On the significant enhancement of the Stern-Gerlach effect for neutron, diffracting in a crystal at Bragg angles close to the right one

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The report is mainly based on:

1. Voronin V.V., Semenikhin S.Yu., Shapiro D.D., Braginets Yu.P., Fedorov V.V., Nesvizhevsky V.V., Jentschel M., Ioffe A., Berdnikov. Ya.A. **7-order enhancement of the Stern-Gerlach effect of neutrons diffracting in a crystal**. Phys. Lett, 2020, **B 809**, 135739.
2. Fedorov V.V., Kuznetsov I.A., Lapin E.G., Semenikhin S.Yu., Voronin V.V. **Diffraction enhancement and new way to measure neutron electric charge and the ratio of inertial to gravitational mass**. Nucl. Instr. Meth., 2008, **A593,** 505–509.
3. Vezhlev E.O., Voronin V.V., Kuznetsov I.A., Seminikhin S.Yu., Fedorov V.V. **Effect of anomalous absorption of neutrons undergoing Laue diffraction at Bragg angles close to π/2**. JETP Lett., **96**, 1–5.

Hystory:

1. Kato N. **Pendellösung fringe in distorted crystals.**
    1. Fermat's principle for Bloch waves. J. Phys. Soc. Jap., 1964, **18**, 1785—1791.
    2. Application to two beam cases. J. Phys. Soc. Jap., 1964, **19**, 67–77.
    3. Application to homogeneously bend crystals. J. Phys. Soc. Jap., 1964, **19**, 971–985.
2. Kato N. **The energy flow of X-rays in an ideally perfect crystal: comparison between theory and experiments.** Acta Cryst., 1960, **13**, 349–356.
3. Zeilinger A., Shull C.G., Horne M.A., Finkelstein K.D. **Effective mass of neutrons diffracting in crystals.** Phys. Rev. Lett., 1986, 57, 3089–3092. .

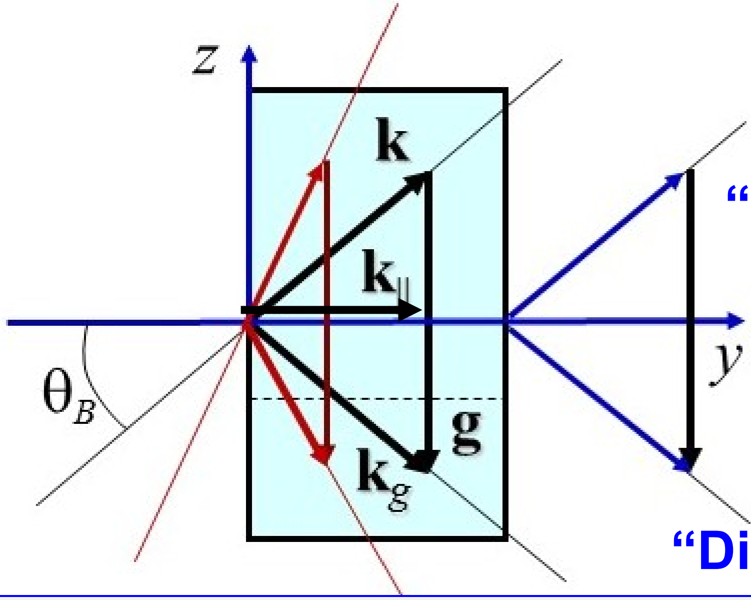
Outline

1. **As an example** of the **significant amplification** of an external force, acting on a neutron in a crystal**, we will discuss the spatial splitting** of a neutron beam into **two beams** with **opposite spin directions** in a **small magnetic field gradient** during Laue diffraction in a crystal **(that is** analogous to **the Stern-Gerlach effect).**

2. **In experiment** the value of the **spatial splitting** of the beam reached **4.1 ± 0.1 cm** at the **flight distance** of **21.8 cm** (crystal thickness) for **field gradient** of **3 G/cm** and **Bragg angle** of **82°**

3. In the **empty** space (crystal is removed), the **splitting** would be ~ **3.8×10^{-7}** cm at the **same distance and gradient**. So the experimental value of the amplification factor is **$\sim 2\times10^{5}\tan^{2}\theta_B$**, **(which is $\sim 10^{7}$ for $\theta_B \sim 82°$)** what **agrees well with the theory**.

We consider diffraction in a **symmetrical Laue scheme** (crystal **boundary is perpendicular to the reflecting planes**), in a "**perfect**", "**non-absorbing**" crystal of **large dimensions** at diffraction **angles close to the right one**.



"Perfect" → mosaicity of the crystal << the Darwin diffraction width

"Non-absorbing" → absorption length ~ crystal size >> extinction length

"Large sizes" → crystal thickness (~ 20 cm in our case) >> extinction length

"Diffraction angles" → $\theta_B = 78 - 82^o$, $\tan\theta_B = 4{,}7 - 7{,}1$, $(\tan 87^o = 19)$

g is a reciprocal lattice vector, $g = 2\pi/d$, $k = 2\pi/\lambda$

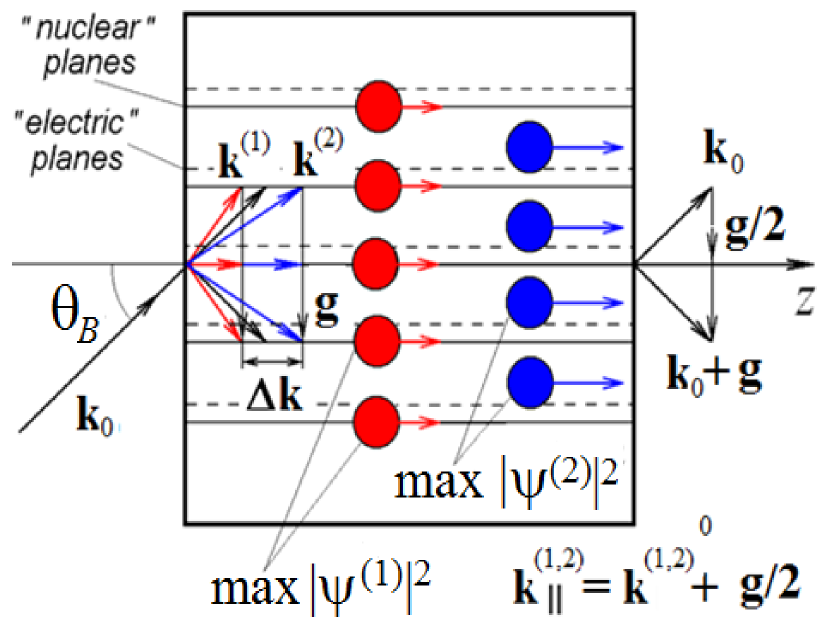
$\mathbf{k}_g = \mathbf{k} + \mathbf{g}$

$|\mathbf{k}+\mathbf{g}| = |\mathbf{k}|$ is **Bragg condition**, it is equal to $\lambda = 2d\sin\theta_B$

The nuclear potential of the system of reflecting planes responsible to diffraction has the form:

It can transfer only the momenta equal to $\pm\hbar\mathbf{g}$

In more detail, we will describe the deviation from the Bragg condition by the parameter:



For exact Bragg condition that is

$$|\mathbf{k}_0+\mathbf{g}|=|\mathbf{k}_0|$$

two types of Bloch waves are formed in a crystal – symmetric and antisymmetric:

The wave vectors $\mathbf{k}^{(1)}$ and $\mathbf{k}^{(2)}$ belong to two branches of the dispersion surface:

Here K is the wave vector of an incident neutron, taking into account refractive index:

So for the case neutrons propagate in crystal **along crystallographic planes** with wave vectors

moreover, the **neutrons in the state (1)** are concentrated mainly **on the nuclear planes** (at the maxima of nuclear potential), and in **the states (2) – between them** (at the minima of nuclear potential :

Thus, neutrons in states (1) and (2) move at slightly different potentials and so have different kinetic energies (different values of wave vectors and velocities), which reflects the equation of the dispersion surface for

The neutron velocities themselves are equal

The difference of wave vectors in a crystal for the exact Bragg condition $\mathbf{k}_0^2 = |\mathbf{k}_0+\mathbf{g}|^2$ is easily calculated from the equation of the dispersion surface:

and

that is, the expression for changing the $\mathbf{k}_0$ contains the angle between $\mathbf{k}_0$ and $\Delta\mathbf{k}_0$ (which is normal to the boundary). In the symmetric **Laue diffraction** scheme, **this angle coincides with the Bragg angle**.

The phase difference of waves (1) and (2) during the passage of a crystal with a thickness L determines so called **pendulum patten:**

the time neutron spends in crystal

is the **extinction length**.

The **different symmetry of the waves** in the crystal leads to **another, so** called, Borman effect. This is the effect of **abnormal crystal transparency** (or **abnormal absorption**) for waves passing through the crystal **under the Bragg conditions**.

The effect is due to the fact that the **wave (1) concentrated on the planes** (atoms) **is absorbed stronger** than the **wave (2) concentrated between them**.

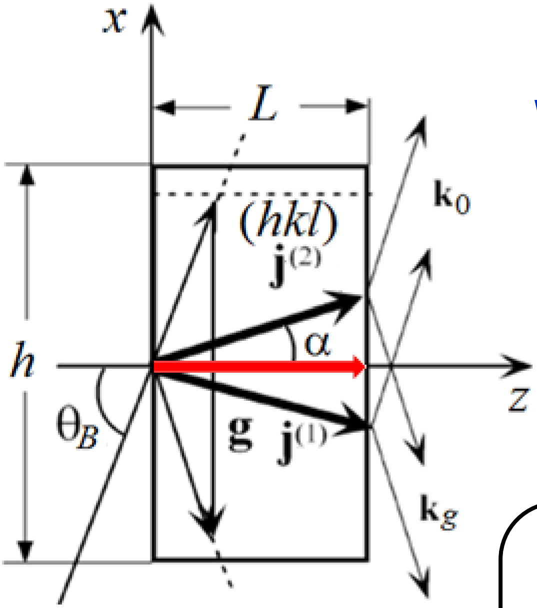
Neutron absorption in a crystal can be described by adding **an imaginary part** to the potential $(-iV')$, $V'(\mathbf{r}) \lll V(\mathbf{r})$ **is real and positive.** It also decomposes into harmonics. As a result, for exact Bragg condition, we will get

and

➡

Is average damping index (for Si, for instance, $L_a \sim 40$ cm)

For neutrons in a monatomic crystal, due to the small size of the nucleus, all harmonics are practically the same, so ε_g can be close to 1.



In the general case of Laue diffraction

Wave functions can be written as

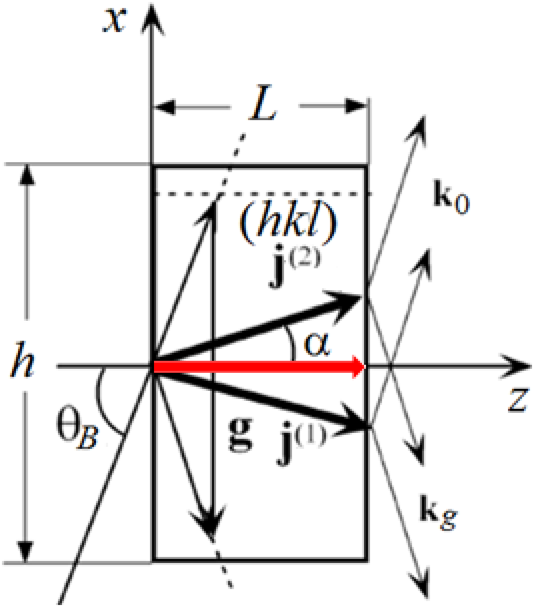
Here

and

At the **exact Bragg condition** $(w_g = 0)$, the waves (1) and (2) **move parallel to the planes**, and **the packets always overlap** for any thickness of the crystal.

If $w_g \neq 0$, then the neutron currents (1) and (2) will diverge, so that at a certain thickness of the crystal and at a finite width of the packets they cease to overlap.

Indeed, averaging over fast oscillations with period d the value



we get

Kato trajectories are the lines **tangent to which coincide with the directions of the current** density at each point. In this case, these are **straight lines**. If neutrons fall on the crystal with a deviation from the Bragg angle they diverge in opposite directions

If the incident wave is a wave packet limited, for example, by an entrance slit, then the Kato trajectory will describe the movement of this packet in crystal if its size is significantly larger than the extinction length.

If deviation from the Bragg angle is small (within the Bragg width, that is $w_g << 1$), then $\mathbf{j}^{(1)}$ and $\mathbf{j}^{(2)}$ will have a simple form

note that

The angles α of the slope of the Kato trajectories are determined by

Note that in the symmetric Laue scheme waves $\psi^{(1)}$ and $\psi^{(2)}$ are excited in the crystal with amplitudes $\cos\gamma$ and $-\sin\gamma$. Therefore, for small deviations from the Bragg condition, i.e. for we have

and both states are excited with almost the same probability ($\cos^2\gamma \approx \sin^2\gamma \approx$1/2).

However, the **directions of the currents** (especially at Bragg angles θ_B close to the right one, when $k_{\|} << g/2$ and $\tan\theta_B = g/2k_{\|} >> 1$) **can change very significantly**, indeed in this case:

the slopes of the Kato trajectories are determined by

we can note

coordinate of the exit point of the Kato trajectory from the crystal

the half-width of the base of the Borrmann triangle (fan)

Thus, at diffraction angles **close to 90°**, even a **small change** in the parameter w_g will lead to a significant change in the direction of the neutron current.

If an **external force** acts on the neutron, the parameter w_g will constantly change – the trajectories will curve, and they will diverge in opposite directions.

The change in the slope of the $x(z)$ curve, describing the Kato trajectory, is determined by the expression

Consider the action of a constant external force $\mathbf{F}$ acting on a neutron in a perfect undeformed crystal. Only the force component $\mathbf{F}_x$ along the vector $\mathbf{g}$ (x axis) leads to a change in the deviation parameter w_g. The force components parallel to the planes (along the y and z axes) do not change it. In this case the derivative of w_g is easily calculated

where

Finally the equation for Kato trajectories will be

Describes classic neutron trajectory

The classical Newton's equation for the neutron trajectory in the vacuum under the action of an external force directed perpendicular to its velocity is

We see that the effect of the force, due to diffraction in crystal, is increased many times compared to the "empty" space, and the coefficient of such **diffraction enhancement** K_D is equal to

For example, for a system of planes (220) of a silicon crystal with an interplane distance $d = 1.92$ Å, which is often used in diffraction experiments with neutrons $(E_n = 5{,}5\cdot10^{-3}$ eV, $V_g = 5{,}2\cdot10^{-8}$ eV$)$ the value of the diffraction gain is:

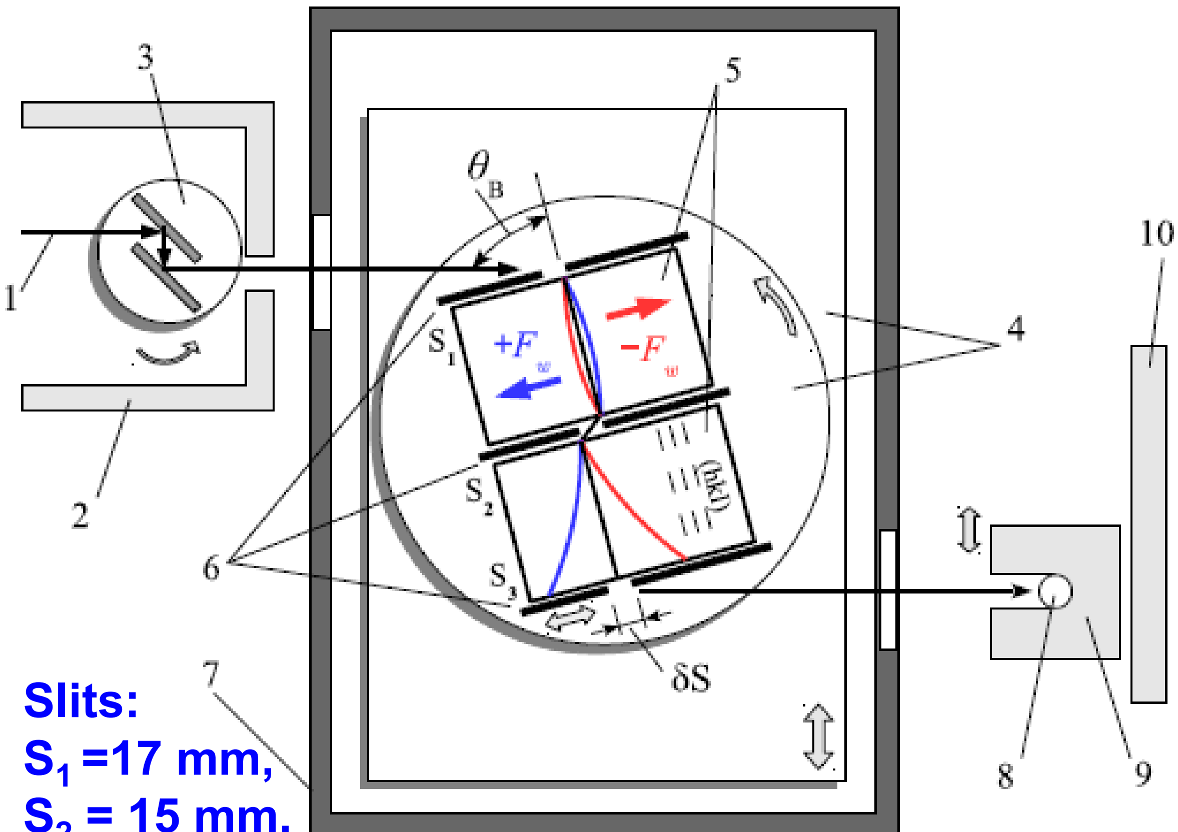
is an additional essential enhancement of the effect, it is associated with an increase of **time** the neutron spends in the crystal, which is proportional to $\tan\theta_B$.

Already at the Bragg angle $\theta_B \sim 82^o$ $(c_0 = 7{,}1)$ the value of K_D is $\sim 10^7$.

(For $\theta_B \sim 88^o$, $c_0 \sim 30$).

Observation of diffraction enhancement of the Stern–Gerlach effect (PF1B, ILL)

Neutrons were deflected in a small gradient of the magnetic field under the action of forces



Slits:
S_1 =17 mm,
S_2 = 15 mm,
S_3 = 18 mm

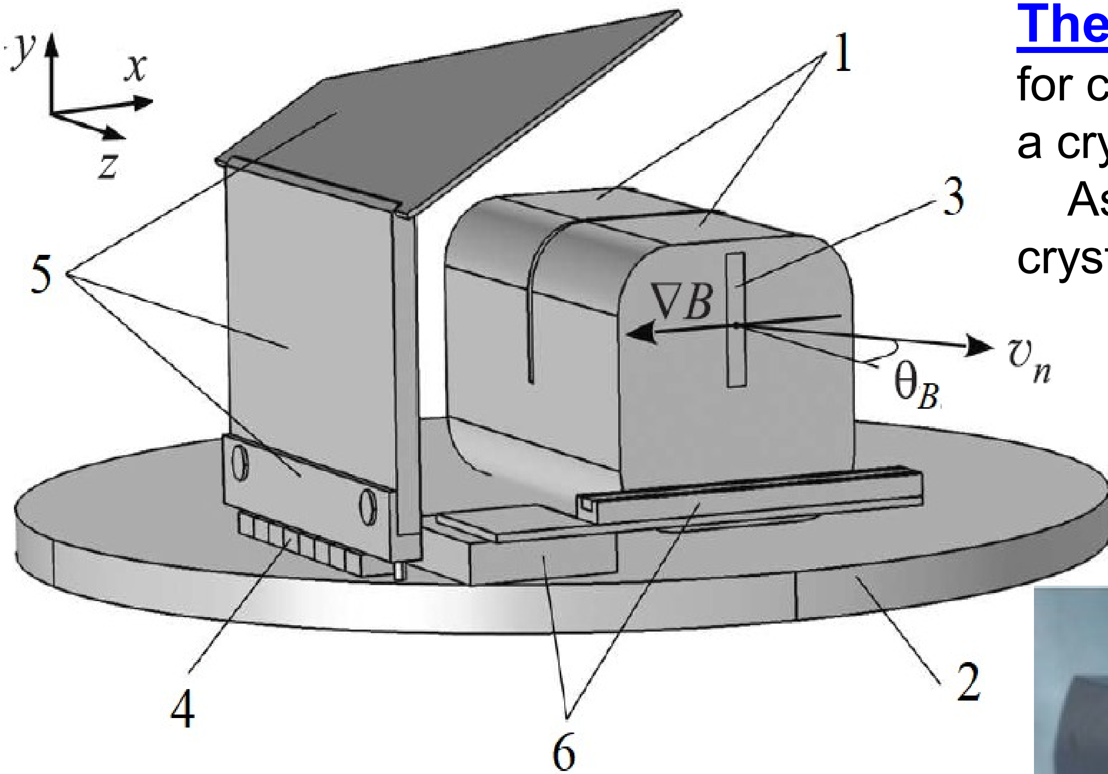
A double crystal scheme with direct beam collimation (top view).

1 – **neutron beam (PF1B, ILL);**
2 – concrete protection (casemate);
3 – double crystal monochromator (pyrographite, d_{PG}=3,35 Å, λ=(3,5−3,9) Å));
4 – translation stage for the crystals including a rotation stage;
5 – two working silicon single crystal;
6 – **collimating** (S1,S2) and **scanning** (S3) **slits**;
7 – thermostat;
8 – detector;
9 – detector shielding;
10 – beam dump

The design of a magnetic system

for creating a magnetic field gradient in a crystal.

As a result, opposite forces act in crystal on neutrons with opposite spins



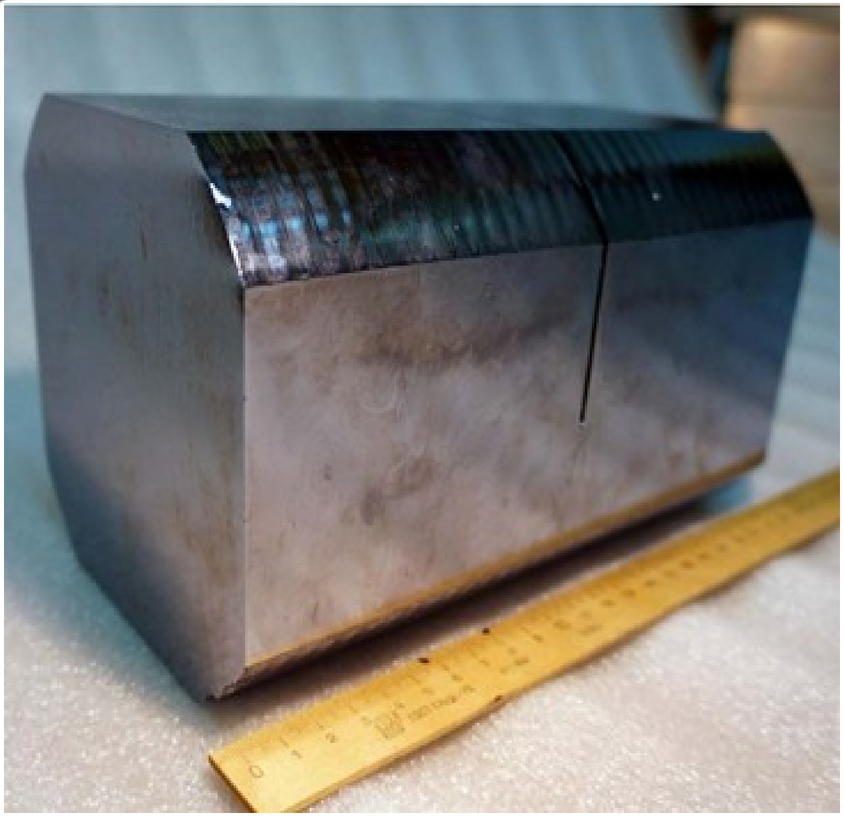
1 – silicon crystal, 2 – rotation stage (also part of field guide), 3 – neutron beam exit area, 4 – permanent magnets, 5 – magnetic field guide, 6 – piezomotor for exit slit S3

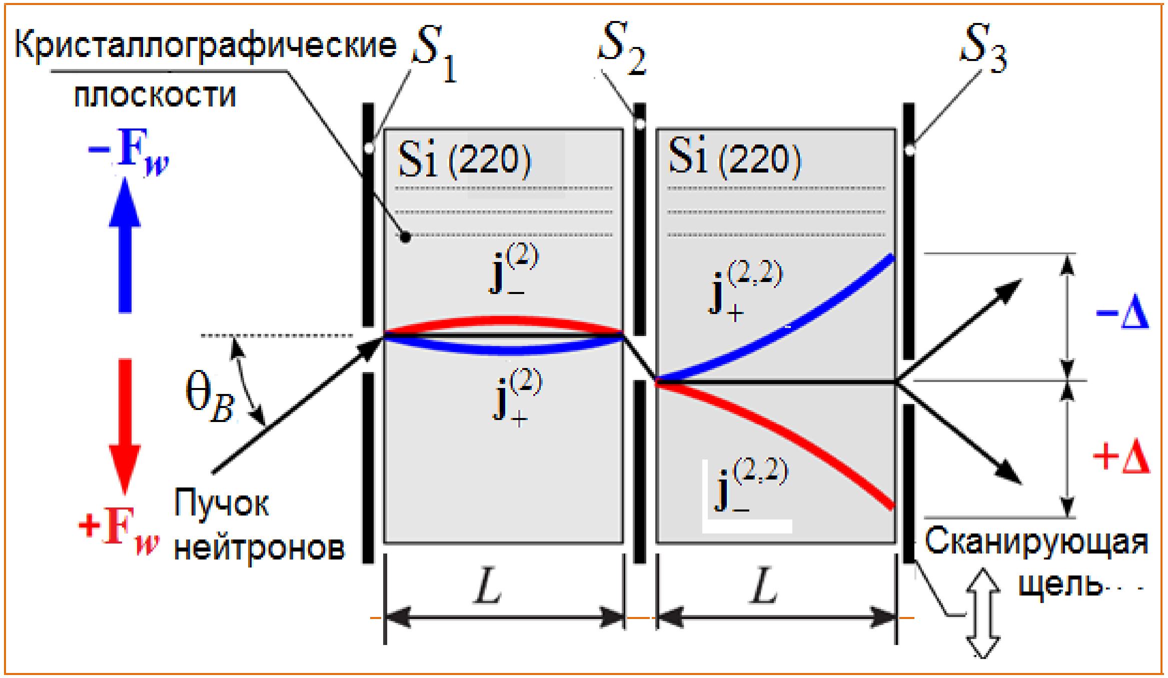
Si working crystal

130×130×218 mm^3,

plane (220) $d = 1.92$ Å,

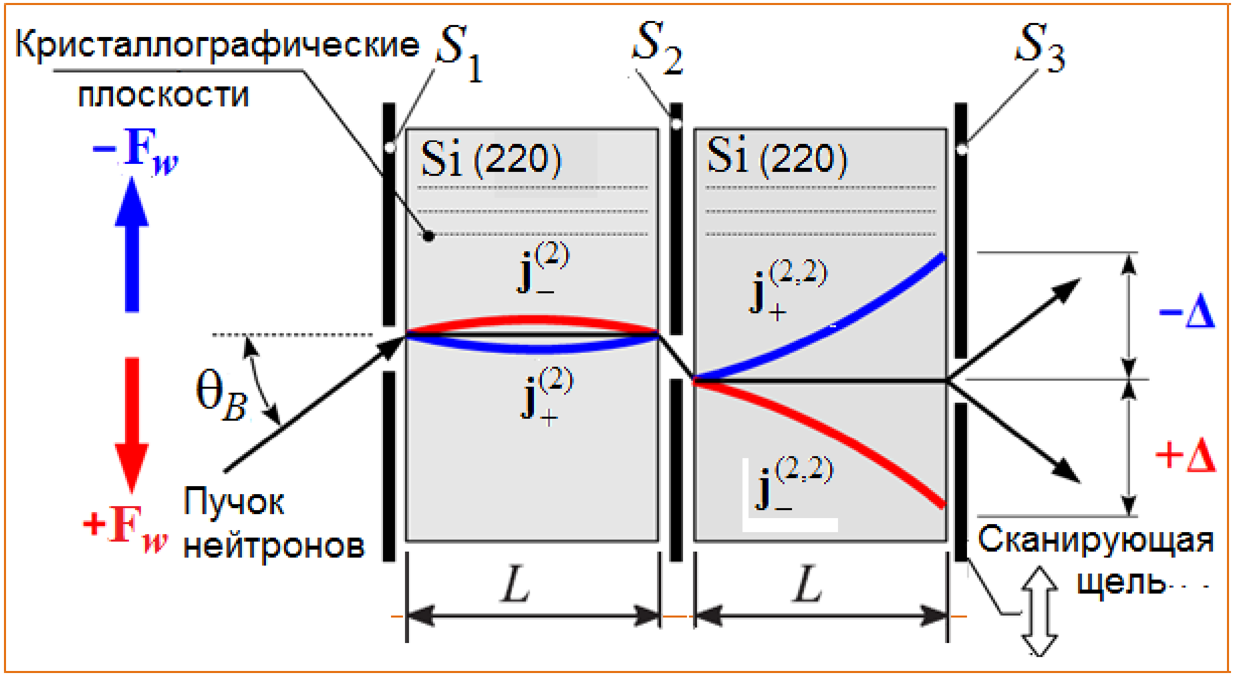
$\Delta d/d \sim 10^{-7}$





In an inhomogeneous magnetic field, inside the crystal, forces of opposite direction will act on the neutrons with opposite spin projections (along and against the field) (as in the Stern–Gerlach experiment). Only the force components perpendicular to the planes along the vector $\mathbf{g}$ (x axis) lead to a change in the direction of the Kato trajectory:

In our case (**large** crystal **thickness** and **Bragg angles**, i.e. $L_{eff} = L \tan \theta_B >> L_a$, only **antisymmetric weakly absorbed waves** (2) for both spins "survive" in crystals. They deviate in different directions (see Fig.)



The slits S_1 and S_2 (at $x = 0$) separate in the first crystal the trajectories of neutrons, which bent in a certain way under the action of forces (set the initial slopes for 2 spin projections). :

In the absence of forces, these would be trajectories with zero inclination (parallel to the planes, i.e. the z axis) of neutrons falling on the crystal exactly at the Bragg angle.

The presence of an external force will bend these trajectories, so only neutrons falling on the first crystal with **fixed (opposite for opposite spins) parameters** of deviation can pass through the second slit.

So these slits determine the **initial angles of inclination** of the trajectories $\pm\alpha_0$.

They can be found from the trajectory equations from the condition $x(0) = x(L) = 0$

As a result, in the first crystal these trajectories corresponding to neutrons with opposite polarizations will be described by curves

In the second crystal, these trajectories will start at opposite angles of inclination, so that the force will continue to bend them in the same direction

shift at the exit face
of the 2nd crystal:

that is, in such a beam collimation scheme, the effect of two crystals (thickness L) doubles (the effect for one crystal in the case of doubling its thickness is quadrupled). Splitting at the exit of the 2nd crystal:

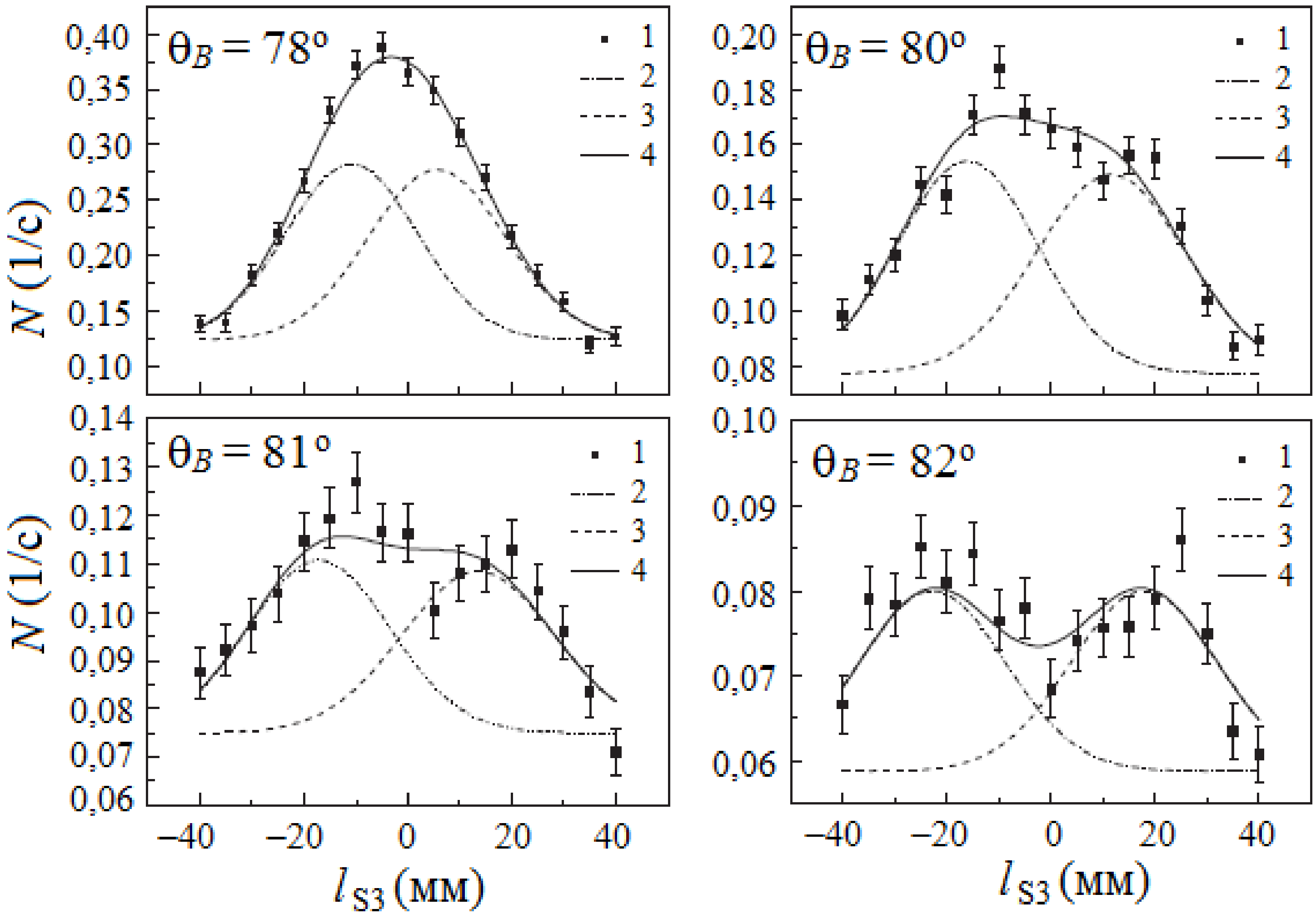
Note that the sensitivity of this experiment to external forces acting on a neutron in a crystal is determined by the magnitude of the force F_w required to shift the neutron beam at the exit from the second crystal by the width of the slit δ_{S3}:

Here K_D is the diffraction gain coefficient, the value $(2E_n\,\delta_{S3})/L^2$ is the force perpendicular to the direction of motion of the neutron and necessary for its displacement by δ_{S3} in vacuum. We noted already that

and can reach 10^7 for maximum Bragg angle of 82^o in experiment.

Measurements were carried out, using neutron beam PF1B at ILL, for Bragg angles θ_B from 78^o to 82^o. The minimum sizes of collimating slits $(\delta_{S1} = 17\text{ mm}, \delta_{S2} = 15\text{ mm}, \delta_{S3} = 18\text{ mm})$ were selected to obtain sufficient statistical accuracy during a limited time of the experiment

Measurement results

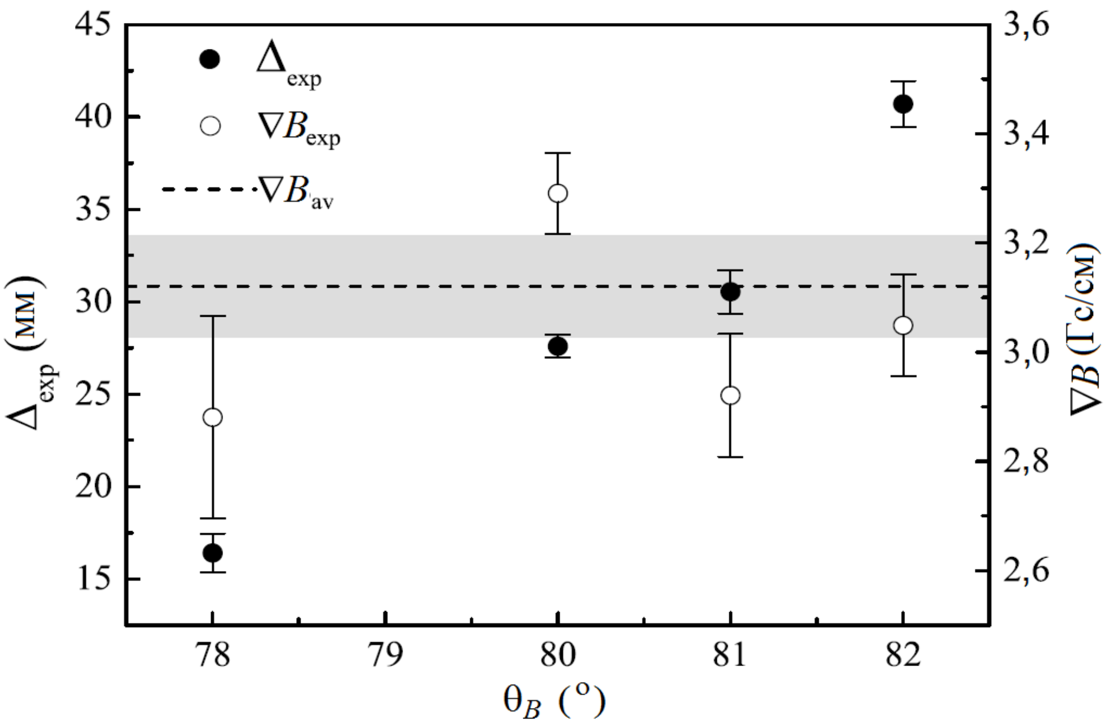


Intensity distribution N over the exit surface of the working crystal (l_{S3} is the position of the scanning slit S_3) at different diffraction angles $\theta_B = (78 - 82)^o$ in the presence of a magnetic field gradient

At a maximum angle of 82°, the splitting value of Δ_{exp} is

$$\Delta_{exp} = (4{,}1 \pm 0{,}1)\ \text{см}$$

From this data we can extract the value of the field gradient (the open circles in Fig.)



The distance between the maxima for the two spin projections and the gradient of the field depending on the Bragg angle. Dotted line is the average value of the magnetic field gradient

The average value of the magnetic field gradient over the neutron beam in the experiment turned out to be

which is consistent with estimates based on magnetometer readings at three points on each side (entrance and exit) of the crystal, which gave

3.0 ± 0.3 Gs/cm

The **calculation of the spatial splitting of a neutron beam** with a wavelength $\lambda = 3.8$ Å, $E_n \approx 5.5$ meV (which corresponds to $\theta_B = 82°$), in **free space** when passing in the same magnetic field gradient through the same 3-slit collimator (21.8 cm), but **without a crystal** (removed from the installation) gives **$3.9 \cdot 10^{-7}$ cm**.

To split into **4.1 cm**, the beam must travel **~ 900 m!**

Thus, the **experimentally measured** coefficient of diffraction enhancement

at $\theta_B = 82°$, which is in good agreement with the theory.

Conclusion

An experimental study of **small effects on a diffracting neutron depending on the Bragg angle** at $\theta_B \sim 90^o$ has been carried out.

The spatial splitting of a neutron beam into **two with opposite spin directions** in a weak magnetic field gradient during Laue diffraction in a crystal (**analogous to the Stern-Gerlach experiment**) was measured

For the first time, a 7 orders enhancement of the Stern–Gerlach effect was observed for a neutron in a crystal and

the total coefficient of diffraction enhancement for the external force acting on the neutron at Bragg angles close to $\pi/2$ **was measured.**

Its value is consistent with the theoretical one..

Based on the count rates in this experiment, it is possible to estimate its **sensitivity** (the error in measuring the external force) achieved per day:

The use of cold neutron sources, such as the planned for PIK reactor with a spectral neutron flux density of $\sim 5 \cdot 10^8$ n/Å·cm^2·c, makes it possible to use slits ~ 0.1 mm in size and Bragg angles up to 88^o , while the neutron count rate from one exit slit can reach 50 n/s, which results in an improvement in sensitivity by about 12 000 times.

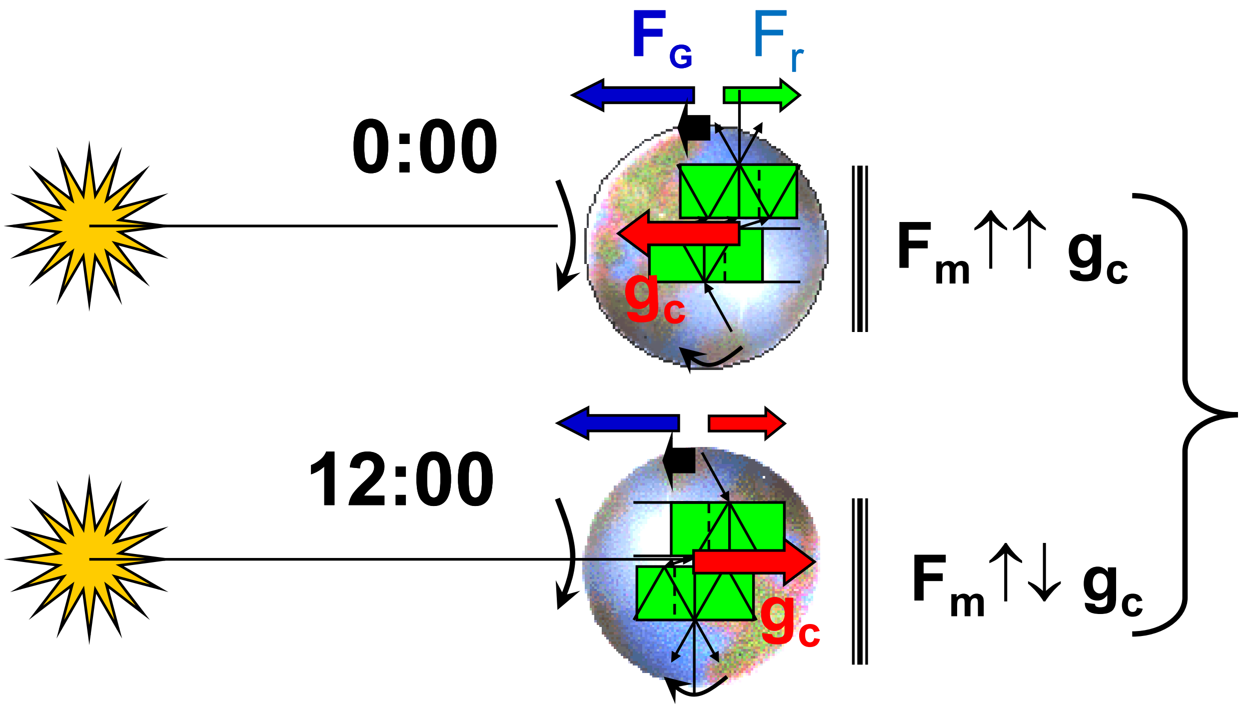
The multi-slit version (for example, 100 slots) gives an increase in sensitivity by an order of magnitude. Thus, in principle, sensitivity at the level

The attraction force of a neutron by the Sun in the Earth's orbit is

So a possible application of such a setup may be related to measuring the ratio of the inertial and gravitational masses of the neutron.

Thank you for attention!

Кристалл-дифракционный метод проверки эквивалентности инертной и гравитационной масс нейтрона



Сила будет менять знак каждые 12 часов

Будем видеть суточные Колебания интенсивности

Чувствительность установки может достичь величины:

$\delta F \sim 5\ 10^{-18}$ eV/cm per day

Современная точность $\delta(m_i/m_G)\sim 1.7\ 10^{-4}$

J. Schmiedmayer (1989)