Transition states, K number and mechanism of nuclear fission

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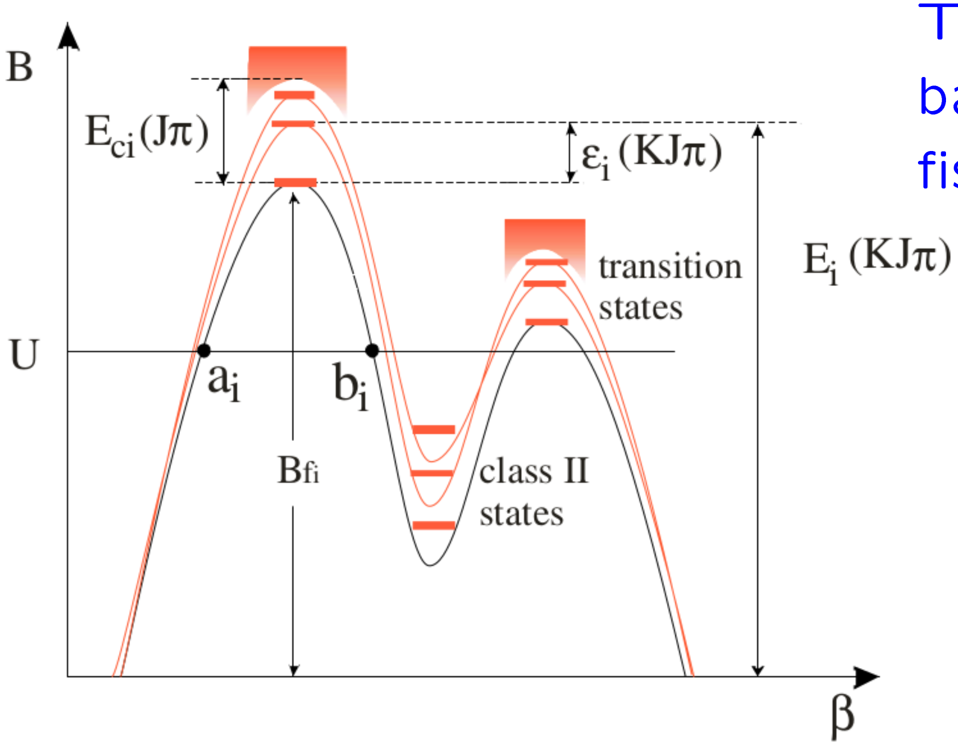
Transition states on the fission barrier: angular distributions and fission cross sections



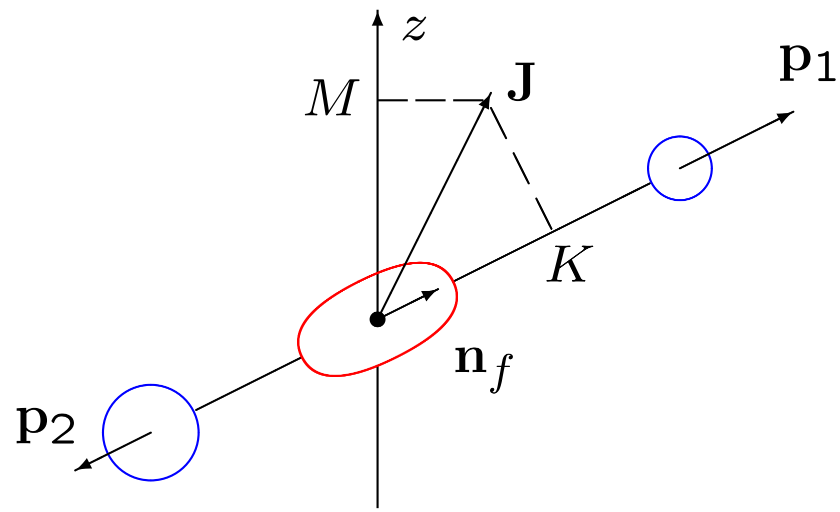


Figure: R.Capote et al. RIPL
Nuclear Data Sheets **110** 3107 (2009)

Wave function:

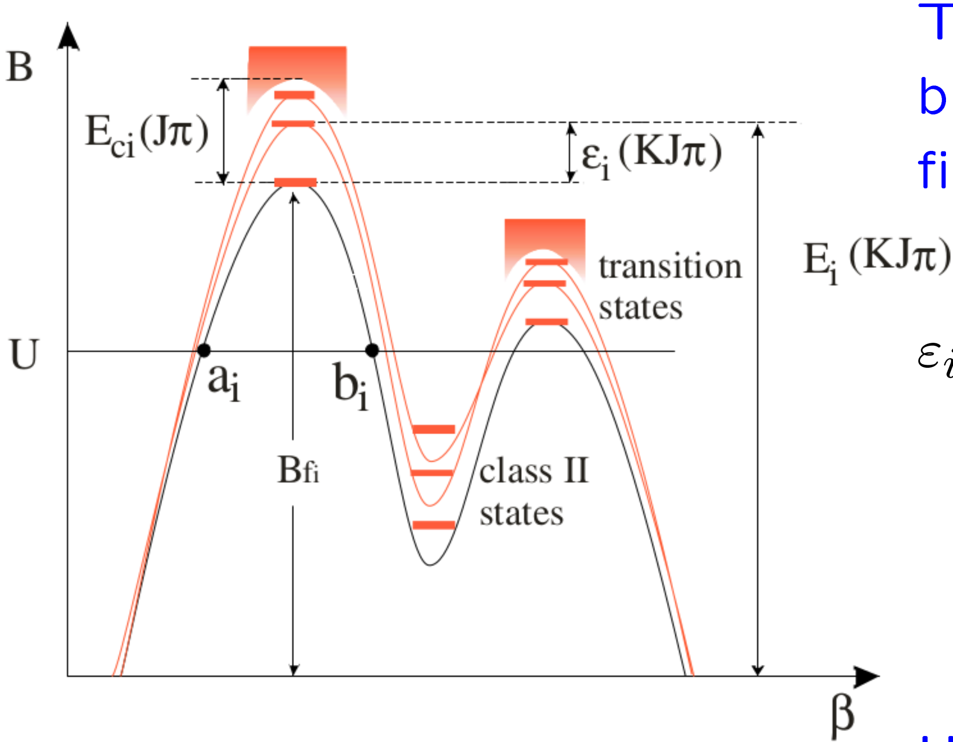
$$\Psi_{JMK} \sim D^J_{MK}(\mathbf{n}_f)$$

Angular distribution for fixed M and K:

$$\frac{dw^J_{MK}(\mathbf{n}_f)}{d\Omega} = \frac{2J+1}{4\pi}\left|D^J_{MK}(\mathbf{n}_f)\right|^2$$

Angular distribution summed over M and K:

$$\frac{dw^{J\pi}(\mathbf{n}_f)}{d\Omega} = \sum_M \eta^{J\pi}(M) \sum_K \rho^{J\pi}(K)\,\frac{dw^J_{MK}(\mathbf{n}_f)}{d\Omega}$$



Transition states on the fission barrier: angular distributions and fission cross sections

$$\varepsilon_i(KJ\pi) = \epsilon_i(K\pi) + \frac{J(J+1) - K(K+1)}{2I_{\perp i}}$$

$$E_i(KJ\pi) = B_i + \varepsilon_i(KJ\pi)$$

$$E_k = E_x - E_i(KJ\pi)$$

Hill-Wheeler: $T_i(E_k) = \dfrac{1}{1 + e^{-2\pi \frac{E_k}{\hbar\omega_i}}}$

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$$T_i^{J\pi}(E_x) = \sum_{K\leq J} T_i(E_x - B_i - \varepsilon_i(KJ\pi)) + \int_{E_{ci}(J\pi)}^{E_x} \rho_i^{\mathrm{def}}(E, J^\pi)\, T_i(E_x - B_i - E)\, dE.$$

Hauser-Feshbach fission cross section for (n, f) reaction:

$$\sigma_f(E) = \pi\lambda^2 \sum_{J\pi} g_J \frac{\sum_{lj} T_{lj}(J^\pi, E)}{\sum_{\alpha\lambda_\alpha} T_{\alpha\lambda_\alpha}(J^\pi, E_\alpha)} T_f(J^\pi, E_x), \qquad T_f(J^\pi, E_x) = \frac{T_1^{J\pi} T_2^{J\pi}}{T_1^{J\pi} + T_2^{J\pi}}$$

Differential fission cross section:

$$\frac{d\sigma_f(E)}{d\Omega} = \frac{\pi\lambda^2}{4\pi} \sum_{J\pi} g_J \sum_{Q=0,2,4,\ldots} (2Q+1) \frac{\sum_{lj} z_Q(ljJ)\, T_{lj}(J^\pi, E)}{\sum_{\alpha\lambda_\alpha} T_{\alpha\lambda_\alpha}(J^\pi, E_\alpha)} \times$$

$$\times\, T_f(J^\pi, E_x)\, b_Q(J^\pi, E_x)\, P_Q(\cos\theta)$$

$z_Q(ljJ)$ — numerical coefficient

fission probability distribution over K: $\beta(J^\pi K) = \dfrac{T_f(J^\pi K, E_x)}{T_f(J^\pi, E_x)}$

discrete function of fission probability distribution: $b_Q(J^\pi, E_x) = \sum_K C^{JK}_{JKQ0}\, \beta(J^\pi K)$

Angular distribution of fission fragments:

$$W(\theta) = \frac{d\sigma_f(E)/d\Omega}{\sigma_f(E)} \simeq \frac{1}{4\pi}\left(1 + A_2 P_2(\cos\theta) + A_4 P_4(\cos\theta)\right), \quad \frac{W(0^\circ)}{W(90^\circ)} \simeq \frac{1+A_2}{1-A_2/2}$$

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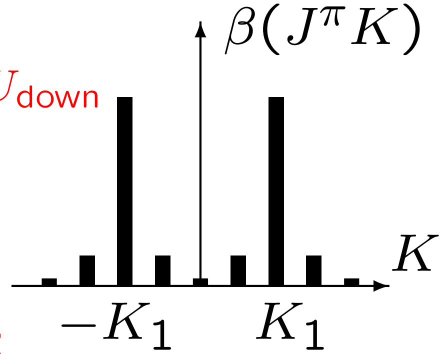
«Low» energies:

$$E^* = E_x - B_f < \Delta + U_{\text{down}}$$

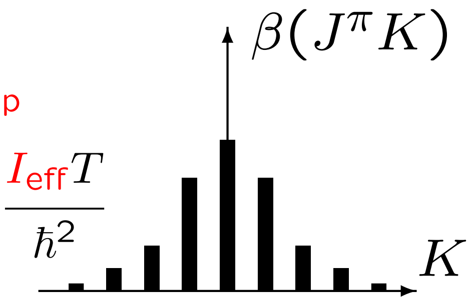
$$\beta(J^\pi K) \sim e^{-\alpha(|K|-K_1)^2}$$



«High» energies:

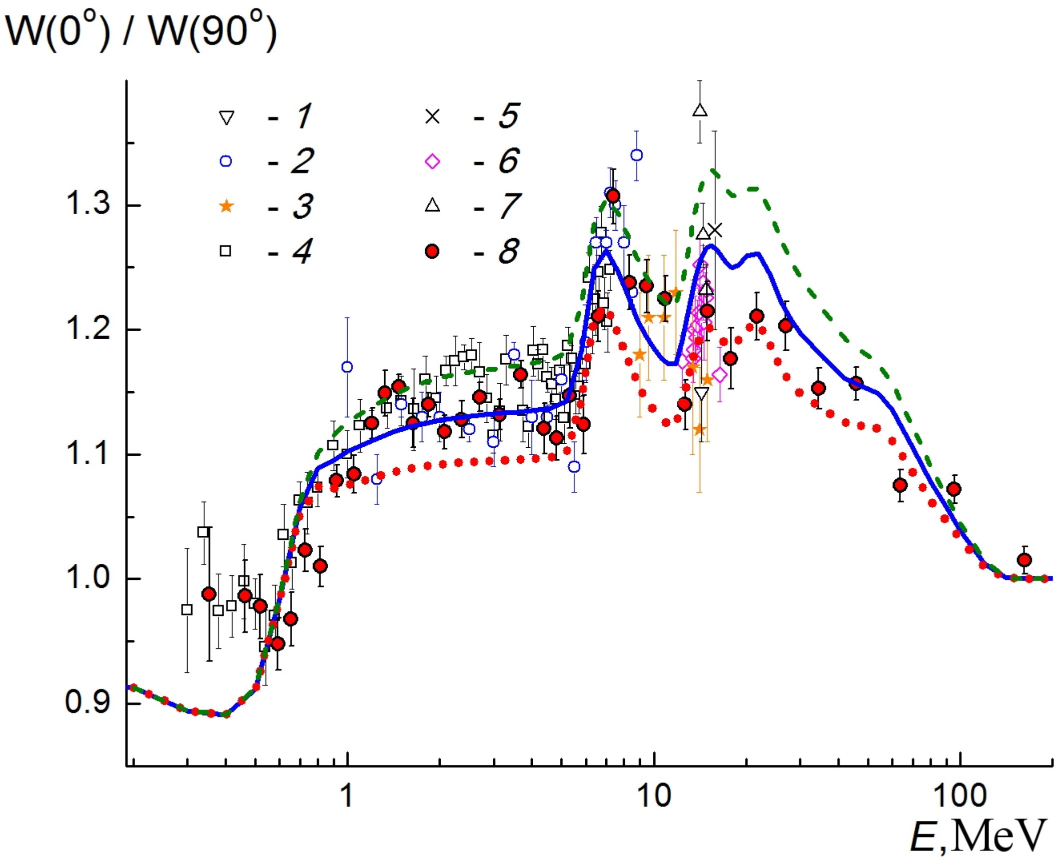
$$E^* = E_x - B_f > \Delta + U_{\text{up}}$$

$$\beta(J^\pi K) \sim e^{-\frac{K^2}{2K_0^2}}, \quad K_0^2 = \frac{I_{\text{eff}} T}{\hbar^2}$$



Additional parameters:

$$U_{\text{up}} = 0.4 \text{ MeV}, \quad U_{\text{down}} = -0.1 \text{ MeV}, \quad \alpha = 0.15, \quad K_1 = \begin{cases} 0.0, & ^{238}\text{Np}, \\ 0.5, & ^{237}\text{Np}, \\ 1.5, & \text{all other isitopes.} \end{cases}$$



$$\frac{\hbar^2}{J_{\text{eff}}} = 0.022 \text{ MeV}$$

$$\frac{\hbar^2}{J_{\text{eff}}} = 0.017 \text{ MeV}$$

$$\frac{\hbar^2}{J_{\text{eff}}} = 0.012 \text{ MeV}$$

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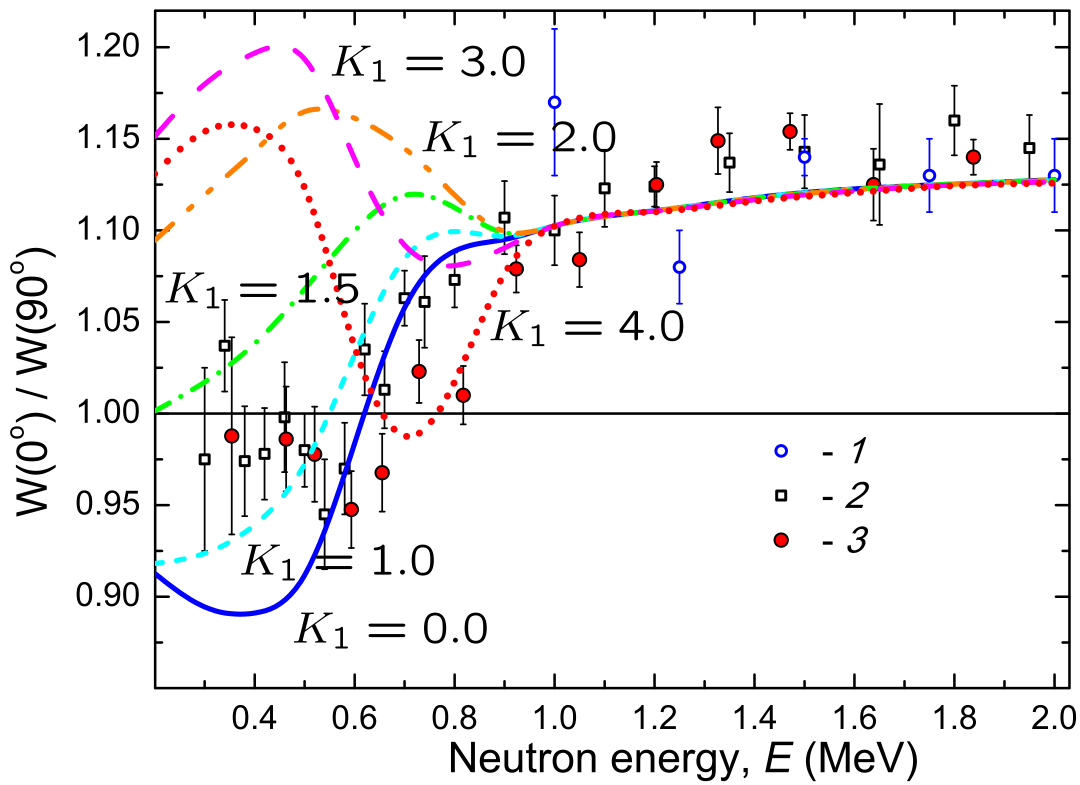
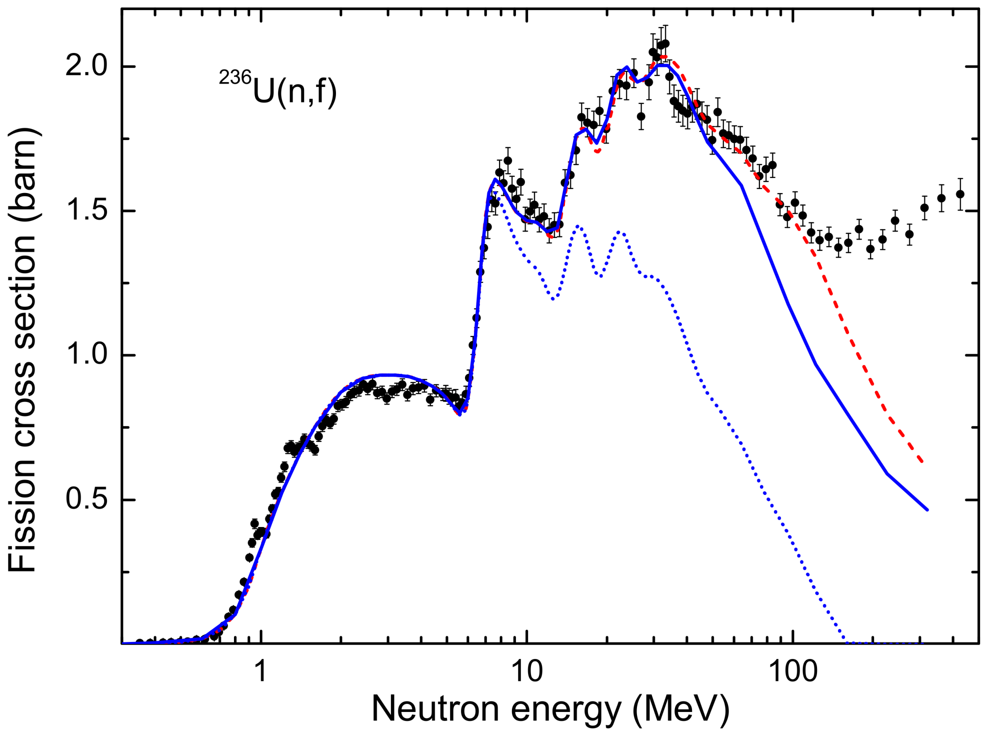
Problems at low energies:

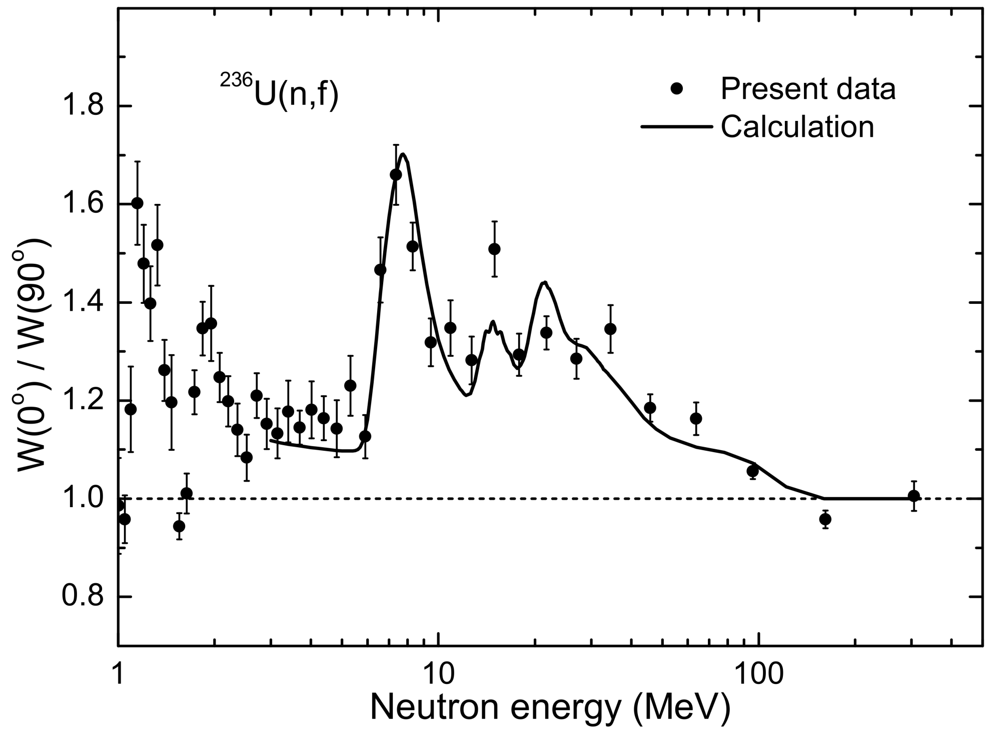
$$\beta(J^\pi K) \sim e^{-\alpha(|K|-K_1)^2}$$



A.S.Vorobyev, A.M.Gagarski, O.A.Shcherbakov, L.A.Vaishnene, A.L.Barabanov, T.E.Kuz'mina.

arXiv:2301.06835 (2023): $^{236}U(n,f)$





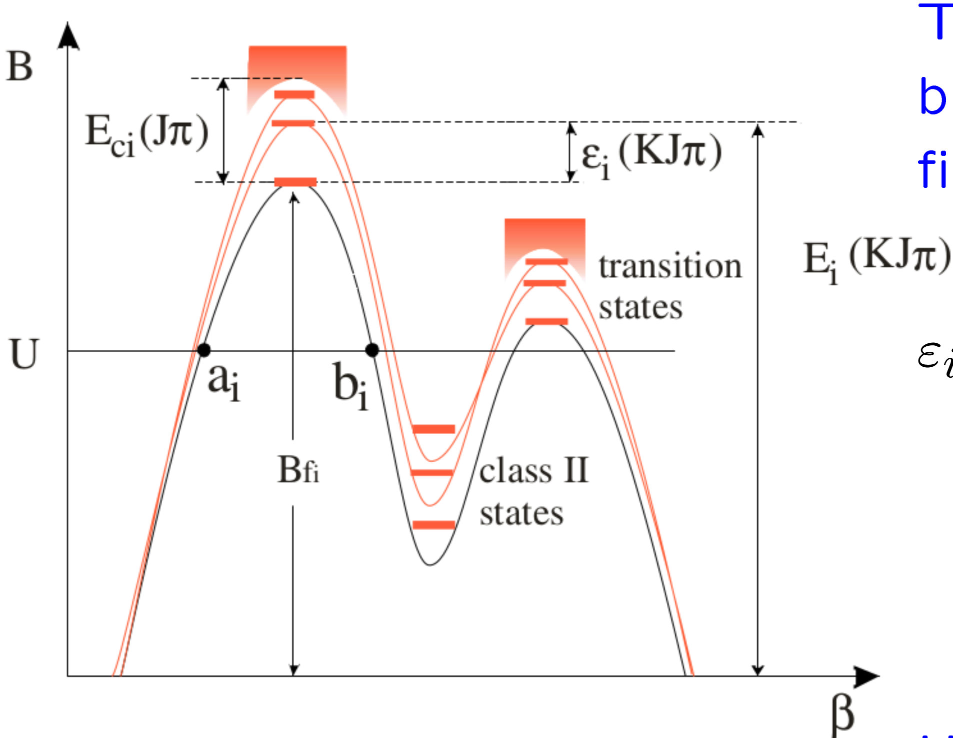


Figure: R.Capote et al. RIPL
Nuclear Data Sheets **110** 3107 (2009)

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$$T_i^{J\pi K}(E_x) = \sum T_i(E_x - B_i - \varepsilon_i(KJ\pi)) + \int\limits_{E_{ci}(J\pi K)}^{E_x} \rho_i^{\text{def}}(E, J^\pi K)\, T_i(E_x - B_i - E)\, dE.$$

T.Ericson.
On the level density of deformed nuclei. — Nucl. Phys. **6** 62 (1958)

Level density for deformed nuclei:

$$\rho_F^{\text{def}}(E, J^\pi K) = \frac{1}{2} K_{\text{coll}}\, X(E, J, K)\, R_{\text{def}}(E, J)\, \rho_F^{\text{tot}}(E),$$

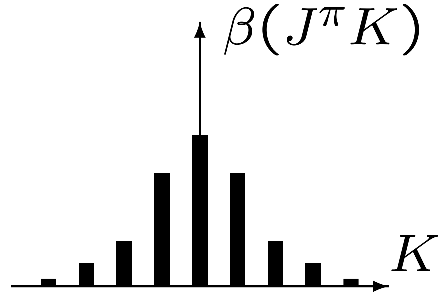
$$R_{\text{def}}(E, J) = \frac{2J+1}{2\sigma_\perp^2}\, e^{-\frac{J(J+1)}{2\sigma_\perp^2}}, \quad \sum_J R_{\text{def}}(E, J) = 1$$

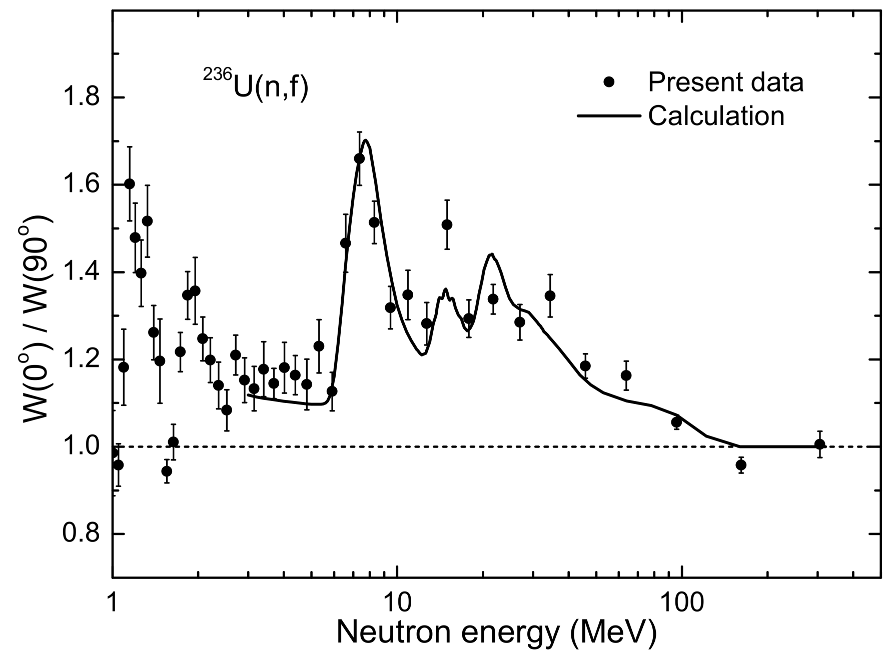
$$X(E, J, K) = \frac{e^{-\frac{\hbar^2 K^2}{2 I_{\text{eff}} T}}}{2J+1}, \quad \sum_K X(E, J, K) \simeq 1, \text{ if } \hbar^2 J^2 \ll I_{\text{eff}} T$$

Angular anisotropy for ^{236}U

«High» energies:

$$\beta(J^\pi K) \sim e^{-\frac{\hbar^2 K^2}{2 I_{\text{eff}} T}}$$

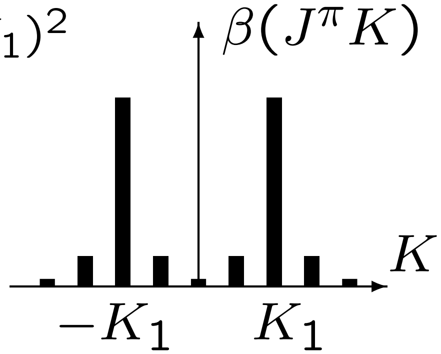


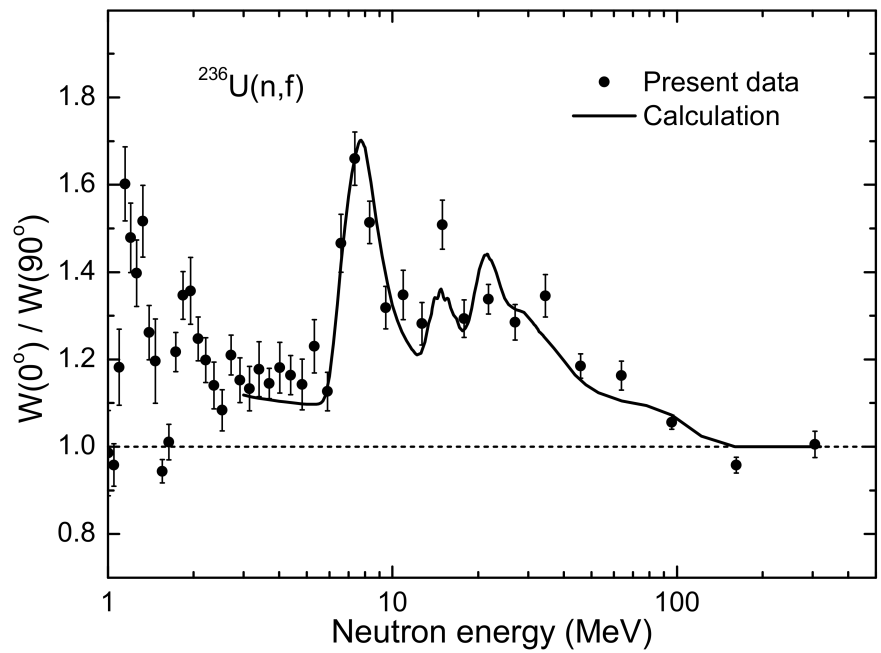


Angular anisotropy for ^{236}U(n,f)

«Low» energies:

$$\beta(J^\pi K) \sim e^{-\alpha(|K|-K_1)^2}$$



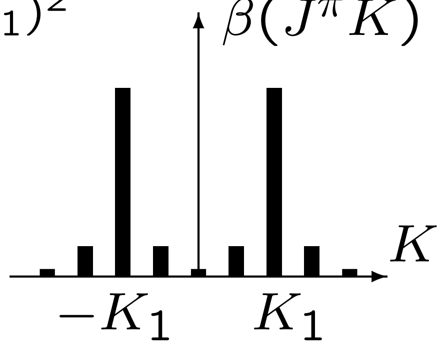


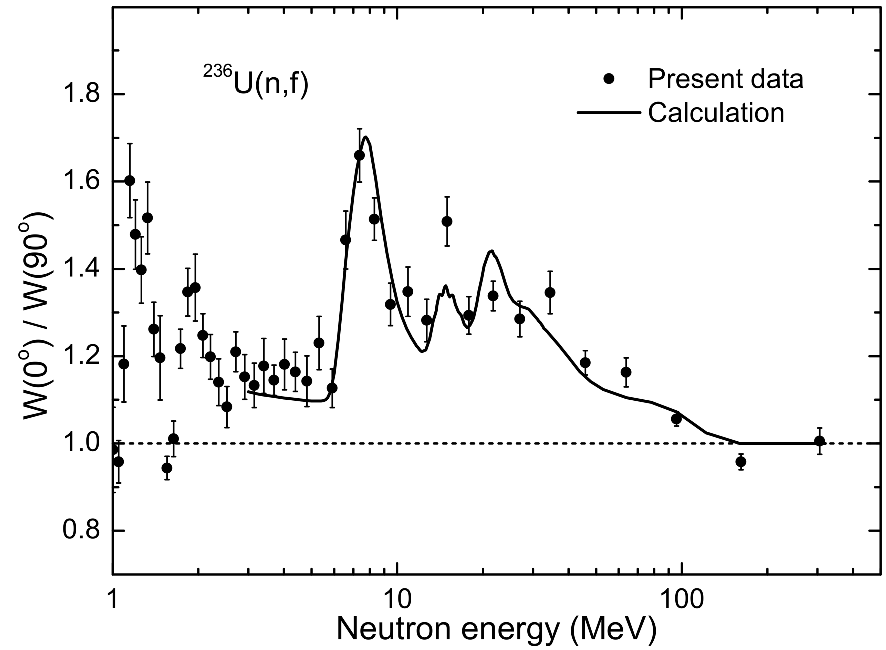
Angular anisotropy for ^{236}U

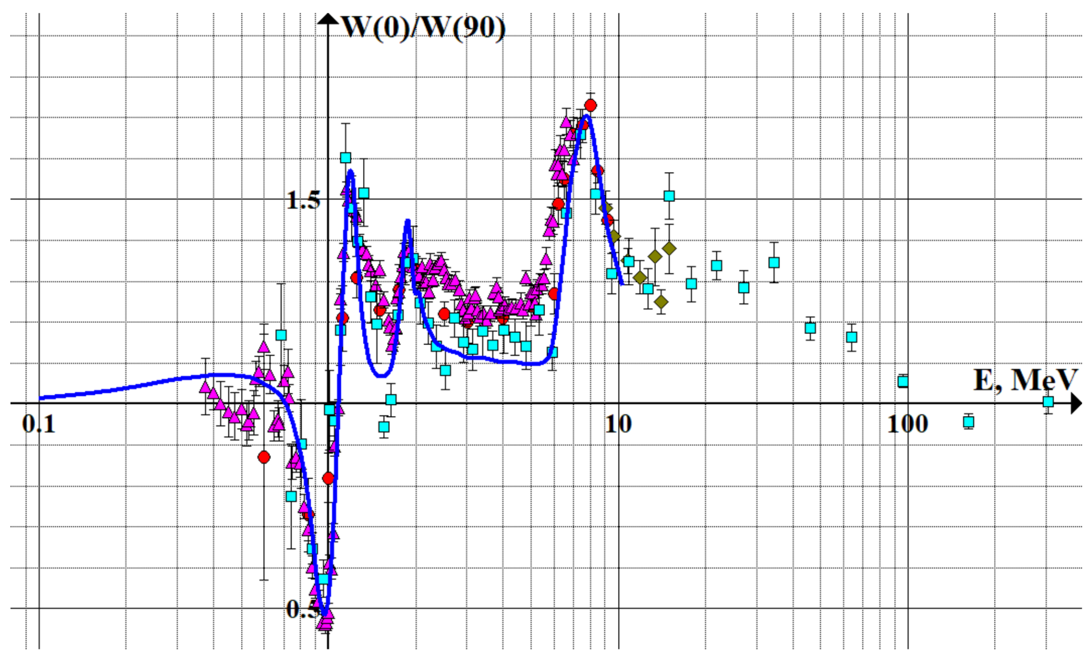
«Low» energies:

$\beta(J^\pi K) \sim e^{-\alpha(|K|-K_1)^2}$

$K_1 = K_1(J^\pi)$







Summary

1. Transition states — states of rotational bands with given J, π and K — determine both the fission probability and the angular distribution of fission fragments

2. To describe the fission mechanism, it is necessary to take into account the dependence of the density of transition states on the quantum number K

3. We are working