

# AB INITIO STUDY OF ENERGIES AND DECAY WIDTHS OF NEUTRON RESONANCES

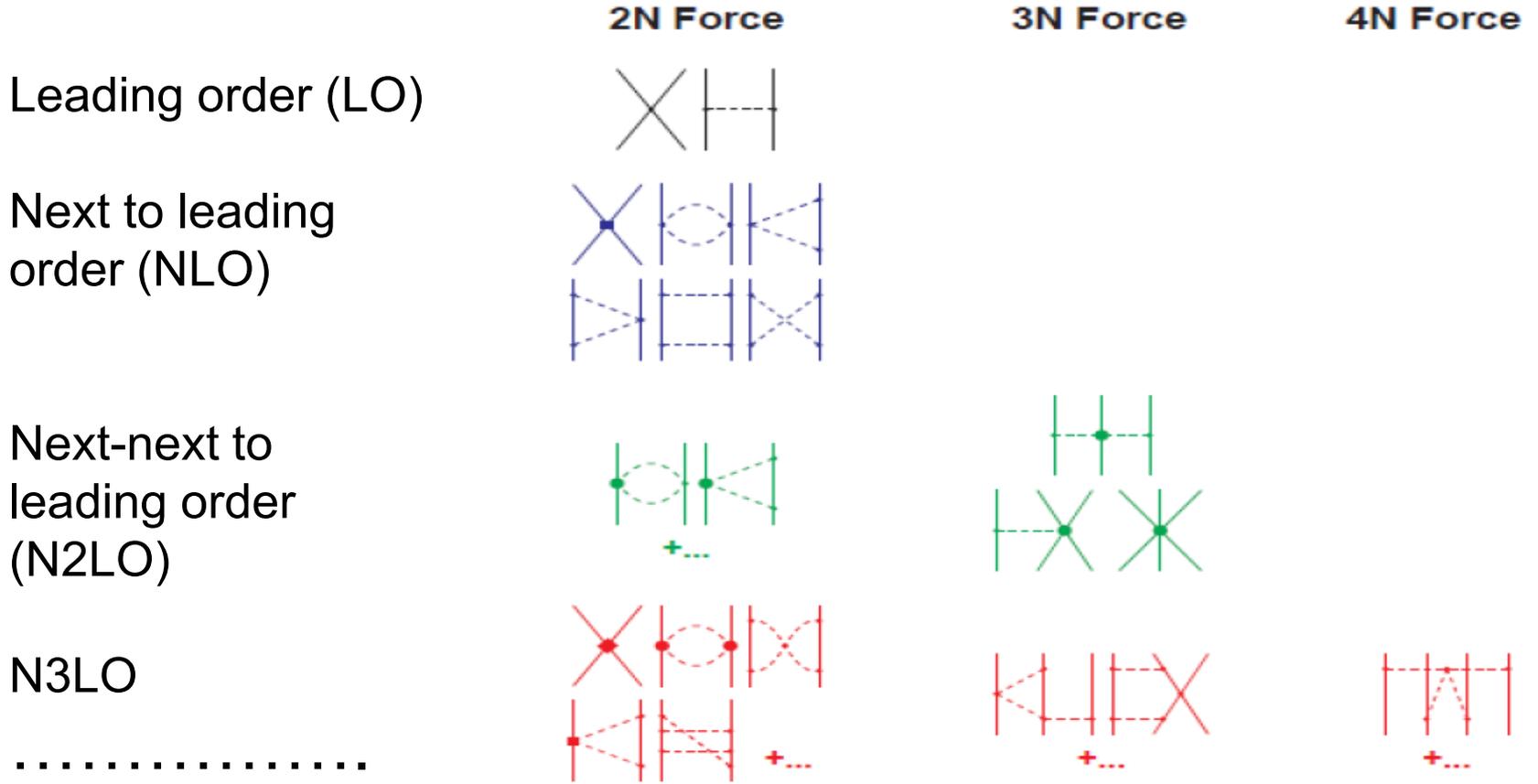
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# CHIRAL EFFECTIVE FIELD THEORY HAMILTONIAN



Diagrams of the chiral effective theory.

The weights of amplitudes of all chosen diagrams are fitted to all known two- and three-nucleon systems data. The similarity renormalization group transformation that preserves two- and three-nucleon data is used to tune the Hamiltonian used in multiparticle calculations. A small number of known levels of various nuclei are used as benchmarks for this transformation.

## TOTAL BINDING ENERGY. TYPICAL RESULTS (MeV)

Nucleus	Nature	Daejeon16		
		Theory	$\hbar\Omega$	$N_{\max}$
$^3\text{H}$	8.482	8.442( $^{+0.003}_{-0.000}$ )	12.5	16
$^3\text{He}$	7.718	7.744( $^{+0.005}_{-0.000}$ )	12.5	16
$^4\text{He}$	28.296	28.372(0)	17.5	16
$^6\text{He}$	29.269	29.39(3)	12.5	14
$^8\text{He}$	31.409	31.28(1)	12.5	14
$^6\text{Li}$	31.995	31.98(2)	12.5	14
$^{10}\text{B}$	64.751	64.79(3)	17.5	10
$^{12}\text{C}$	92.162	92.9(1)	17.5	8
$^{16}\text{O}$	127.619	131.4(7)	17.5	8

$\hbar\omega$  – oscillator parameter (MeV),  $N_{\max}$  – the basis limitation

# NO-CORE SHELL MODEL (NCSM) AS A GROUND OF AB INITIO APPROACHES OF LIGHT NUCLEI

The dynamics of canonic NCSM is described by  $A$ -nucleon Hamiltonian with realistic NN- (+NNN-) interaction. The variational problem is solved by diagonalization of the Hamiltonian matrix on the basis of  $A$ -nucleon Slater determinants (so-called M-scheme):

(1)

which, as a rule, consist of the spherical oscillator one-nucleon wave functions. All matrix elements are taken into account.

# NCSM-BASED REFINED APPROACHES

An interesting strategy is to get rid of at least a part of inessential matrix elements before a calculation.

For these purposes the SU(3)-NCSM, which uses natural symmetries of a nuclear system (Dreyfuss A. C., Launey K. D., Dytrych T. PRC 95 044312 (2017)), No-Core Monte-Carlo Shell Model (NCMCSM) (Abe T., Maris P., Otsuka T. et al. PRC 86 05430, (2012) ), etc. were created.

Various extrapolating procedures are used to refine the results. Recently the maximal bases may be  $2 \cdot 10^{10} \times 2 \cdot 10^{10}$

Calculations within the NCSM with accurately calibrated potentials reproduce quite well not only the total binding energies of nuclei up to  $^{16}\text{O}$  and excitation energies of their lower levels. The magnetic moments of nuclei and the probabilities of M1-transitions are also reproduced well.

The accuracy of NCSM calculations of nuclear radii, quadrupole moments and probabilities of E2 and E1 transitions is slightly lower, but, nevertheless, satisfactory.

As a result, for undetected levels and loosely measured transitions, one can be reasonably based on these theoretical data. **So, in some areas of nuclear spectra, ab initio calculations are already becoming competitive with experiment.**

# NUCLAR DECAY WITH THE EMISSION OF NUCLEONS OR CLUSTERS

Decays into nucleon and cluster channels play a role in nuclear spectroscopy of light nuclei which is comparable or even superior with the role of electromagnetic transitions. Alpha, neutron, proton, deuteron, helion and triton decays were observed.

So the next step on the way to creation of the theoretical nuclear spectroscopy is to build more or less universal (suitable for a lot of resonances and decay channels) schemes of the theoretical description of these processes. **Naturally these schemes are based on the theory of cluster channels.**

# AB INITIO STUDIES OF CONTINUOUS SPECTRUM AND RESONANCES

The No-Core Shell Model / Resonating Group Model (NCSM / RGM) and **No-Core Shell Model with Continuum (NCSMC)** P. Navratil et al – a lot of papers.

Another approach is Fermionic Molecular Dynamics (T. Neff and H. Feldmeier, Int. J. Mod. Phys. E 17, 2005 (2008)).

The combination of NCSM and SS-HORSE methods (A.M. Shirokov, A.I. Mazur, I.A. Mazur et al. Phys. Rev. C. 94, 064320 (2016)) also looks an interesting scheme to explore this problem.

# CLUSTER CHANNEL ORTHOGONAL FUNCTIONS METHOD

Two-fragment clustering in bound and decay of resonance states can be considered. To describe the virtual and actual decay Cluster Channel Orthogonal Functions Method (CCOFM) was proposed. The cluster-channel terms of CCOFM basis are built in the form:

(2)

is the antisymmetrizer,

- translationally-invariant WF of the fragment;
- the oscillator WF of the relative motion.

A technique has been developed for transforming such WFs to superpositions of Slater determinants (SDs).

Projecting of an eigenvector of ab initio Hamiltonian on the functions of the cluster basis defines the cluster characteristics of the corresponding nuclear state.

However some problems appear. In general case WFs (2) are not orthogonal one to another and to the shell-model components.

Thus, the next step in creating of a basis is to build orthonormalized wave functions (WFs). The WFs are obtained by diagonalization of the overlap matrix

Eigenvectors of the matrix normalized by its eigenvalues shape the desirable basis of CCOFM taking the form of SD linear combinations.

# CLUSTER FORM FACTORS AND SPECTROSCOPIC FACTORS

The value CFF of the channel  $A_1 + A_2$  is defined as:

where the norm (overlap) kernel takes the form:

Let us denote its eigenvalues and eigenfunctions  
respectively.

By representing of the delta function as

the eigenfunctions turn out to be expressed through the oscillator wave functions

And the eigenvalues take the form:

The final forms of CFF and its norm (SF) are the following:

(5)

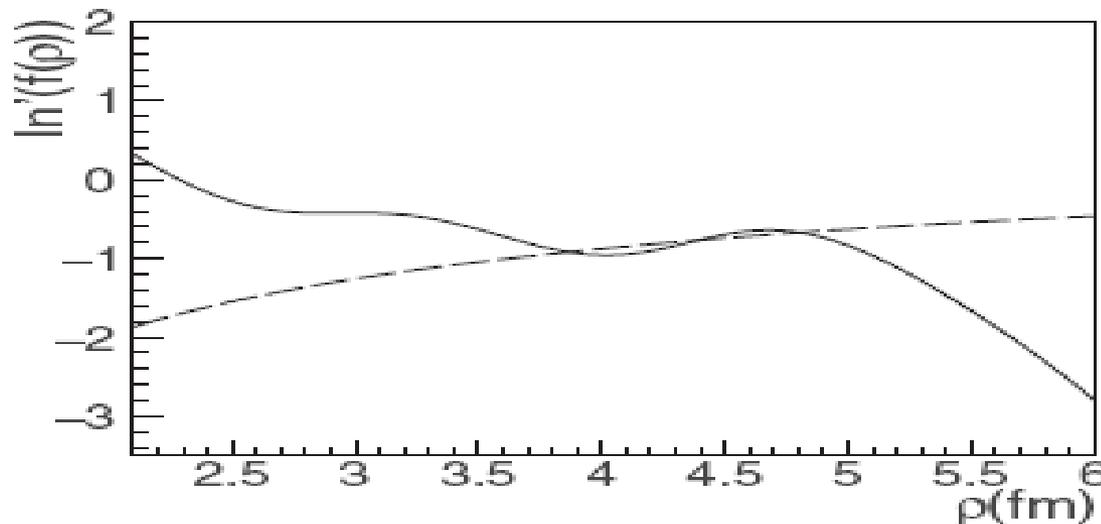
where

Superposition  
of SDs

This definition of CFF and SF plays an important role in the theory of nuclear reactions. The authors of the idea (T. Fließbach, and H. J. Mang, Nucl. Phys. A **263** 75 (1976)) called the values “new” CFF and SF.

# ASYMPTOTIC CHARACTERISTICS (ANC, PARTIAL AND TOTAL WIDTHS)

The asymptotic characteristics are deduced using the matching procedure **described in textbooks on quantum mechanics**. To determine the position of the matching point  $R_p$  of the CFF and the asymptotic WF, the condition of equality of the logarithmic derivatives is used:



Logarithmic derivatives of CFF (solid line) and function  $G_1(\rho)$  (dashed line) for  $7/2^-$  state of  $7\text{Li}$  nucleus for  $4\text{He}+3\text{H}$  channel.

Therefore, the asymptotic normalization coefficient (ANC) the decay width of this resonance is given by the expression:

If the resonance is wide, then the partial width is calculated using prescription of the R-matrix theory

For determining the decay width of subthreshold resonance (neutron physics terminology), we used the formulation of (Mukhamedzhanov and Tribble, 1999):

Our ab initio approach is in fact the sole allows one to obtain the partial decay widths in multy-channel cases.

# METHOD OF CALCULATIONS

The results of all calculations presented below are obtained by use of advanced NN-potential **Daejeon16**. It is built starting from N3LO forces of the Effective Field Theory with the Similarity Renormalization Group flow parameter  $\lambda = 1.5\text{fm}^{-1}$ . The novel is that various phase equivalent transformations are apply to it (A.M. Shirokov, I.J. Shin, Y. Kim et al, PLB **761** 87 (2016)).

The Daejeon16 interaction is well-tested. The set of parameters of it is obtained by the fit to binding energies of  $^3\text{H}$ ,  $^4\text{He}$ ,  $^6\text{Li}$ ,  $^8\text{He}$ ,  $^{10}\text{B}$ ,  $^{12}\text{C}$  and  $^{16}\text{O}$  nuclei and to excitation energies of a few narrow excited states: the two lowest excited states ( $3+$ , 0) and ( $0+$ , 1) in  $^6\text{Li}$ , the first excited state ( $1+$ , 0) in  $^{10}\text{B}$ , and ( $2+$ , 0) in  $^{12}\text{C}$ .

Code **Bigstick** is used for shell-model computing of the polarization terms and the internal WFs of clusters.

Our possibilities were limited by the dimension of the A-nucleon Hamiltonian matrix of approximately  $10^9 \times 10^9$ .

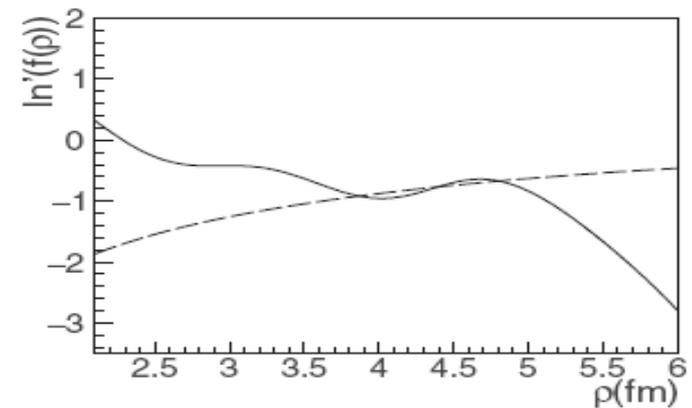
So-called A5 extrapolation procedure is applied to determine the values of level energies for infinite basis.

# AB INITIO STUDY OF NEUTRON CHANNELS

The widths of the neutron decay channels were studied for the wide list of states of the following compound nuclei:  ${}^7\text{Li}$  (i. e.  ${}^6\text{Li} + n$ ),  ${}^8\text{Be}$ ,  ${}^{10}\text{Be}$ ,  ${}^{10}\text{He}$  and  ${}^{10}\text{Li}$ . The last two are unstable nuclei.

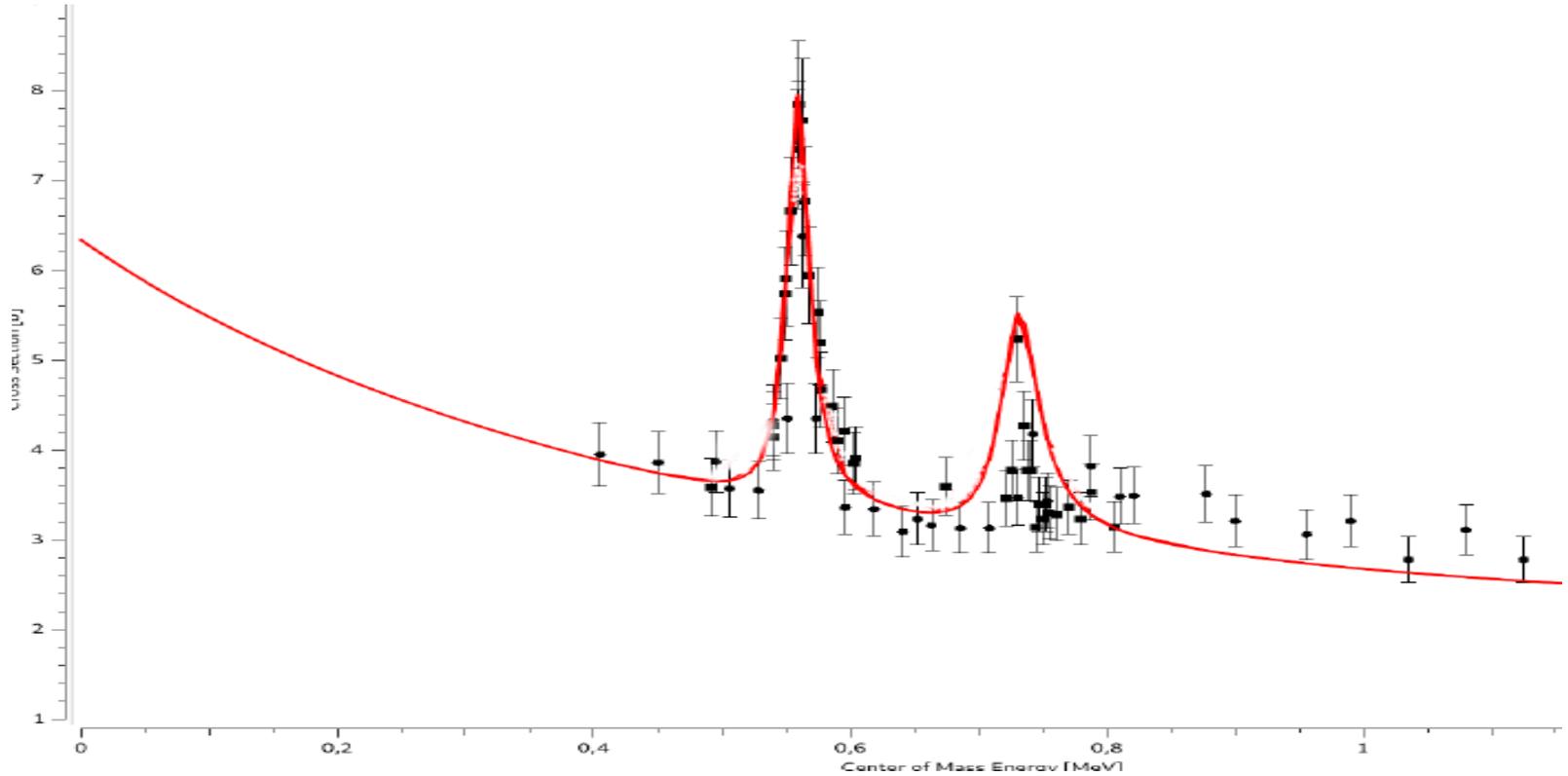
For some of these states other decay channels were also investigated.

To date, the AZURE2 code has been mastered and the first results of calculating the cross sections of resonance reactions have been obtained. The sole problem is that AZURE2 input requires to use one and the same matching point for all channels under study.



# LOW ENERGY ${}^9\text{Be}$ (n,e) PROCESS

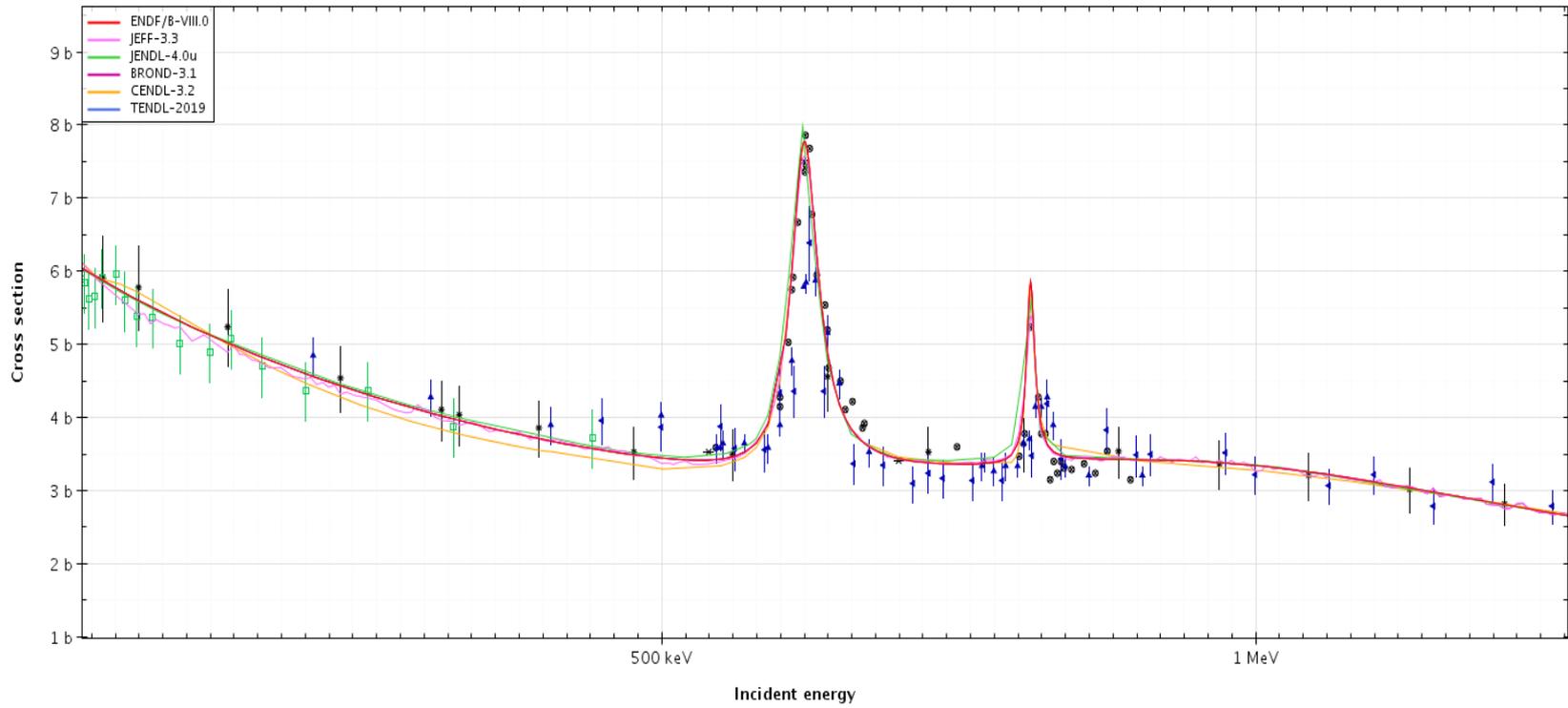
$J^\pi$	$E_{th}$	$E_{exp}$	L(S)	$2g\Gamma(E_{exp})$ or $\Gamma$	$2g\Gamma^0_{exp}$ or $\Gamma$
1-	-0.74	-0.85	0(1)	1.00	0.51
			2(1)	<	
			2(2)	<	
2+	-0.33	-0.85	1(1)	<	
			1(2)	<	
2-	-0.12	-0.55	0(2)	1.03	0.94
			2(1)	<	
			2(2)	<	
3-	1.11	0.62	2(1)	9.1	15.7
			2(2)	11.3	
2+	1.89	0.73	1(1)	2.1	6.3
			1(2)	33.6	
			$\alpha$ 0(2)	0.05 eV	



## EXPERIMENT

1. J. P. Chien, A. B. Smith J. Nuclear Science and Engineering vol. 26 p. 500, 1966
2. R. O. Lane, J. E. Monahan, Bulletin of the American Physical Society, vol. 1 p. 187 K1, 1956

# Be9 (n,e)



# SPECTRUM of $^8\text{Be}$ NUCLEUS (positive parity)

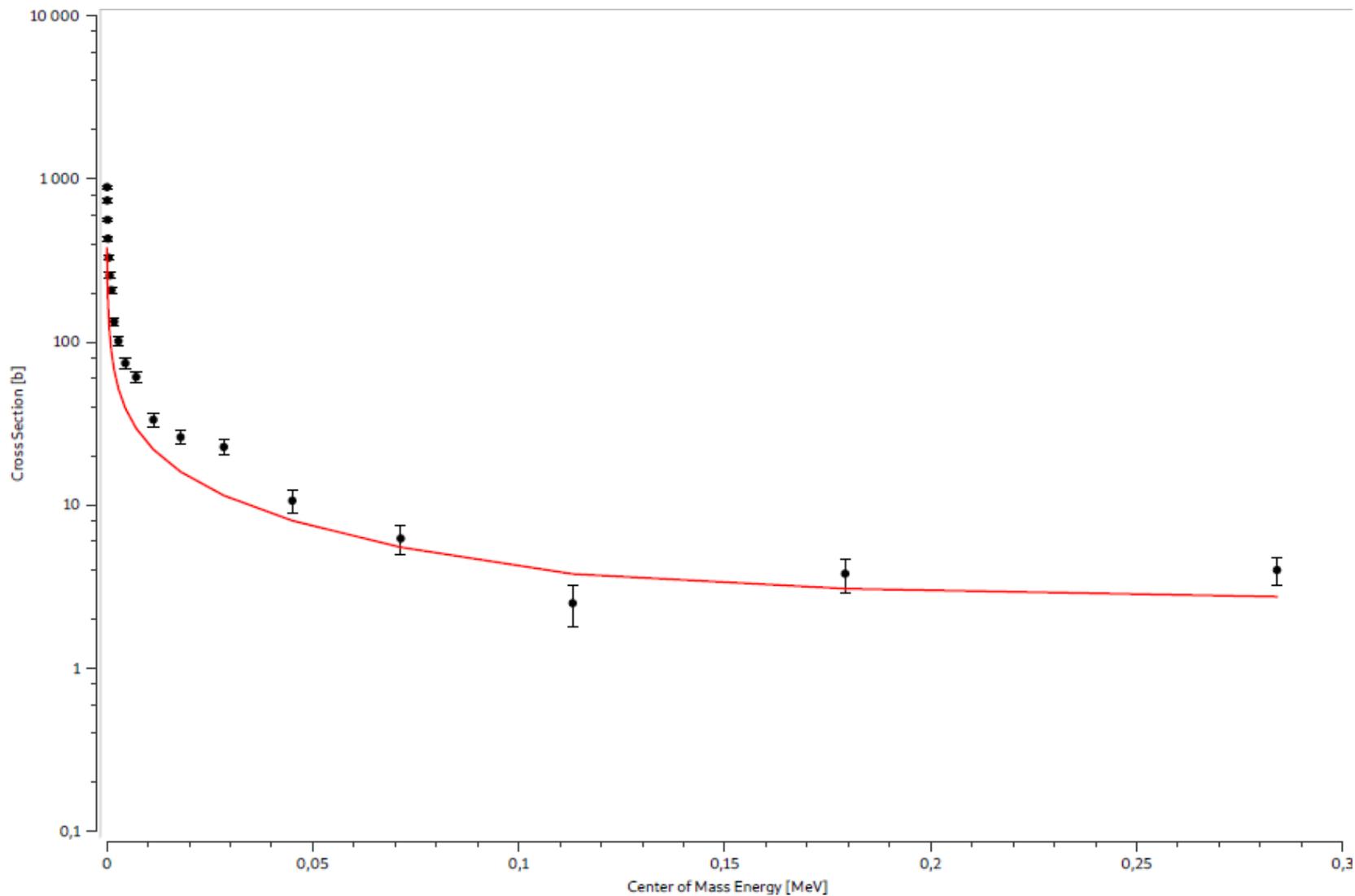
J_P	TBE exp	TBE th	E_4He	$\Gamma_{4\text{He}}$	E_Li+p	$\Gamma_{\text{Li+p}}$	E_Be+n	$\Gamma_{\text{Be+n}}$	decay	$\Gamma_{\text{exp}}$
0 <sub>+</sub>	56.50	56,50	0,091	6.73 eV	-17,25	-----	-18,9	-----	alpha	5.57 eV
2 <sub>+</sub>	53.47	53,39	3,121	1.20 MeV	-14,22	-----	-15,9	-----	alpha	1.51 MeV
4 <sub>+</sub>	45.15	45,52	11,44	2.43 MeV	-5,90	-----	-7,55	-----	alpha	3.50 MeV
2 <sub>+</sub>	39.87	39,99	16,72	395 keV	-0.63	-----	-2,27	-----	alpha	108 keV
2 <sub>+</sub>	39.58	40,34	17,01	28.5 keV	-0,333	-----	-1,98	-----	alpha	74 keV
1 <sub>+</sub>	38.86	38,66	-----	-----	0,385	13.9 keV	-1,26	-----	proton	10.7 keV
1 <sub>+</sub>	38.35	37,79	-----	-----	0,895	208 keV	-0,75	6.6e-4 eV	proton	138 keV
0 <sub>+</sub>		37,92	18,83	1.45 MeV	1,49	238 keV	-0.016	-----	-----	-----
3 <sub>+</sub>	37.26	37,68	-----	-----	1,98	449 keV	0,335	119 keV	n, p	227 keV
4 <sub>+</sub>	36.64	38,46	19,95	2.50 MeV	2,60	-----	0,96	-----	p, alpha	700 ± 100 keV
2 <sub>+</sub>	36.4	36,08	20,19	400 keV	2,84	316 keV	1,2	52.7 keV	n, p, alpha	880 keV
0 <sub>+</sub>	36.3	35,3	20,29	87.6 keV	2,94	501 keV	1,3	672 keV	alpha	720±20 keV
0 <sub>+</sub>		36,07	20,68	54.6 keV	3,34	740 keV	1,83	6.95 keV	-----	-----
1 <sub>+</sub>		36,03	-----	-----	3,38	1.46 MeV	1,87	1.50 MeV	-----	-----
1 <sub>+</sub>		35,69	-----	-----	3,72	74.1 keV	2,21	20.0 keV	-----	-----
2 <sub>+</sub>		35,11	21,63	2.09 keV	4,23	306 keV	2,79	748 keV	-----	-----
2 <sub>+</sub>	34.3	34,48	22,29	130 keV	4,94	250 keV	3,3	301 keV	n, p, d, alpha	800 keV
1 <sub>+</sub>		33,94	-----	-----	5,47	45.7 keV	3,99	72.2 keV	-----	-----
0 <sub>+</sub>		30,43	26,31	1.53 MeV	8,97	469 keV	7,46	243 keV	-----	-----
0 <sub>+</sub>	29.0	29.60	27,59	11.1 keV	10,24	-----	8,59	0.951 keV	n, p, d, t, $^3\text{He}$ , $^4\text{He}$ T=2 (!)	5.5 ± 2. keV

# NEGATIVE PARITY STATES of $^8\text{Be}$ NUCLEUS AND $^7\text{Be}(n,p)^7\text{Li}$ REACTION

J_P	TBE	E_res Li+p	l(S)	G Li+p	E_res Be+n	G Be+n	decay	G exp
<b>2<sub>-</sub>(1)</b>	<b>-37,59</b>	1.655	0(2)	<b>899 keV</b>	0,01	<b>51.4 keV</b>	n, p	<b>122 keV</b>
			2(1)	<b>4.81 keV</b>		<b>0.02 eV</b>		
			2(2)	<b>24.8 keV</b>		<b>0.16 eV</b>		
<b>0<sub>-</sub></b>	<b>-37,435</b>	<b>1.974</b>	2(2)	<b>107 keV</b>	0,4644	<b>6.78 keV</b>	-----	-----
<b>1<sub>-</sub></b>	<b>-37,1</b>	2.145	0(1)	<b>1.17 MeV</b>	0,5	<b>332 keV</b>	n, p	<b>645 keV</b>
			2(1)	<b>33.1 keV</b>		<b>1.64 keV</b>		
			2(2)	<b>30.8 keV</b>		<b>0.79 keV</b>		
<b>2<sub>-</sub>(2)</b>	<b>-36,301</b>	3.1085	0(2)	<b>995 keV</b>	1,5988	<b>1.33 MeV</b>	-----	-----
			2(1)	<b>28.3 keV</b>		<b>15.9 keV</b>		
			2(2)	<b>26.5 keV</b>		<b>26.3 keV</b>		
<b>0<sub>-</sub></b>	<b>-35,739</b>	<b>3.6702</b>	2(2)	<b>64.3 keV</b>	2,1605	<b>78.2 keV</b>	-----	-----
<b>1<sub>-</sub></b>	<b>-35,714</b>	3,6955	0(1)	<b>1.85 MeV</b>	2,1858	<b>28 keV</b>	-----	-----
			2(1)	<b>15.4 keV</b>		<b>71.8 keV</b>		
			2(2)	<b>19.8 keV</b>		<b>0.331 keV</b>		

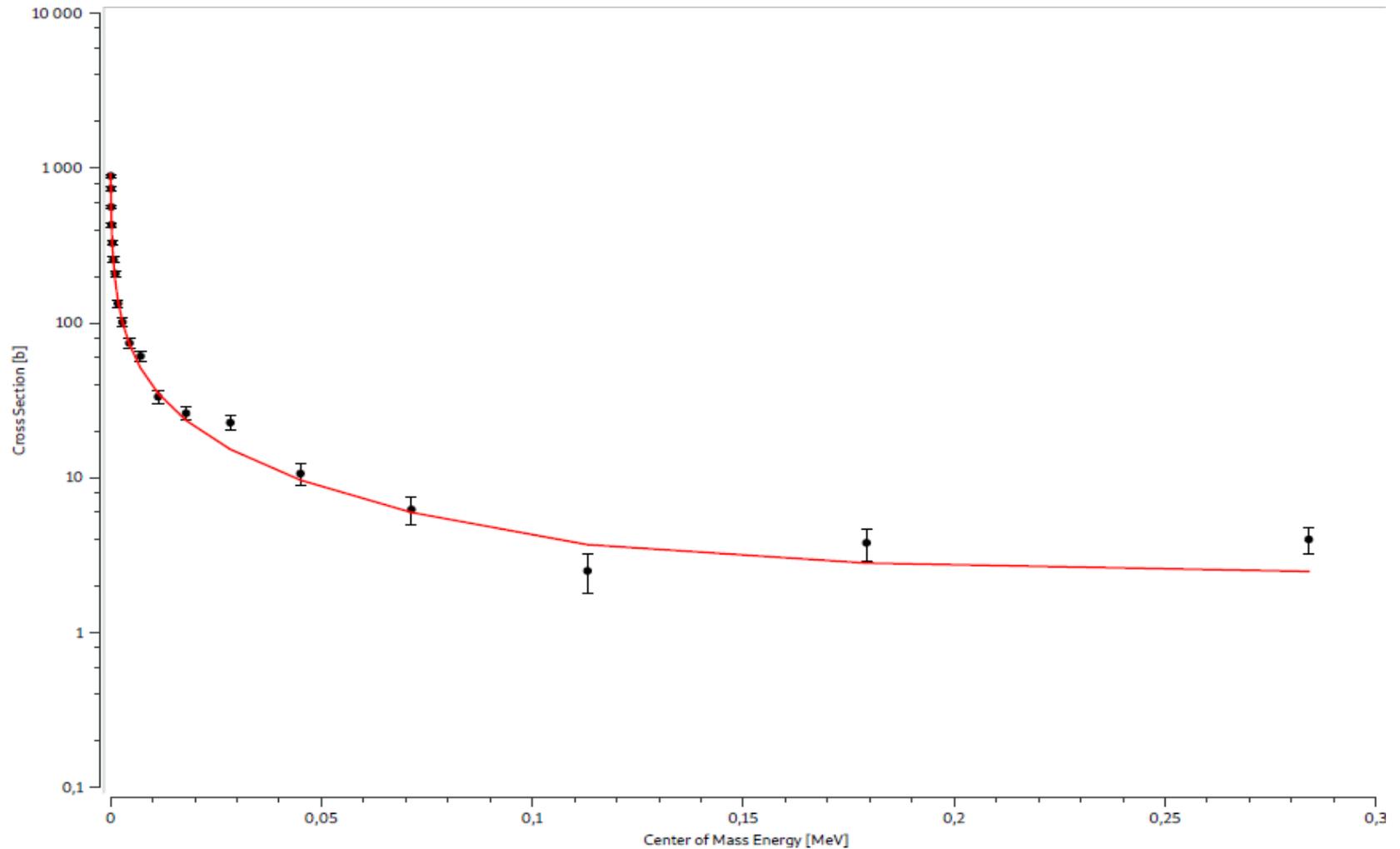
Another fit of 2-(1) state (Adahchour A., Descouvemont P.):

$$\Gamma_p = 1409 \text{ keV}. \Gamma_n = 225.$$



$\sigma_{\text{exp}}(\text{therm})=38400 \pm 800 \text{ bn}$ ,  $\sigma_{\text{adah}}(\text{therm})=37290 \text{ bn}$ ,  $\sigma_{\text{th}}(\text{therm})=7370 \text{ bn}$

Results obtained by identifying the known 2- with obtained by as 2-(2) and shifting 2-(1) down by 1.3 MeV.



$$\sigma_{\text{th(therm)}} = 20440 \text{ bn}$$

# CONCLUSION

**Ab initio theoretical approaches have become now an instrument of equal strength in the spectroscopy of light nuclei.**



**THANK YOU FOR YOUR ATTENTION!**

# SPECTRUM OF ${}^7\text{Li}$ NUCLEUS

$J^\pi$	$E_{\text{expt}}$	$E_{\text{theor}}$	$E_{\alpha+t}^{\text{expt}}$	$E_{\alpha+t}^{\text{theor}}$	$\text{ANC}_\alpha$ [66]	$\text{ANC}_\alpha$ theor.	$E_n^{\text{expt}}$	$E_n^{\text{theor}}$	$l$	$J_n$	$\text{ANC}_n$ theor.	$\text{ANC}_n$ [78]
$3/2^-$	39.245	39.110	-2.467	-2.529	$3.57 \pm 0.15$	3.44	-7.25	-7.639	1	1/2	-1.618	1.652
									1	3/2	1.317	1.890
$1/2^-$	38.768	38.279	-1.99	-1.69	$3.0 \pm 0.15$	2.95	-6.77	-6.81	1	1/2	-0.531	-0.540
									1	3/2	1.979	-2.540

$J^\pi$	$E_{\text{expt}}$	$E_{\text{theor}}$	$E_{\alpha+t}^{\text{expt}}$	$E_{\alpha+t}^{\text{theor}}$	$\Gamma_\alpha$ [74]	$\Gamma_\alpha$ theor.	$E_n^{\text{expt}}$	$E_n^{\text{theor}}$	$\Gamma_n$ [67]	$l$	$J_c$	$\Gamma_n(\text{ANC}_n)$ theor.	$\Gamma$ theor. [42]
$7/2^-$	34.593	34.409	2.195	2.172	0.069	0.065	-2.59	-2.938		3	1/2	0.013 <sup>a</sup>	$\Gamma_\alpha = 0.214$
$1/2^+$	32.804	28.921	3.984	7.66	3.15 [67]	7.4	-0.81 [67]	2.55	0.295 keV <sup>b</sup>	0	1/2	0.54 keV <sup>b</sup>	
$5/2^-$	32.641	31.610	4.147	4.971	0.918 <sup>c</sup>	0.564	-0.65	-0.139		1	3/2	0.199 <sup>a</sup>	$\Gamma_\alpha = 0.785$
$5/2^-$	31.791	30.816	4.997	5.765	0.033 [67]	0.797	0.2	0.655	0.058	1	3/2	0.053	$\Gamma_n = 0.210$
										1	1/2	0.088	
$3/2^-$	30.495	28.175	6.293	8.406	4.7 <sup>c</sup>	0.873	1.5	3.296	0.867 [79]	1	3/2	1.0	$\Gamma_n = 1.70$
										1	1/2	0.23	
$1/2^-$	30.155	27.280	6.633	9.301	2.7 <sup>c</sup>	0.282	1.84	4.191		1	3/2	1.0	$\Gamma_n = 2.44$ $\Gamma_\alpha = 0.435$
$7/2^-$	29.675	28.489	7.133	8.092	0.437 <sup>c</sup>	0.453	2.32	2.982		3	1/2	0.72 keV	$\Gamma_n = 0.039$
$3/2^-$		27.047		9.60		1.25		4.424		1	1/2	0.785	

<sup>a</sup>ANCs ( $\text{fm}^{-1/2}$ ).

<sup>b</sup> $\Gamma$  ( $E_n = 1$  eV).

<sup>c</sup>Total decay width.

$J^\pi$	$T$	$E_{\text{expt}}$	$E_{\text{theor}}$	$E_{\text{He}}^{\text{expt}}$	$E_{\text{He}}^{\text{theor}}$	$\Gamma_{\text{tot}}$ [74]	$SF_{\text{He}}$	$\Gamma_{\text{He}}(\text{ANC}_{\text{He}})$ theor.	$E_{\text{Li}^*}^{\text{expt}}$	$E_{\text{Li}^*}^{\text{theor}}$	$SF_{\text{Li}^*}$	$\Gamma_{\text{Li}^*}(\text{ANC}_{\text{Li}^*})$ theor.
$1/2^-$	$1/2$	30.155	27.280	-0.885	1.538		0.1465	0.343 <sup>a</sup>	-1.727	0.638	0.0453	0.259 <sup>a</sup>
$3/2^-$	$3/2$	28.005	27.247	1.265	1.571	$0.260 \pm 0.35$	0.1638	0.111	0.433	0.671	0.3770	0.117

<sup>a</sup>ANCs ( $\text{fm}^{-1/2}$ ).