental multiplicities;
- energy and angular distributions of neutrons and gamma quanta evaporated from thermalized fragments of spontaneous and low-energy forced fission of actinide nuclei, isomeric ratios of the yields of final fragments, as well as the characteristics of delayed neutrons emitted during beta decay of final fragments;
- isomeric ratios of the yields of final fragments;
- characteristics of delayed neutrons emitted during  $\beta$ -decay of fission fragments.

Attempts to explain this fact through the Coulomb interaction of strongly deformed fission fragments emitted from the fissioning nucleus turned out to be unsatisfactory. This happened because the considered Coulomb interaction can change the values of the relative orbital momenta  $\mathbf{L}$  and spins  $\mathbf{J}_1, \mathbf{J}_2$  of fragments only by small values  $\Delta L, \Delta J_1, \Delta J_2 \leq 2$  in comparison with their average values.

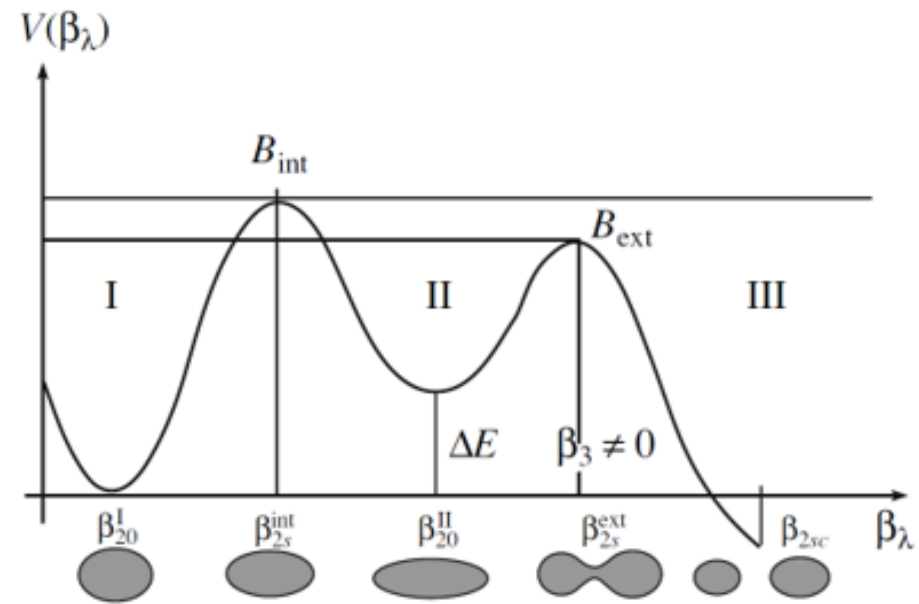
## **Purpose of the work**

1. Analysis of the connection between the angular and spin distributions of fragments of low-energy double fission of nuclei with collective wriggling - and bending - vibrations of a fissile nucleus in the vicinity of the point of its scission.
2. Estimation of average values of relative orbital moments of fission fragments formed by wriggling - vibrations.
3. Proof of the non-statistical nature of the distribution of fission fragments spins due to the simultaneous influence of wriggling - and bending - vibrations.

# 1. The nature of the spin distribution of double fission fragments

At all stages of the fission process, the axial symmetry of both the specified nucleus and the fragments of its fission is preserved. The evolution of a composite fissile nucleus is due to the collective deformation modes of its motion in the field of the deformation potential of the nucleus  $V(\beta_\lambda)$ , which has a two-humped nature, where  $\beta_\lambda$  are the parameters of the deformation of the nucleus.

Regions I and II correspond to the first and second pits of the deformation potential  $V(\beta_\lambda)$ , at the minima of which the nucleus has the equilibrium parameters of the quadrupole deformation  $\beta_{20}^I$  and  $\beta_{20}^{II}$ . Region III is associated with the descent of the specified nucleus from the external fission barrier and reaching the point of rupture into primary fission fragments with deformation parameters  $\beta_{2sc}$ .



## 1. The nature of the spin distribution of double fission fragments

Gibbs distributions  $\rho_i(E_i^*, J_i)$  over excitation energy  $E_i^*$  and spin  $J_i$  for the  $i$ -th fission fragment ( $i = 1, 2$ ):

$$\rho_i(E_i^*, J_i) = \rho_i(E_i^*)\rho_i(J_i), \quad (1)$$

where the energy  $\rho_i(E_i^*)$  and spin  $\rho_i(J_i)$  distributions of the  $i$ -th fission fragment with temperature  $T_i$  and moment of inertia  $\mathfrak{I}_i$  have the form:

$$\rho_i(E_i^*) \sim \exp(-E_i^*/kT_i), \quad (2)$$

$$\rho_i(J_i) \sim (2J_i + 1) \exp[-\hbar^2 J_i(J_i + 1)/\mathfrak{I}_i kT_i]. \quad (3)$$

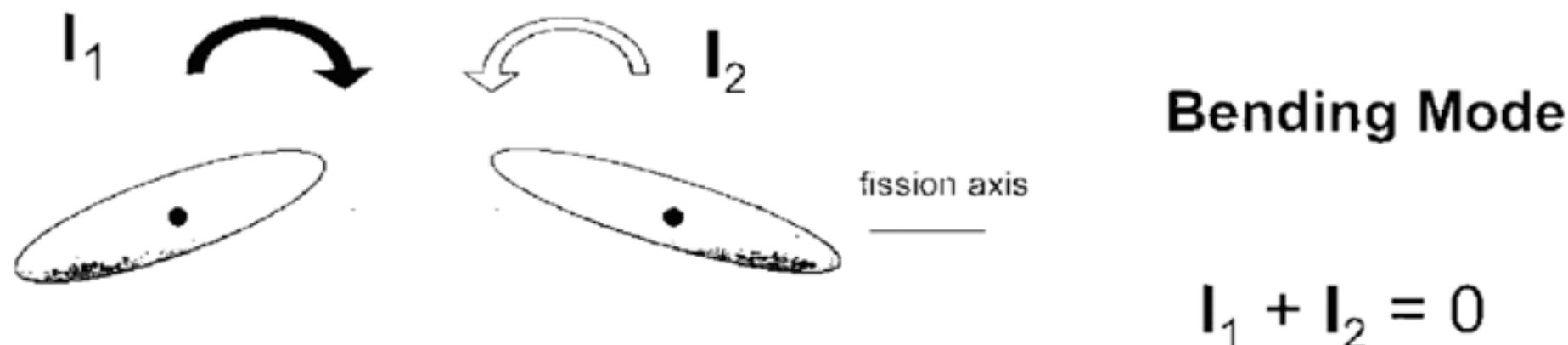
In this case,  $\rho_i(J_i)$  is presented in «standard» form:

$$\rho_i(J_i) \sim (2J_i + 1) \exp[-J_i(J_i + 1)/B^2], \quad (4)$$

where  $B^2$  can differ significantly from  $\mathfrak{I}_i kT$  appearing in (3).

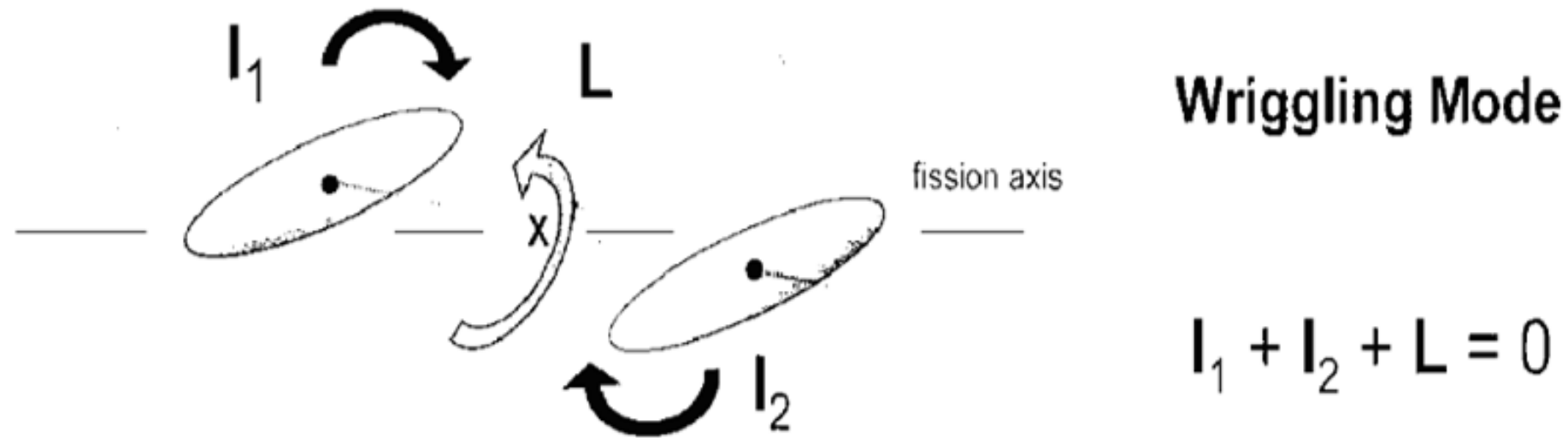
The appearance of nonequilibrium spin distributions of fission fragments, similar to (4), is due to transverse wriggling - and bending - vibrations of a fissioning nucleus in the vicinity of its scission point.

## 1. The nature of the spin distribution of double fission fragments



The first type includes bending - vibrations associated with rotations in opposite directions around a certain axis perpendicular to the axis of symmetry of the fissioning nucleus  $Z$ , two strongly deformed prefragments of fission, touching their vertices in the region of the neck of the fissile nucleus and turning into fission fragments after the rupture of the specified neck. Due to the conservation law of the total spin of a fissile nucleus, the spins of the fragments emitted from the fissioning nucleus due to bending - vibrations satisfy condition  $\mathbf{J}_{b1} = -\mathbf{J}_{b2}$ .

## 1. The nature of the spin distribution of double fission fragments



The second type of considered transverse vibrations of a fissioning nucleus is associated with wriggling - vibrations, for which rotations of prefission fragments occur in one direction around an axis also perpendicular to the symmetry axis of the fissioning nucleus, which leads to the appearance of equally directed and large spins of emitted fission fragments  $\mathbf{J}_{w1}$  and  $\mathbf{J}_{w2}$ . Compensation of the nonzero total spin of these pre-fragments  $\mathbf{L}_w = \mathbf{J}_{w1} + \mathbf{J}_{w2}$  is carried out by rotating the entire fissile nucleus around the same axis in the opposite direction.

## 1. The nature of the spin distribution of double fission fragments

The moments are associated with the projections of spins  $\mathbf{J}_1$  and  $\mathbf{J}_2$  of fission fragments on the  $X, Y$  axes perpendicular to the symmetry axis of the fissioning nucleus  $Z$

$$J_{w_x} = J_{1x} + J_{2x}, J_{w_y} = J_{1y} + J_{2y}; J_{b_x} = J_{1x} - J_{2x}, J_{b_y} = J_{1y} - J_{2y}; J_1^2 = J_{1x}^2 + J_{1y}^2, J_2^2 = J_{2x}^2 + J_{2y}^2. (5)$$

As a result,  $\Psi_0(J_{w_x})$  and  $\Psi_0(J_{b_x})$  are represented as:

$$\Psi_0(J_{w_x}) = (\pi C_w)^{-1/4} \exp\left(-\frac{J_{w_x}^2}{4C_w}\right); \quad \Psi_0(J_{b_x}) = (\pi C_b)^{-1/4} \exp\left(-\frac{J_{b_x}^2}{4C_b}\right), \quad (6)$$

where  $C_w = M_w \hbar \omega_w$ ,  $C_b = M_b \hbar \omega_b$ , frequencies  $\omega_w$  and  $\omega_b$  of wriggling - and bending - vibrations are determined by the classical formulas  $\omega_w = \sqrt{K_w / M_w}$  and  $\omega_b = \sqrt{K_b / M_b}$ , where  $K_i$  is the stiffness parameter and  $M_i$  is the mass parameter.



## 1. The nature of the spin distribution of double fission fragments

$$W(\mathbf{J}_1, \mathbf{J}_2) = |\Psi_0(J_{w_x})|^2 |\Psi_0(J_{w_y})|^2 |\Psi_0(J_{b_x})|^2 |\Psi_0(J_{b_y})|^2 \quad (7)$$

it is possible to obtain an explicit form of the distribution (7):

$$W(\mathbf{J}_1, \mathbf{J}_2) = \frac{4J_1 J_2}{\pi C_b C_w} \exp \left[ -\frac{1}{2} \left( \frac{1}{C_b} + \frac{1}{C_w} \right) (J_1^2 + J_2^2) + \left( \frac{1}{C_b} - \frac{1}{C_w} \right) J_1 J_2 \cos \phi \right], \quad (8)$$

where  $\phi (0 \leq \phi \leq 2\pi)$  is the angle between two-dimensional vectors of spins of fragments  $\mathbf{J}_1$  and  $\mathbf{J}_2$  lying in plane  $xy$ .

$$W(J_1) = \frac{4J_1}{C_b + C_w} \exp \left[ -\frac{2J_1^2}{C_b + C_w} \right]. \quad (9)$$

Equation (9) coincides with (4) when choosing the value of constant  $B^2$ :

$$B^2 = (C_b + C_w)/2. \quad (10)$$

## 1. The nature of the spin distribution of double fission fragments

Using (9), one can calculate the average values of spin  $\bar{J}_1$  of one of the fission fragments:

$$\bar{J}_1 = \int_0^{\infty} J_1 W(J_1) dJ_1 = \frac{1}{2} \sqrt{\frac{\pi}{2}} (C_b + C_w)^{1/2} = \frac{\sqrt{\pi}}{2} B. \quad (11)$$

From the estimates of the work for a fissile nucleus  $^{236}\text{U}$  at the values of the deformation parameters of the prefragments of fission  $\beta_2 \approx 0.2$ , it follows that  $M_w = 1.6 \cdot 10^6 \text{ MeV} \cdot \text{Fm}^2 \cdot \text{s}^2$ ;  $M_b = 2.0 \cdot 10^6 \text{ MeV} \cdot \text{Fm}^2 \cdot \text{s}^2$ ;  $K_w = 295 \text{ MeV} \cdot \text{rad}^{-2}$ ;  $K_b = 52 \text{ MeV} \cdot \text{rad}^{-2}$ ;  $\hbar\omega_w = 2.3 \text{ MeV}$ ;  $\hbar\omega_b = 0.9 \text{ MeV}$ ;  $C_w = 132\hbar^2$  and  $C_b = 57\hbar^2$ , i.e. stiffness  $K_i$ , energy of quanta  $\hbar\omega_w$ , and coefficients  $C_w$  for wriggling - vibrations turn out to be noticeably larger than those for bending - vibrations.

## **1. The nature of the spin distribution of double fission fragments**

The approach developed above to describe the spin distribution of fission fragments, based on the concept of the coldness of a fissile nucleus at the scission point and taking into account zero-point transverse vibrations of a fissile nucleus, is fundamentally different from the approach of works in which the assumption of noticeable thermalization of fission fragments in the vicinity of the scission point of a fissile nucleus is used. when the temperature of  $T$  fission fragments exceed 1 MeV. However, since the fissile nucleus remains in a cold state near its rupture into fission fragments, the presentations of the work are not realized, and the non-Gibbsian character of the distribution of the spins of fission fragments has been established, due to the simultaneous influence of wriggling - and bending - vibrations.

## 2. The nature of the angular distributions of double fission fragments of nuclei

Almost all modern calculations of the angular distributions of fragments of spontaneous and low-energy forced fission of nuclei are based on the hypothesis of O. Bohr about the collinearity of the directions of emission of fragments to the direction of the symmetry axis of a fissile nucleus. However, from the quantum mechanical relation for uncertainties  $\Delta L$  and  $\Delta\theta'$

$$\Delta L \cdot \Delta\theta' \sim 1. \quad (12)$$

In the general case, when the axial symmetry of the fissile nucleus is taken into account, the angular distribution of fragments of double fission  $P(\Omega')$  in the v.s.c. can be represented as

$$P(\Omega') = |A(\Omega')|^2 = \left| \sum_L \psi_L Y_{L0}(\Omega') \right|^2, \quad (13)$$

where  $\psi_L$  is the wave function normalized to unity, which describes the distribution of fragments of fission  $W(L) = |\psi(L)|^2$ .

## 2. The nature of the angular distributions of double fission fragments of nuclei

$$P_{MK}^J(\Omega) = \frac{2J+1}{16\pi^2} \int d\omega \left[ |D_{MK}^J(\omega)|^2 + |D_{M-K}^J(\omega)|^2 \right] P(\Omega'). \quad (14)$$

In (14),  $D_{MK}^J(\omega)$  is a generalized spherical function depending on the Euler angles  $\omega = \alpha, \beta, \gamma$ , which determine the orientation of the v.s.c. axes in relation to the l.s.c. axes.

The amplitude of the angular distribution  $A(\Omega)$ , associated with the wave function  $\psi_L$ , which determines the distribution of fission fragments by their relative orbital angular momenta  $L$  in the vicinity of the break point of the fissile nucleus. This function should be related only to the wriggling - vibrations of the fissioning nucleus, since only for such vibrations are the indicated orbital moments taken into account.

## 2. The nature of the angular distributions of double fission fragments of nuclei

To prove this statement, we transform the spin distribution of fission fragments (7) to the form

$$W(\mathbf{L}, \mathbf{J}') = \frac{1}{\pi^2 C_w C_b} \exp \left[ -\frac{\mathbf{L}^2}{2C_w} - \frac{\mathbf{J}'^2}{2C_b} \right], \quad (15)$$

where the definitions of the relative orbital angular momentum  $\mathbf{L}$  and the relative spin  $\mathbf{J}'$  of fission fragments through the spins of the first and second fission fragments  $\mathbf{J}_1$  and  $\mathbf{J}_2$  are introduced

$$\begin{aligned} \mathbf{L} &= -(\mathbf{J}_1 + \mathbf{J}_2); & \mathbf{J}' &= (\mathbf{J}_1 - \mathbf{J}_2)/2; \\ \mathbf{J}_1 &= -\mathbf{L}/2 + \mathbf{J}'; & \mathbf{J}_2 &= -\mathbf{L}/2 - \mathbf{J}'. \end{aligned} \quad (16)$$

## 2. The nature of the angular distributions of double fission fragments of nuclei

Taking into account that the elements of the phase volume  $d\mathbf{L}$ ,  $d\mathbf{J}'$ , taking into account the two-dimensionality of the vectors  $\mathbf{L}$ ,  $\mathbf{J}'$  in the cylindrical coordinate system, are represented as

$$d\mathbf{L} = LdLd\varphi_{\mathbf{L}}, \quad d\mathbf{J}' = J'dJ'd\varphi_{\mathbf{J}'}, \quad (17)$$

and integrating distribution (15) over  $dJ'$ ,  $d\varphi_{\mathbf{J}'}$ ,  $d\varphi_{\mathbf{L}}$ , we can obtain the distribution  $W(L)$  normalized when integrating over  $dL$  per unit:

$$W(L) = \frac{L}{C_w} \exp\left[-\frac{L^2}{2C_w}\right]; \quad \psi_L = \sqrt{\frac{L}{C_w}} \exp\left(-\frac{L^2}{4C_w}\right). \quad (18)$$

## 2. The nature of the angular distributions of double fission fragments of nuclei

The average value of  $\bar{L}$  modulus of the relative orbital angular momentum of  $L$  fission fragments, defined as

$$\bar{L} = \int_0^{\infty} L |\psi(L)|^2 dL = \frac{1}{C_w} \int_0^{\infty} L^2 \exp\left(-\frac{L^2}{2C_w}\right) dL = \sqrt{\frac{\pi}{2}} (C_w)^{1/2}, \quad (19)$$

and for the average value  $\overline{C_w} \approx 132\hbar^2$ , found for fissile actinide nuclei using the parameters of work [J. R. Nix, and W. J. Swiateski, Nucl. Phys. A 71, 1 (1965).], and for the average takes on a sufficiently large value of  $\bar{L} = 14.4$ , which ensures a good accuracy of the implementation of Bohr's hypothesis.



## Conclusion

1. The relationship between the angular and spin distributions of fragments of low-energy double fission of nuclei with collective wriggling - and bending - vibrations of a fissile nucleus in the vicinity of its scission point is demonstrated.
2. The estimation of the average values of the relative orbital moments of fission fragments formed by wriggling - vibrations, which took a sufficiently large value of  $\bar{L} = 14.4$ , was carried out, which ensures a good accuracy of the implementation of Bohr's hypothesis.
3. The non-Gibbsian character of the distribution of spins of fission fragments was established, due to the simultaneous influence of wriggling - and bending - vibrations.

Thank you for attention!