A. BOHR’S HYPOTHESIS FOR ANGULAR DISTRIBUTIONS OF FRAGMENTS OF LOW-ENERGY NUCLEAR FISSION AND WRIGGLING VIBRATIONS OF THE FISSILE NUCLEI

Titova Larisa

Voronezh State University, Nuclear Physics Department

E-mail: titova_lv@phys.vsu.ru

INTRODUCTION

The hypothesis of A. Bohr [***A.Bohr, B.Mottelson, Nuclear Structure, V.2 (Benjamin, N-Y, 1974)***] – **all primary fission fragments irrespective of their composition and structure emerge only in the direction of or opposite to that of axis of symmetry Z′ of the fissile nucleus.**

$$T(\Omega') = \frac{1}{4\pi}\left[\delta(\cos\theta' - 1) + \delta(\cos\theta' + 1)\right],$$

$$T_{MK}^{J}(\Omega) = \frac{2J+1}{16\pi^2}\left[\left|D_{MK}^{J}(\omega)\right|^2 + \left|D_{M-K}^{J}(\omega)\right|^2\right]_{\beta=\theta,\alpha=\varphi}.$$

As follows from the **quantum-mechanical uncertainty relation** for and corresponding to the definition of relative orbital momentum L of fission fragments and their emission angle θ'.

Normalized per unit fragments' angular distribution in LCS [***Kadmensky S.G.* // Phys. At. Nucl. 2002. V. 65. P. 1424; Phys. At. Nucl. 2002. V. 65. P. 1833 // Phys. At. Nucl. 2003. V. 67. P. 1259; Phys. At. Nucl. 2005. V. 68. P. 1491**]:

$$T^J_{MK}(\Omega) = \frac{2J+1}{16\pi^2}\int d\omega\left[\left|D^J_{MK}(\omega)\right|^2 + \left|D^J_{M-K}(\omega)\right|^2\right]T(\Omega'),$$

For axially symmetric nucleus in ICS:

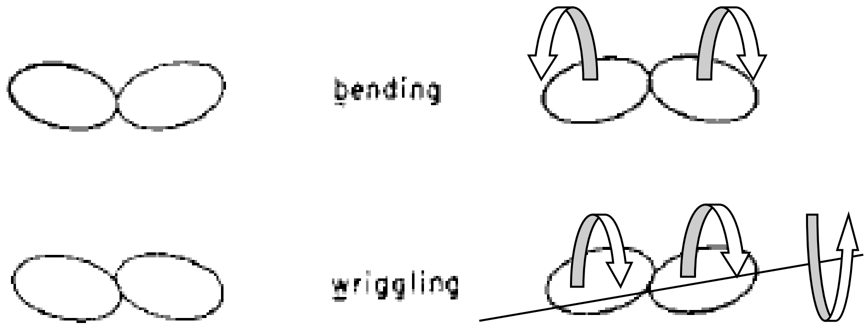
$$T(\Omega') = \left|F(\Omega')\right|^2, \qquad F(\Omega') = \sum_L \psi_L Y_{L0}(\Omega'),$$

ψ_L is the wave function, describing distribution W(L) of fission fragments over their relative orbital momenta L in the vicinity of scission point of the fissile nucleus.

What mechanism can explain the appearance of big values of orbital momenta and spins of fission fragments?

Rusmussen J.O. et al. // Nucl. Phys. A. 1969. V. 136. P. 465; Rusmussen J.O., Sugawara-Tanabe K. // Nucl. Phys. A. 1971. V. 171. P. 497. – subbarrier α-decay of the deformed nuclei, non-sphericity of the Coulomb potential for fission prefragments ($\Delta L \approx 2$).

Nix J.R. and Swiatecki W.J. // Nucl. Phys. A. 1965. V. 71. P. 1. – collective transverse zero bending- and wriggling-vibrations of the fissile nucleus in the vicinity of its scission point.



(e) *Bending*: This mode is doubly degenerate (occurs twice), corresponding to rotations in the *x-z* plane and the *y-z* plane. For a given plane, one of the spheroids rotates clockwise about an axis through its centre perpendicular to the plane, and the other counterclockwise through the same angle; the spheroids remain touching.

(f) *Wriggling*: This mode is also doubly degenerate, corresponding to rotations in the *x-z* and *y-z* planes. For a given plane, both spheroids rotate through the same angle either clockwise or counterclockwise about axes through their centres perpendicular to the plane; they remain touching. The entire system rotates in the opposite direction, ensuring conservation of the *x* and *y* components of total angular momentum.

Rusmussen J.O. et al. **// Nucl. Phys. A. 1969. V. 136. P. 465.**
Rusmussen J.O., Sugawara-Tanabe K. **// Nucl. Phys. A. 1971. V. 171. P. 497.**
Shneidman T.M. et al. **// Phys. Rev. C. 2002. V. 65. P. 064302.**

Bending-vibrations: spins $\vec{J}_1$ and $\vec{J}_2$ are parallel so $\vec{J}_1 + \vec{J}_2 \approx -\vec{L} = 0$.

Kadmensky S.G., Lubashevsky D.E., Titova L.V. **// Bull. Russ. Acad. Sci.: Phys., 2015, vol. 79, p. 879.**

The main source of the big values is zero wriggling-vibrations: spins $\vec{J}_1$ and $\vec{J}_2$ are parallel: $\vec{J}_1 + \vec{J}_2 \approx -\vec{L}$.

DISTRIBUTION OF FISSION FRAGMENTS ACCORDING TO THEIR RELATIVE ORBITAL MOMENTA WITH ALLOWANCE FOR WRIGGLING VIBRATIONS

$$W(L) = \frac{L}{C_w} \exp\left(-\frac{L^2}{2C_w}\right), \qquad \psi_L\left(C_w\right) = \sqrt{\frac{L}{C_w}} \exp\left(-\frac{L^2}{4C_w}\right).$$

The amplitude of the fission fragments angular distribution in ICS:

$$F(\Omega') = \sum_{L=0} \psi_L\left(C_w\right) Y_{L0}\left(\cos\theta'\right) \left(1 + \left(-1\right)^L\right) \Big/ 2 \ ,$$

where $C_w = M_w \hbar\omega_w$, $C_b = M_b \hbar\omega_b$, $\omega_w = \sqrt{\frac{K_w}{M_w}}$, $\omega_b = \sqrt{\frac{K_b}{M_b}}$, K_w, K_b and M_w, M_b are stiffness and mass parameters.

For ^{236}U $C_w = 131.8\hbar^2$ и $C_b = 57.3\hbar^2$ [***Nix J.R. and Swiatecki W.J. // Nucl. Phys. A. 1965. V. 71. P. 1.***]

ANISOTROPIES IN ANGULAR DISTRIBUTIONS OF FRAGMENTS IN SUBTHRESHOLD PHOTOFISSION OF ACTINIDE NUCLEI TAKING INTO ACCOUNT WRIGGLING VIBRATIONS OF A FISSILE NUCLEUS

Differential cross-section of the double fission photofission reaction

$$\frac{d\sigma_f(\theta)}{d\Omega} = \sum_J \sum_{M=-J}^{J} \sigma(E_\gamma, JM) \sum_{K=0}^{J} \frac{\Gamma_f(JK)}{\Gamma(J)} T_{MK}^{J}(\Omega) .$$

$$\frac{d\sigma_f(\theta)/d\Omega}{d\sigma_f(90^\circ)/d\Omega} = a + b\sin^2\theta + c\sin^2(2\theta).$$

$$a = \sum_{JK} \alpha_{JK} P(JK) \Big/ \sum_{JK} (\alpha_{JK} + \beta_{JK}) P(JK)$$

$$b = \sum_{JK} \beta_{JK} P(JK) / \sum_{JK} (\alpha_{JK} + \beta_{JK}) P(JK); \quad c = \sum_{JK} \gamma_{JK} P(JK) \Big/ \sum_{JK} (\alpha_{JK} + \beta_{JK}) P(JK),$$

$$P(JK) = \sigma(E_\gamma, J)\Gamma_f(JK) / \Gamma(J),$$

Below, we consider the subthreshold photofission of a group of even–even actinide nuclei (^{234}U, ^{236}U, ^{238}U, ^{238}Pu, ^{240}Pu, ^{242}Pu) by gamma-quanta with energies $E_\gamma \leq 7$ MэB, with taking into account transitional fission states: $J^\pi K = 1^-0;\ 1^-1$, and 2^+0. [***Ostapenko Yu. B. et al. // PEPAN. 1981. V. 12. P. 1364.***].

Barrier penetrability factors $\tilde{P}_{A(B)}\left(JK\right)$ in parabolic approximation:

$$\tilde{P}_{A(B)}\left(JK\right) = \left\{1 + \exp\left[\frac{2\pi\left(E_{A(B)}\left(JK\right) - E_\gamma\right)}{\hbar\omega_{A(B)}\left(JK\right)}\right]\right\}^{-1},$$

where $E_{A(B)}\left(JK\right)$ is energy of transition fission state on internal (A) and external (B) fission barriers, $\hbar\omega_{A(B)}\left(JK\right)$ – fission barriers curvature.

Transition fission barriers energies for ^{234}U, ^{236}U, ^{238}U and ^{238}Pu, ^{240}Pu, ^{242}Pu

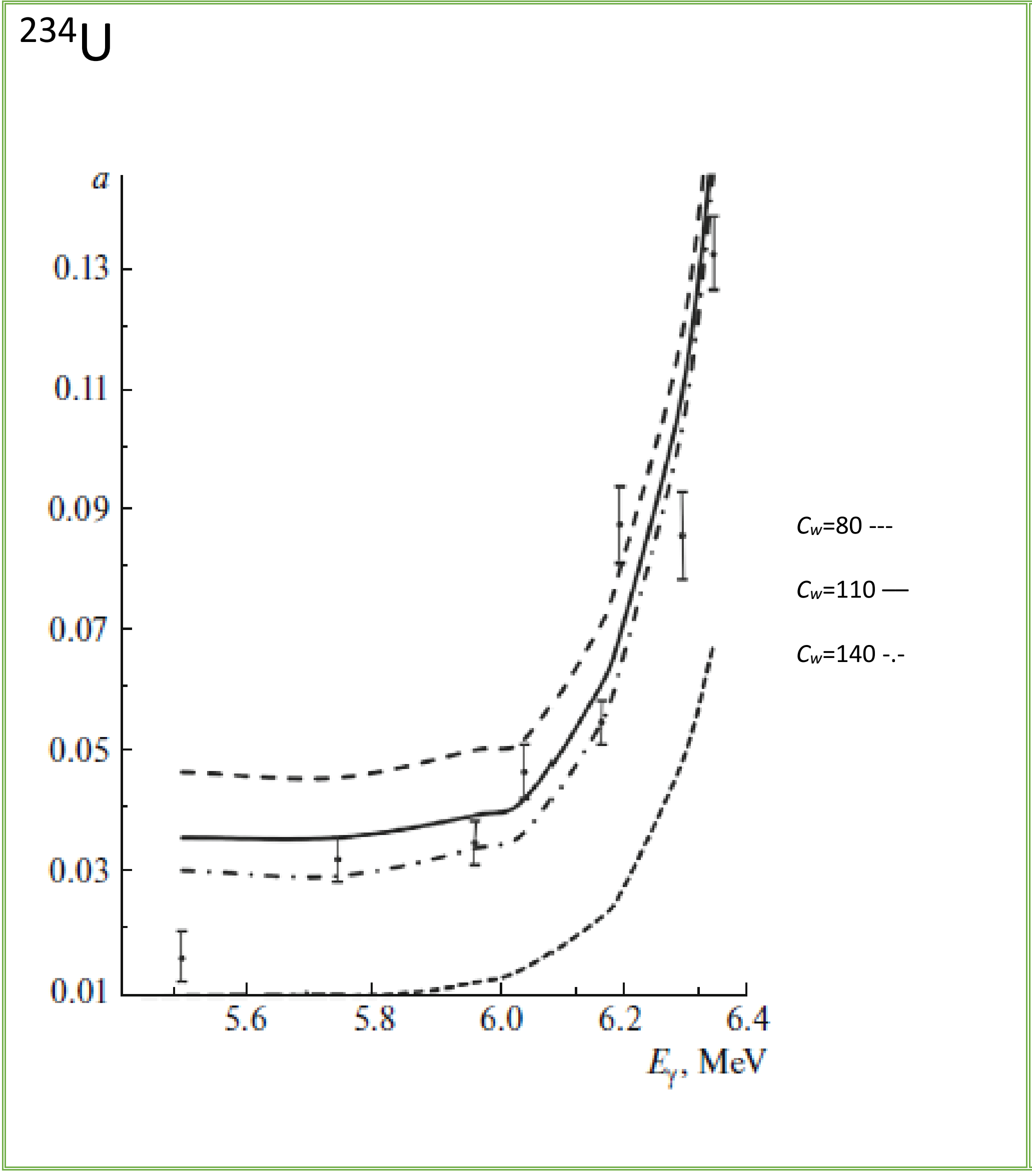
1. ***Dowdy E.J. and Krysinsky* // Nucl. Phys. A. 1971. V. 175. P. 501.**
2. ***Lindgren L.J. et al.* // Nucl. Phys. A. 1978. V. 298. P. 43.**
3. ***Rabotnov N.S.* // Sov. J. Nucl. Phys. 1970. V.** 11**. P. 285.**
4. ***Ignatyuk A.V. et al.* // Sov. Phys. JETP** 34**, 684 (1971)**
5. ***A. S. Soldatov et al.* // Phys. At. Nucl. 2000. V.** 63**. P. 31.**
6. ***Pauli H.C., Ledergerber T. // Nucl. Phys. A. 1971. V. 175. P.545.***

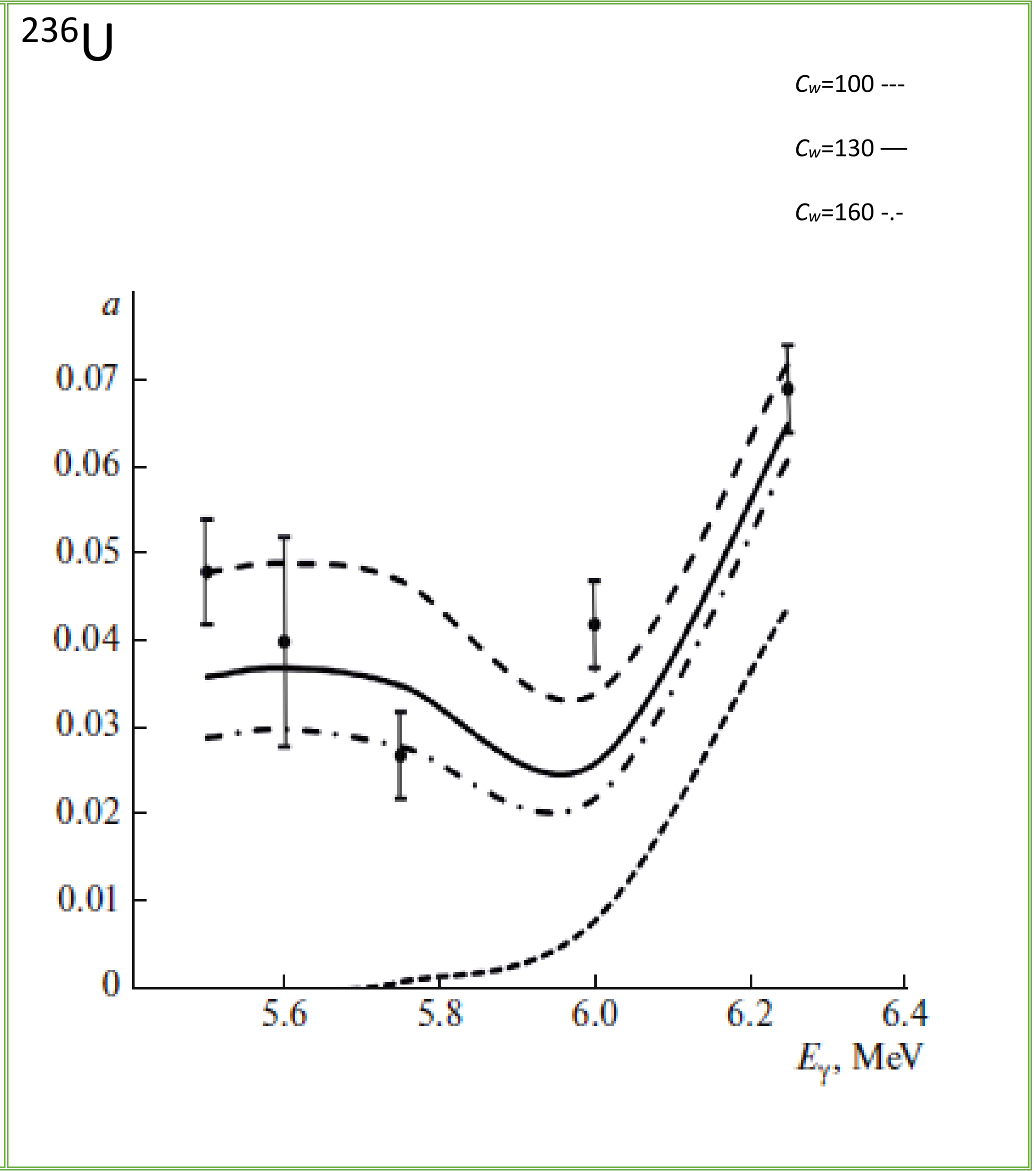
Table 1. Energies $E_i(JK)$ of TFS $|1^-0\rangle, |1^-1\rangle, |2^+0\rangle$ at internal $(i-A)$ and external $(i-B)$ fission barriers, and curvature $\hbar\omega_i(JK)$ of the corresponding fission barriers of uranium isotopes

$\|J^\pi K\rangle$	$\|1^-0\rangle$		$\|1^-1\rangle$		$\|2^+0\rangle$		$\frac{\sigma(E_\gamma,2)}{\sigma(E_\gamma,1)}$
$E_i(JK)$, $\hbar\omega_i$	$E_A(JK)$, $\hbar\omega_A$, MeV	$E_B(JK)$, $\hbar\omega_B$, MeV	$E_A(JK)$, $\hbar\omega_A$, MeV	$E_B(JK)$, $\hbar\omega_B$, MeV	$E_A(JK)$, $\hbar\omega_A$, MeV	$E_B(JK)$, $\hbar\omega_B$, MeV	
^{234}U	6.85, 1.41	5.70, 0.58	6.85, 1.37	6.65, 0.73	6.00, 1.40	5.80, 0.70	0.040
^{236}U	6.55, 0.92	5.60, 0.47	6.70, 0.91	6.50, 0.57	5.80, 1.10	5.60, 0.50	0.040
^{238}U	6.70, 1.00	6.00, 0.52	6.80, 1.00	6.50, 0.60	5.95, 1.00	6.15, 0.65	0.034

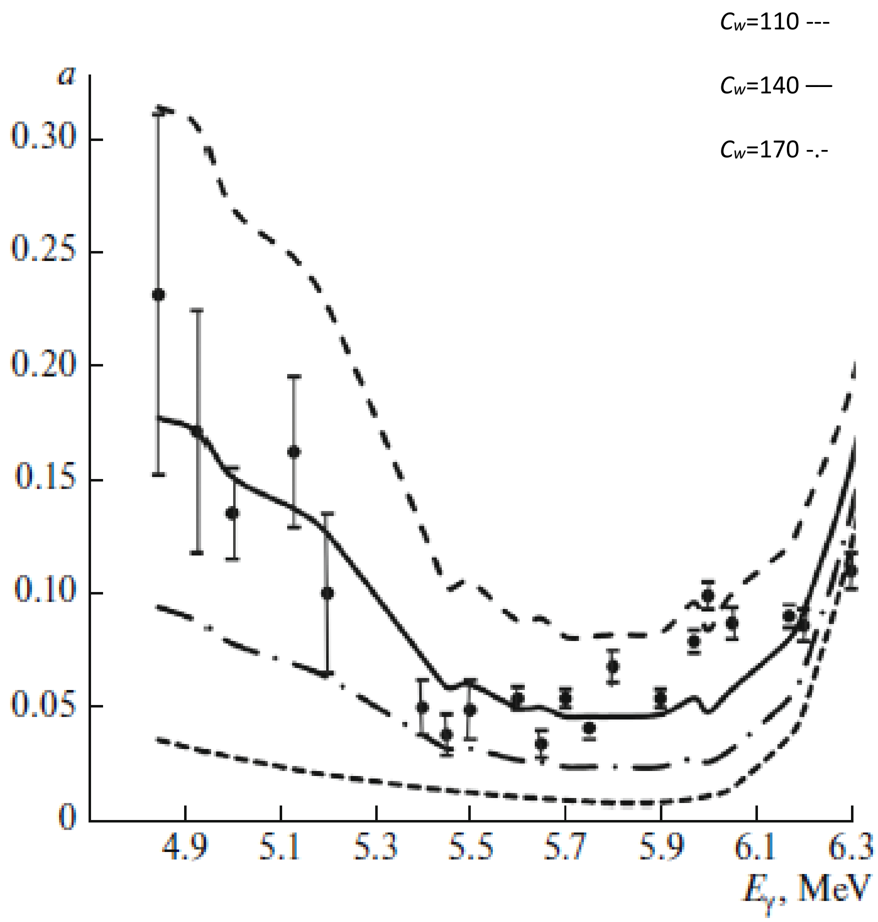
Table 2. Same as in Table 1; TFS $|1^-0\rangle, |1^-1\rangle$ for uranium isotopes

$\|J^\pi K\rangle$	$\|1^-0\rangle$				$\|1^-1\rangle$				$\frac{\sigma(E_\gamma,2)}{\sigma(E_\gamma,1)}$
$E_i(JK)$, $\hbar\omega_i$	$E_A(JK)$, $\hbar\omega_A$, MeV		$E_B(JK)$, $\hbar\omega_B$, MeV		$E_A(JK)$, $\hbar\omega_A$, MeV		$E_B(JK)$, $\hbar\omega_B$, MeV		
^{238}Pu	6.45, 0.74	7.00, 1.35	5.30, 0.74	6.10, 0.60	6.45, 0.63	6.90, 1.35	5.37, 0.63	6.30, 0.63	0.10
^{240}Pu	6.25, 0.70	7.00, 1.35	5.05, 0.70	6.10, 0.60	6.25, 0.57	6.90, 1.35	5.15, 0.57	6.30, 0.60	0.67
^{242}Pu	6.30, 0.76	7.00, 1.35	5.20, 0.76	6.10, 0.60	6.30, 0.65	6.90, 1.35	5.28, 0.65	6.30, 0.67	

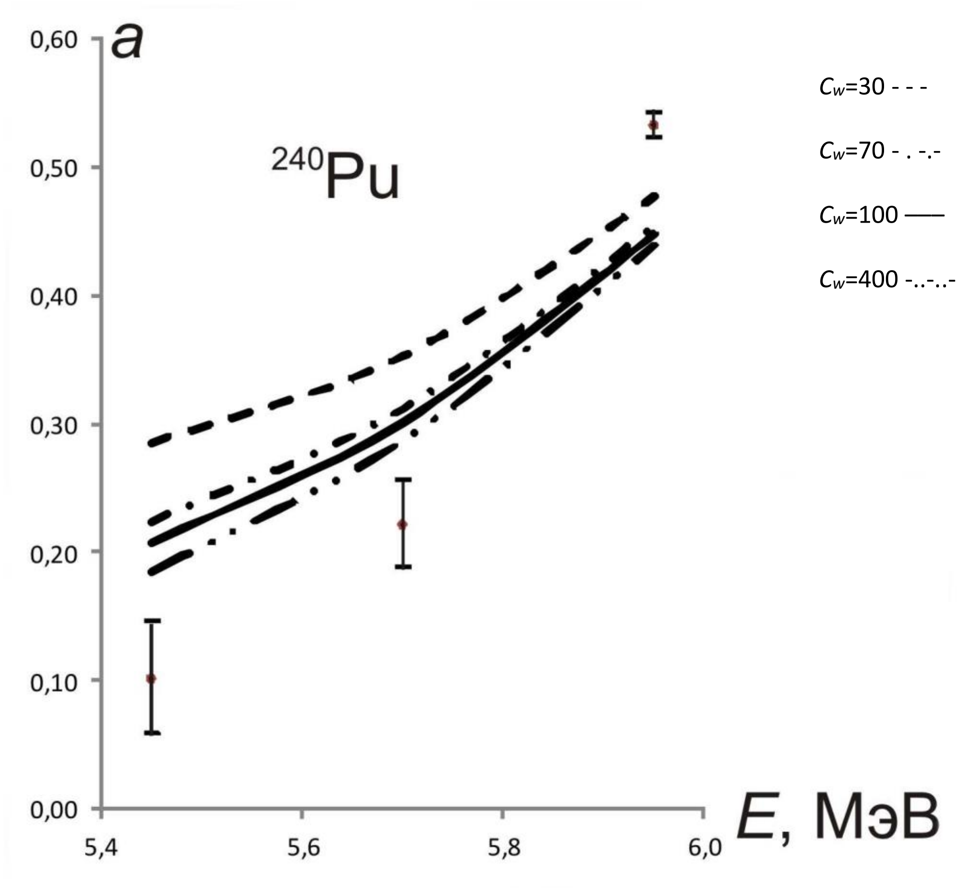

^{234}U
a
0.13
0.11
0.09
0.07
0.05
0.03
0.01
5.6
5.8
6.0
6.2
6.4
E_γ, MeV
C_w=80 ---
C_w=110 —
C_w=140 -.-


^{236}U
C_w=100 ---
C_w=130 —
C_w=160 -.-
a
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
5.6
5.8
6.0
6.2
6.4
E_γ, MeV

^{238}U



C_w for ^{234}U, ^{236}U, ^{238}U: 112; 130; 147.


a
240Pu
0,60
0,50
0,40
0,30
0,20
0,10
0,00
5,4
5,6
5,8
6,0
E, МэВ
Cw=30 - - -
Cw=70 - . -.-
Cw=100 ——
Cw=400 -..-..-

EFFECT OF WRIGGLING VIBRATIONS ON P-ODD ASYMMETRIES IN THE ANGULAR DISTRIBUTIONS OF LOW-ENERGY NUCLEAR FISSION BY POLARIZED NEUTRONS

Using the methods [***Sushkov, O.P. and Flambaum, V.V., Sov. Phys. Usp., 1982, vol. 25, p. 1.; Bunakov, V.E. and Kadmensky, S.G., Bull. Russ. Acad. Sci.: Phys., 2008, vol. 71, p. 2030.***], and taking into account the uniform mixture of projections K for the s-neutron resonances of the first fissile nucleus deformation potential well [***Kadmensky, S.G., Markushev, V.P., and Furman, W.I., Sov. J. Nucl. Phys., 1982, vol. 35, p. 166.***]:

$$\frac{d\sigma^{\text{odd}}}{d\Omega}=\frac{1}{2\pi k_{\text{n}}^{2}}\sum_{sJ_s s'J_{s'}}\frac{(2J_s+1)(2J_{s'}+1)}{16\pi\sqrt{2(2I+1)}}\sum_{pK}D_{pK}^{J_sJ_{s'}}\Gamma_K(-1)^{J_{s'}-K+1}C_{J_sJ_{s'}K-K}^{L0}C_{J_sJ_{s'}M_s-M_{s'}}^{L(M_s-M_{s'})}\times$$

$$\times\sum_{L-\text{неч}}D_L(C_{\text{w}})\,p_{\text{n}}P_L(\cos\theta),$$

$$D_{pK}^{sJ_s s'J_{s'}} = b_{sK}^{J_s} b_{s'K}^{J_{s'}} \left\{ u_{sK}^{J_s} u_{s'K}^{J_{s'}*} \left[\alpha_{sp}^{J_s} + \alpha_{s'p}^{J_{s'}\,*} \right] + u_{sK}^{J_s*} u_{s'K}^{J_{s'}} \left[\alpha_{sp}^{J_s*} + \alpha_{s'p}^{J_{s'}} \right] \left(1 - \delta_{J_s J_{s'}}\right) \right\},$$

$$u_s^{J_s\pi} = \frac{\sqrt{\Gamma_{sn}^{J_s}}}{E - E_s^{J_s} + \frac{i\Gamma_s^{J_s}}{2}}.$$

$$D_L(C_\mathrm{w}) = \sum_{ll'} \psi_l(C_\mathrm{w}) \psi_{l'}(C_\mathrm{w}) \left(C_{ll'00}^{L0}\right)^2 \sqrt{(2l+1)(2l'+1)} \Big/ (2L+1).$$

P-odd asymmetries with $L = 1$ like $\left(\vec{p}_\mathrm{n}, \vec{k}_\mathrm{LF}\right)$:

$$\frac{d\mathrm{P}^\mathrm{odd}}{d\Omega} = \alpha_0^\mathrm{odd} D_1(C_\mathrm{w}) P_1(\cos\theta),$$

where α_0^odd is P-odd asymmetry coefficient.

Table 1. Values of coefficients $D_L(C_w)$ and their deviations from coefficients $D_L(C_w \to \infty)$ at $L = 1, 2$

C_w	10	20	30	40	50	60	70	80	90
$D_1(C_w)$	0.953	0.974	0.982	0.987	0.989	0.991	0.992	0.993	0.994
$\Delta D_1(C_w)$, %	4.7	2.6	1.8	1.3	1.1	0.9	0.8	0.7	0.6

Comparing the relative errors of the experimental measurements of asymmetry coefficients

$\alpha^{odd} = (3.67 \pm 0.06) \cdot 10^{-4}$ for ^{233}U and $\alpha^{odd} = (0.84 \pm 0.06) \cdot 10^{-4}$ for ^{235}U

[***Alfimenkov V.P. et al. // Phys. At. Nucl. 1995. V. 58. P. 799.***]

and the theoretical values of the relative deviations $\Delta D_1(C_w)$ of these coefficients from A. Bohr hypothesis gives $C_w \leq 30$ and $C_w \leq 15$ for ^{233}U and ^{235}U.

ANISOTROPIES IN THE ANGULAR DISTRIBUTIONS OF BINARY FISSION OF ALINGED NUCLEI BY UNPOLARIZED NEUTRONS WITH TAKING INTO ACCOUNT WRIGGLING VIBRATIONS

$$\frac{dP^{J\pi}(\Omega)}{d\Omega} = \frac{1}{2\pi k_{\mathrm{n}}^{2}} \sum_{K\geq 0} T^{J\pi} \frac{\Gamma_{\mathrm{f}}^{J\pi K}}{\Gamma^{J\pi}} \sum_{MM'} \rho_{MM'}^{J} T_{MK}^{J}(\Omega), \qquad (20)$$

where k_{n} **is** neutron wave vector, $\Gamma_{\mathrm{f}}^{J\pi K}$ and $\Gamma^{J\pi}$ are fission and total widths, $T^{J\pi}$ optical neutron penetrability coefficients **[*Gonin, N.I., et al., Yad. Fiz., 1994, vol. 57, p. 1235.*]** Anisotropy $W^{J}(\theta)$ of the angular distribution:

$$W^{J\pi K}(\theta) = \left(1 + \sum_{L=2,4,6,\ldots} f_{L}^{J} A_{L} P_{L}(\cos\theta)\right), \qquad (21)$$

where f_{L}^{J} is parameter of nuclear orientation,

$$A_{L} = D_{L}(C_{\mathrm{w}}) \sum_{K} \frac{\Gamma_{\mathrm{f}}^{J\pi K}}{\Gamma^{J\pi}} A_{L}(J,K), \qquad (22)$$

$$A_L(J,K) = (-1)^{J-K} C^{L0}_{JJK-K} \frac{(2J+1) J^L (2L)!}{(L!)^2} \left[\frac{(2L+1)(2J-L)!}{(2J+L+1)!} \right]^{1/2}.$$

Table 1. Coefficients $D_2(C_w)$ and their deviations $\Delta D_2(C_w)$ from coefficients $D_2(C_w \to \infty)$

C_w	10	20	30	40	50	60	70	80	90	100	120	200
$D_2(C_w)$	0.835	0.913	0.941	0.955	0.964	0.970	0.974	0.977	0.980	0.982	0.985	0.991
$\Delta D_2(C_w)$, %	16.5	8.7	5.9	4.5	3.6	3.0	2.6	2.3	2.0	1.8	1.5	0.9

Kuiken R., Pattenden N. J. and Postma H. // Nucl. Phys. A. 1972. V. 190. P. 401; Kuiken R., Pattenden N. J., Postma H. // Nucl. Phys. A. 1973. V. 200. P. 392; Kuiken R., Pattenden N. J., Postma H. // Nucl. Phys. A. 1972. V. 196. P. 389

Neutron energy (0.735-2.840) eV for ^{233}U $C_w \leq 80$

(1.14 – 12.39) eV for ^{235}U $C_w \leq 60$

(39.2 – 41.4) eV for ^{237}Np $C_w \leq 30$

for which the deviations of angular distributions from A. Bohr's formula can be obtained.

Conclusion

The conditions under which it is possible to observe deviations from the A. Bohr formula in the fragments' angular distributions of low energy fission are derived. Parameters C_w are calculated from the analysis of fragments' angular distributions deviations from A. Bohr formula

for **photofission of even-even uranium nuclei**: $C_w \leq 112$, $C_w \leq 130$ and $C_w \leq 147$ for ^{234}U, ^{236}U, ^{238}U;

for **P-odd asymmetries coefficients**: $C_w \leq 15$ for ^{233}U and $C_w \leq 30$ for ^{235}U,

for **fragments' angular distribution anisotropy coefficients of aligned nuclei fission by resonant neutrons**: $C_w \leq 80$ and $C_w \leq 80$ for ^{233}U and ^{235}U.

Parameters C_w, which are obtained for photofission, are consistent with the values of this parameter in [***Nix J.R. and Swiatecki W.J. // Nucl. Phys. A. 1965. V. 71. P. 1.***].

To obtain more accurate estimations of the C_w parameter based on the coefficients of P-odd asymmetries and the coefficients of anisotropies of fragments' angular distributions in fission of aligned nuclei with resonant neutrons, a higher experimental accuracy of measuring these coefficients is required.

THANK FOR YOUR ATTENTION!