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Orbital momenta of fragments in binary asymmetric fission of actinide nuclei

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Angular distribution of fission fragments for low-energy induced and spontaneous fission

$$d\sigma \sim \sum_{M,K} a_M b_K \{ | D^J_{MK}(\theta) |^2 + | D^J_{M-K}(\theta) |^2 \} \sin\theta d\theta, \quad (1)$$

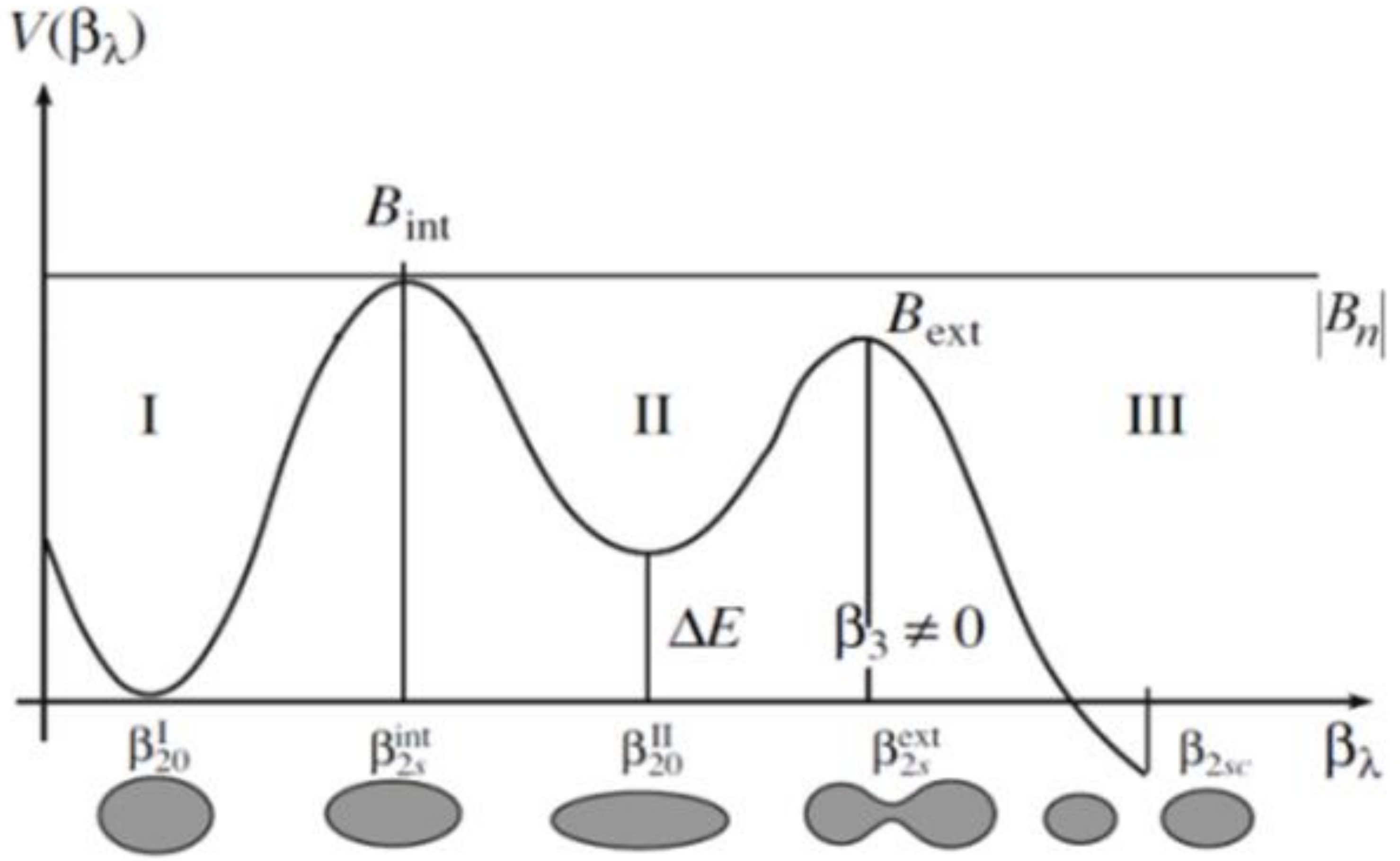
- where θ is the angle that is measured from the axis of symmetry of the fissile nucleus in the direction of the fission fragments flight in the laboratory coordinate system; M and K are the projections of the spin direction on the axis of laboratory frame and on the fissile nucleus symmetry axis
- For the anisotropic nature of the directions of fragments flight only a few values of the quantum number K should contribute to the sum (1)

$$P^J_{MK}(\Omega) = \frac{2J+1}{16\pi^2} \int d\omega \left[\left| D^J_{MK}(\omega) \right|^2 + \left| D^J_{M-K}(\omega) \right|^2 \right] P(\Omega'). \quad (2)$$

- The angular distribution of the fission fragments in intrinsic coordinate system

$$P(\Omega') = \left| A(\Omega') \right|^2 = \left| \sum_L \psi_L Y_{L0}(\Omega') \right|^2, \quad (3)$$

The fissile nucleus deformation potential



Coriolis interaction: Kadmensky S.G., Markushev V.P., Furman W.I. // Phys. At. Nucl. 1982. V. **35**. P. 300.

- The hypothesis of O. Bohr [1]: the directions of fission fragments flight close to the axis of symmetry
- It allows us to represent the angular distribution of fragments in the form of a smeared delta function determined by large relative orbital momenta L.
- To observe anisotropies in the angular distributions of binary fission fragments, it is necessary that the nucleus remains cold until it breaks into fission fragments
- In order to explain large L values it is necessary to analyze the appearance of zero collective transverse vibrations of the fission prefragments: wriggling-vibrations.

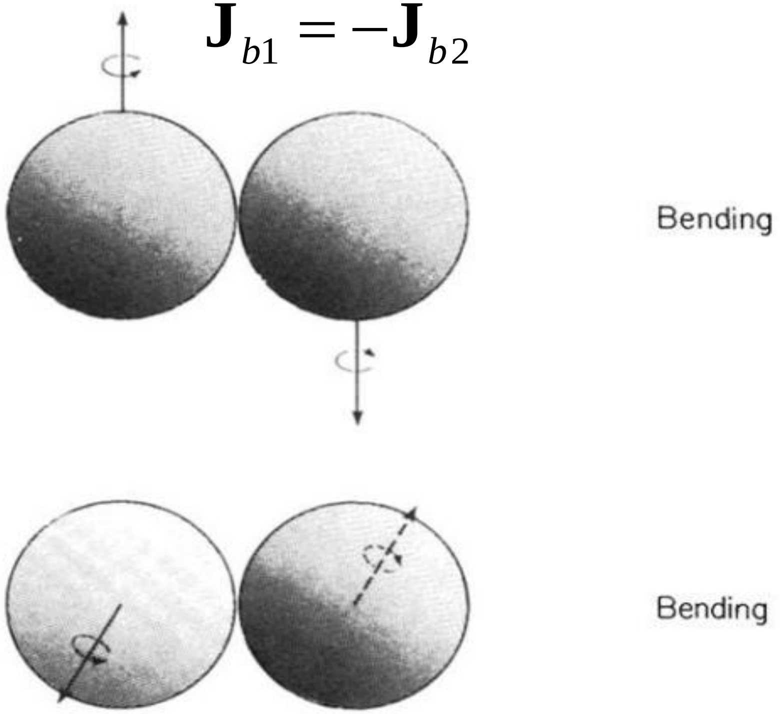
[1] *Bohr A., Mottelson B.* Nuclear Structure. Benjamin. N-Y. 1974. **2**

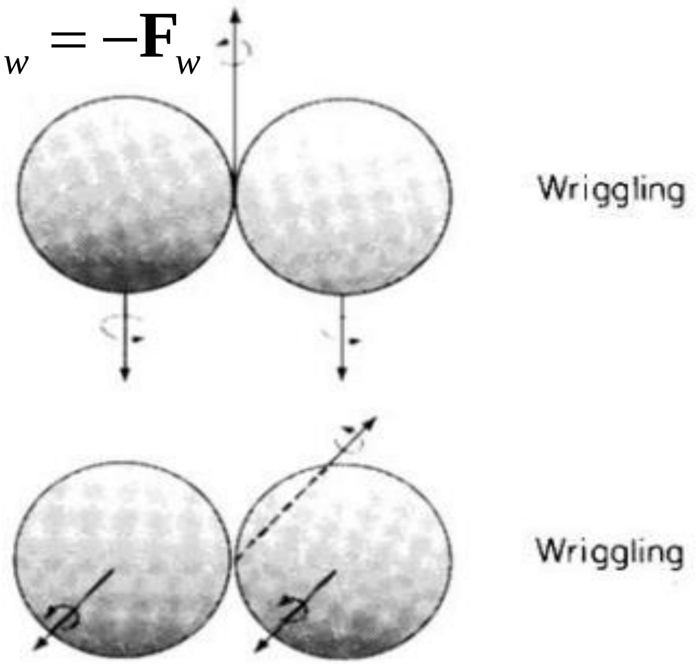
Transverse vibrations [2]

$$\mathbf{F}_w = \mathbf{J}_{w1} + \mathbf{J}_{w2}$$

$$\mathbf{J}_{b1} = -\mathbf{J}_{b2}$$

$$\mathbf{L}_w = -\mathbf{F}_w$$





Bending-vibrations

Wriggling-vibrations

[2] *Nix J.R. and Swiatecki W.J.* // Nucl. Phys. 1965. **71**, P. 1.

The fission fragments spin distribution In polar coordinates [5], [6]:

$$P(J_1,J_2,\phi)=\frac{2J_1J_2}{\pi I_w\hbar\omega_w I_b\hbar\omega_b}\exp\left[\begin{array}{l}-J_1^{\,2}\left\{aI_2^2+b\right\}-J_2^{\,2}\left\{aI_1^2+b\right\}+\\ +2J_1J_2\cos\phi\left\{aI_1I_2-b\right\}\end{array}\right]$$

$a=\dfrac{1}{I_b\hbar\omega_b(I_1+I_2)^2}$; $b=\dfrac{1}{I_w\hbar\omega_w}$, ϕ - angle between fragments' spins

Moment of the inertia for wriggling-vibrations[4]:

$$I_w=\frac{(I_1+I_2)I_0}{I}\qquad I_0=\frac{M_1M_2}{M_1+M_2}\left(R_1+R_2+d\right)^2;I=I_0+I_1+I_2$$

$$R_{1,2}=r_oA^{1/3}\left[1-\beta_{1,2}^2/(4\pi)+\sqrt{5/(4\pi)}\beta_{1,2}\right]$$

[4] *Randrup J.* // Phys. Rev. C.106. L051601 2022

[5] *Lyubashevsky et al.* // ISINN – 2024

[6] *Kadmensky S.G., Bunakov V.E., Lyubashevsky D.E.* // *Phys. Atom. Nuclei*, 80, № 5 (2017).

Relative orbital moment and relative fission fragments spin

$$\mathbf{L} = -(\mathbf{J}_1 + \mathbf{J}_2); \quad \mathbf{G} = \left(\mathbf{J}_1 - \mathbf{J}_2\right);$$

$$\mathbf{J}_1 = -\mathbf{L}/2 + \mathbf{G}; \quad \mathbf{J}_2 = -\mathbf{L}/2 - \mathbf{G}.$$

$$\mathbf{L}^2 = (\mathbf{J}_1 + \mathbf{J}_2)^2 = (J_{1x} + J_{2x})^2 + (J_{1y} + J_{2y})^2;$$

$$\mathbf{G}^2 = (\mathbf{J}_1 - \mathbf{J}_2)^2 = (J_{1x} - J_{2x})^2 + (J_{1y} - J_{2y})^2.$$

The relative orbital momenta distribution

$$P(\mathbf{L}) = \frac{1}{\pi I_w \hbar \omega_w} \exp\left[-\frac{\mathbf{L}^2}{I_w \hbar \omega_w}\right]$$

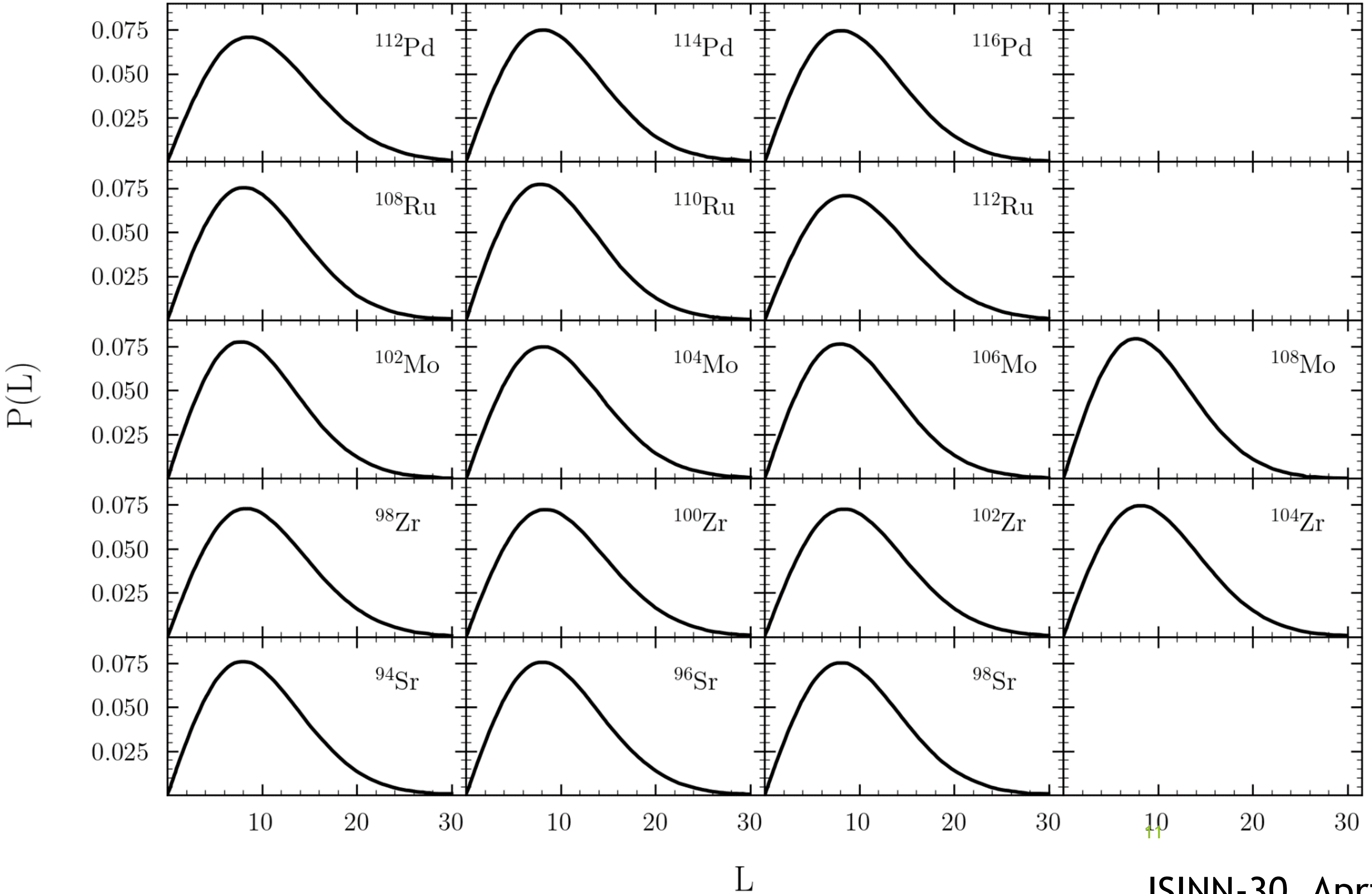
Integrating in the polar coordinate system, obtain a distribution by L

$$P(L) = \frac{2L}{I_w \hbar \omega_w} \exp\left[-\frac{L^2}{I_w \hbar \omega_w}\right] = \frac{2L}{C_w} \exp\left[-\frac{L^2}{C_w}\right];$$

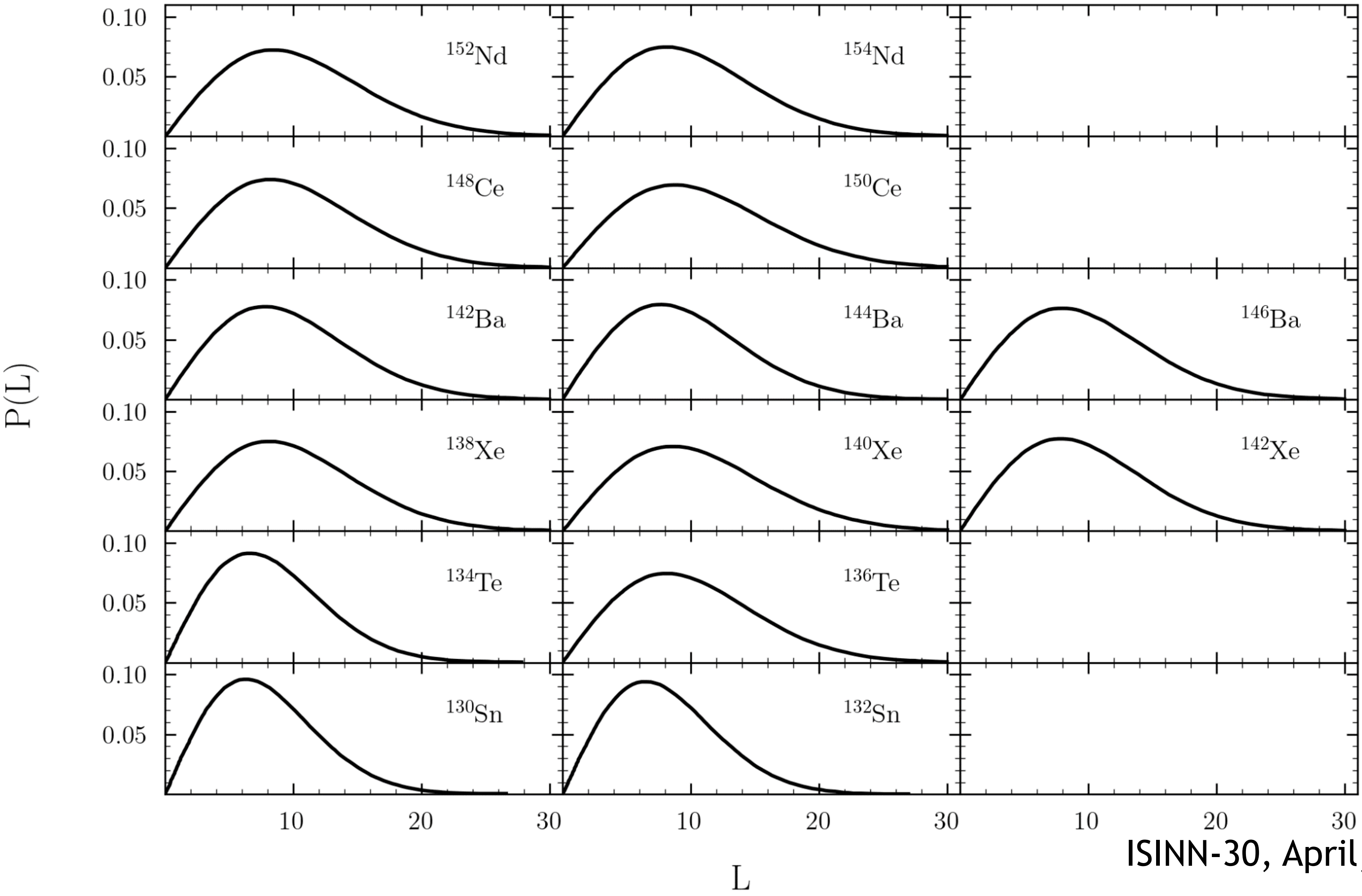
$$\bar{L} = \int_0^\infty \frac{2L^2}{I_w \hbar \omega_w} \exp\left[-\frac{L^2}{I_w \hbar \omega_w}\right] dL = \frac{\sqrt{C_w \pi}}{2}.$$

$$C_w = I_w \hbar \omega_w$$

Relative orbital momenta L distribution for $^{252}Cf(s,f)$



Relative orbital momenta L distribution for $^{252}Cf(s,f)$



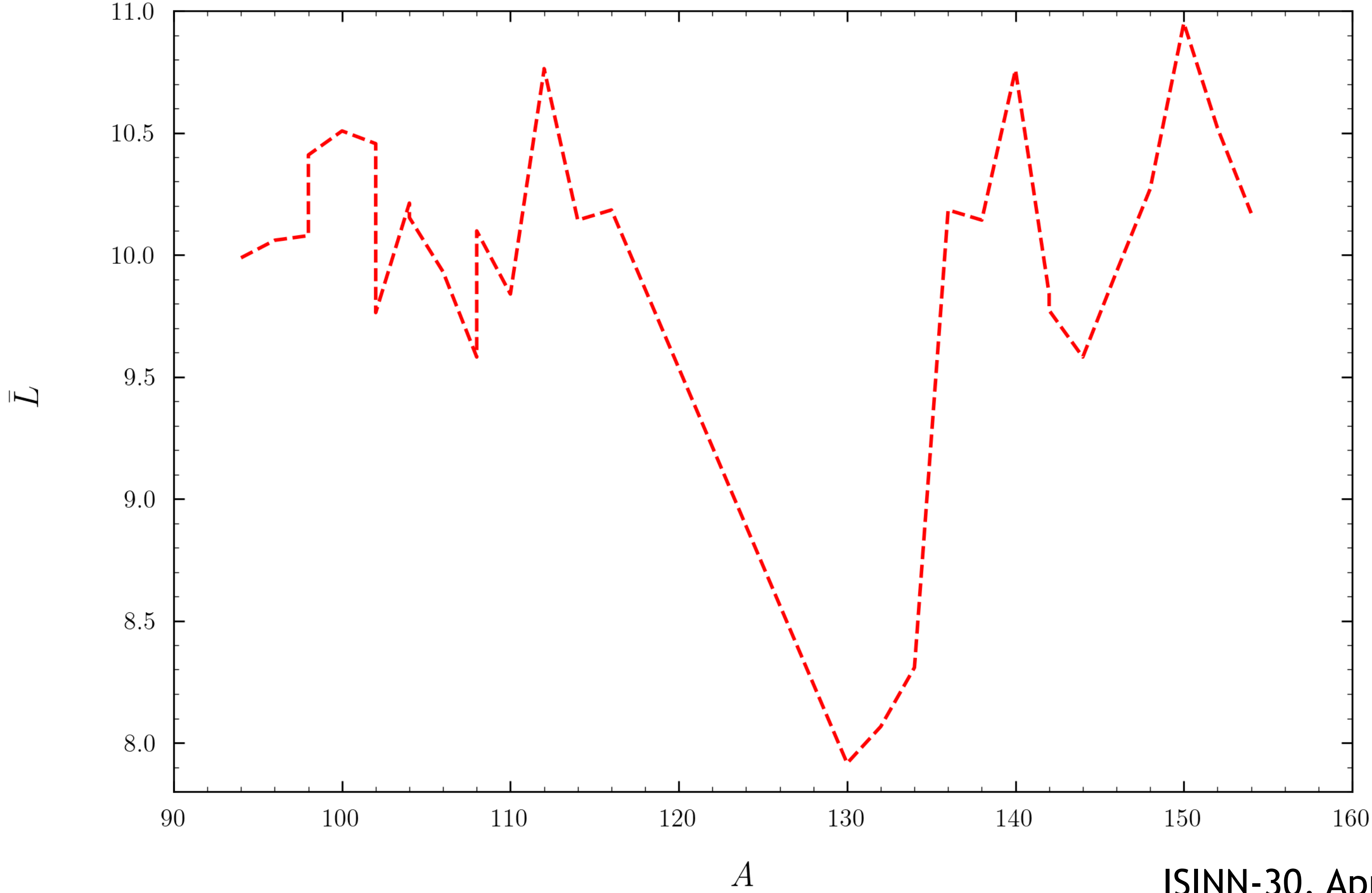
Dependence of the <L> from the mass number A_1 in $^{252}Cf(s,f)$

A_1	<L>	Cw, ħ
^{94}Sr	9.99	61.06
^{96}Sr	10.06	61.95
^{98}Sr	10.08	62.18
^{98}Zr	10.41	58.96
^{100}Zr	10.51	60.08
^{102}Zr	10.46	59.48
^{104}Zr	10.21	56.74
^{102}Mo	9.76	51.87
^{104}Mo	9.12	56.09
^{106}Mo	9.93	53.66
^{108}Mo	9.58	49.95
^{108}Ru	10.10	49.93
^{110}Ru	9.84	47.41
^{112}Ru	10.76	47.23
^{112}Pd	10.76	47.28
^{114}Pd	10.14	41.98
^{116}Pd	10.18	42.32

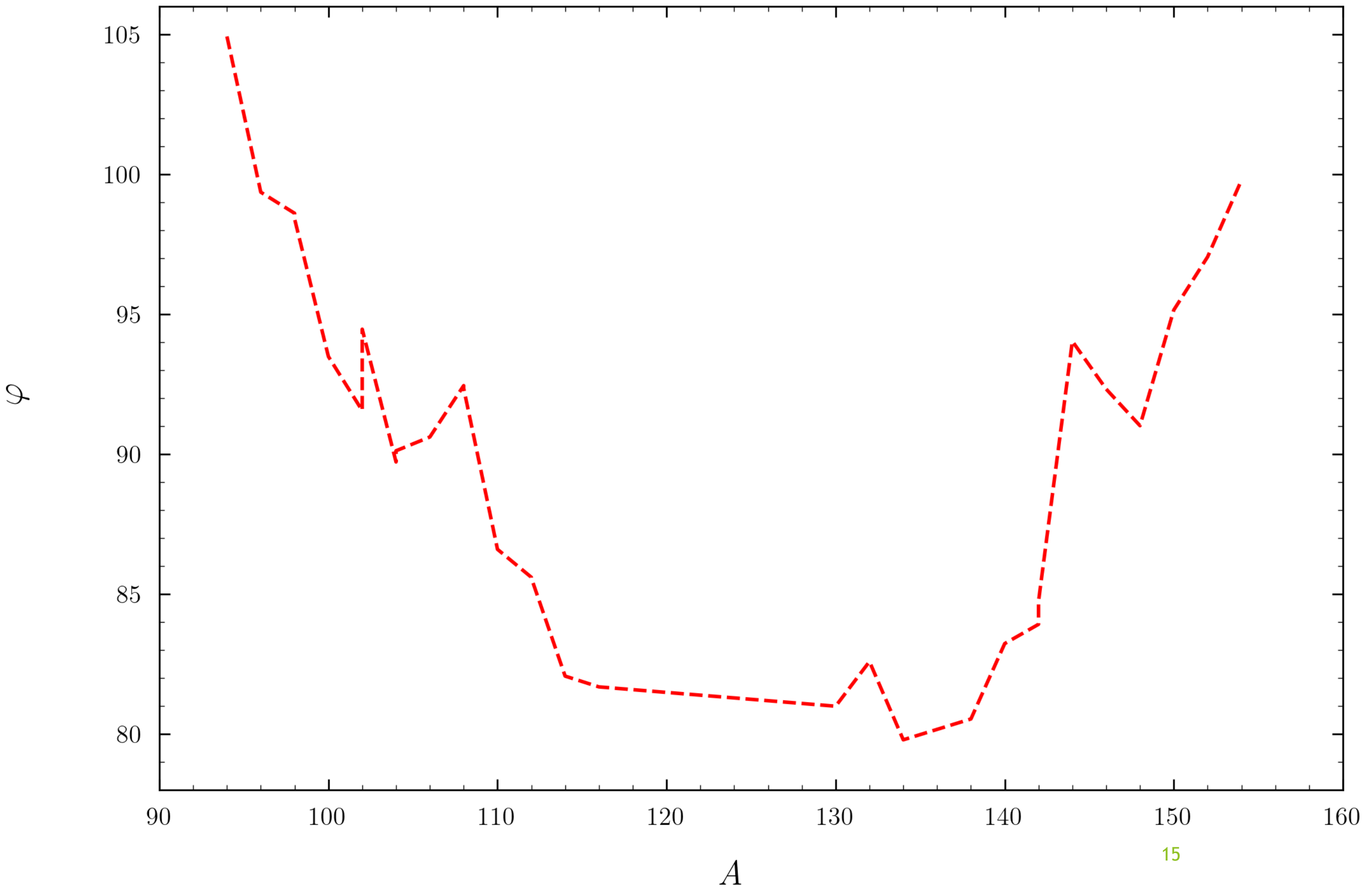
A_1	<L>	Cw, ħ
^{130}Sn	7.92	27.90
^{132}Sn	8.07	28.97
^{134}Te	8.31	30.73
^{136}Te	10.18	42.32
^{138}Xe	10.14	41.98
^{140}Xe	10.76	47.23
^{142}Xe	9.84	47.41
^{142}Ba	9.77	51.96
^{144}Ba	9.58	49.95
^{146}Ba	9.93	53.66
^{148}Ce	10.27	57.39
^{150}Ce	10.95	65.24
^{152}Md	10.52	60.21
^{154}Md	10.17	63.32

$$C_w = I_w \hbar \omega_w$$

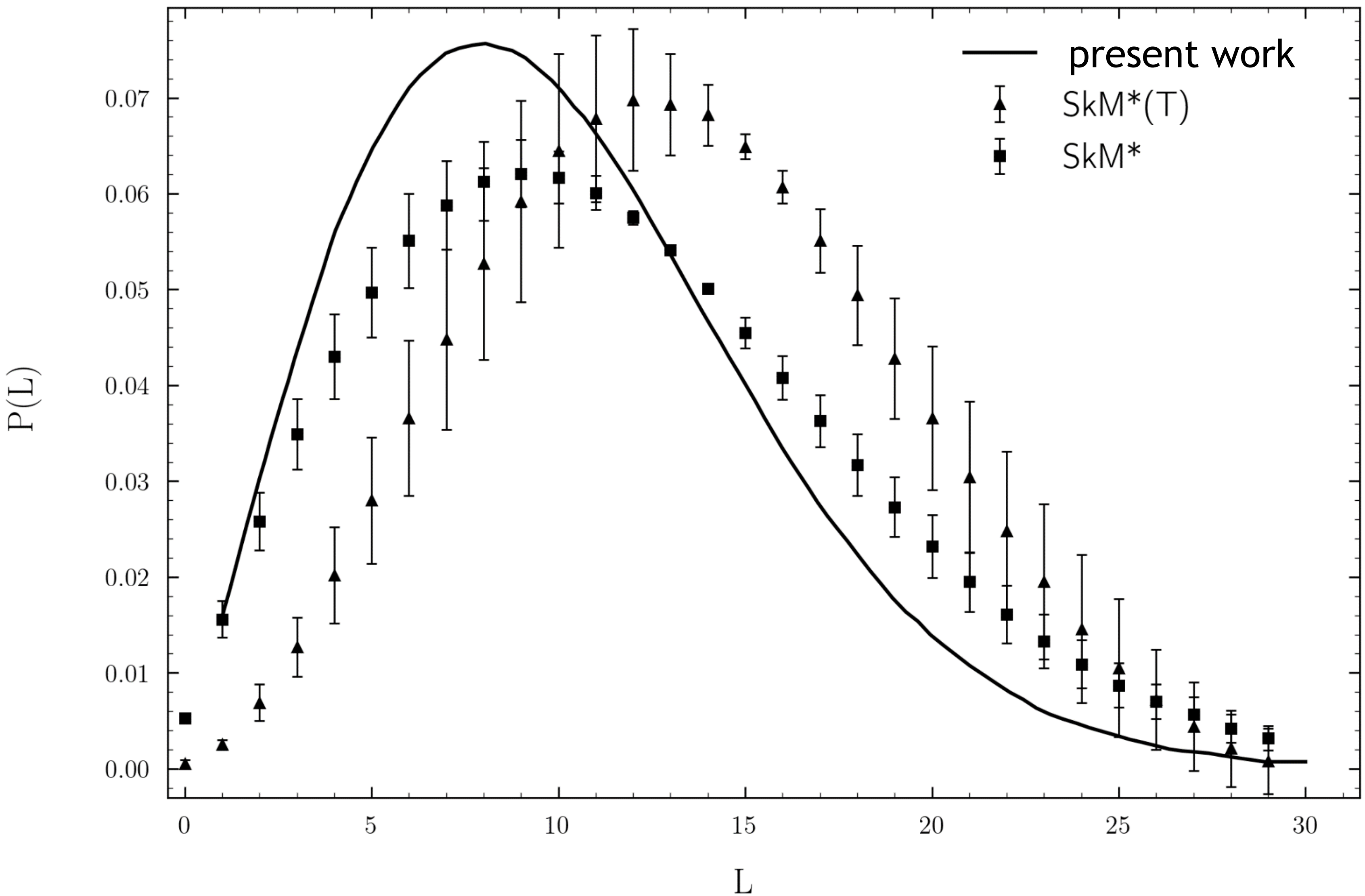
Dependence of the average value of the orbital moment on the fragments' mass number $^{252}Cf(s,f)$



Dependence of the angle between the spins of the 1st and 2nd fission fragments on the fragments’ mass number A



The distribution of the relative orbital momenta for ^{252}Cf(s,f)



Bulgac A., Abdurrahman I., Godbey K., and Stetcu I. // Phys. Rev. Lett. 2022. **128**, 022501.

Conclusion

- Taking into account the "coldness" of the fissile nucleus in its' rupture point and zero transverse wriggling vibrations of the nucleus, the appearance of large relative orbital momenta of fragments in spontaneous and low-energy binary fission of nuclei is demonstrated
- Using the obtained spin distributions of the fission fragments, as well as the moments of inertia of the fission fragments within the framework of the superfluid nucleus model, the average values of the relative orbital momenta of the fission fragments were calculated, which range is from 8 to 11 ħ for ^{252}Cf(s,f).
- A reasonable agreement was obtained between the angular momentum distribution calculated by obtained formula and the theoretical approach [*Bulgac A. et al,* // Phys. Rev. Lett. 2022. **128**, 022501]. For the ^{252}Cf the obtained agreement indicates the importance of taking into account quantum effects in the analysis of nuclear fission.

Thank you for attention!