Fundamental differences in theoretical approaches to describing of the observed characteristics of spontaneous and induced binary and ternary (with the emission of nucleons and light nuclei as third particles) nuclear fission

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1. Introduction.

Widely used theoretical approaches [J. Wilson et al., Nature, 2021. **590**. 566] to the description of the observed characteristic of spontaneous and induced binary fission of fissile nuclei (FN) (A, Z) into light (A_L, Z_L) and heavy (A_H, Z_H) fragments of decay although take into account quantum mechanical concepts associated, for example, with the use of transition fission states of these nuclei [A. Bohr and B. Mottelson, Nuclear Structure (W.A. Benjamin, NY, Amsterdam, 1969)], are largely macroscopic in nature, associated with hydrodynamic (liquid drop model of the nucleus taking into account shell corrections) and thermodynamic (Gibbs distributions taking into account various temperatures) characteristics of fission fragments. In this work we will demonstrate the possibility of successfully describing these observed characteristics of nuclear binary fission using developed in [A. Bohr and B. Mottelson, Nuclear Structure (W.A. Benjamin, NY, Amsterdam, 1969); J. R. Nix and W. J. Swiatecki, Nucl. Phys. A. 1965. **71**. 1; S.G. Kadmensky, V.E. Bunakov, D.E. Lyubashevsky, Phys. of At. Nucl. 2016. **80**. 441; S.G. Kadmensky, L.V. Titova, D.E. Lyubashevsky, Phys. of At. Nucl. 2020. **83**. 581; S.G. Kadmensky, L.V. Titova, Bull. Russ. Acad. Sci., Phys. 2021. **85**. 732; S.G. Kadmensky et. al., Bull. Russ. Acad. Sci., Phys. 2022. **86**. 1332; S.G. Kadmensky, D.E. Lyubashevsky, Bull. Russ. Acad. Sci., Phys. 2022. **86**. 991] the consistent quantum mechanical theory of nuclear binary fission.

To describe the observed characteristics of spontaneous and induced ternary fission of nuclei with the emission of nucleons and light nuclei as the third particles from prescission configuration of FN the quantum mechanical presentations [S.G. Kadmensky, L.V. Titova, D.E. Lyubashevsky, Phys. of At. Nucl. 2020. **83**. 581; S.G. Kadmensky, L.V. Titova, Bull. Russ. Acad. Sci., Phys. 2021. **85**. 732; S.G. Kadmensky et. al., Bull. Russ. Acad. Sci., Phys. 2022. **86**. 1332; S.G. Kadmensky, D.E. Lyubashevsky, Bull. Russ. Acad. Sci., Phys. 2022. **86**. 991] are used about the virtual mechanism of this emission. This mechanism can be presented as the two-stage process, when in the first stage the third light charged particle with kinetic energy T_3, close to the height of its Coulomb barrier, is emitted from the FN with the formation of the virtual state of the intermediate nucleus, which at the next stage undergoes binary fission. This mechanism made it possible to successfully describe the observed characteristics of ternary nuclear fission.

2. Quantum mechanisms of binary spontaneous and induced (with the participation of thermal neutrons) fission of nuclei

A description of the observed characteristics of binary spontaneous and induced fission of FN can be carried out using the representations of quantum fission theory in which binary fission is considered by analogy with alpha decay of nuclei as the decay of the parent nucleus (A, Z) into light (A_L, Z_L) and heavy (A_H, Z_H) fragments of decay under the condition $A=A_L+A_H$; $Z=Z_L+Z_H$. In this case the relative yield of a fixed pair of fission fragments L and H is determined by the transitional fission state [A. Bohr and B Mottelson, Nuclear Structure, Benjamin. **1, 2** (1977)] of FN, associated with the collective deformation movement of FN in the direction of the appearance of the analyzed pair. It is accepted that in binary fission processes during the evolution of the transitional fission state, the shape of FN passes from a shape close to spherical, through an elongated spheroid to a dumbbell shape with a neck for the prescission configuration of FN, which is transformed to the shape of two separated initial fission fragments (see . Fig. 1).

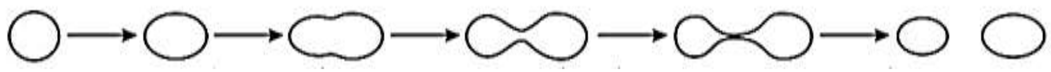

Fig. 1. Successive stages of evolution of FN form during binary fission

To describe this evolution, we can introduce the potential deformation energy $E\left(\beta_\lambda\right)$ of FN using formula [A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, 1977) Vol. 1,2]:

$$E(\beta_\lambda) = E(\beta_\lambda) + E_{sh}(\beta_\lambda), \tag{1}$$

where $E(\beta_\lambda)$ – is the binding energy of FN, calculated in the droplet model of the nucleus [C.F. Weizssacker, Zs. f. Phys. **96**, 431(1935)] at values of the nucleus deformation parameters β_λ, and $E_{sh}(\beta_\lambda)$ – is the Strutinsky shell correction [V.M. Strutinsky, JETP. **37**, 613 (1960); Nucl. Phys. **3**, 614 (1965)[JETP **37**, 613 (1959)]]. Then we can introduce [A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, 1977) Vol. 1,2] the deformation potential $V(\beta_\lambda)$ of FN: $V(\beta_\lambda) = E(\beta_\lambda) - E(\beta_{20})$, where β_{20} – is the ground state quadrupole deformation parameter FN, presented in Fig. 2.

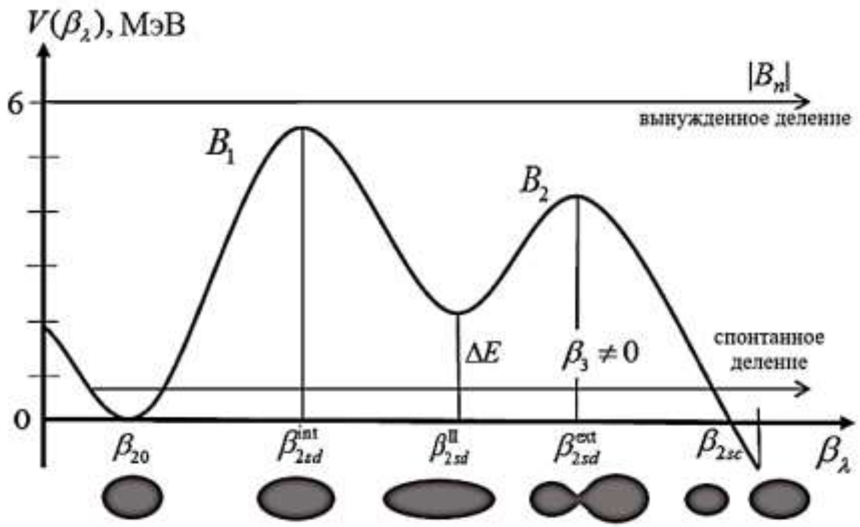

Fig. 2. Deformation potential $V(\beta_\lambda)$ for actinide nuclei

Spontaneous binary fission of actinide nuclei is usually observed when FN (A, Z) is even-even, is in the ground state with full spin J and its projections M, K, equal to zero, and is described by a collective deformation wave function $\psi_j(\beta_\lambda)$, corresponding to zero deformation vibrations of FN and corresponding to that introduced in [A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, 1977) Vol. 1,2] transitional fission state of this nucleus. The indicated transitional fission state passes through potential deformation barriers into the prescission configuration shown in Fig. 2 (see bottom arrow).

Binary fission is induced if FN, formed, for example, during the capture of a thermal neutron with very low kinetic energy $T_n \approx 0.025 эB$ by a target nucleus $(A-1, Z)$, located in the ground state, appears in an excited state with excitation energy $|B_n| \approx 6 MэB$, where B_n – is the binding energy of the captured neutron in FN (A, Z). For nuclear times $T_0 \approx 10^{-22} c$ this excited state transforms into the neutron resonance state of FN, the wave function ψ_K^{JM} of which is represented using the Wigner random matrix method [E. P. Wigner, Ann. Math. **62**, 548 (1955); **65**, 203 (1958); **67**, 325 (1958)] by the formula:

$$\psi_K^{JM} = \sum_i b_i \psi_{iK}^{JM} + \sum_j b_j \psi_{jK}^{JM}(\beta_\lambda), \tag{2}$$

where ψ_{iK}^{JM} corresponds to the i-quasiparticle excited state of FN, and $\psi_{jK}^{JM}(\beta_\lambda)$ describes the transition fission state j with an excitation energy equal to the energy $|B_n|$. The squared coefficients b_i and b_j in (2) have average values $1/N$, where N – is the total number of states involved in the formation of the wave function (2). Induced binary fission of FN occurs with a high probability if the energy $|B_n|$ is close to the maximum height of the internal B_1 and external B_2 deformation barriers of fission, which is realized, for example, for the induced fission of target nuclei ^{233}U and ^{235}U by thermal neutrons and is reflected by the upper arrow in Fig. 2.

Note that the experimental charge-mass distribution of fission fragments pairs has an extreme character with a maximum at certain values ($A_{L(0)}$, $Z_{L(0)}$). It is logical to associate the appearance of this maximum with the energies of transition fission states E(L, H), which for $A_L = A_{L(0)}$, $Z_L = Z_{L(0)}$; $A_H = A_{H(0)}$, $Z_H = Z_{H(0)}$ should be closest to the energy values E(B), corresponding to the maximum deformation fission barriers.

Important experimental characteristics of binary nuclear fission also include the kinetic energies of fission fragments, the excitation energies of these fragments, the multiplicity and energy of prompt neutrons and γ-rays, and the spin distributions of fission fragments. All these characteristics, in contrast to the relative yields of fission fragments, depending on (A_L, Z_L) have an approximately linear character in the vicinity of ($A_{L(0)}$, $Z_{L(0)}$).

The angular and spin distributions of binary fission fragments are described using two fundamental hypotheses proposed in the work [A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, 1977) Vol. 1,2]. The first of them assumes that during the decay of FN its axial symmetry is preserved, which is confirmed by the analysis of existing experimental data. The second hypothesis is that at all stages of fission, starting with the descent of the FN from the outer saddle point of its deformation potential, the projection K of the spin J of the FN onto its symmetry axis is preserved. The main obstacle to maintaining the projection of the spin K of the transition fission state onto its symmetry axis during fission is the perceptible heating of both the FN before it breaks into fission fragments, and the fission fragments themselves in the early stages of their evolution. Indeed, when an axially symmetric nucleus is heated to sufficiently high temperatures, the effect of dynamic amplification of the Coriolis interaction occurs [A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, 1977) Vol. 1,2; S.G. Kadmensky, V.P. Markushev, V.I. Furman, Phys. Atom. Nucl. **65**, 1785 (2002)], taking into account which leads to uniform statistical mixing of all possible values of the projections K of the spin J of FN onto its symmetry axis at not too large values. J. This mixing of projections in the vicinity of the FN decay into fission fragments leads [S.G. Kadmensky, V.E. Bunakov, D.E. Lyubashevsky, Phys. Atom. Nucl. **79**, 304 (2016); Phys. Atom. Nucl. **80**, 850 (2017)] to the fission nucleus "forgetting" the values K, and to the complete disappearance of any kind of asymmetries in the angular distributions of nuclear fission products, including P-odd, P-even и T-odd asymmetries. Since such anisotropies for low-energy fission induced in reactions with polarized neutrons, γ-rays and other particles have been reliably established experimentally [A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, 1977) Vol. 1,2; P. Carruthers, M. Nieto, Rev. Mod. Phys. **40**, 441 (1968)], we can come to the fundamental conclusion that the fissile system remains "cold" at all stages of fission, starting with the descent of the FN from the outer saddle point and ending with the formation of angular distributions of the initial fission products. The conclusion obtained allows us to take a new look at the physics of the fission process and the relationship between the quantum and thermodynamic characteristics of the fissile system for various stages of this process.

Almost all modern calculations of the angular distributions of fragments of spontaneous and low-energy induced fission of nuclei are based on the hypothesis of O. Bohr [A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, 1977) Vol. 1,2] about the proximity of the directions of fragment emission to the direction of the symmetry axis of the FN. However, from the quantum mechanical uncertainty relation for the relative orbital moments ΔL and emission angles $\Delta\theta$ of fission fragments with respect to the direction of the symmetry axis of FN it follows [S.G. Kadmensky, V.E. Bunakov, D.E. Lyubashevsky Phys. Atom. Nucl. **79**, 304 (2016); Phys. Atom. Nucl. **80**, 850 (2017)], that the smallness of the values $\Delta\theta$ requires the appearance of large values of ΔL , and, consequently, the L themselves. At the same time, estimates of the experimental values of the spins J_i of the light $(i=1)$ and heavy $(i=2)$ fragments of binary spontaneous and induced fission lead to the conclusion that these spins have quite large values and significantly exceed the spins J of FN.

The large values of the spins J_i and relative orbital angular moments L of the fission fragments can be understood by incorporating [J.R. Nix, W.J. Swiatecki, Nucl. Phys. **71,** 1 (1965).] transverse bending- и wriggling-vibrations that arise in the prescission configuration of FN. In Fig. 3 shows the modes of these oscillations [A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, 1977) Vol. 1,2]. During bending-vibrations, the spins of the primary fragments $\mathbf{J}_1$ and $\mathbf{J}_2$ are antiparallel to each other, so the total spin $(\mathbf{J}_1+\mathbf{J}_2)$ of these fragments is zero. In wriggling- vibrations the spins of the primary fragments $\mathbf{J}_1$ and $\mathbf{J}_2$ are parallel to each other, so the total spin $(\mathbf{J}_1 + \mathbf{J}_2)$ of these fragments is not zero. Therefore, due to the law of conservation of total spin FN, wriggling oscillations require the appearance of a relative orbital momentum of fission fragments $\mathbf{L}$, equal in magnitude to $\mathbf{L} = -(\mathbf{J}_1 + \mathbf{J}_2)$. Using the methods of [J.R. Nix, W.J. Swiatecki, Nucl. Phys. **71,** 1 (1965)], we can introduce stiffness coefficients K_b, K_w and mass coefficients M_b, M_w for bending и wriggling- vibrations and with their help restore the wave functions of these oscillations not only depending on the angles of rotation, but also in the impulse representation, and on their basis calculate the distribution functions of spins $W(J_i)$ and relative orbital moments $W(L)$ of fission fragments.

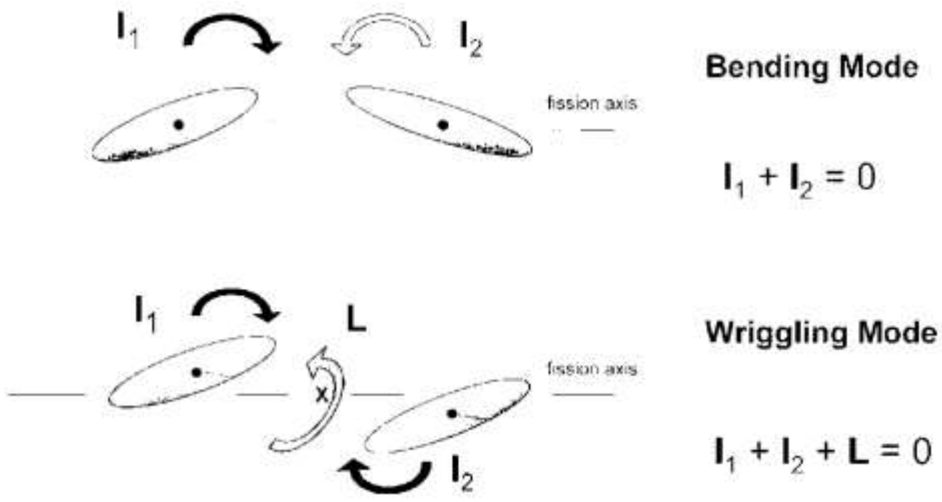

Fig. 3. Transverse bending- и wriggling- vibrations of FN in the vicinity of the breaking into fission fragments point

In this case, due to the above-mentioned coldness of the induced configuration FN, when obtaining $W(J_i)$ and $W(L)$ only zero bending- and wriggling-vibrations should be taken into account [S.G. Kadmensky, L.V. Titova, D.E. Lyubashevsky, Phys. Atom. Nucl. **83,** 581 (2020); S.G. Kadmensky, D.E. Lyubashevsky, Bull. Russ. Acad. Sci., Phys. **85**, 1160 (2021)]. Then, in the case of symmetric binary decay, the functions $W(J_i)$ and $W(L)$ are represented as

$$W\left(J_1\right)=\frac{4J_1}{C_b+C_w}\exp\left[-\frac{4J_1^2}{C_b+C_w}\right], \tag{3}$$

$$W\left(L\right)=\frac{L}{C_w}\exp\left[-\frac{L^2}{2C_w}\right]. \tag{4}$$

In formulas (3 - 4), the constants C_b, C_w and frequencies of bending- ω_b and wriggling- ω_w vibrations are determined by [J.R. Nix, W.J. Swiatecki, Nucl. Phys. **71,** 1 (1965)] through the stiffness coefficients K_b, K_w and mass coefficients M_b, M_w for these vibrations by the relations $C_b = M_b\hbar\omega_b$; $C_w = M_w\hbar\omega_w$ and $\omega_b = \sqrt{\frac{K_b}{M_b}}$; $\omega_w = \sqrt{\frac{K_w}{M_w}}$. From evaluations of [J.R. Nix, W.J.

Swiatecki, Nucl. Phys. **71,** 1 (1965)] for the FN ^{236}U it follows that $M_w = 1,6 \cdot 10^6$ MeV·Fm^2·sec^2, $M_b = 2,0 \cdot 10^6$ MeV·Fm^2·sec^2, $K_w = 295$ MeV·rad^{-2}, $K_b = 52$ MeV·rad^{-2}, so that the oscillation frequencies of bending-vibrations $\hbar\omega_b = 0,9$ MeV, turn out to be significantly lower, than the frequencies of wriggling- vibrations $\hbar\omega_w = 2,3$ MeV, and the constant $C_w = 132\hbar^2$ for wriggling-vibrations turns out to be noticeably larger than the constant $C_b = 57\hbar^2$ for bending-vibrations. This leads, when using formula (3), to the average fission fragment spin value $\bar{J} \approx 8,6$, which correlates well with the ranges of average fission fragment spin values $\bar{J}$ (7-9), obtained in [K. Skarsvag, K. Bergheim, Nucl. Phys. **45**, 72 (1963)]. At the same time, as follows from formula (4), the average value of the orbital momentum L turns out to be quite large and approximately 2 times greater than the average value of the spin of one of the fission fragments, which makes it possible to confirm the implementation of O. Bohr's hypothesis about the smallness of the angles of deviation of the direction of emission of fission fragments from the direction of the axis of symmetry FN.

The approach developed above to describing the spin distribution of fission fragments, based on the idea of the coldness of the fissile nucleus at the scission point and taking into account its zero transverse bending- and wriggling-vibrations, is fundamentally different from the approach of [E. S. Troubetzkoy, Phys. Rev., **122**, 212 (1961); M. Brack et al., Rev. Mod. Phys., **44**, 320 (1972)], in which, in principle, such vibrations are not taken into account, and the spin distributions of the formed fragments are entirely determined by their transition to equilibrium thermalized states on the path from their birth to the transition to the formed states, in which the temperatures T_i of the fission fragments exceed 1 MeV. This difference casts doubt on the possibility of using classical statistical physics to describe the states of formed fission fragments of a closed system FN, for which, in principle, there is no concept of a thermostat.

3. Quantum mechanisms of spontaneous and induced ternary nuclei fission with the emission of prescission nucleons and light nuclei

The spontaneous and induced ternary fission of FN (A, Z) with the emission of the third light prescission particle p (A_p, Z_p) at the initial stage is similar to the binary fission of FN, which is associated with the transition of FN due to its collective deformation oscillations into the prescission configuration shown in Fig.1. Then particle p flies out of this configuration of FN with the appearance of a virtual state of an intermediate nucleus (A-A_p, Z-Z_p), which then undergoes binary fission with the formation of light (A_{LF}, Z_{LF}) and heavy (A_{HF}, Z_{HF}) fission fragments. In this case, the experimental angular distribution of the prescission particle p during spontaneous and induced ternary fission has an anisotropic character with a maximum at emission angles in the direction perpendicular to the direction of emission of light fission fragments [M. Mutterer, J.P. Theobald, Dinuclear Decay Modes, **1**, 12 (1996); M. Mutterer, Yu.N. Kopatch, P. Jesinger et al., Nucl. Phys. **738**, 122 (2004)]. This allows us to conclude that the emitted p-particles are formed in the neck of the prescission configuration FN. The experimental energy distribution $W(T_p)$ of a *p*-particle with kinetic energies T_p has a maximum value at energies $T_{p\max}$, noticeably exceeding the heat of p-decay Q_p^A of the ground state FN, which has the value:

$$Q_p^A = E(A,Z) - E(A - A_p, Z - Z_p) - E(A_p, Z_p), \tag{5}$$

where $E(A,Z)$ – is the nucleus binding energy (A,Z) in the liquid-droplet model taking into account V.M. Strutinsky's shell corrections.

Using the results of work on the theory of virtual 2p and 2β decays of atomic nuclei [S.G. Kadmensky, U.V. Ivankov, Phys. Atom. Nucl. **77**, 1019 (2014); Phys. Atom. Nucl. **77**, 1532 (2014); S.G. Kadmensky, A. O. Bulichev, Bull. Russ. Acad. Sci., Phys. **80**, 921 (2016); S.G. Kadmensky, L.V. Titova, D.E. Lyubashevsky, Phys. Atom. Nucl. **83**, 581 (2020)] we can obtain a formula for the width Γ_{pf}^A of the virtual spontaneous ternary fission FN with the emission of prescission third light particles p:

$$\Gamma_{pf}^{\mathrm{A}} = \frac{1}{2\pi} \int_0^{Q_f^{A-A_p}} \frac{\Gamma_p^{\mathrm{A}}(T_p)\Gamma_f^{(A-A_p)}(Q_f^{A-A_p} - T_p + Q_p^A)}{(Q_p^A - T_p)^2} dT_p \ , \tag{6}$$

where $\Gamma_p^{A}(T_p)$ – is the width of the p-decay of FN(A,Z) with the emission of the third p-particle (A_p, Z_p) in the ground state and with the formation of a virtual state daughter nucleus (A-A_p, Z-Z_p), $\Gamma_f^{(A-A_p)}(Q_f^{A-A_p} - T_p + Q_p^A)$ – the width of binary nuclear fission (A-A_p, Z-Z_p) with the heat of this fission $Q_f^{A-A_p}$:

$$Q_f^{A-A_p} = E(A - A_p, Z - Z_p) - E(A_{LF}, Z_{LF}) - E(A_{HF}, Z_{HF}). \tag{7}$$

It is worth noting that the width of the induced ternary fission FN (A, Z), formed when a thermal neutron is captured by a target nucleus in the ground state (A-1, Z), is also determined by formula (6), since the excitation energy, equal to $|B_n|$, is conserved in the corresponding prescission configuration of this nucleus and does not participate in the formation of the kinetic energy T_p of the prescission third particle p. Now we can imagine the yield of p particles from ternary fission FN in relation to the number of binary fission fragments, defined as:

$$\delta_{pf} = \frac{\Gamma_{pf}^{A}(T_p)}{\Gamma_f^A(Q_f^A)} = \frac{1}{2\pi} \int_0^{Q_f^{A-A_p}} \frac{\Gamma_p^{A}(T_p)\Gamma_f^{(A-A_p)}(Q_f^{A-A_p} - T_p + Q_p^A)}{(Q_p^A - T_p)^2 \Gamma_f^A(Q_f^A)} dT_p \ . \tag{8}$$

From here we can obtain the energy distribution of the output of emitted p-particles $W_{pf}(T_p)$:

$$W_{pf}(T_p) = \frac{1}{2\pi} \frac{\Gamma_p^{\mathrm{A}}(T_p)\Gamma_f^{(A-A_p)}(Q_f^{A-A_p} - T_p + Q_p^A)}{(Q_p^A - T_p)^2 \Gamma_f^A(Q_f^A)}. \tag{9}$$

Considering that the energy Q_f^{A-4} or actinide nuclei reaches 170 MeV, which significantly exceeds the energy $(T_p - Q_p^A)$ and proximity of the widths $\Gamma_f^{(A-A_p)}(Q_f^{A-A_p})$ and $\Gamma_f^A(Q_f^A)$, formula (9) can be transformed into the form:

$$W_{pf}(T_p) = W_p(T_p) = \frac{1}{2\pi} \frac{\Gamma_p^{\mathrm{A}}(T_p)}{(Q_p^A - T_p)^2}. \tag{10}$$

Then the width value $\Gamma_p^{\mathrm{A}}(T_p)$ can be calculated through values $W_{pf}(T_p)$ (10) as

$$\Gamma_p^{\mathrm{A}}(T_p) = 2\pi W_p(T_p)(Q_p^A - T_p)^2. \tag{11}$$

The width $\Gamma_p^{\mathrm{A}}(T_p)$ of the p-decay FN can be approximately represented by Gamow's formula:

$$\Gamma_p^{A}(T_p) = \omega_p \frac{\hbar\sqrt{2T_p}}{2r_{neck}\sqrt{m_p}} P\left(T_p\right), \qquad (12)$$

where $P(T_p)$ – is the permeability coefficient of the Coulomb barrier for particle p, close to one at $T_p = T_{p\max}$, where $T_{p\max}$ – is the position of the maximum in the energy distribution (9), m_p – is the mass of the p-particle, and ω_p – is the probability of the formation of a *p*-particle from FN nucleons neck with radius r_{neck}. Using experimental values of widths $\Gamma_p^{A}(T_{p\max})$ and averaged values of radii r_{neck}, calculated in [O. Serot, N. Carjan, C. Wagemans, Eur. Phys. **8**, 187 (2000)], using formula (12), one can calculate the formation probabilities ω_p for such charged third prescission particles p as α-particles, protons, as well as light nuclei ^2H, ^3H and ^6He, and compare their values with similar values, calculated within the framework of the core cluster model [S.G. Kadmensky, V. I. Furman, *Alpha-decay and related nuclear reactions* (Energoatomizdat, 1985)].

4. Spontaneous and induced ternary nuclei fission with emission of prescission neutrons

It is well known [A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, 1977) Vol. 1,2], that during spontaneous and low-energy induced (with the participation of thermal neutrons) binary fission of actinide nuclei, neutrons appear n with short characteristic emission times $\tau \leq 10^{-4}$ sec. The number of these neutrons, normalized to their yield δ_{nf}, is determined by the normalized sum of evaporating neutrons n_{ev}, which are emitted from fission fragments fully accelerated by their mutual Coulomb field, and prescission neutrons n_{pr}, emitted from prescission FN configurations at times close to the moment of scission into fission fragments. The angular and energy spectrum $n(T_n, \theta_n)$ of these neutrons n, where T_n and θ_n – are the asymptotic kinetic energies of the emitted neutron and the angle between the directions of emission of the neutron and the light fission fragment, can be represented as:

$$n(T_n, \theta_n) = n_{pr}(T_n, \theta_n) + n_{ev}(T_n, \theta_n). \qquad (13)$$

Experimental spectra $n(T_n, \theta_n)$ were found for spontaneous binary fission of ^{252}Cf in [J. Chwaszczewska, Phys. Lett. **24B**, 87 (1967); A. S. Vorobiev et al., JETF **125**, 619 (2017)] and for induced binary fission of ^{233}U and ^{235}U in [A. S. Vorobiev et al., JETF

125, 619 (2017); A. S. Vorobiev et al., Bull. Russ. Acad. Sci., Phys. **82**. 1245 (2018)]. The theoretical spectrum $n_{ev}(T_n, \theta_n)$ of evaporation neutrons was calculated using the representation in [J. Chwaszczewska, Phys. Lett. **24B**, 87 (1967)].

5. Conclusion

The new approaches to describing the characteristics of spontaneous and induced binary and ternary fission of FN, demonstrated above within the framework of quantum fission theory, provide the basis for a consistent understanding of the processes that play a decisive role in the nuclear energy sector.