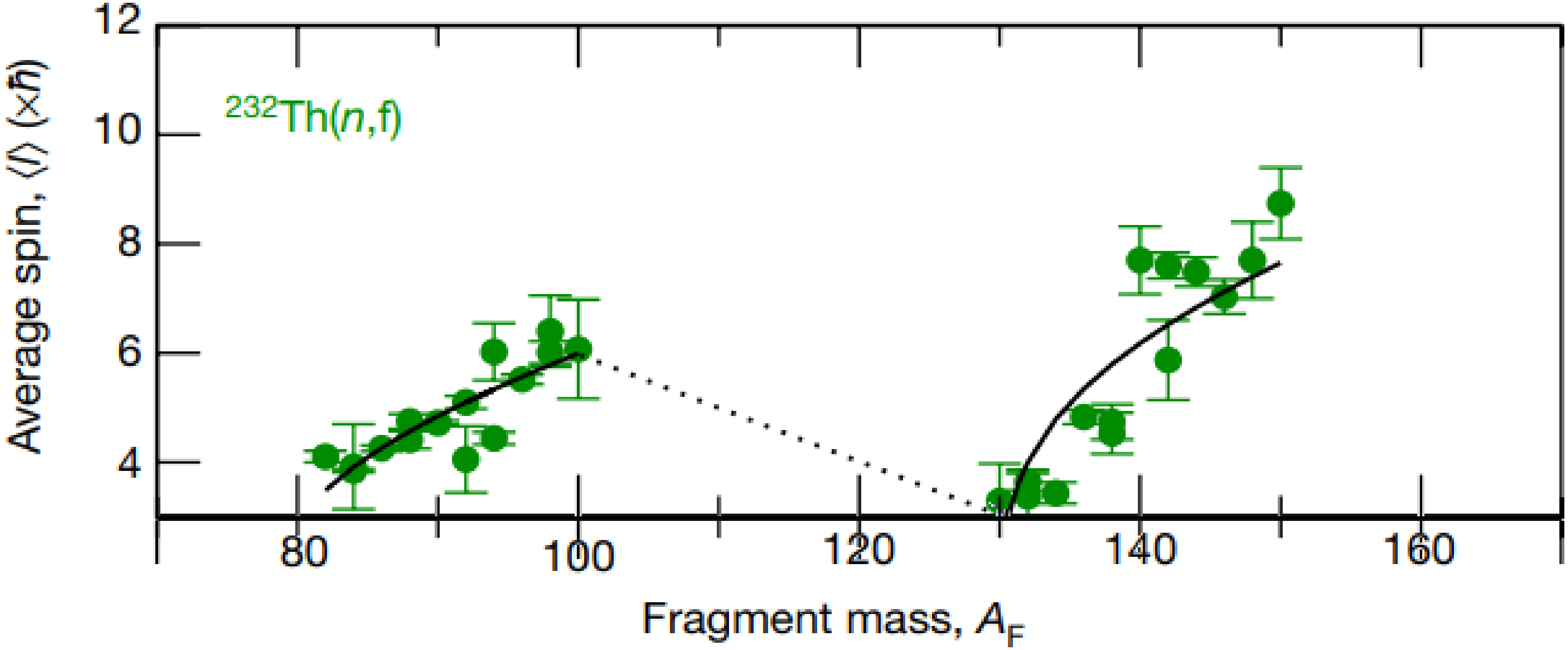
Voronezh State University

Spin distributions of fragments in binary asymmetric nuclear fission

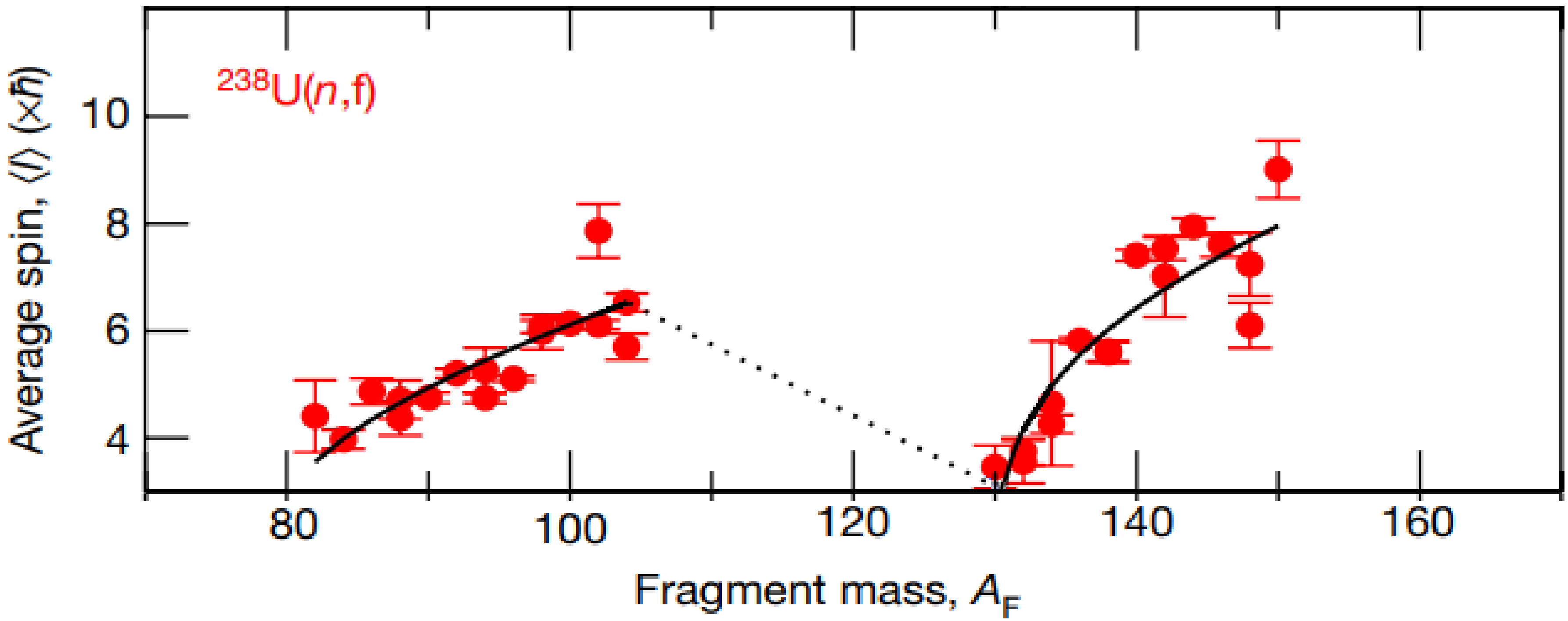
Lyubashevsky D.E.

2024



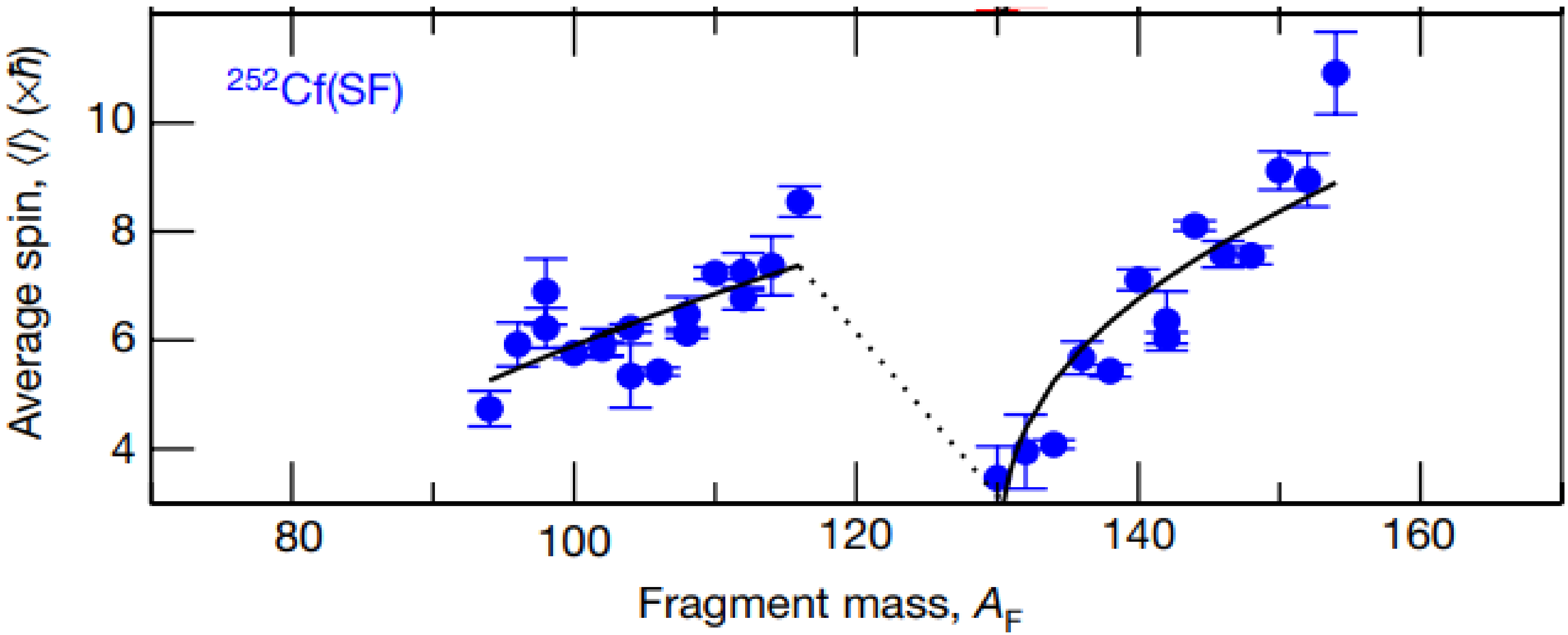
Dependence of average values of fragment spins on the fragment mass number in [1] for $^{232}Th(n,f)$

[1] *Wilson J. N. et al*. // Nature (London) 590, 566 2021



Dependence of average values of fragment spins on the fragment mass number in [1] for ^{238}U(n,f)

[1] *Wilson J. N. et al*. // Nature (London) 590, 566 2021



Dependence of average values of fragment spins on the fragment mass number in [1] for ^{252}Cf(sf)

[1] *Wilson J. N. et al*. // Nature (London) 590, 566 2021

The formula for calculating the fragment spin in [1] is based on the Gibbs distribution:

$$P(J_i) = \frac{2J+1}{2\sigma^2} \exp\left(-\frac{(J+1/2)^2}{2\sigma^2} \right)$$

If we assume that the excitation energy of the nucleus is proportional to the mass number from the nucleus neck, we obtain the final parameterization:

$$J = cA_N^{1/4} A_F^{7/12}$$

Jørgen Randrup

Ramona Vogt

[2] *Randrup J., Vogt R.//* Phys. Rev. C 103, 014610 2020.
[3] *Vogt R., Randrup J //* Phys. Rev. Lett. 127, 062502 2021.
[4] *Randrup J., Døssing T., Vogt R.//* Phys. Rev. C 106, 014609 2022.
[5*] Randrup J.* // Phys. Rev. C.106. L051601 2022

Determination of fragment spins [2-5]:

$$\vec{J}_1 = \frac{I_1}{I_1 + I_2}\vec{j}_w + \vec{j}_b \qquad \vec{J}_2 = \frac{I_2}{I_1 + I_2}\vec{j}_w - \vec{j}_b$$

Distribution for wriggling- and bending- oscillations [6] taking into account the establishment of thermal equilibrium of the fissile system with the environment:

$$P(J_{w/b}) = \frac{1}{\sqrt{\pi C_{w/b}}}\exp\left(-\frac{J_{w/b}^2}{C_{w/b}}\right)$$

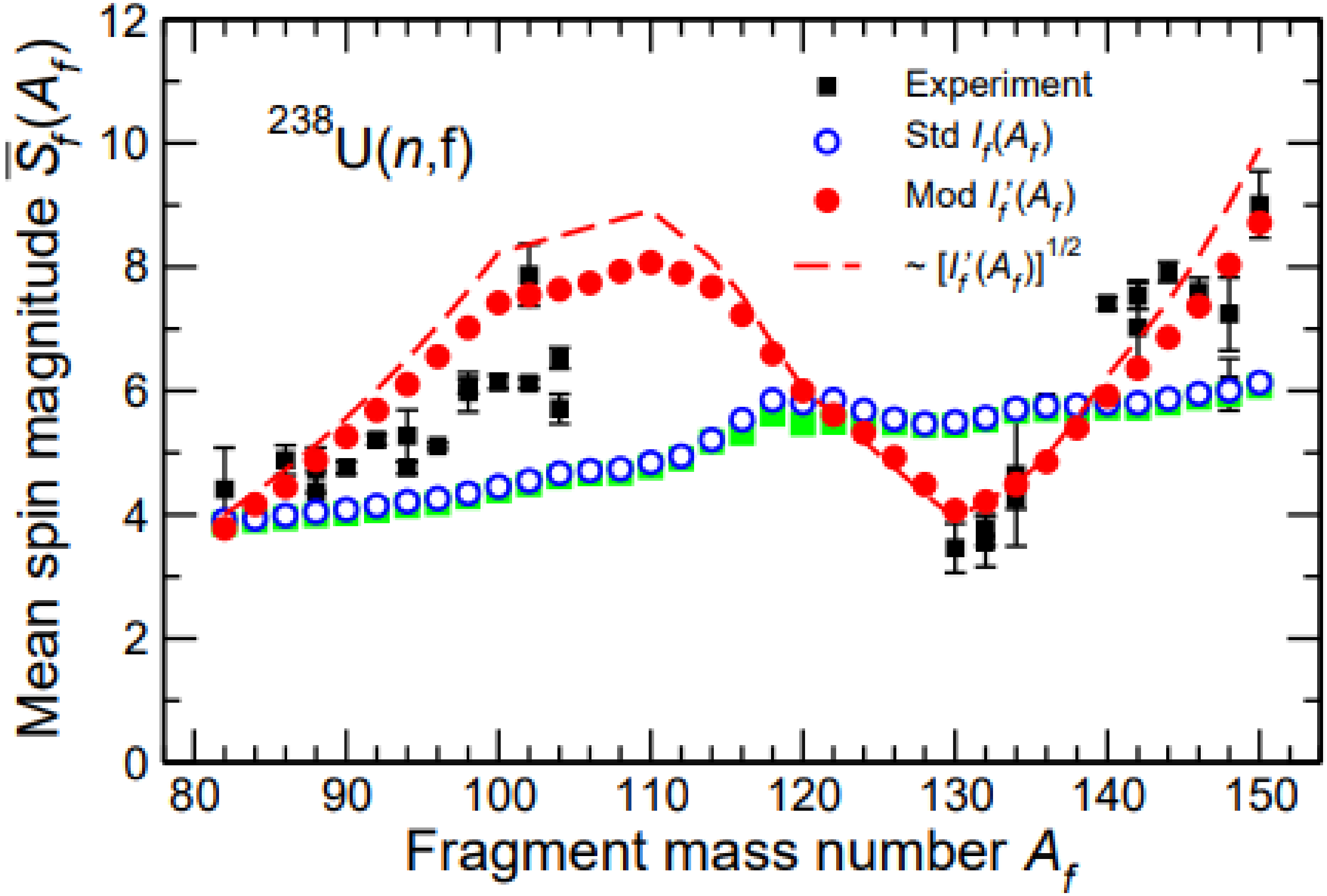
[6]. *Nix J. R., Swiatecki W. J.* // Nucl. Phys. A 71, 1 (1965).

$$C_{w/b} = I_{w/b} \coth\left(\frac{\hbar\omega_{w/b}}{2T_{w/b}}\right) \rightarrow \begin{cases} 2I_{w/b}T_{w/b}, & T_{w/b} >> \hbar\omega_{w/b} \\ I_{w/b}\hbar\omega_{w/b}, & T_{w/b} << \hbar\omega_{w/b} \end{cases}$$

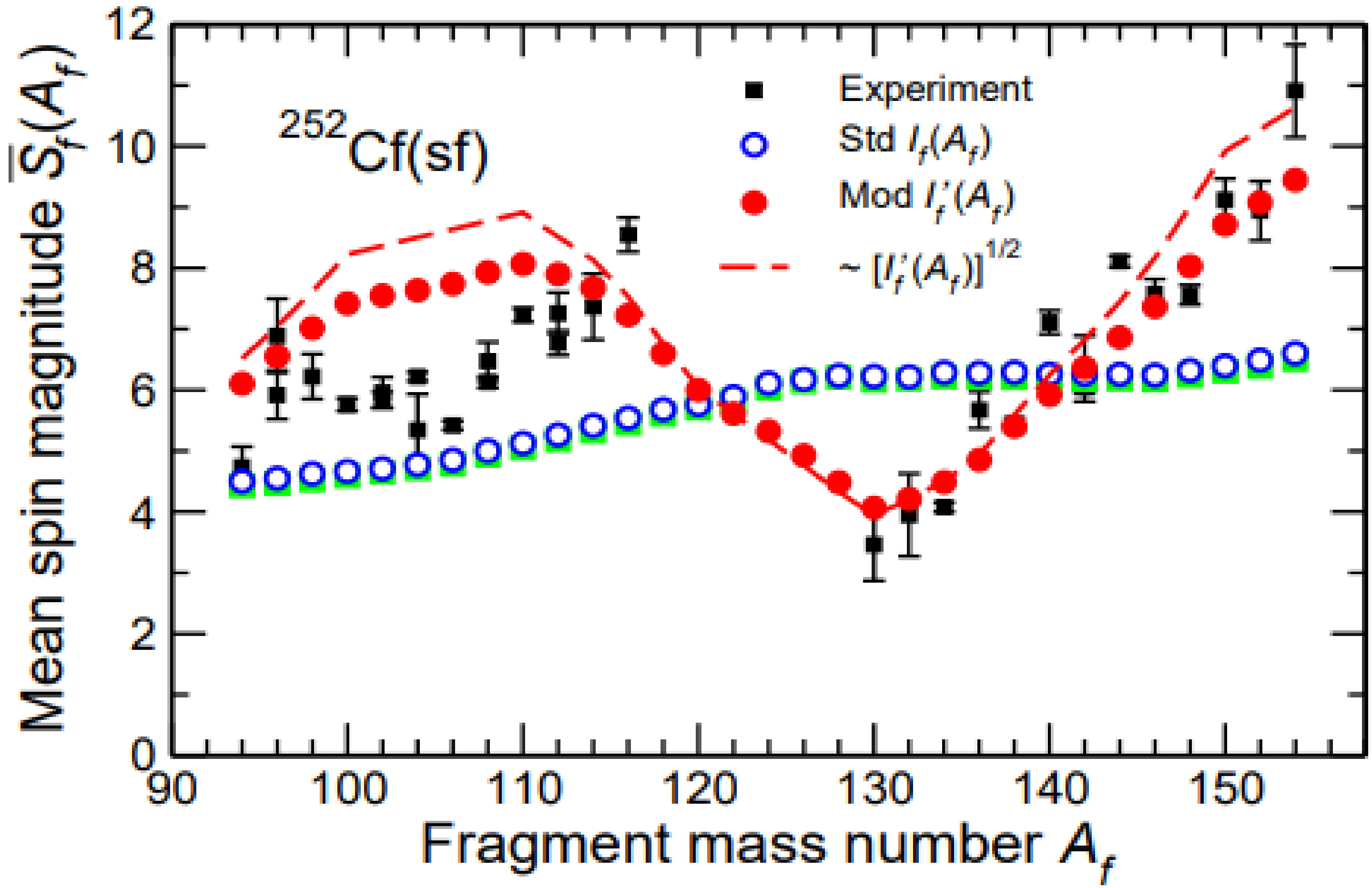
The upper limit is realized at $T_{w/b} >> \hbar\omega_{w/b}$

However $T \approx 0,85 - 1,15\,\mathrm{MeV}$, and frequencies of wriggling oscillations for the indicated nuclei $\hbar\omega_w \approx 2,3\ \mathrm{MeV}$ which leads to a contradiction.

Therefore, in our approach the lower approximation will be realized

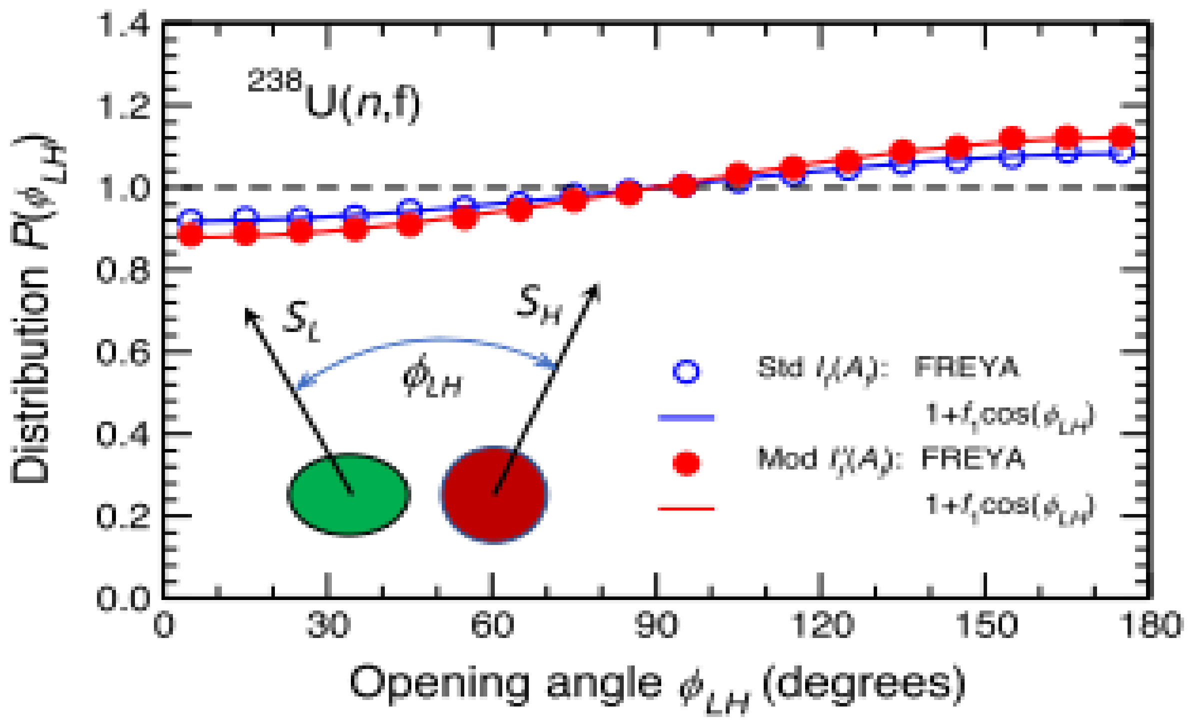


Dependence of the average value of the fragment spin on the mass number in [3] for ^{238}U(n,f)



Dependence of the average value of the fragment spin on the mass number in [3] for $^{252}Cf(sf)$

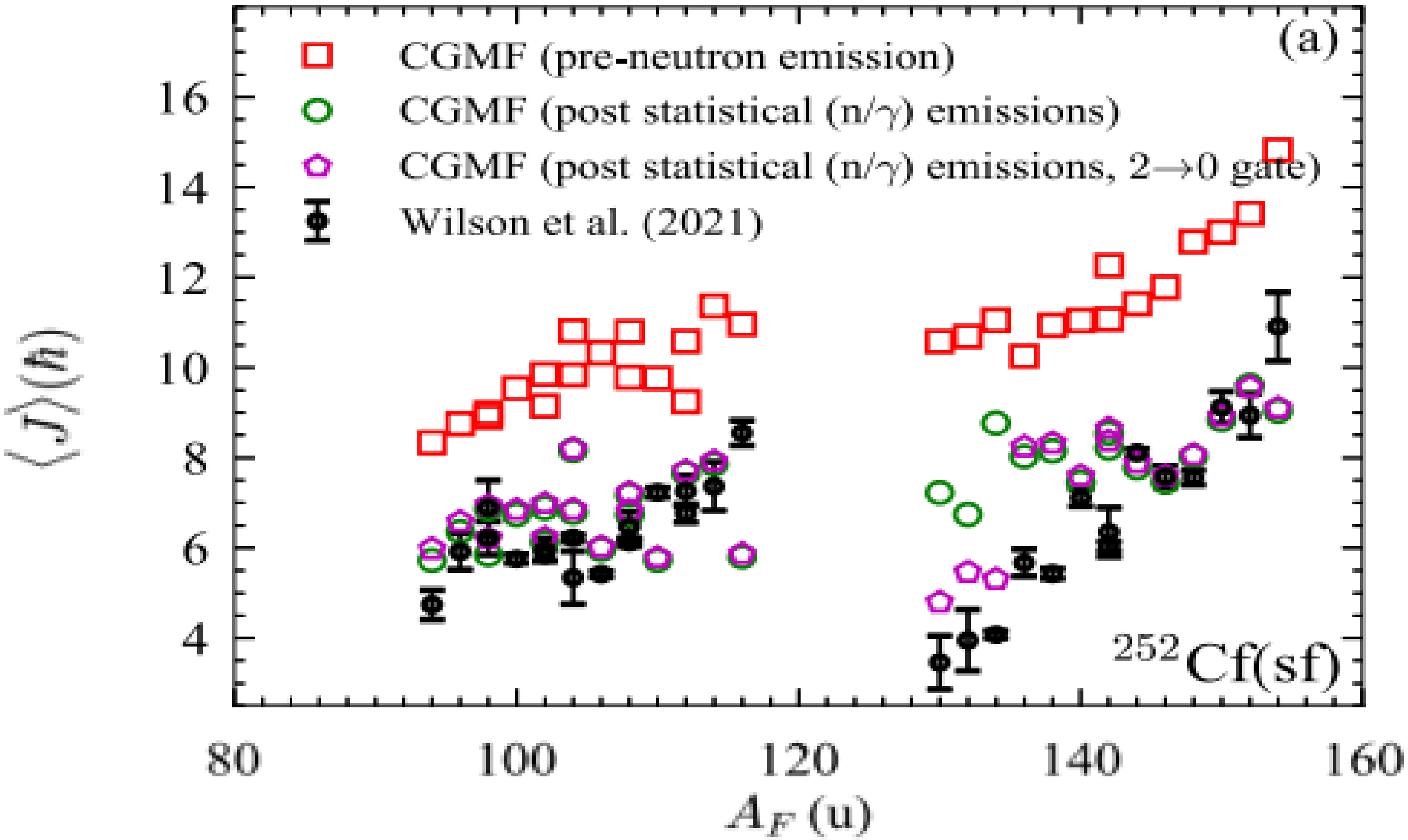
Correlations of angles between the spins of fragments



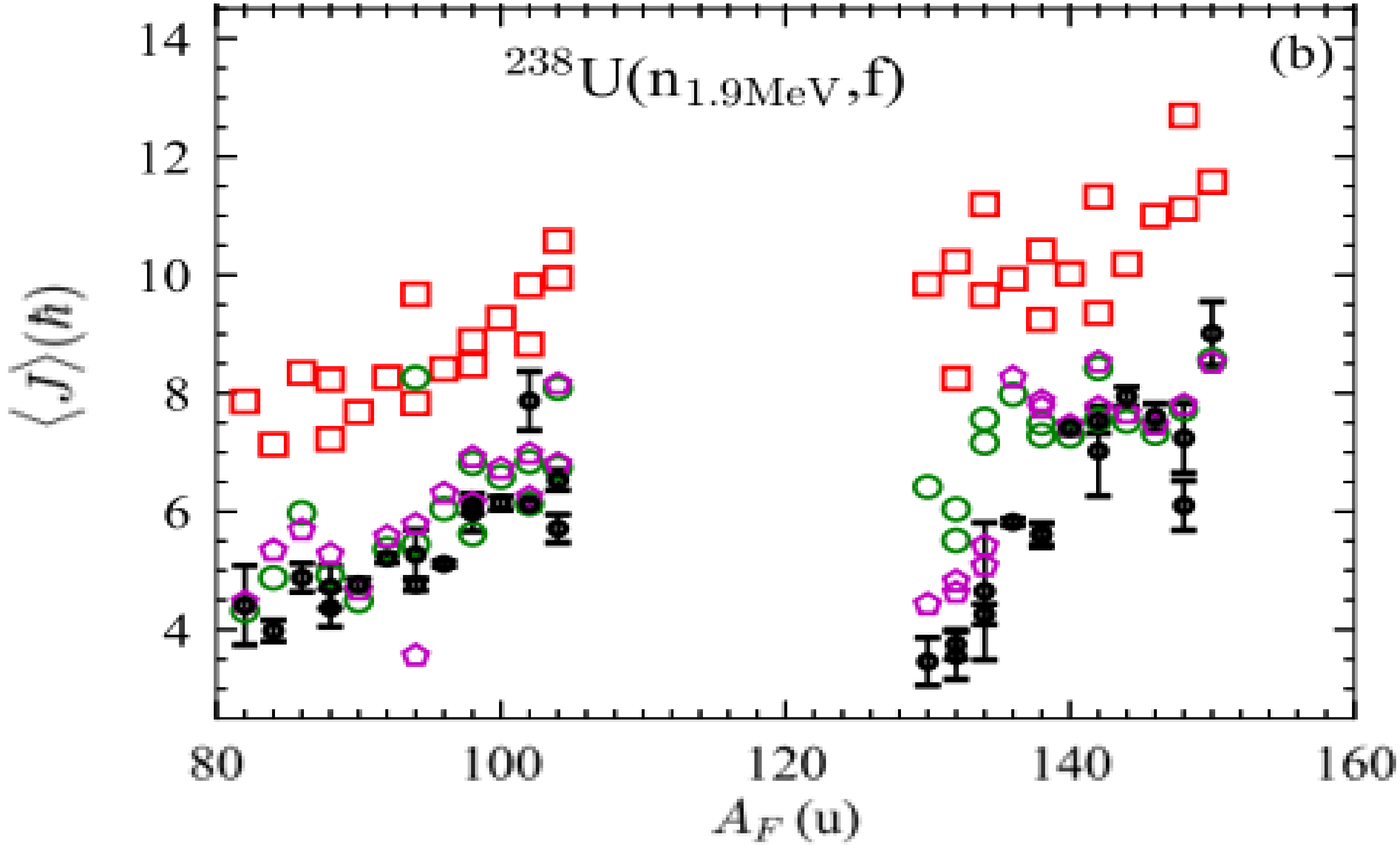
Distribution for the angle between the fragment spins in the case of ^{252}Cf(sf) fission

Aurel Bulgac

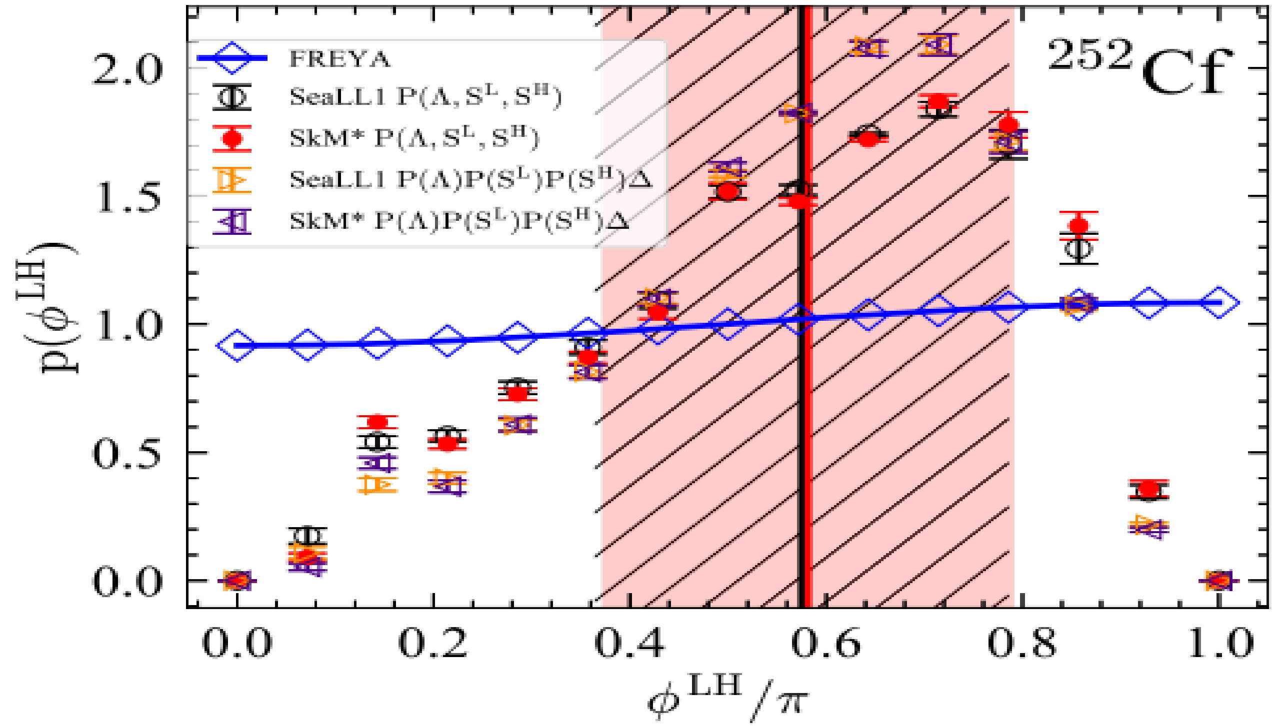
[7] Bulgac A., Magierski P., Roche K. J., Stetcu I.// Phys. Rev. Lett. 116, 122504 2016.
[8] Bulgac A., Abdurrahman I., Godbey K., Stetcu I.// Phys. Rev. Lett. 128, 022501 2022
[9] Bulgac A.// Phys. Rev. C 106, 014624 , 2022.



Distribution of spins of light and heavy fragments for ^{252}Cf(sf)



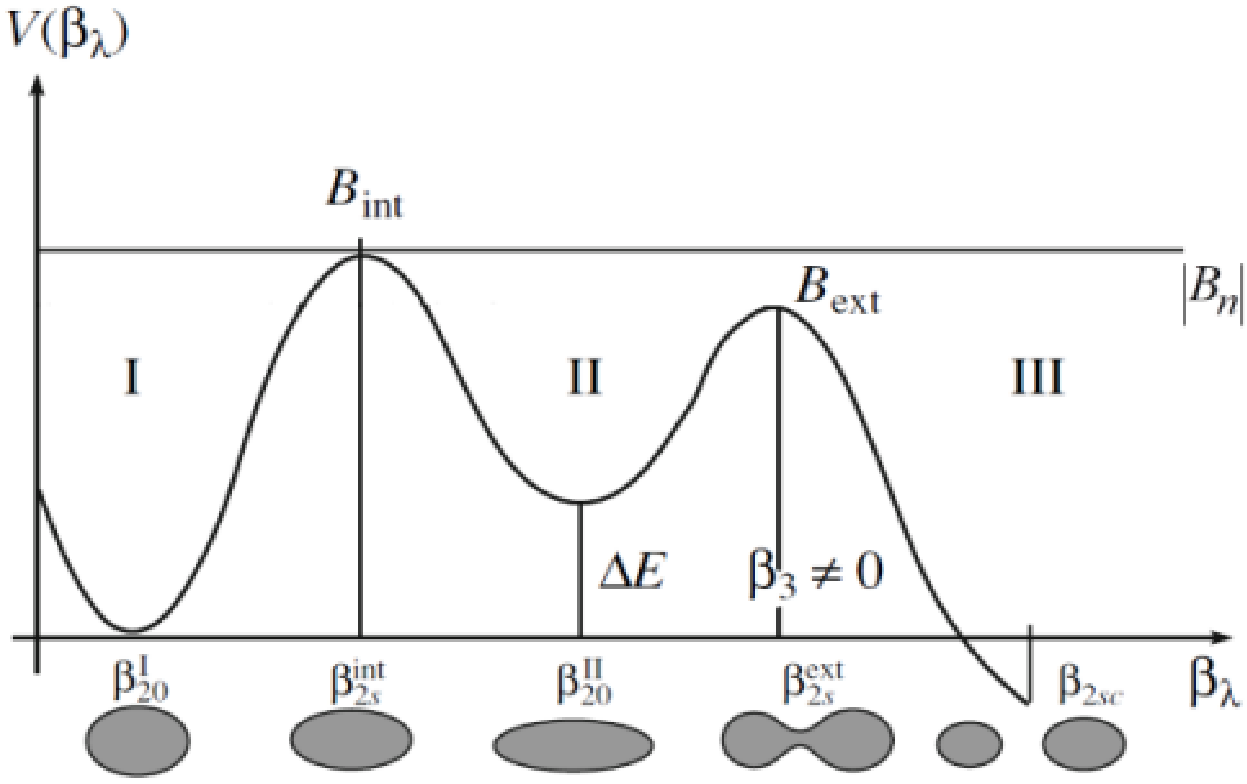
Distribution of spins of light and heavy fragments for $^{238}U(n,f)$



Distribution for the angle between the spins of the fragment in the case of fission ^{252}Cf(sf)

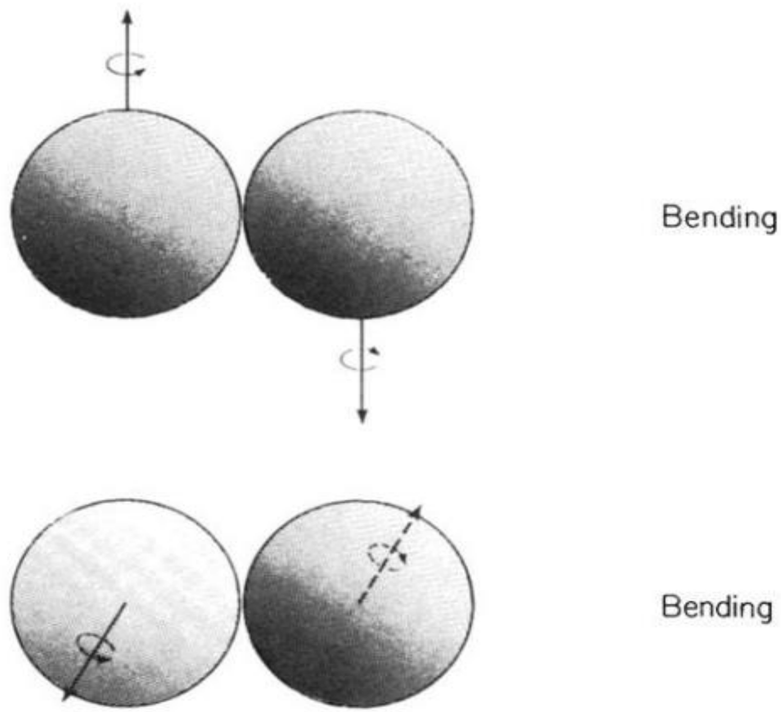
Cold nuclear spontaneous and low-energy induced fission

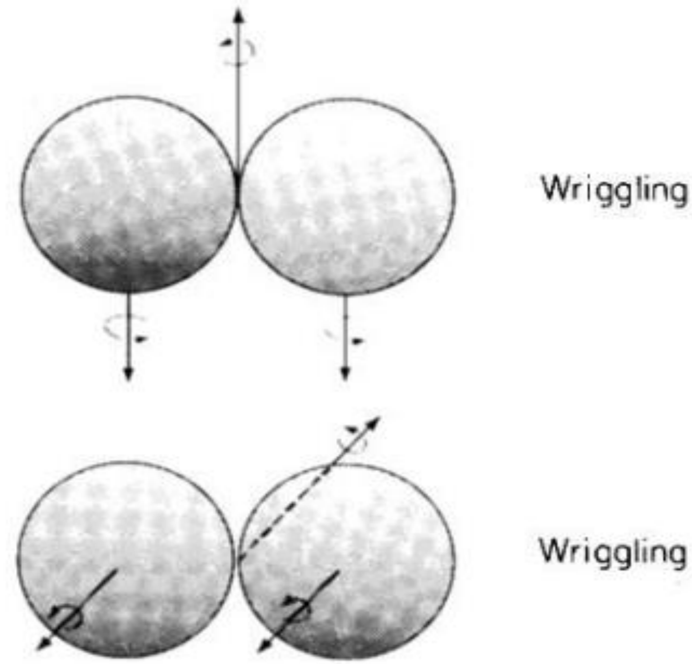
$$\psi_K^{JM} = \sum_{i \neq 0} b_i \psi_{iK}^{JM} + b_0 \psi_{0K}^{JM} (\beta_\lambda).$$



Deformation potential based on the liquid drop model of the nucleus taking into account shell corrections

Transverse oscillations





Bending-oscillations

Wriggling-oscillations

For the considered spontaneous and low-energy induced fission, the fissile nucleus and the emitting fragments must be only in cold nonequilibrium states[10]

$$P\left(J_{b_x}, J_{b_y}\right) = P\left(J_{b_x}\right) P\left(J_{b_y}\right) = \frac{1}{\pi I_b \hbar \omega_b} \exp\left[-\frac{\left(J_{b_x}^2 + J_{b_y}^2\right)}{I_b \hbar \omega_b}\right]$$

$$P\left(J_{w_x}, J_{w_y}\right) = P\left(J_{w_x}\right) P\left(J_{w_y}\right) = \frac{1}{\pi I_w \hbar \omega_w} \exp\left[-\frac{\left(J_{w_x}^2 + J_{w_y}^2\right)}{I_w \hbar \omega_w}\right]$$

[10] *Kadmensky S.G., Bunakov V.E., Lyubashevsky D.E. // Phys. Atom. Nucl.*, 80, № 5 (2017).

Moment of inertia of wriggling-oscillations[5]:

$$I_w = \frac{(I_1 + I_2)I_0}{I}$$

$$I_0 = \frac{M_1 M_2}{M_1 + M_2}\left(R_1 + R_2 + d\right)^2; I = I_0 + I_1 + I_2$$

$$R_{1,2} = r_o A^{1/3}\left[1 - \beta_{1,2}^2 / (4\pi) + \sqrt{5/(4\pi)}\beta_{1,2}\right]$$

Moment of inertia of bending-oscillations[11]:

$$I_b = I_1 + \left(\frac{R_1}{R_2}\right)^2 I_2$$

[11] Shneidman T. M., *et al.* // Phys. Rev. C 65, 064302 (2002).

Relation of spins of wriggling and bending vibrations with projections of spins of fission fragments on X and Y axes perpendicular to the symmetry axis of the fission nucleus Z

$$J_{1x} = \frac{I_1}{I_1 + I_2} J_{w_x} + J_{b_x} \qquad J_{1y} = \frac{I_1}{I_1 + I_2} J_{w_y} + J_{b_x}$$

$$J_{2x} = \frac{I_2}{I_1 + I_2} J_{w_x} - J_{b_x} \qquad J_{2y} = \frac{I_2}{I_1 + I_2} J_{w_y} - J_{b_y}$$

Let us express the projections of spins of wriggling- and bending-oscillations

$$J_{1x}+J_{2x}=\frac{I_1+I_2}{I_1+I_2}J_{w_x}=J_{w_x}$$

$$J_{1y}+J_{2y}=\frac{I_1+I_2}{I_1+I_2}J_{w_y}=J_{w_y}$$

$$J_{b_x}=J_{1x}-\frac{I_1}{I_1+I_2}J_{w_x}=J_{1x}-\frac{I_1(J_{1x}+J_{2x})}{I_1+I_2}=\frac{I_2J_{1x}-I_1J_{2x}}{I_1+I_2}$$

$$J_{b_y}=J_{1y}-\frac{I_1}{I_1+I_2}J_{w_y}=J_{1y}-\frac{I_1(J_{1y}+J_{2y})}{I_1+I_2}=\frac{I_2J_{1y}-I_1J_{2y}}{I_1+I_2}$$

The probability distribution of the spins of two independent oscillations can be represented as a product of the distributions of these oscillations:

$$P\left(J_{b_x}, J_{w_x}, J_{b_y}, J_{w_y}\right) = P(J_{b_x}, J_{b_y})P(J_{w_x}, J_{w_y})$$

Let's pass to distribution on projections of spins of the first and second fragments

$$P\left(J_{1x}, J_{2x}, J_{1y}, J_{2y}\right) = P\left(J_{bx}, J_{wx}, J_{by}, J_{wy}\right) \cdot \left| \frac{\partial\left(J_{bx}, J_{wx}, J_{by}, J_{wy}\right)}{\partial\left(J_{1x}, J_{2x}, J_{1y}, J_{2y}\right)} \right|$$

$$P\left(J_{1x},J_{2x},J_{1y},J_{2y}\right)=\frac{1}{\pi^2 I_w\hbar\omega_w I_b\hbar\omega_b}\exp\left[-\frac{J_{b_x}^2+J_{b_y}^2}{I_b\hbar\omega_b}-\frac{J_{w_x}^2+J_{w_y}^2}{I_w\hbar\omega_w}\right]\cdot\left|\frac{\partial\left(J_{bx},J_{wx},J_{by},J_{wy}\right)}{\partial\left(J_{1x},J_{2x},J_{1y},J_{2y}\right)}\right|=$$

$$=\frac{1}{\pi^2 I_w\hbar\omega_w I_b\hbar\omega_b}\exp\left[\begin{array}{l}-\frac{1}{I_b\hbar\omega_b\left(I_1+I_2\right)^2}\left\{(I_2J_{1x}-I_1J_{2x})^2+(I_2J_{1y}-I_1J_{2y})^2\right\}-\\ \qquad -\frac{1}{I_w\hbar\omega_w}\left\{(J_{1x}+J_{2x})^2+(J_{1y}+J_{2y})^2\right\}\end{array}\right]$$

Passing to polar coordinates, we obtain the distribution:

$$P\left(J_1,J_2,\phi\right)=\frac{2J_1J_2}{\pi I_w\hbar\omega_w I_b\hbar\omega_b}\exp\left[\begin{array}{l}-J_1{}^2\left\{aI_2^2+b\right\}-J_2{}^2\left\{aI_1^2+b\right\}+\\ +2J_1J_2\cos\phi\left\{aI_1I_2-b\right\}\end{array}\right]$$

where $a=\frac{1}{I_b\hbar\omega_b\left(I_1+I_2\right)^2}$; $b=\frac{1}{I_w\hbar\omega_w}$;

ϕ is angle between the fragments

Integrating by projections to the spins of the second fragment and passing to polar coordinates, we obtain the distribution for the first fission fragment (a similar distribution can be obtained for the second fragment):

$$P\left(J_1\right)=\frac{2J_1}{d_1}\exp\left[-\frac{J_1{}^2}{d_1}\right] \qquad P\left(J_2\right)=\frac{2J_2}{d_2}\exp\left[-\frac{J_2{}^2}{d_2}\right]$$

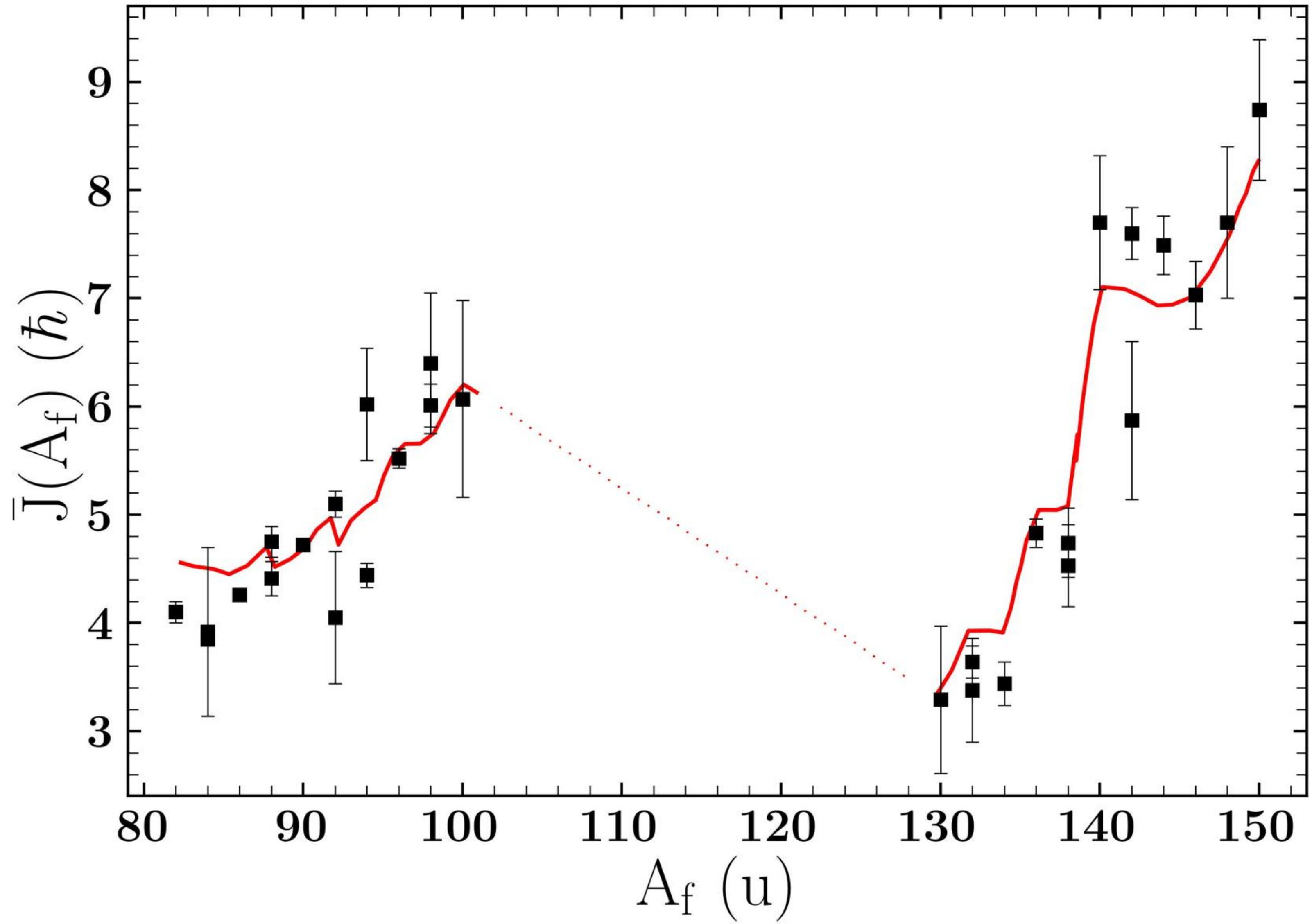
$$d_1=\frac{I_1{}^2 I_w \hbar\omega_w + I_b\hbar\omega_b\left(I_1+I_2\right)^2}{\left(I_1+I_2\right)^2} \qquad d_2=\frac{I_2{}^2 I_w \hbar\omega_w + I_b\hbar\omega_b\left(I_1+I_2\right)^2}{\left(I_1+I_2\right)^2}$$

From $P(J_1)$ and $P(J_2)$, a convenient expression for calculating the average value of the fragment spin can be obtained:

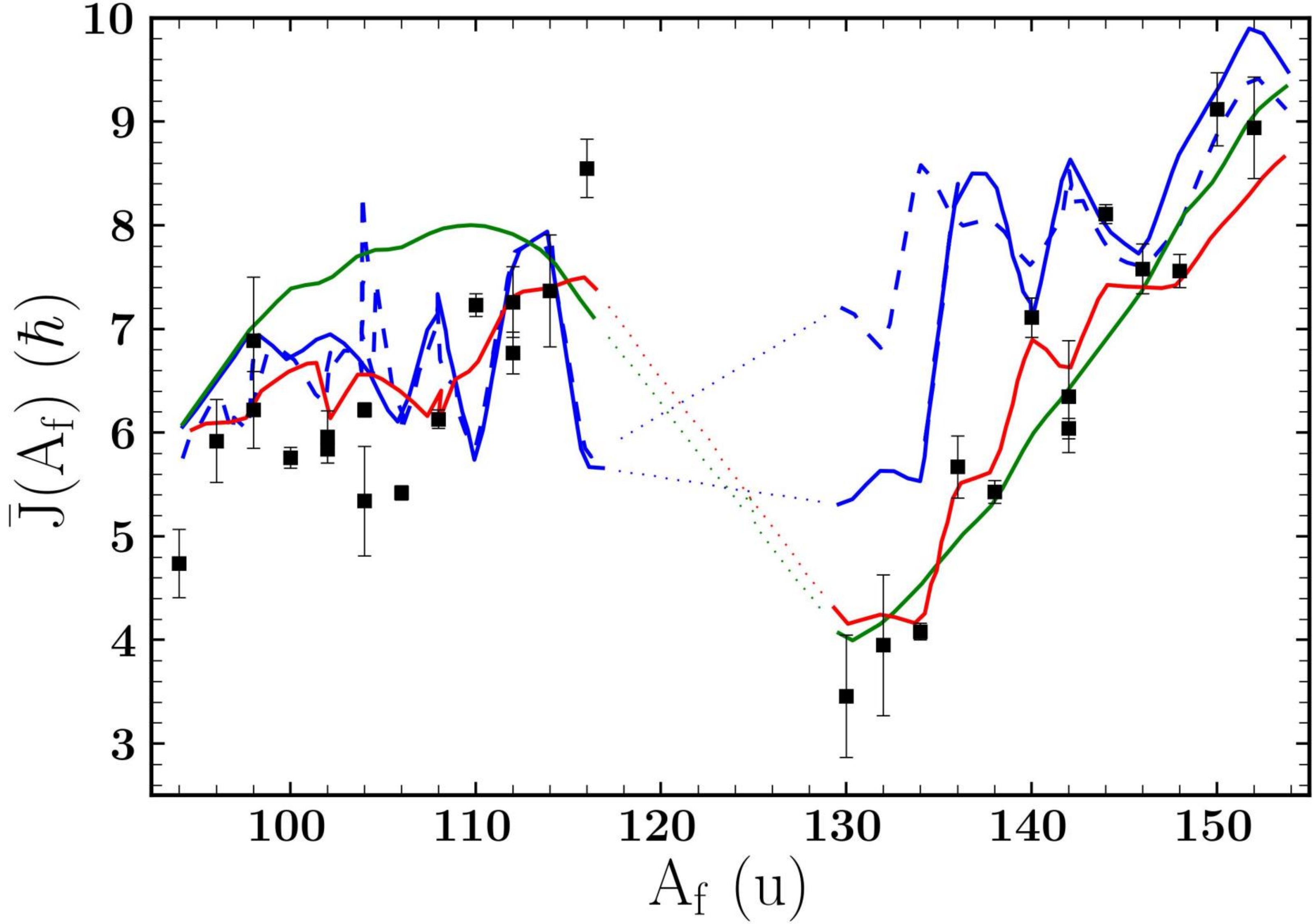
$$\overline{J_1} = \int_0^\infty J_1 \frac{2J_1}{d_1} \exp\left[-\frac{J_1{}^2}{d_1}\right] dJ_1 = \frac{\sqrt{\pi d_1}}{2}$$

$$\overline{J_2} = \int_0^\infty J_2 \frac{2J_2}{d_1} \exp\left[-\frac{J_2{}^2}{d_1}\right] dJ_2 = \frac{\sqrt{\pi d_2}}{2}$$

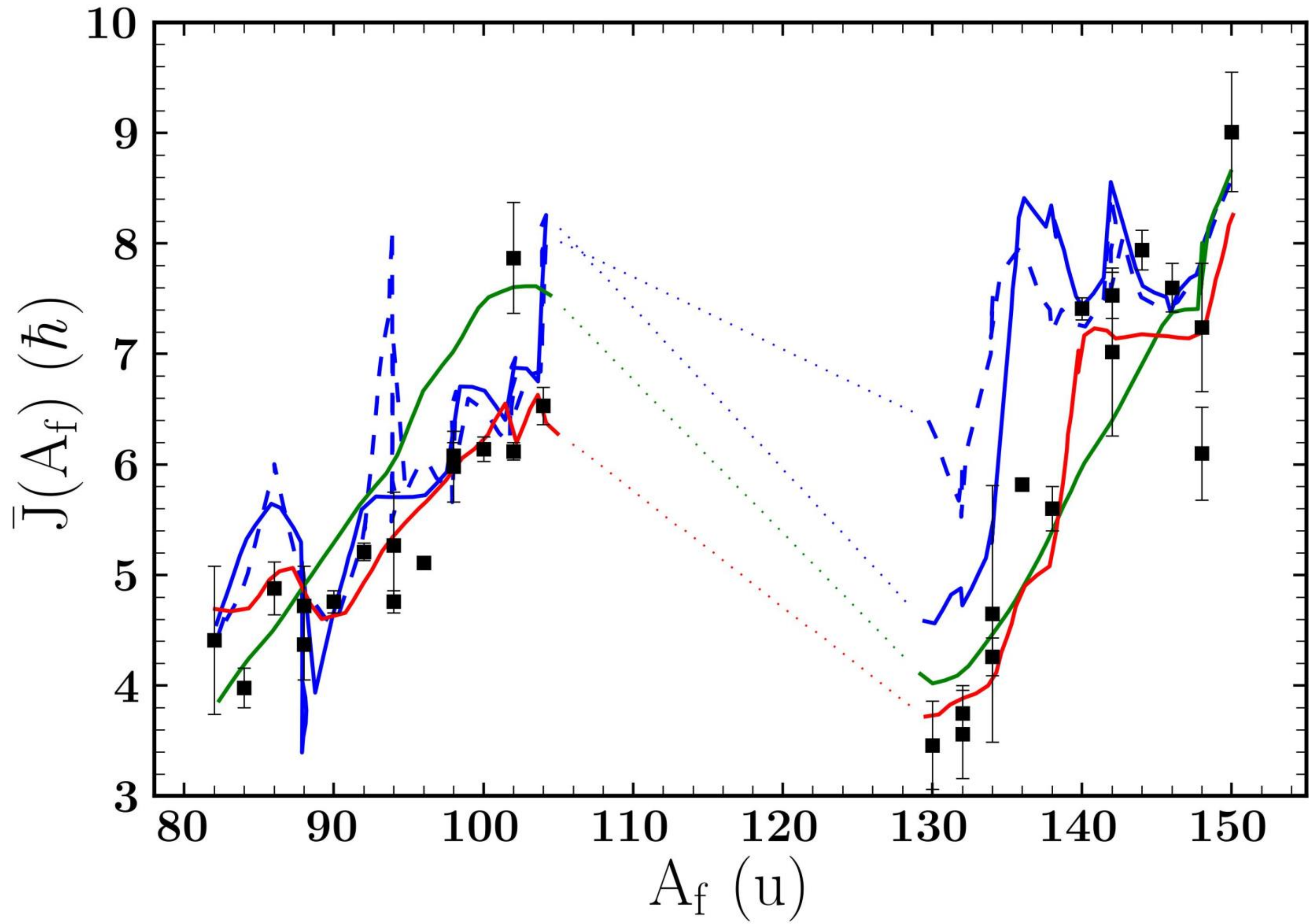
Average values of spins of binary fission fragments for ^{232}Th(n,f)



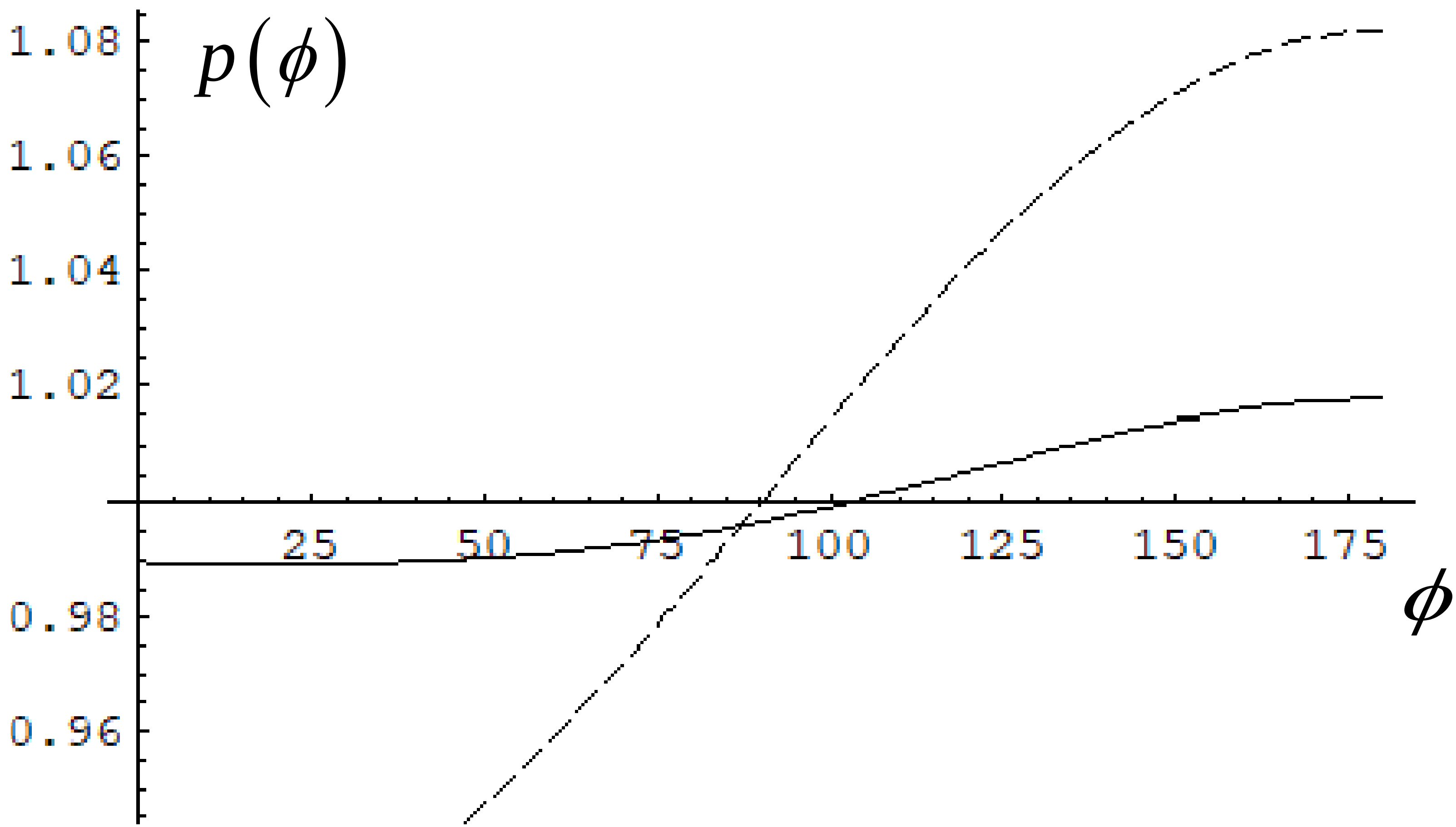
Average values of spins of binary fission fragments for $^{252}Cf(sf)$



Average values of spins of binary fission fragments for $^{238}U(n,f)$



Comparison with the results of [3]. The solid line is the distribution within the framework of the present work; the dashed line is the distribution within the framework of [3] for $^{232}Th(n,f)$



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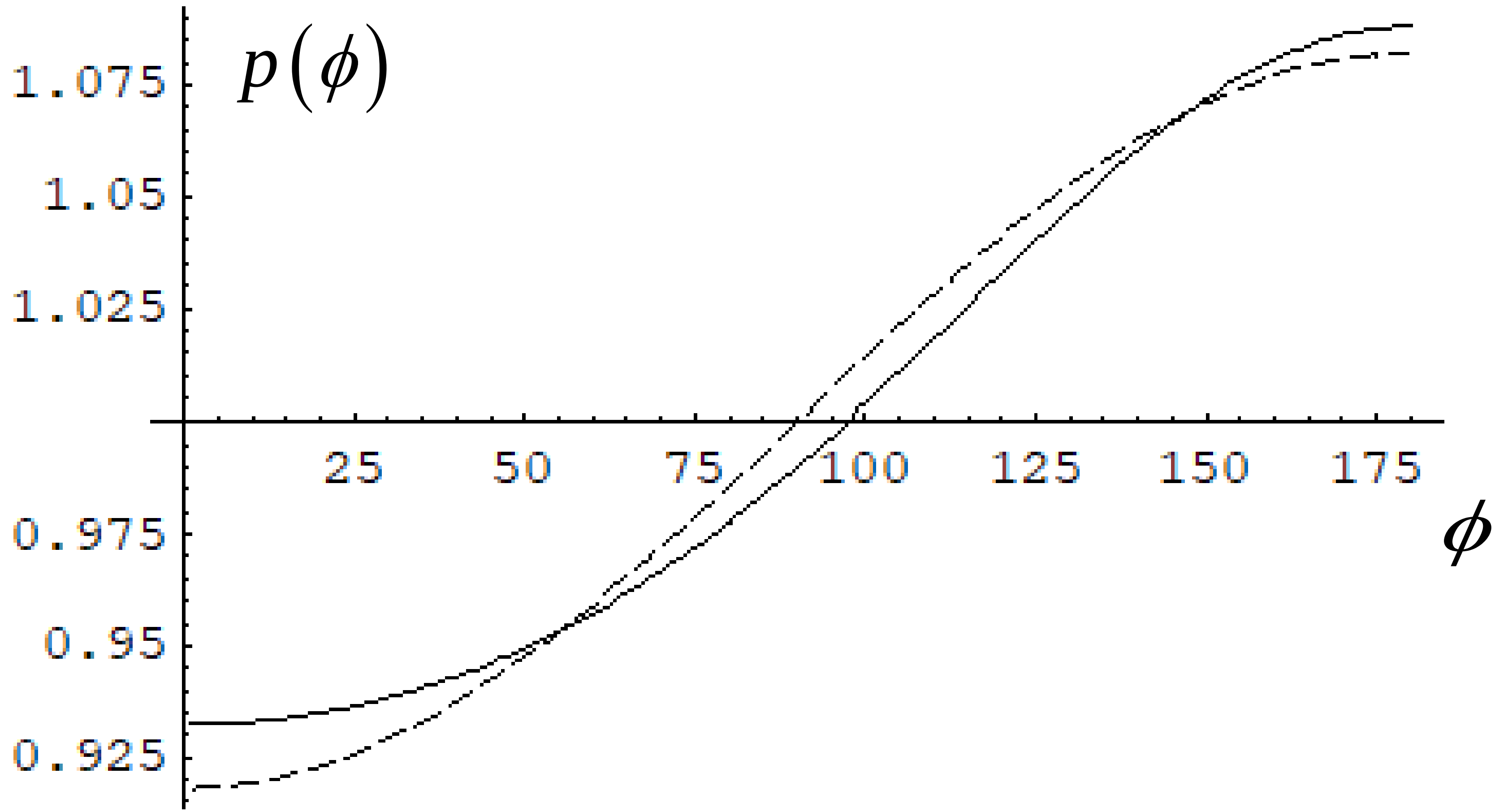

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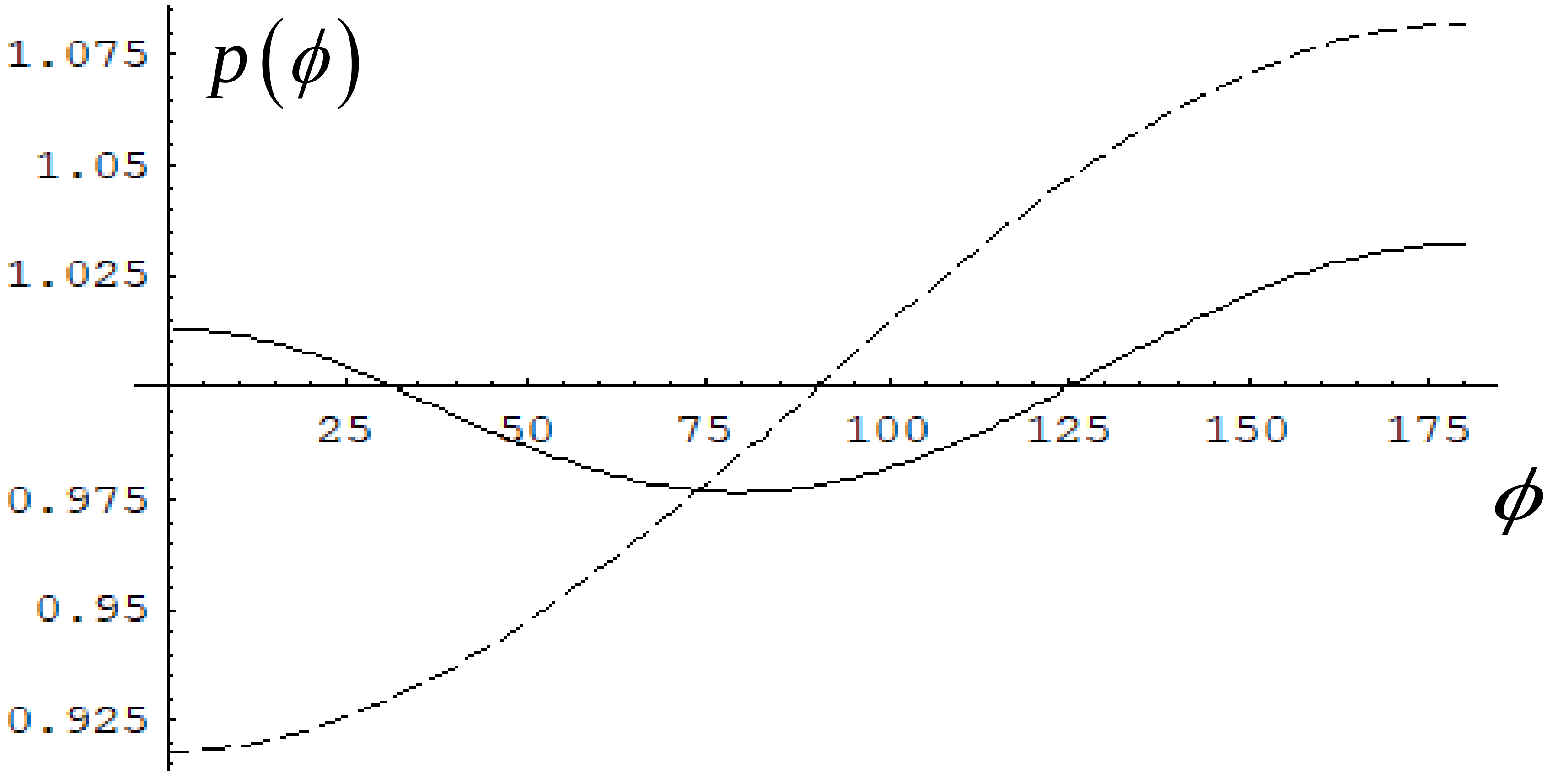

Conclusions

- Spin distributions of spontaneous and low-energy binary fission fragments are obtained;
- Theoretical provisions are taken into account: coldness of the fissile nucleus at the scission point; consideration of zero transverse bending- and wriggling-oscillations of the nucleus; appearance of large relative orbital moments of the fissile nucleus system; accounting of the total momentum of motion conservation law;

Conclusions

- The use of the momentum distributions allowed us to move from the generalized temperature parameter to the frequencies of wriggling- and bending-oscillations;
- This work established a weak angular correlation between the spins of the fission fragments (< 10%);
- The moments of inertia of the fission fragments were calculated within the framework of the liquid drop model of the nucleus.

Thank you for your attention!