**Al-Azhar University**
**Faculty of Science**
**Department of Physics**

# Description of Superdeformed Bands in A ~ 190 Mass Region in Framework of Suggested Three Parameters Nuclear Collective Model

**By**

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30th International Seminar on Interaction of Neutrons with Nuclei: Fundamental Interactions & Neutrons, Nuclear Structure, Ultracold Neutrons, Related Topics (ISINN-30)

# *Problems and unexpected features in SD nuclei*

- Spin Assignments of SDRB ’s
- Behavior of the dynamic and kinematic moments of inertia for different bands and rational frequency
- The phenomenon of the staggering like ($\Delta I=1$), ($\Delta I = 2$), staggering energy
- The phenomena of identical bands (IB's) in ND,SD bands.

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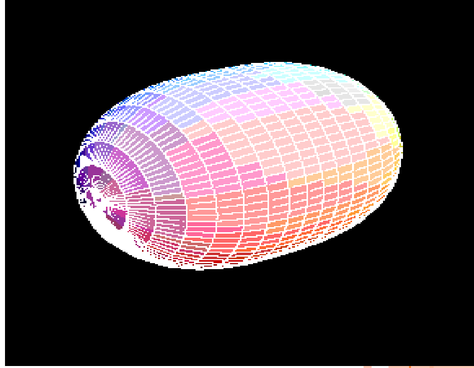
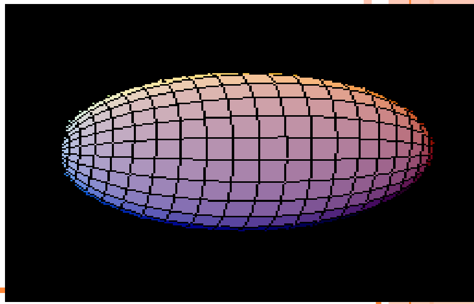
# How Practical We Can Predict The Shape Of The Nucleus





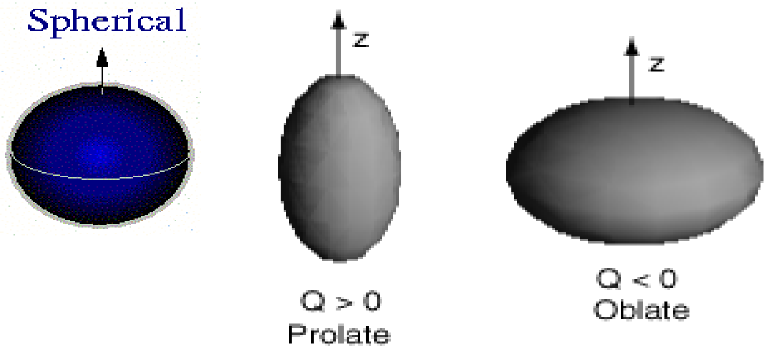
## Tests To Predict Shape of the Nucleus

- Electric Quadrupole
- Gamma transitions probabilities
- Deformation Parameter

**Electric quadrupole Q**

Spherical shape (Q=0)

Deformed shape




Classical definition

$$Q_0 = \int \rho(3z^2 - r^2)dV$$

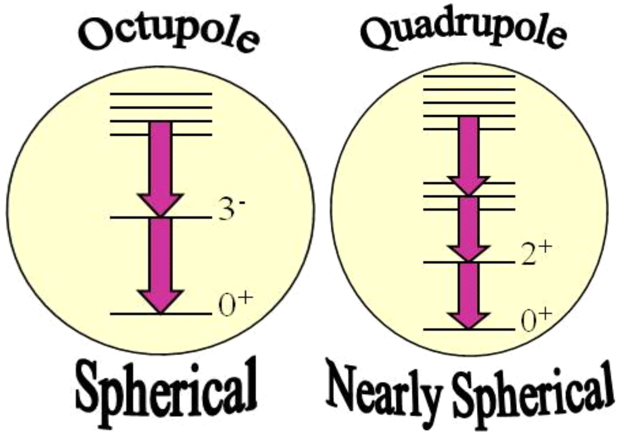
$$Q = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} Q_0$$

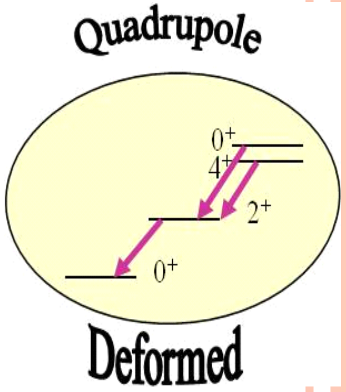
Quantum measurement

**Gamma transitions probabilities**

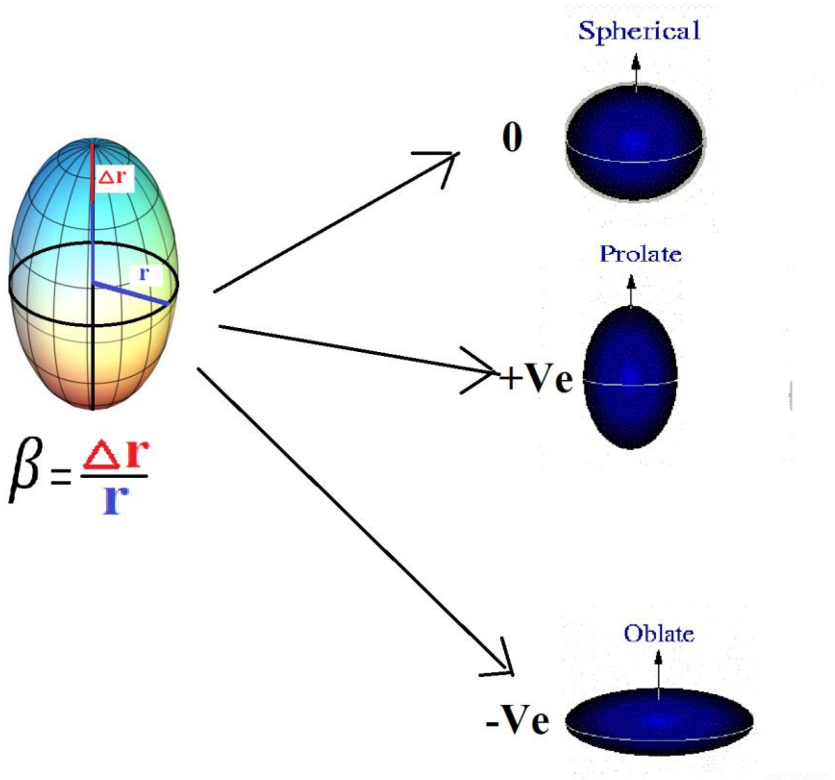
Spherical shape Evenly spaced levels

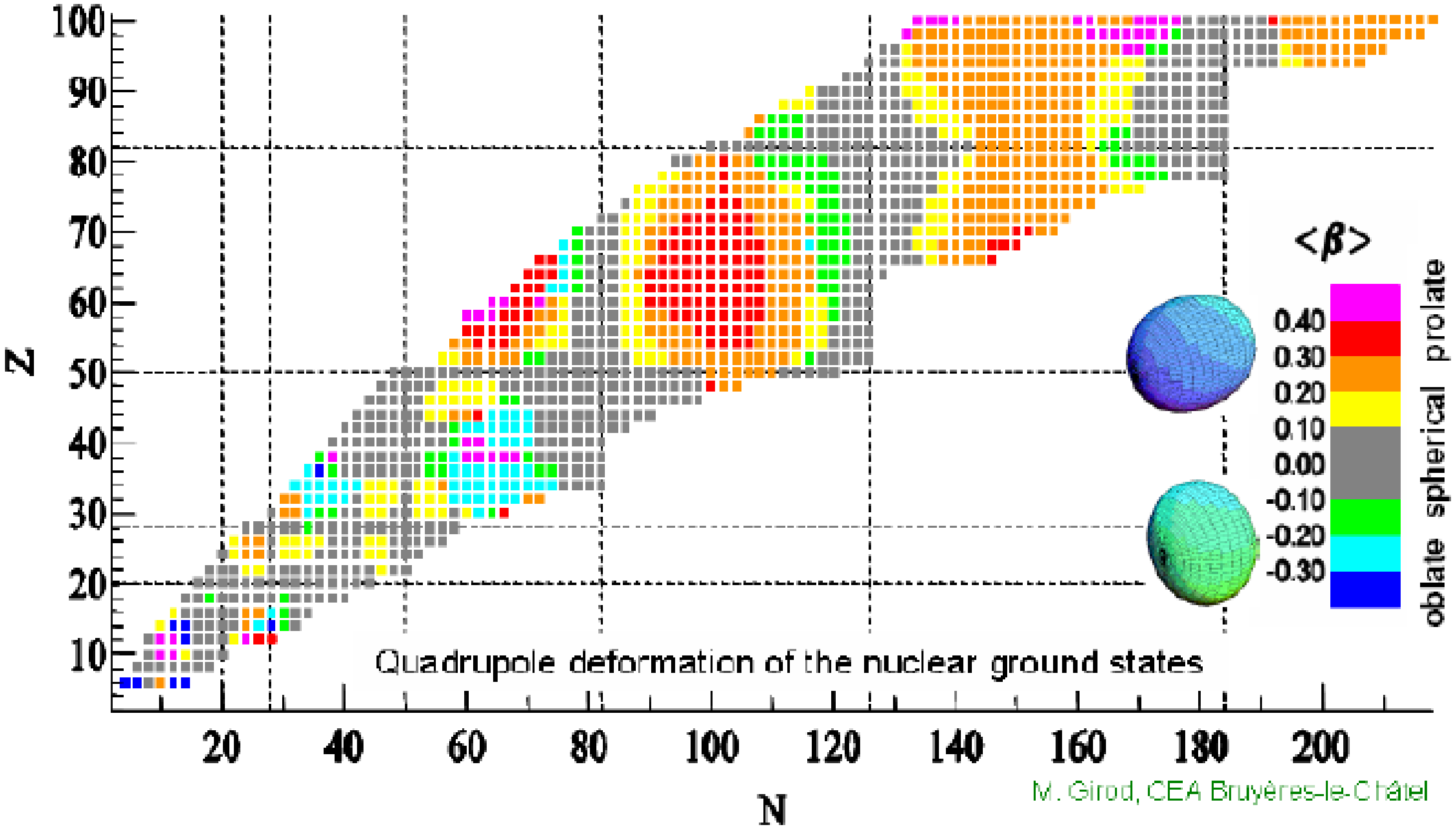
Deformed shape Varied spaced levels



# Deformation Parameter which measure Deformation from Sphere




Normal deformed (ND) bands with deformation β ~ 0.3 (axis ratio 1.3:1:1) and superdeformed (SD) bands with deformation β ~ 0.6 (axis ratio 2:1:1)

# How Can We Theoretical Predict The Shape of the Nucleus

In **(ND)** bands, Spins and State excitation energies are known,

In **(SD)** bands γ-ray transition energies are the only information available. The spins and excitation energies are not determined because of the non observation of the transition energies linking the (SD) states and the normal deformed states.

## Phase Transition

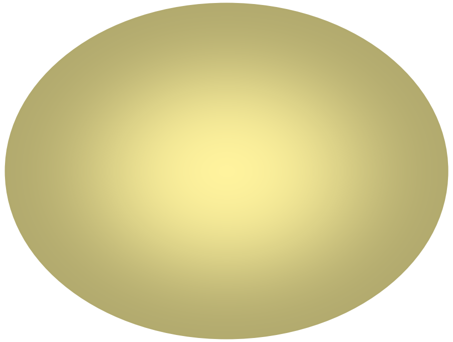
Isotopic series of some nuclei show different phase transitions

The phase transition can be investigated using different methods

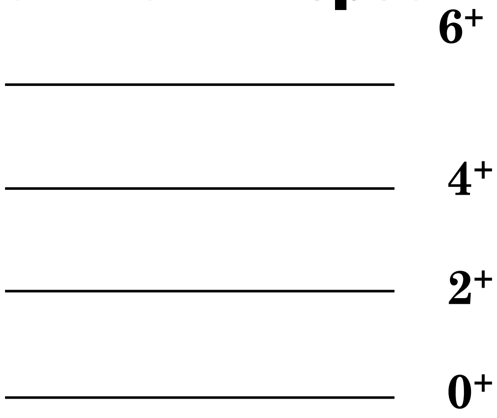
- Tests To Predict Phase Transitions In Series Of Isotopes Of Some Elements
  - Energy Ratios
  - Transition Energy Over Spin (EGOS)
  - Electric Transition Rates

# The nuclear shape : spectrum ?

**Spherical nuclei**

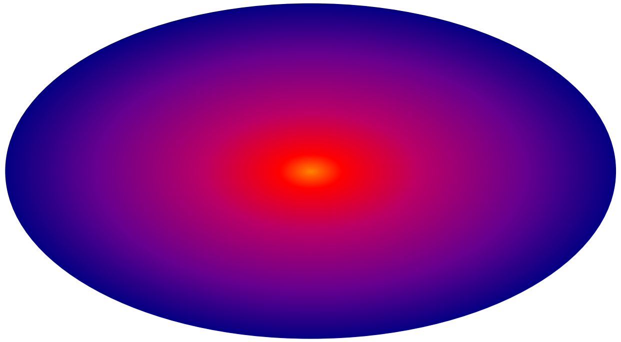


**«vibrational» spectrum**

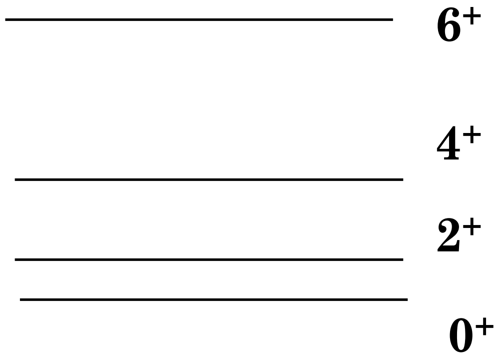



$E(I) \propto I$

**Deformed nuclei**



**«rotational» spectrum**



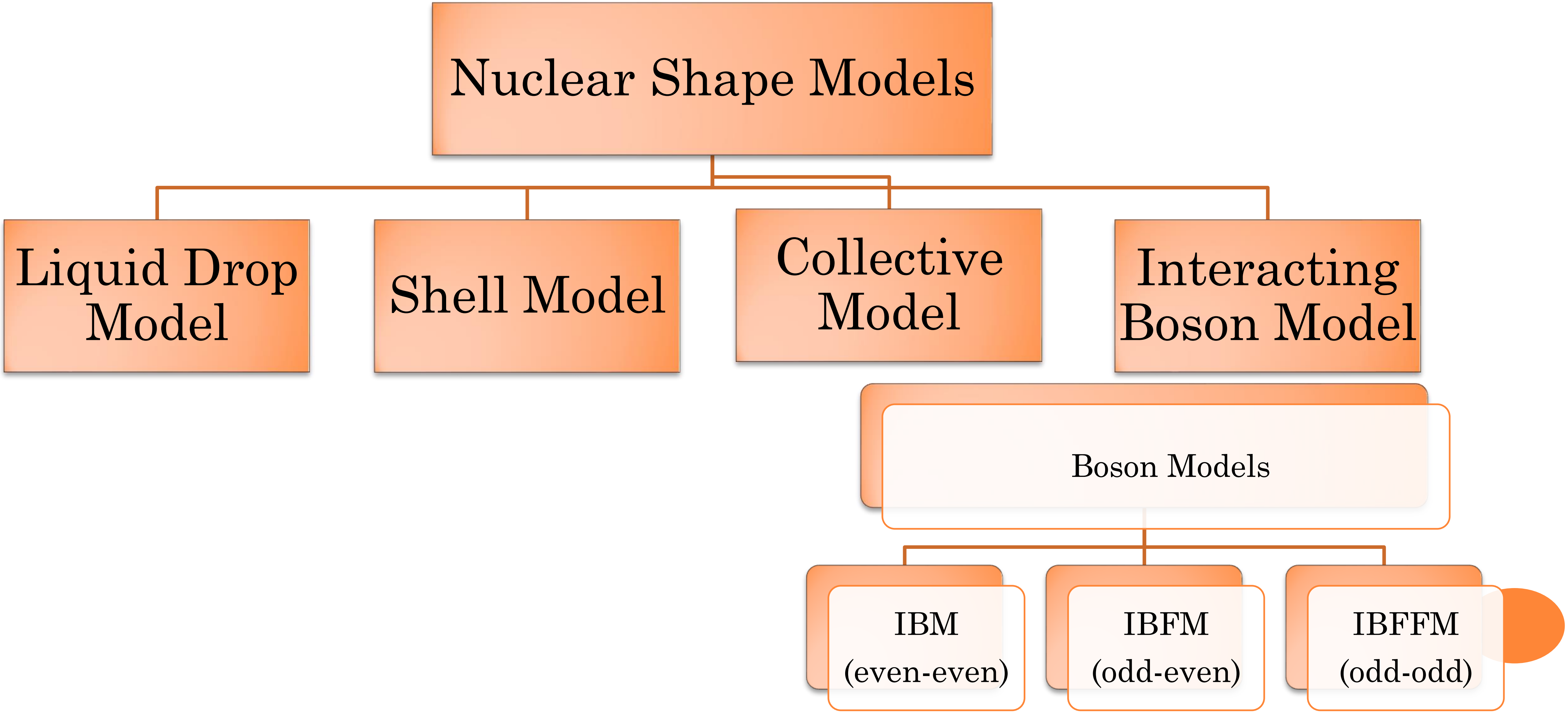

$E(I) \propto I(I+1)$

# Nuclear shapes

- **Spherical**
  - $\beta = 0, \gamma = 0$
  - Ratio of lengths
  - **1 : 1 :1**
- **Deformed**
  - **Axially Symmetric**
    - $x = y \neq z$
    - **Normal Deformed**
      - $\beta < 0.35$
      - **1 : 1 :1.3**
    - **Superdeformed**
      - **A~190** $\beta = 0.46$
      - **1 : 1 : 1.65**
      - **A~150** $\beta = 0.6$
      - **1 : 1 : 2**
      - **A~130** $\beta = 0.5$
      - **2 : 2 :3**
    - **Hyperdeformed**
      - $\beta > 0.8$
      - 1 : 1 : 3
  - **Triaxial**
    - $X \neq y \neq z$ 3:4:6
    - $\beta > 0, \quad 0 < \gamma < 60$

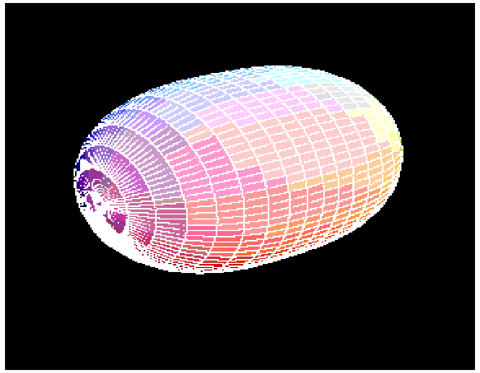
# Why We Need the Model?

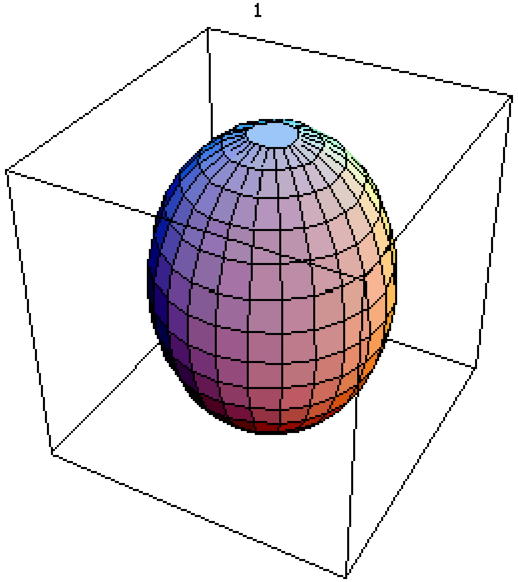
**To describe and predict nuclear shapes and properties associated with the structure.**



# Nuclear models

| Vibrator | Rotator |
|---|---|
| The transition energies between energy levels are constant. | The transition energies between levels are increase. |
| The excitation energy is given by E(I)= B.I | The excitation energy is given by E(I)= A.I(I+1). |
| Energy ratio = E(4)/E(2) = 2. | Energy ratio = E(4)/E(2)= 3.3333. |





- ***The moments of inertia***

which are related to the first (kinematic) and second (dynamic) order derivatives of the excitation energy with respect to the spin.

- The **kinematic** moments of inertia $J^{(1)}$.

$$\frac{J^{(1)}}{\hbar^2} = \hat{I}[\frac{dE(I)}{d\hat{I}}]^{-1}, \qquad \hat{I} = \sqrt{I(I+1)}$$

- The **dynamic** moment of inertia $J^{(2)}$.

$$\frac{J^{(2)}}{\hbar^2} = [\frac{d^2E(I)}{d\hat{I}^2}]^{-1} = [\frac{d}{d\hat{I}}\frac{dE(I)}{d\hat{I}}]^{-1} = [\frac{d\omega}{d\hat{I}}]^{-1}$$

# Experimentally

- **The rotational frequency ħω is**

$$\hbar\omega(I) = \frac{1}{4}\left[E_\gamma(I+2) + E_\gamma(I)\right]$$

The moments of inertia, which are related to the first and second order derivatives of the excitation energy with respect to the spin.

**The first order derivative is the kinematic moments of inertia $J^{(1)}$**

$$J^{(1)}(I) = \frac{2I-1}{E_\gamma(I)}$$

**The second order derivative is the dynamic moment of inertia $J^{(2)}$**

$$J^{(2)}(I) = \frac{4}{E_\gamma(I+2) - E_\gamma(I)}$$

## ○ ***Staggering***

A regular $\Delta I = 2$ staggering pattern of the transition energies was observed It manifests itself in systematic shifts of energy levels which are alternately pushed up and down with respect to rotational sequence.

Where the behavior of $\Delta I = 1$ staggering is shown in two different bands in signature partner pairs.

# $\Delta I = 1$ Staggering

The behavior of ΔI = 1 staggering in signature partner pairs, one may calculate the differences between the average transition I + 2 → I → I - 2 energies in one band and the transition I + 1 → I - 1 energies in its signature partner

$$S^{(2)}(I) = \tfrac{1}{2}\left\{\tfrac{1}{2}[E_\gamma(I+2 \to I) + E_\gamma(I \to I-2)] - E_\gamma(I+1 \to I-1)\right\}$$

$$= \tfrac{1}{4}[E_\gamma(I+2) - 2E_\gamma(I+1) + E_\gamma(I)]$$

The EGOS staggering function. It represents the gamma transition energy over the spin

$$EGOS(I) = \frac{E_\gamma(I)}{2I}$$

To show the staggering phenomenon the three functions have been calculated and illustrated

$$EGos\,(I) = \frac{E_{\gamma 1}(I)}{2I}$$

$$e(I) = (\mathrm{EGOS1}) - (2A + 4BI^2).$$

$$\Delta^2 E_\gamma(I) = \frac{1}{2}[E_{\gamma 2}(I+2) - 2E_{\gamma 2}(I+1) + E_{\gamma 2}(I)]$$

$$Y(I) = \left(\frac{2I-1}{2I}\right)\frac{E_{\gamma 1}(I)}{E_{\gamma 2}(I)} - 1$$

where

$$E_{\gamma 1}(I) = E(I) - E(I-1)$$

$$E_{\gamma 2}(I) = E(I) - E(I-2)$$

# Nature of cross-talk transitions and $\Delta I = 1$ energy staggering in signature partners of odd mass SD nuclei

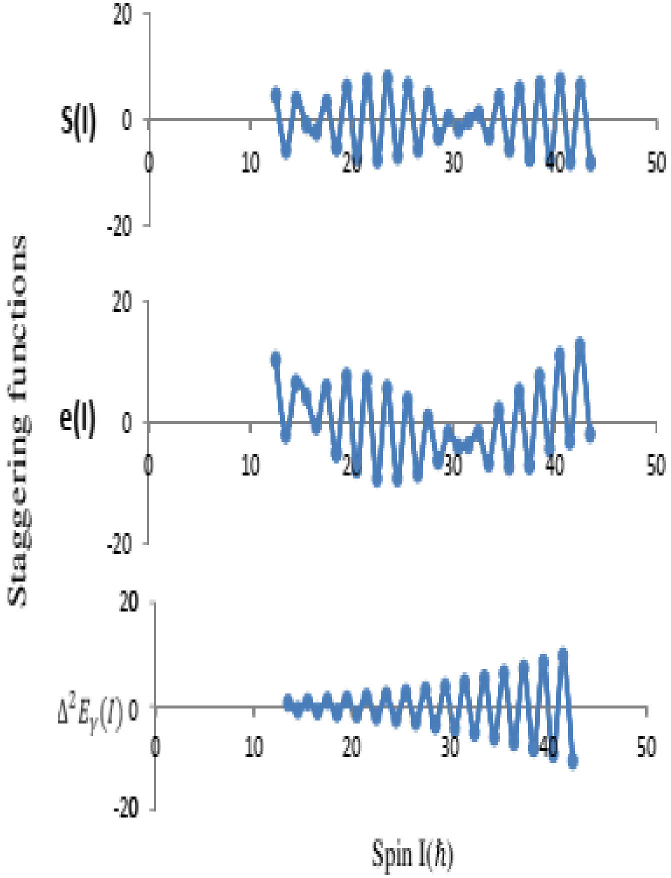



Fig. 3. Staggering functions $S(I)$, $e(I)$ and $\Delta^2 E_\gamma(I)$ as a function of the spin $I$ for the signature partner pair $^{191}$Hg(SD2,SD3).

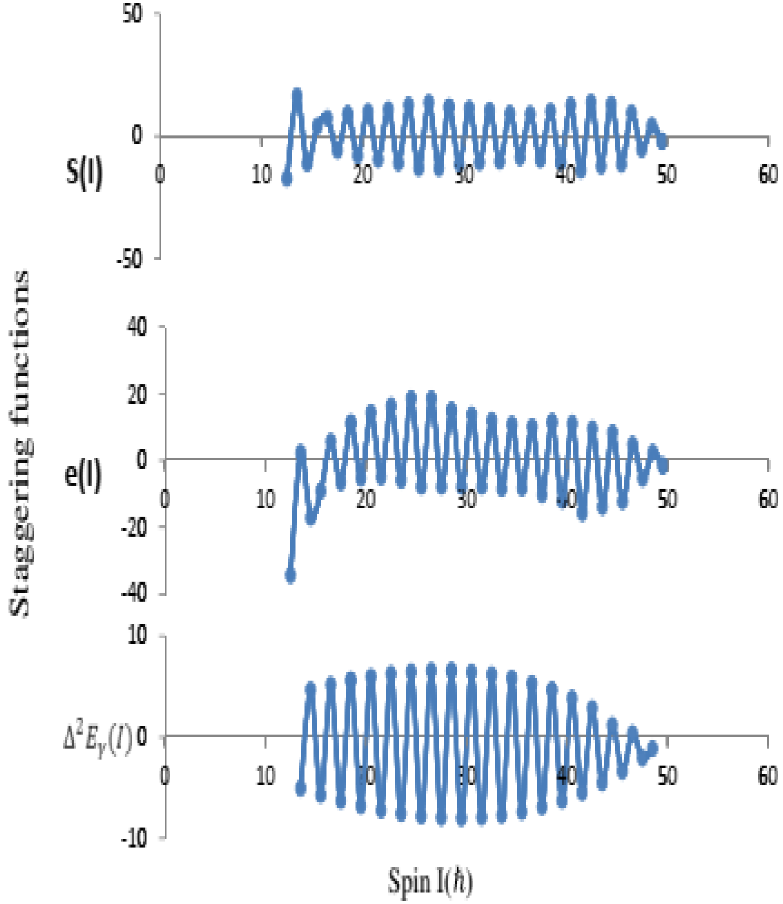



Fig. 4. Like Fig. 3 but for the signature partner pair $^{193}$Hg(SD3,SD4).

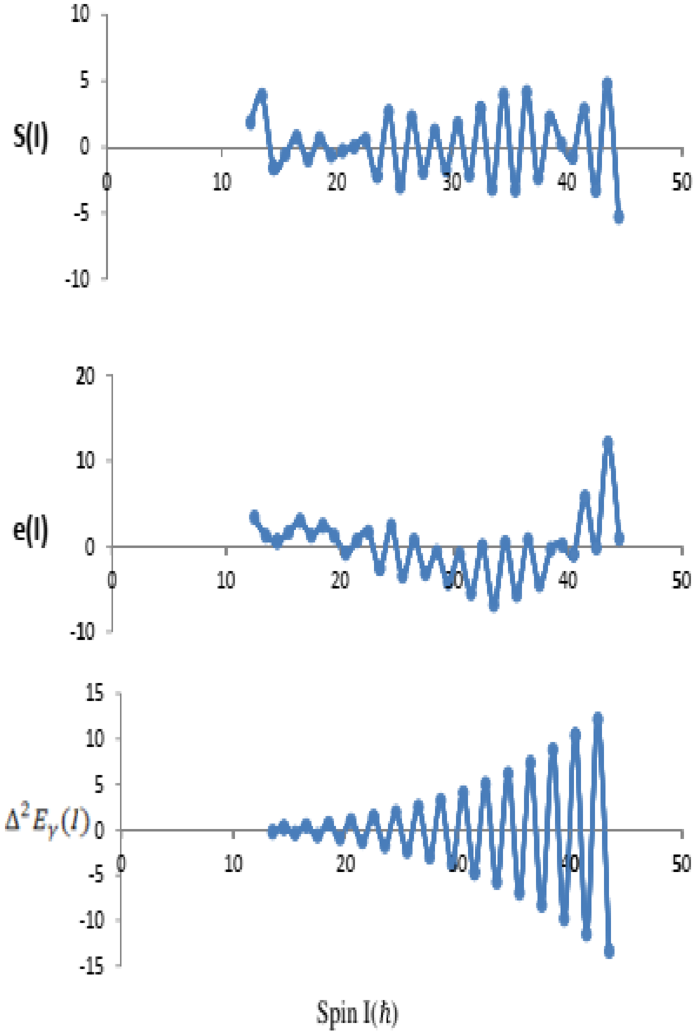



Fig. 5. Like Fig. 3 but for the signature partner pair $^{193}$T1(SD1,SD2).

# APPEARANCE OF ΔI=1 STAGGERING EFFECTS IN SIGNATURE PARTNERS OF ODD SD Tl AND Pb NUCLEI

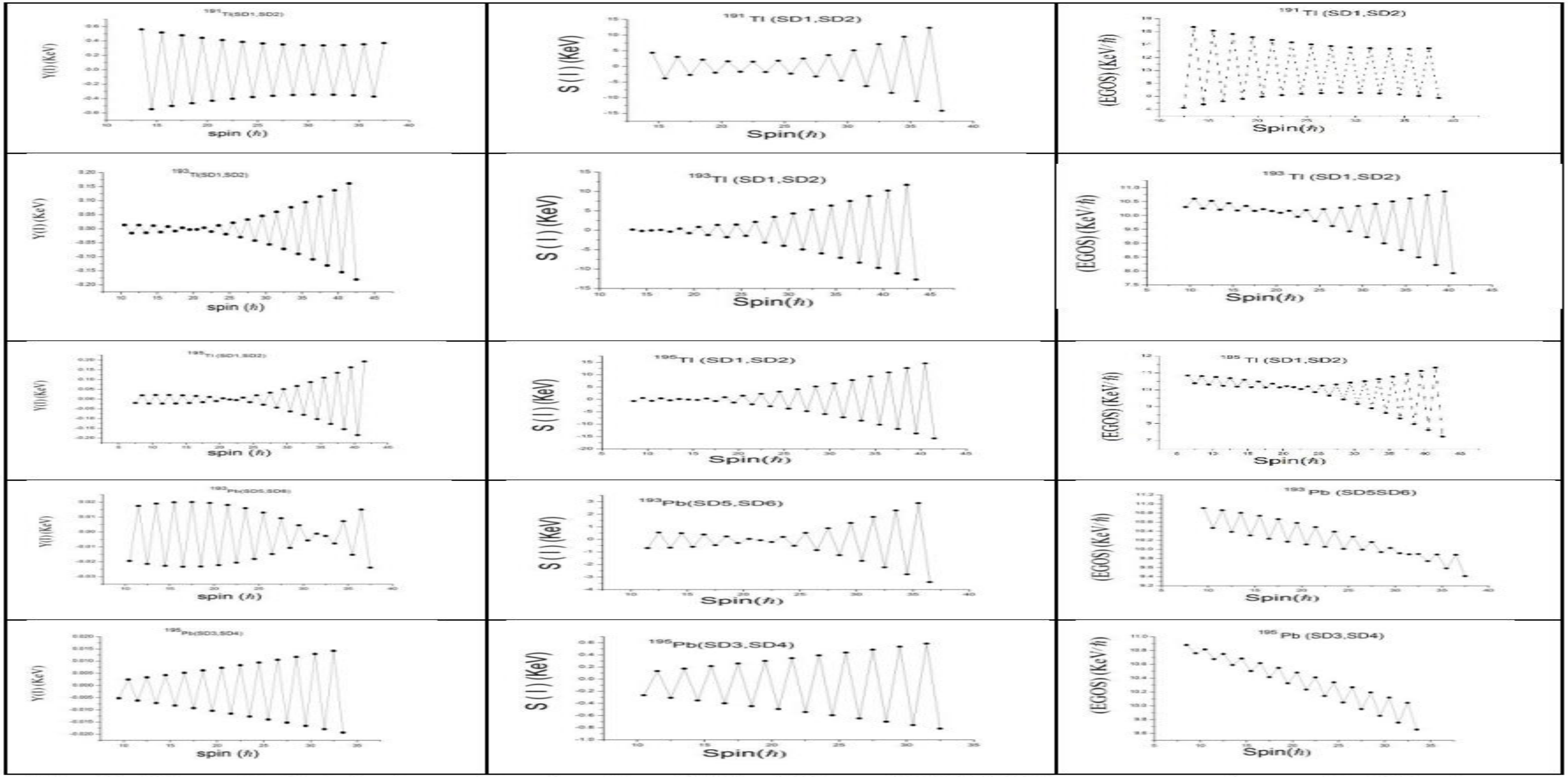



Fig. 3. The calculated staggering function Y(I) , S(I) and EGOS(I)) versus nuclear spin I for the studied signature partner SD bands observed in Tl and Pb nuclei.

# $\Delta I = 2$ Staggering

Band sequence is split into two branches separated by $\Delta I = 4$ shift up and down in energy (bifurcation), spin I, I+4, I+8, is displaced from the sequence I+2, I+6, I+10

The finite difference approximation to the fourth order derivative of transition energies is calculated and denoting as staggering quantity $\Delta I = 2$ staggering (five-point formula)

$$\Delta^4 E\gamma(I) = \frac{1}{16}\left[E\gamma(I-4) - 4E\gamma(I-2) + 6E\gamma(I) - 4E\gamma(I+2) + E\gamma(I+4)\right]$$

## The Δ I = 2 Energy Staggering

$$S^{(1)}(I) = \frac{1}{2}\left[E_\gamma(I+2) - E_\gamma(I)\right]$$

$$S^{(2)}(I) = \frac{1}{4}\left[E_\gamma(I-2) - 2E_\gamma(I) + E_\gamma(I+2)\right]$$

$$S^{(3)}(I) = \frac{1}{8}\left[-E_\gamma(I-2) + 3E_\gamma(I) - 3E_\gamma(I+2) + E_\gamma(I+4)\right]$$

$$S^{(4)}(I) = \frac{1}{16}\left[E_\gamma(I-4) - 4E_\gamma(I-2) + 6E_\gamma(I) - 4E_\gamma(I+2) + E_\gamma(I+4\right.$$

# Extended Exponential Model with Pairing Attenuation and Investigation of Energy Staggering and Identical Bands Effects in SD Thallium Nuclei

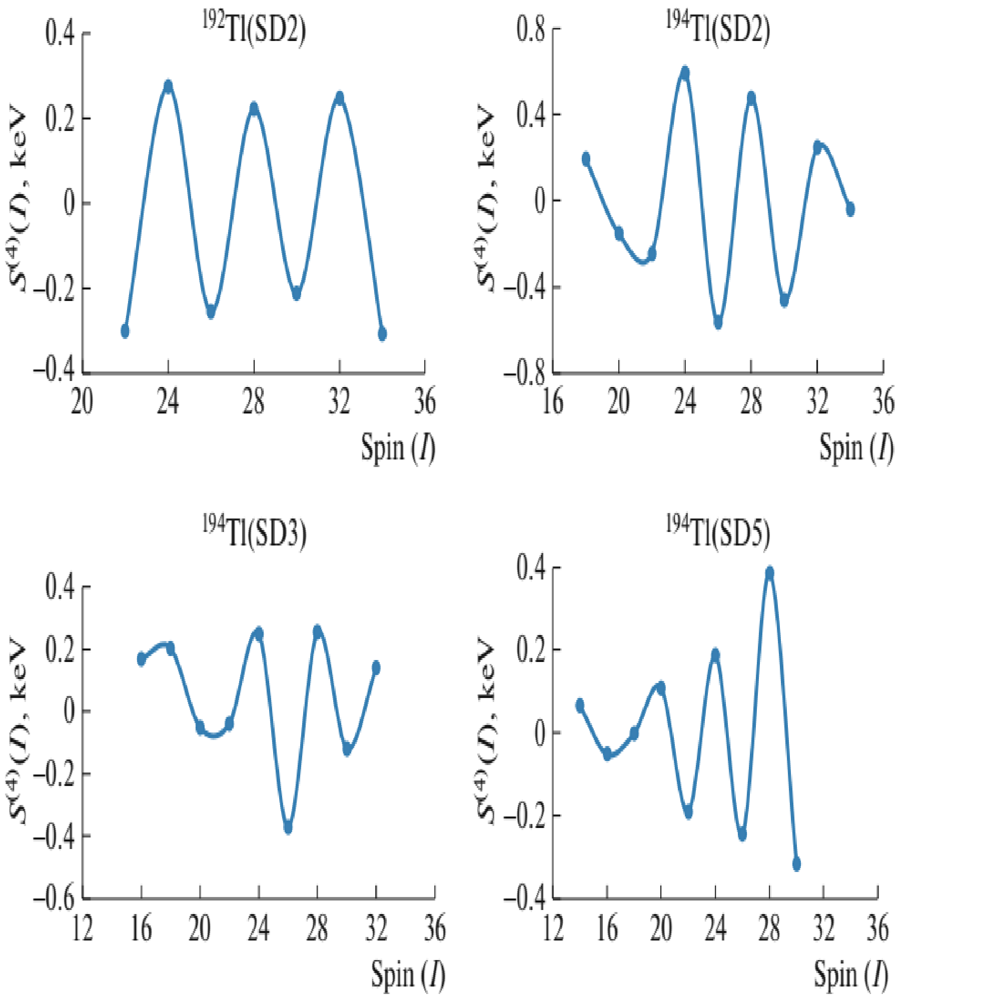



Fig. 3. The $\Delta I = 2$ staggering parameter $S^{(4)}(I)$ (in keV) as a function of nuclear spin $I$ for the SDRB's $^{192}$Tl(SD2), $^{194}$T (SD1, SD3, SD5).

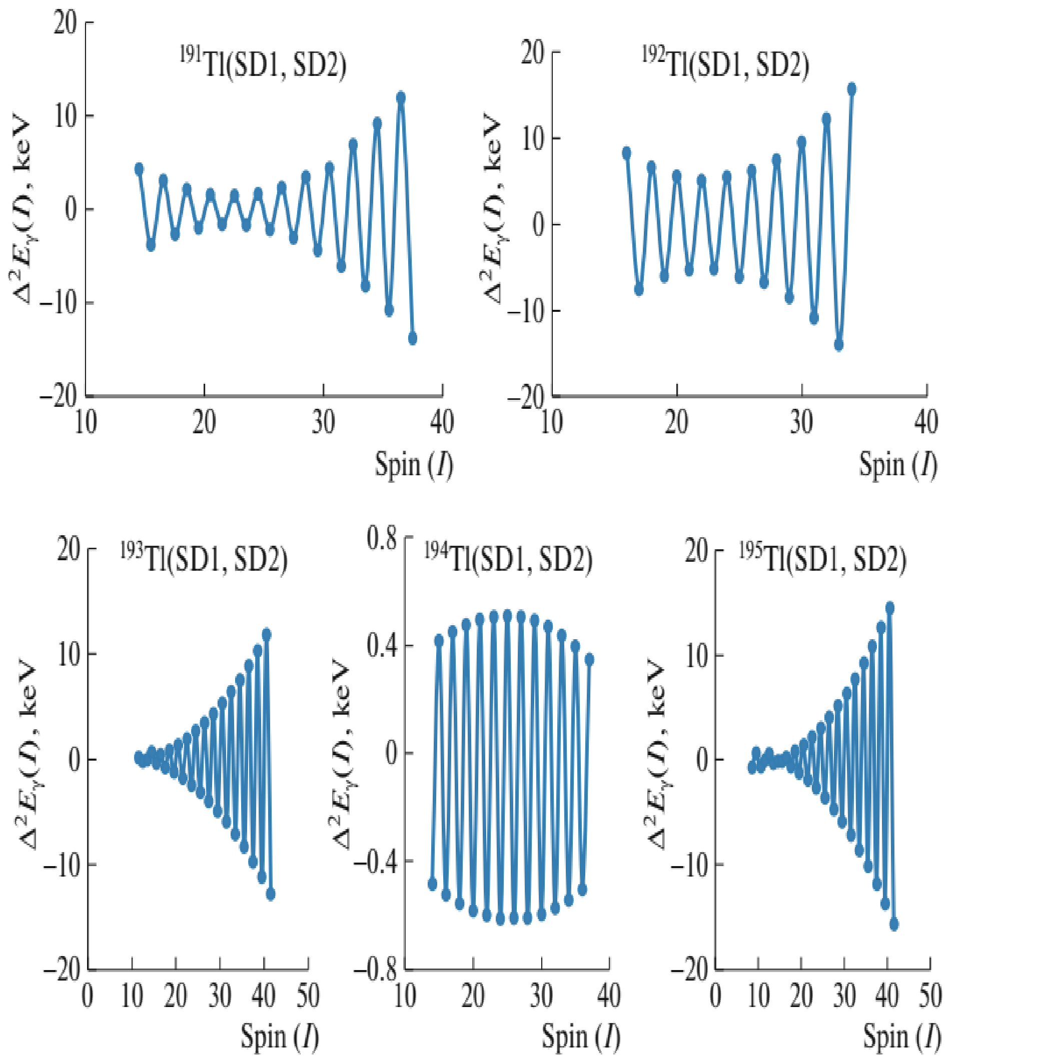



Fig. 4. The $\Delta I = 1$ staggering parameter $\Delta^2 E_\gamma(I)$ (in keV) as a function of nuclear spin $I$ for the five pairs of signature partners in Tl nuclei.

# Description of SD Bands of the Isotones $N = 113$ for Nuclear Mass Region $A \sim 190$

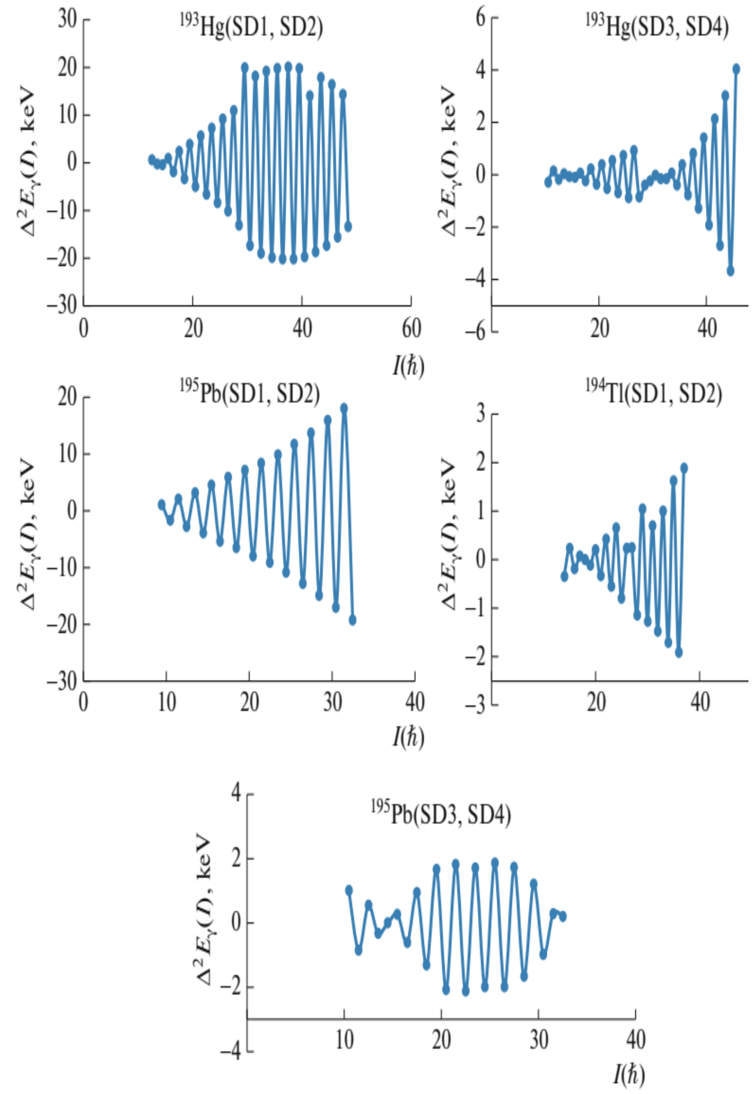



**Fig. 3.** The calculated $\Delta I = 1$ staggering parameter $\Delta^2 E_\gamma(I)$ as a function of nuclear spin $I$ for the sign
$^{193}$Hg(SD1, SD2), $^{193}$Hg(SD3, SD4), $^{194}$Tl(SD1, SD2), $^{195}$Pb(SD1, SD2), and $^{195}$Pb(SD3, SD4).

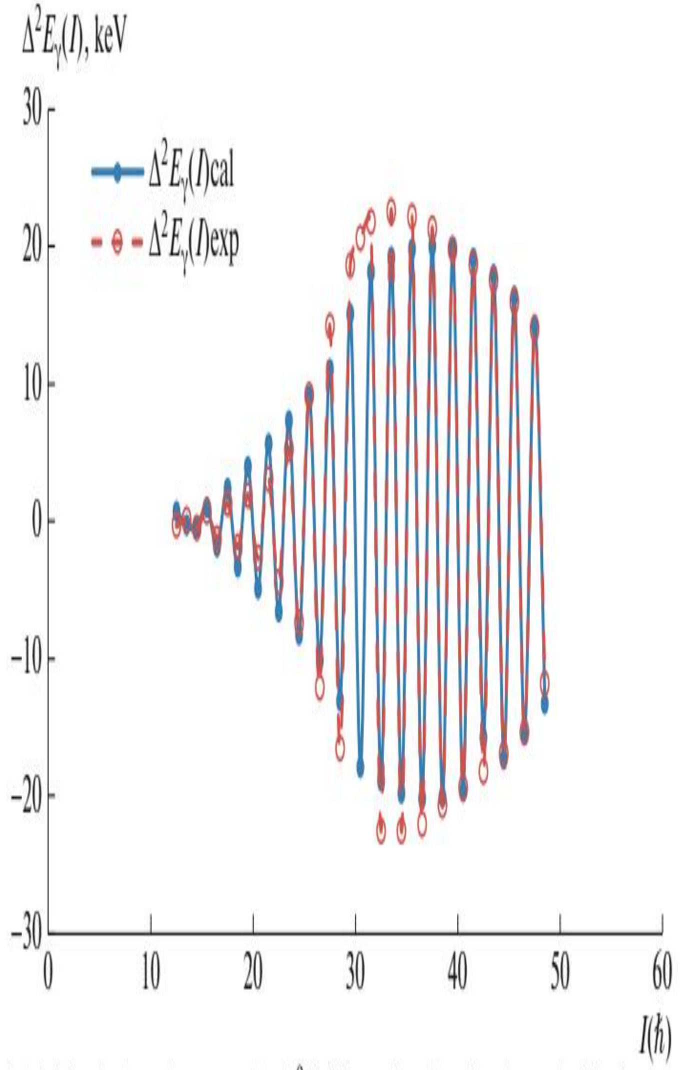
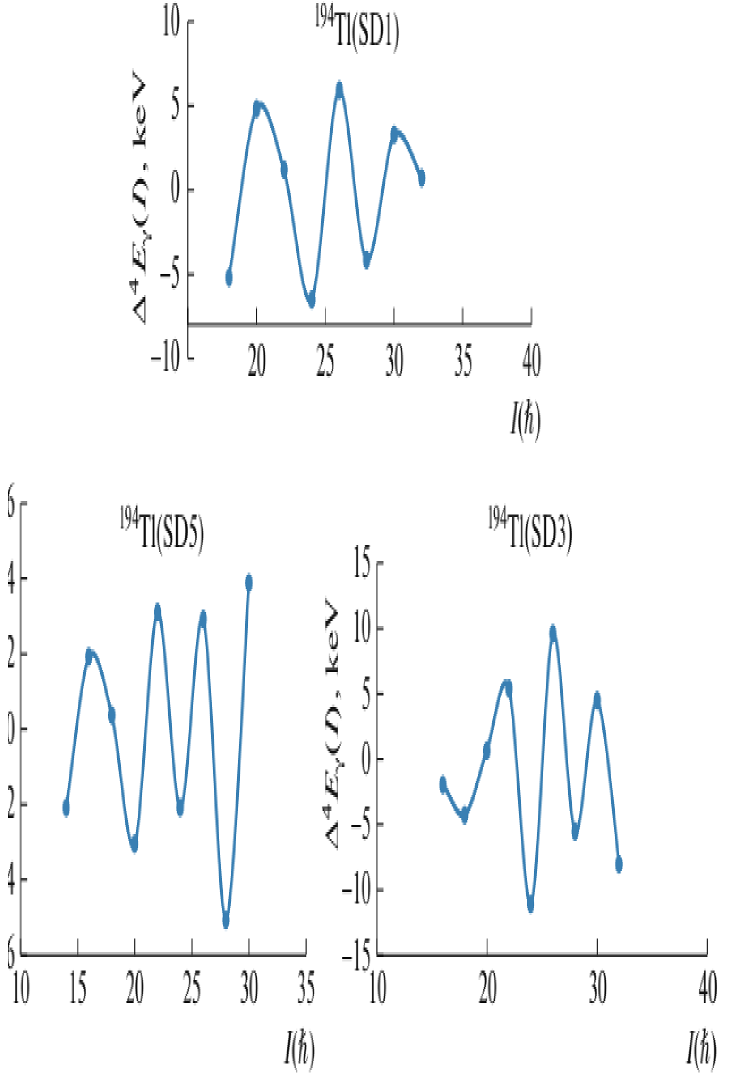



**Fig. 5.** The calculated $\Delta I = 1$ staggering parameter $\Delta^2 E_\gamma(I)$ as a function of nuclear spin $I$ for the signature partner pairs $^{193}$Hg(SD1, SD2) and comparison with experimental values.




culated $\Delta I = 2$ staggering index $\Delta^4 E_\gamma(I)$ as a function of nuclear spin $I$ for $^{194}$Tl(SD1, SD3, SD5).

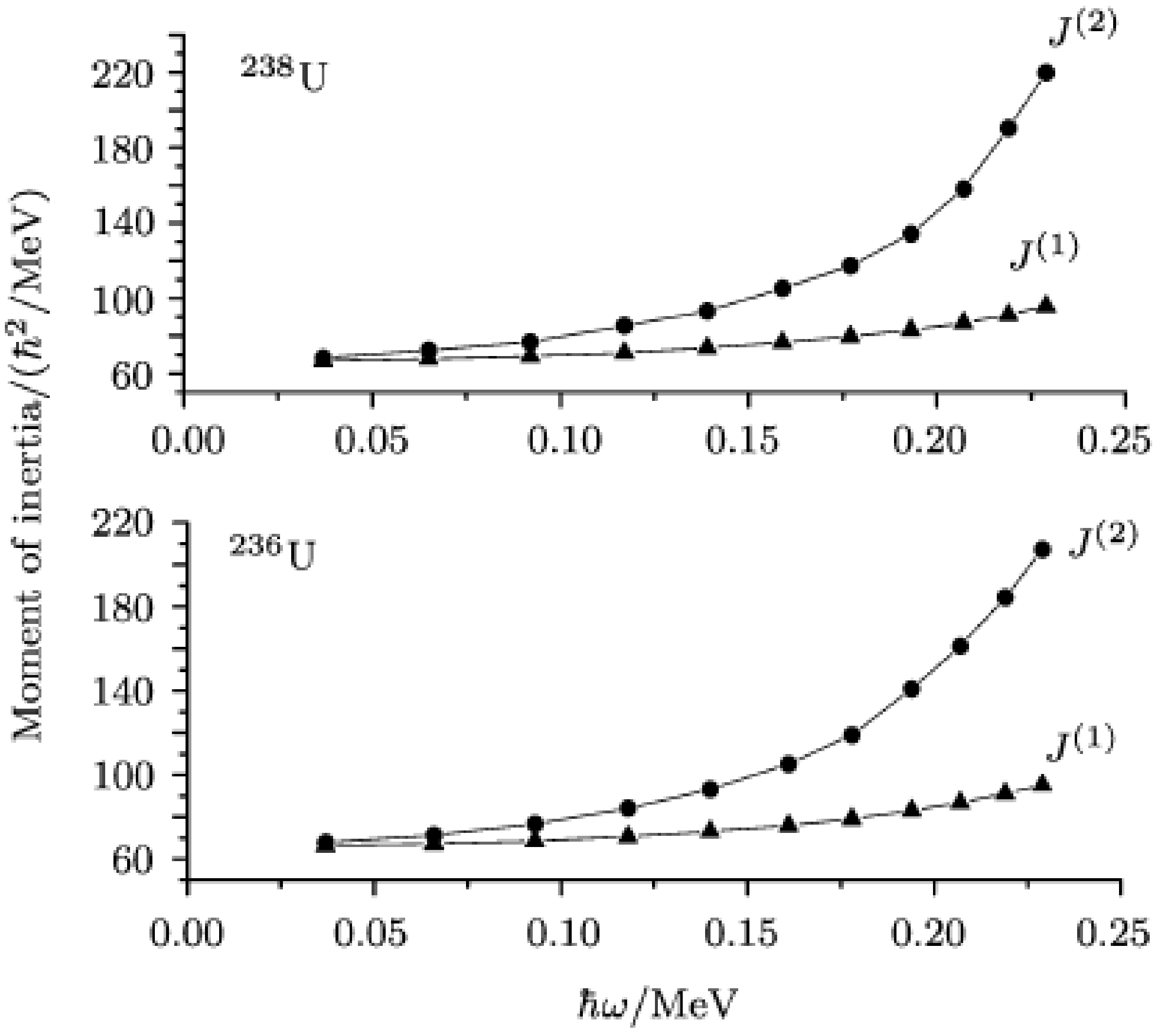
# ***Identical Bands***

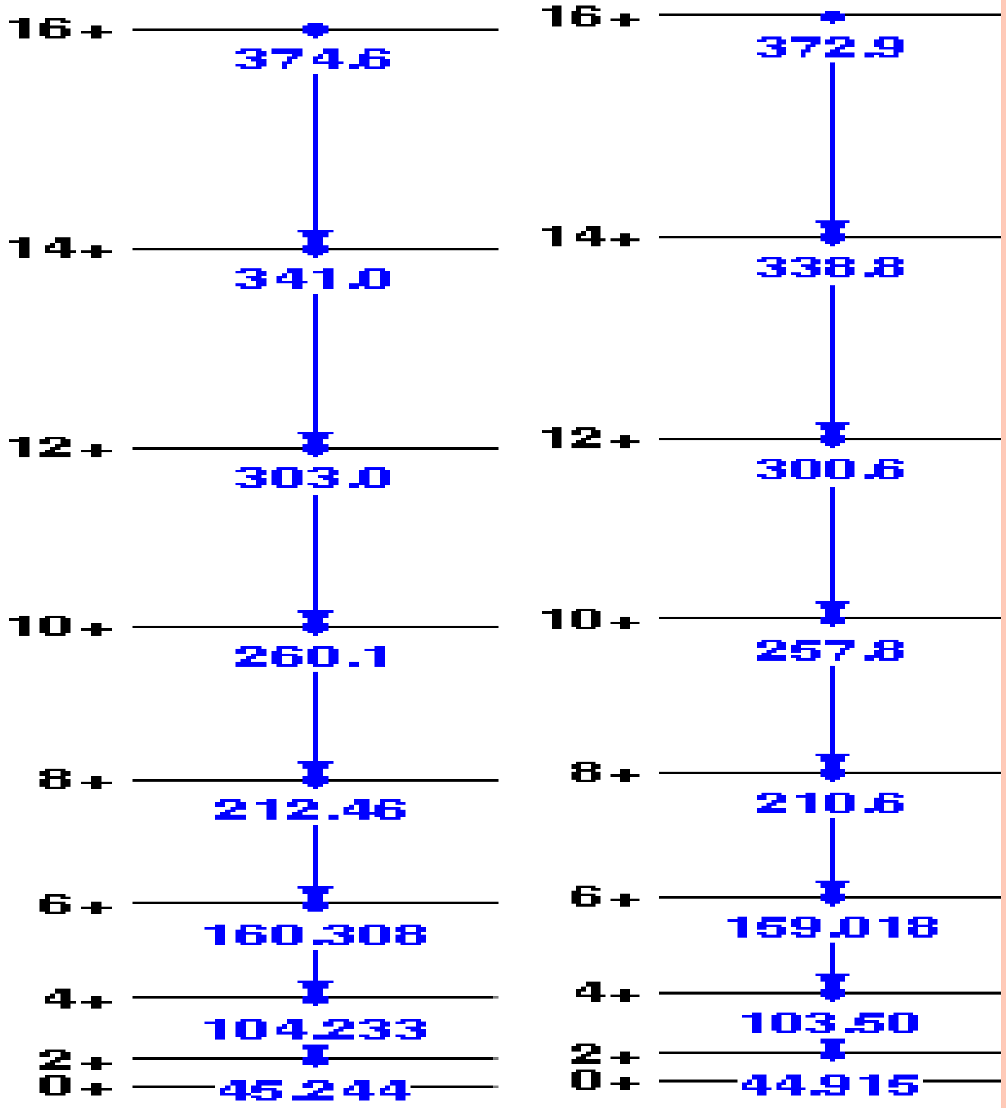
The identical bands(IB's) or twin bands in SD bands, bands with nearly identical transition energies and identical dynamical moment of inertia in neighboring nuclei with different mass number.

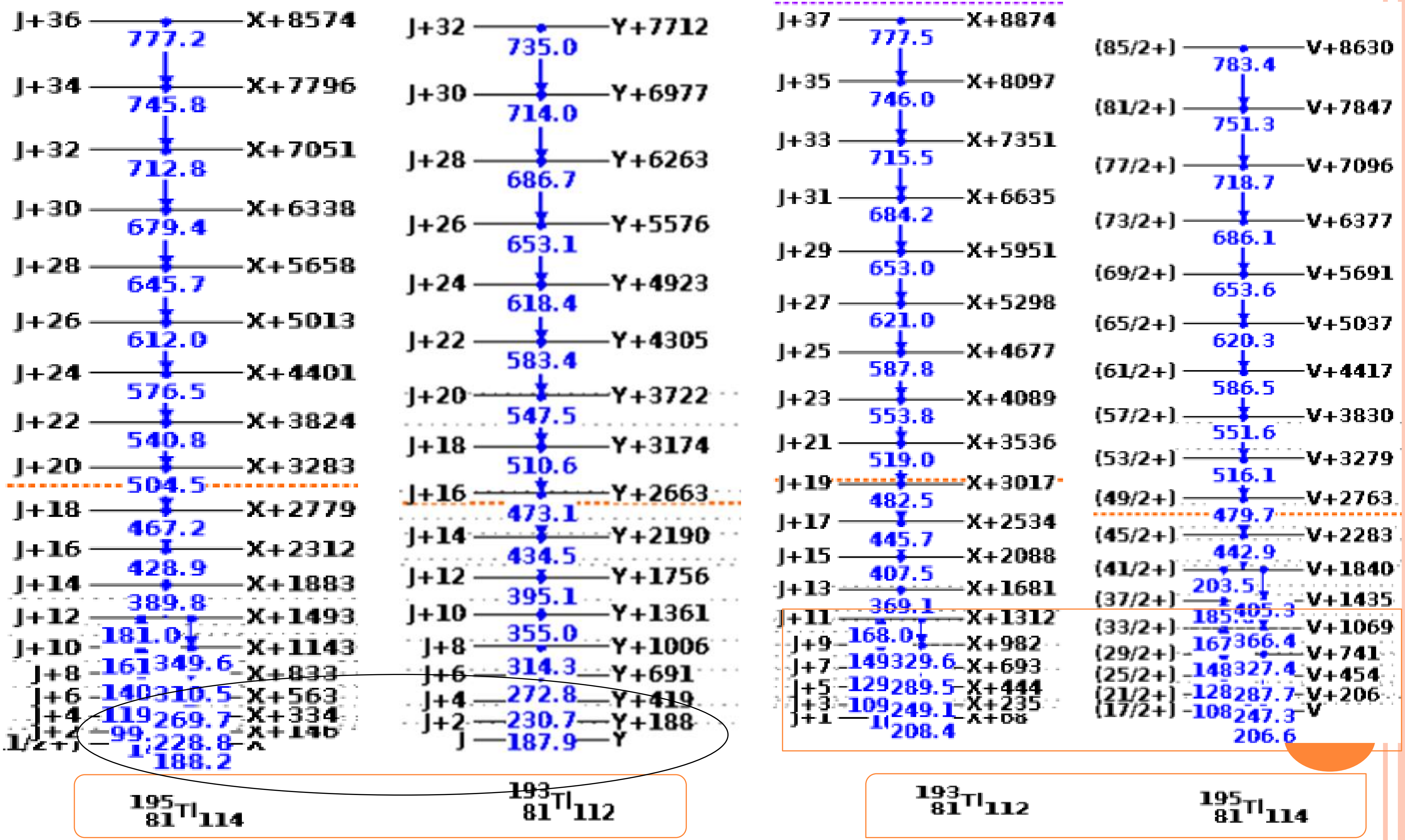
The difference in γ-ray transition energies ΔEγ was found to be only 1-3 Kev.

In order to determine whether a pair of SD bands is identical or not, one must compare the γ-ray transition energies or the dynamical moment of inertia.

We see that both $J^{(1)}$ and $J^{(2)}$ of the two bands in the two isotopes are almost identical, and thus their γ- transition energies are equal.





J+36 X+8574
777.2
J+34 X+7796
745.8
J+32 X+7051
712.8
J+30 X+6338
679.4
J+28 X+5658
645.7
J+26 X+5013
612.0
J+24 X+4401
576.5
J+22 X+3824
540.8
J+20 X+3283
504.5
J+18 X+2779
467.2
J+16 X+2312
428.9
J+14 X+1883
389.8
J+12 X+1493
181.0
J+10 X+1143
349.6
J+8 X+833
310.5
J+6 X+563
269.7
J+4 X+334
228.8
188.2
195 81 Tl 114
J+32 Y+7712
735.0
J+30 Y+6977
714.0
J+28 Y+6263
686.7
J+26 Y+5576
653.1
J+24 Y+4923
618.4
J+22 Y+4305
583.4
J+20 Y+3722
547.5
J+18 Y+3174
510.6
J+16 Y+2663
473.1
J+14 Y+2190
434.5
J+12 Y+1756
395.1
J+10 Y+1361
355.0
J+8 Y+1006
314.3
J+6 Y+691
272.8
J+4 Y+419
230.7
J+2 Y+188
187.9
J Y
193 81 Tl 112
J+37 X+8874
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168.0
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149 329.6
J+7 X+693
129 289.5
J+5 X+444
109 249.1
J+3 X+235
208.4
193 81 Tl 112
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783.4
(81/2+) V+7847
751.3
(77/2+) V+7096
718.7
(73/2+) V+6377
686.1
(69/2+) V+5691
653.6
(65/2+) V+5037
620.3
(61/2+) V+4417
586.5
(57/2+) V+3830
551.6
(53/2+) V+3279
516.1
(49/2+) V+2763
479.7
(45/2+) V+2283
442.9
(41/2+) V+1840
203.5
(37/2+) V+1435
185 405.3
(33/2+) V+1069
167 366.4
(29/2+) V+741
148 327.4
(25/2+) V+454
128 287.7
(21/2+) V+206
108 247.3
(17/2+) V
206.6
195 81 Tl 114

# *Papers*

**Identical Bands Around the Isobaric Rare-Earth Even–Even Nuclei with the Mass Number of A=160**
***Physics of Atomic Nuclei***, 86(6), 946–961, 2023 **DOI:** 10.1134/S1063778824010010

**Nuclear Shape Transition, Triaxiality and Energy Staggering of Y -Band States for Even–Even Xenon Isotopic Chain**
***Physics of Atomic Nuclei***, 86(4), 356–369, 2023 **DOI:** 10.1007/s13538-023-01357-y

**Nuclear Shape-Phase Transition From Spherical U(5) to Deformed γ -Unstable O(6) Dynamical Symmetries of Interacting Boson Model Applied to Ru, Pd, and Xe Isotopic Chains**
***Brazilian Journal of Physics***, 53(6), 148, 2023 **DOI:** 10.1134/S1063778823040208

**Deviation From Rigid Rotational Behavior of Superdeformed Nuclear Bands in Tl and Pb Signature Partners**
Khalaf, A.M., Taha, M.M., Sirag, M.M.
***Physics of Atomic Nuclei***, 86(2), 87–104 , 2023 **DOI:** 10.1134/S1063778823020102

**Energy Staggering and Identical Superdeformed Bands in $^{191-195}$ Tl Nuclei within Modified Perturbed SU(3) Limit of The sdg IBM**
*Physics of Atomic Nuclei*, 85(6), 606–618, 2022 **DOI:** 10.1134/S1063778823010271

**Description of Some Superdeformed Bands in Mercury, Thallium, and Lead Nuclei Using a Suggested Collective Rotation–Vibration Model Plus Fluctuation Energy Term**
*Physics of Atomic Nuclei*, 85(5), pp. 434–445, 2022 **DOI:** 10.1134/S1063778822050088

**Occurrence of Staggering and Identical Energies in Ground State Rotational Bands in Some Actinide Isotopes within Two-Parameter Rotational Model**
*Physics of Atomic Nuclei*, 85(3), pp. 263–274, 2022 **DOI:** 10.1134/S1063778822030103

**Application of Suggested Three Parameters Collective Rotational Model to Superdeformed Thallium Odd-Mass Nuclei**
*Journal of Physics: Conference Series*, 2304(1), 2022 **DOI:**10.1088/1742-6596/2304/1/012014

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## Theoretical study of nuclear collective phenomena and broken SU(3) symmetry of even-even $^{238-244}$Pu isotopes

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Received 17 July 2020; received in revised form 22 November 2020; accepted 2 December 2020
Available online 10 December 2020

**Abstract**

Theoretical nuclear study of the even-even Plutonium Isotopes $^{238-244}$Pu was carried out. The positive-parity rotational state band was calculated in the framework of the interacting Boson model-1 (IBM-1) and modified soft rotor formula (MSRF), while the negative-parity band was calculated in the framework of the new modified negative-parity formula (NMF). Furthermore, the energy levels for the $\beta$- and $\gamma$-vibrational energy bands were calculated. The odd-even staggering effect between the ground and octupole bands, which is a function of spin, was analyzed. The intrinsic coherent state was used to find the potential energy surfaces (PES). In addition, electric transition probabilities $B(E1)$, $B(E2)$, and $B(E3)$ for these isotopes were calculated. The results obtained by applying the IBM-1, MSRF, and the NMF were inferred to be in good agreement with the corresponding experimental data for most of the nuclear states.


*Keywords:* Theoretical nuclear study; Interacting boson model-1; Potential energy surfaces; Staggering effect

### 1. Introduction

Nuclear properties of heavy nuclei show a wide variety of collective phenomena such as rotational energy bands, $\beta$- and $\gamma$-vibrational energy bands, electric transition probability $B(E2)$, and odd-even staggering. These collective effects are described in several frameworks.

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https://doi.org/10.1016/j.nuclphysa.2020.122117
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Bohr and Mottelson (BM) [1] gave the simplest well-known expression describing the ground-state rotational bands. A major drawback of this expression is the rapid increase of the nuclear moments of inertia as a function of the nuclear deformation. Moreover, several different factors aggravate the simple structure of the rotational spectrum, which makes it deviate from the experimental data, such as the presence of Coriolis coupling, centrifugal stretching, and antipairing. Therefore, huge numbers of endeavors have been undertaken to generate a simple expression for rotational energy [2–9]. Brentano et al. [10] obtained one of these interesting expressions called soft rotor formula (SRF), which applies to all collective even-even nuclei, and gives suitable fitting data up to known highest Yrast spins.

Algebraic models of the nuclear structure have been quite successful to describe the nuclear properties. Recently, for instance, the interacting boson model (IBM) [11] has been strongly effective in giving a phenomenological depiction of spectroscopic information over a wide scope of nuclei indicating collective structures. Based on the Hilbert space, the IBM Hamiltonian carries an irreducible representation of the group U(6) of three dynamical symmetries corresponding to different geometrical shapes, as the U(5) vibrator chain and two other possibilities, namely the axially symmetric deformed rotor SU(3) and the $\gamma$-unstable model O(6) [12–17]. Furthermore, only a few nuclei can be regarded as these three dynamical symmetries, while most nuclei may have treated as transitional nuclei in terms of a symmetry triangle [18].

In even nuclei, there exist octupole deformation in the ground state band, which contains energy levels $I^\pi = 0^+, 2^+, 4^+, \ldots$, which is joined to a negative parity band containing energy levels with $I^\pi = 1^-, 3^-, 5^-, \ldots$. After the first few values of angular momentum $I$ the two bands become interlaced, forming a single octupole band with levels characterized by $I^\pi = 0^+, 1^-, 2^+, 3^-, 4^+, \ldots$. The odd levels do not lie at the energies predicted but all of them lie systematically above or all of them lie systematically below the predicted energies. This is a case of odd-even staggering or $\Delta I = 1$ staggering [19].

Recently, several experimental and theoretical investigations of the nuclear structure have been conducted [20–26]. H. S. Elgendy [27] investigated some nuclear structures of the Ytterbium isotopes $^{166-176}$Yb, wherein, modified soft rotor formulas (MSRF) were used to calculate rotational bands. Moreover, the correction formula for the BM-formulas (CBMFs) was used to calculate the $\beta$- and $\gamma$-vibrational energy bands. In addition, Doma and Elgendy [28–30] discussed some nuclear phenomenological study as Spectra, potential energy surface (PES), and electromagnetic transitions for some even-even nuclei.

Moreover, A. M. Khalaf et al. [31] calculated the odd-even staggering effect in Thorium isotopes within the framework of the interacting vector Boson model. Also, A. M. Khalaf et al. [32] considered the phase transition of even–even ruthenium isotopic chain within the dynamical symmetry $\gamma$-unstable model O(6) of interacting boson model with the three-body quadrupole interaction. Furthermore, Al-Jubbori et al. [33] introduced a new empirical formula that calculated rotational bands for even–even rare-earth $Er - Os$ isotopes for $N = 102$ and discussed the properties of gamma, beta Yrast band. Moreover, the potential energy surface and reduced transition probabilities $B(E2)$ have been also calculated for these rare-earth isotopes. The authors concluded that the $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W nuclei show a rotational dynamical symmetry SU(3) and $^{178}$Os shows dynamical symmetry $X(5)$.

The aim of this paper is to consider some nuclear features of the Plutonium isotopes $^{238-244}$Pu, wherein we study the rotational bands in the framework of nuclear softness formula (NSF) [6,34], modified soft rotor formulas (MSRF), and (IBM). Furthermore, we introduce a New modified negative-parity formula (NMF) through which the negative-parity band has been calculated. We will also calculate the $\beta$- and $\gamma$-vibrational energy bands in the framework of





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# Nuclear shape phase transition in even-even $^{158-168}$Hf isotopes

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Received 15 March 2020; received in revised form 16 June 2020; accepted 19 June 2020

**Abstract**

The interacting *sdf*-boson-approximation model is adopted to describe the low-lying positive and negative parity states in even-even $^{158-168}$Hf isotopes. The negative parity states are described using the *sdf*-IBM model by adding the L = 3 *f*-bosons nucleon pairs to the standard *sd*-boson model space. To determine the deformation of the nuclear structure of these isotopes the potential energy surfaces are calculated as functions of the deformation parameters. The reduced transition probabilities B(E2) in N = 94 are compared to the critical point symmetry X(5) predictions. In this chain, the *sdf*-boson interacting parameters are investigated and plotted against a number of neutrons. The shape phase transition from spherical $^{158}$Hf to well-deformed $^{168}$Hf nuclei is observed.





abilities were given in the X(5) and X(5)-$\beta^2$ models. Casten and McCutchan [8] used the idea of quantum phase transitions to study in detail the critical point symmetries X(5) and E(5). They also made a comparison between the theoretical results and the data in some of the isotopes.

Nomura et al. [9] presented low-lying spectra for the neutron-rich Hf and Yb isotopes. They calculated excitation energies, B(E2) ratios, and correlation energies in the ground state and observed a transition from prolate to oblate ground state shapes in the chain. Within the framework of the interacting boson model, Subber [10] described the structure of neutron-rich deformed $^{176,178}$Hf isotopes and calculated energy levels, monopole transitions and B(E2) ratios of these isotopes. Sun et al. [11] studied the multi-quasiparticle and collective excitations of the $^{178}$Hf isotope with an axially symmetric basis. From energy surface calculations, it is suggested that in relation to $^{178}$Hf isotope, there is significant smoothness to the axially asymmetric shapes, which can highly adapt to the level distribution. Gupta [12] investigated the collective band structure of Hf isotopes using IBM and the dynamic pairing plus quadrupole model (DPPQ) models. The author used the IBM-1 model to reproduce the structure evolution of $^{166,168}$Hf isotopes and found out that $^{166}$Hf does correspond to the N = 94 as X(5) nuclei. Further, he predicted that $^{168}$Hf isotope represents a shift towards the SU(3) limit.

McCutchan et al. [13] calculated low-lying, positive parity excitations of the Z = 64 to 72, N = 86 to 104 even-even rare-earth nuclei. In the phase transition region, larger Z values show increased $\gamma$-softness. The $\gamma$-softness increases for the Yb and Hf nuclei which lie very close to the U(5)-O(6) leg of the triangle. Wiederhold et al. [14] determined mean lifetimes of excited states of Hf isotopes. From these lifetimes, the B(E2) transition strengths between the yrast states and the $B(E2; 4_1^+ \to 2_1^+)/B(E2; 2_1^+ \to 0_1^+)$ ratios were calculated. The authors determined the mean lifetimes of the $2_1^-$ and the $3_1^-$ states of $^{176}$Hf isotopes. Khalaf et al. [15] investigated the characteristics of energy ratios and B(E2) values of even-even $^{160-180}$Hf isotopes. The values are analyzed with respect to the increase in the total number of bosons and by using the coherent state formalism.

The aims of the present study include the following:

1) To carry out a systematic $sdf$-IBM calculation of the energy spectra of the even-even Hf isotopes with A = 158-168. The shape phase transition and critical-point within this Hf-chain are of special importance as well.

2) To study the interband and intraband electric transition probabilities $E1$, $E2$ and $E3$. Special attention is to be given to the alternating-parity transitions.

3) Identification of the appropriate potential energy surface values in the deformation space to explore the role of quadrupole and octupole deformation in studying the nuclear structure.

## 2. The interacting boson model

Arima and Iachello [16] have suggested a nuclear model called the interacting boson model (IBM). It is used to describe the low lying collective states in many medium to heavy even-even nuclei. In the calculation of energy levels, the most general Hamiltonian of IBM-1 was used [17,18]:

$$H_{sd} = \varepsilon_d \hat{n}_d + a_0 P^\dagger.P + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4, \quad (1)$$

where $\varepsilon_d$ is the d-boson energy, $\hat{n}_d = (d^\dagger.\tilde{d})$ is the d-boson number operator, $P^\dagger = 1/2[s^\dagger.s^\dagger - d^\dagger.d^\dagger]$ is the paring operator, $\hat{L} = \sqrt{10}[d^\dagger \times \tilde{d}]^{(1)}$ is the angular momentum operator, $\hat{Q} = [d^\dagger \times s + s^\dagger \times \tilde{d}]^{(2)} - \chi[d^\dagger \times \tilde{d}]^{(2)}$ is the quadrupole operator ($\chi$ is the quadrupole parameter), $\hat{T}_3 = [d^\dagger \times \tilde{d}]^{(3)}$ is the octupole operator and $\hat{T}_4 = [d^\dagger \times \tilde{d}]^{(4)}$ is the hexadecapole operator.





of the minimum on the $(\beta_2, \beta_3)$ energy surface. We suppose that the system is allowed to vibrate with respect to the quadrupole $\beta_2$ and octupole $\beta_3$ axial deformation variables [27,28]. The PES values as function of the quadrupole and octupole deformation parameters are plotted in Fig. 14. In this figure, the energy surfaces are symmetric around the $\beta_3 = 0$ axis. With the increase of the mass number, the potential energy surfaces becomes steeper in $\beta_2$ direction and $\beta_{2min}$ shifts away from the origin. For $^{168}$Hf isotope, IBM PES has a pronounced sharp minimum exhibits the SU(3) limit shape. We see that for a fixed physically typical value of $\beta_2$ the barrier in the quadrupole space of $\beta_3$ is large. For $^{160,162}$Hf the energy surfaces maximum begins to increase to (1.53, 1.95) MeV, respectively. And the energy surface of $^{166}$Hf isotope is quit similar to the deformation shape of $^{168}$Hf isotope. An overview of the PES values as a function of the $\beta_2$ and $\beta_3$ is given in Fig. 14. There is a quadrupole deformation in the shape of $^{166,168}$Hf isotopes while there is no octupole deformation in the ground state of all isotopes.

## 7. Conclusion

In the present paper, the low-lying spectra including the positive and negative parity bands of even-even Hf isotopes are investigated within the framework of the $sdf$-IBM model. The transition between the three limit symmetries U(5) to SU(3) is observed in comparison to different nuclear shapes. The potential energy surfaces as functions of the axial quadrupole and the octupole deformation parameters are analyzed. A good agreement is found between the model results and experimental data and show X(5) characters to $^{166}$Hf nucleus. For electromagnetic transition probabilities, future calculations will be possible if an additional or new experimental data become available.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Turkish Journal of Physics
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Research Article

Turk J Phys
(2019) 43: 1 – 10
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doi:10.3906/fiz-1802-8

# Study of A = 100–150 superdeformed mass region by using two-parameter formula

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**Received:** 06.02.2018 • **Accepted/Published Online:** 04.12.2018 • **Final Version:** 22.02.2019

**Abstract:** Empirical formulae of rotational spectra consisting two parameters, such as single-term energy formula, $E = aJ^b$ for spin $J$, and $ab$ formula, were used to study the different features of superdeformed band in A = 100–150 mass region nuclei. The nuclear kinematic and dynamic moment of inertia for the ground-state rotational bands were calculated for this purpose and both showed gradual rise with rotational frequency. The study of $\Delta I = 2$ staggering effects in the $\gamma$-ray energies, where the two sequences $J = 4i, 4i+1$ and $J = 4i+2$, $(i = 0, 1, \ldots)$ are bifurcated, was also done. We also calculated the variation of the gamma ray energies from a smooth reference using the fourth derivative of the gamma ray energies at a given spin. The excellent agreement between the observed and calculated transition energies are in good support of the two-parameter formula.

**Key words:** Superdeformed band, two parameter formula, staggering index, identical bands

## 1. Introduction

A superdeformed nucleus is a nucleus that is predicted to occur at specific magic numbers and at deformations corresponding to the integer ratios of the axes about 2:1:1. Generally, the normal deformation of the nucleus is about 1.3:1:1. Superdeformed structures have been found mostly in nuclei of the A = 150 and 240 mass regions, i.e. in the fission isomers low-spin states of elements in the actinide and lanthanide series. Recently, it has also been discovered in other mass regions, such as $A = 60, 80, 130$, and 190. In the past few years, much effort has been devoted to study the underlying physics of superdeformed bands and other interesting facts and issues, such as the identical bands [1], $\Delta I = 1, 2$ staggering [2, 3] and the multipole correlation and exotic structure of nuclei [4].

A general understanding in the properties of superdeformed nuclei has been attained, but still there are open problems that need to be further studied. In this paper, we used empirical formula of rotational spectra consisting two parameters, i.e. single-term energy formula, $E = aJ^b$, and $ab$ formula, to study the different features of superdeformed band in A = 100–150 mass region.

The moment of inertia is one of the most significant quantities to characterize the nuclear rotational band. There are generally two kinds of moment of inertia ($\Im$) for illustrating the high-spin phenomena, i.e. the dynamic moment of inertia,

$$\Im^2 = \hbar \frac{dI_x}{d\omega} = \hbar^2 \left[ \frac{d^2 E}{dI_x^2} \right], \qquad (1)$$

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International Journal of Modern Physics E
Vol. 27, No. 5 (2018) 1850040 (14 pages)

DOI: 10.1142/S0218301318500404

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# A nuclear phenomenological study of the even–even Thorium Isotopes $^{228-232}$Th

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Received 16 December 2017
Revised 11 April 2018
Accepted 17 April 2018
Published 17 May 2018

The rotational and vibrational energies and the electric transition probability B ($E2$) of the even–even $^{228-232}$Th isotopes are studied empirically in framework of a nuclear phenomenological approach by using the SU(3) dynamical symmetries of the Interacting Boson Model-1 (IBM-1). Furthermore, the potential energy surfaces for these isotopes are plotted as functions of the deformation parameters $\beta$ and $\gamma$. Moreover, we introduce empirical fit formulas for rotational and vibrational energies, which used to calculate these energies for the thorium isotopes. The obtained results by applying the IBM-1 and the authors' formulas are in good agreement with the corresponding experimental data for most of the nuclear states.

*Keywords*: Rotational energy; vibrational energy; electric transitions probability; interacting boson model.

PACS Number(s): 23.40.−s, 21.10.Tg, 21.10.ky



## 1. Introduction

The Interacting Boson Model (IBM), as proposed by Arima and Iachello,[1] has been successful in phenomenological studies for describing low-lying, quadrupole collective states of medium-heavy nuclei. The major assumption of IBM is to describe a system of mutually interacting bosons; a boson can be either in an $L = 0$ ($s$) or in an $L = 2$ ($d$) state. Furthermore, since the bosons are thought of as collective states of nucleon pairs, the total number of bosons $N$ ($s$ and $d$) is a conserved quantity. The IBM Hamiltonian can be regarded as a general transformation acting on a six-dimensional space, spanned by the $s$-boson and the five components of the $d$, leaving the total number of bosons invariant. Consequently, the group structure underlying the IBM-model is U(6). In the U(6) group, three different chains of subgroups can be distinguished, if one requires that the O(3) (angular momentum) be a subgroup. The three dynamica1 symmetries[2–5] are labeled by the first subgroup





of each chain, SU(5), SU(3) and O(6), respectively. The solutions for these three limits show many similarities with three different cases in the geometrical picture,[6,7] namely the anharmonic vibrator, the axially symmetric deformed rotor and the $\gamma$-unstable model, respectively. In contrast to the geometrical picture, the three limits in the IBM are merely special cases of a more general Hamiltonian, which can be diagonalized numerically. The IBM can therefore provide a detailed description not only of the limits, but also for intermediate cases.

The IBM has been frequently used in recent years for the description of the structure of medium and heavy nuclei away from closed shells. Energy spectra, quadrupole moments, electromagnetic transition rates, equilibrium shapes and shape transitions have been studied for a large number of nuclei, and encouraging agreement with the experimental data has been obtained with only a small number of parameters that vary smoothly with mass number.

Gupta[8] studied the capabilities of IBM-1 and gave an insight on the variation of the nuclear structure of $^{122-134}$Ba with neutron number N. A detailed study of the energy systematics of $^{122-134}$Ba and the $E2$ transition rates in their decay has been done. In addition, Gupta studied the shape transition of light Xe, Ba and Ce isotopes in comparison with the predictions of the various analytical symmetries for this region.

Moreover, Khalaf and Taha[9] adopted a simplified two-parameter IBM-1 Hamiltonian which is an intermediate Hamiltonian between the three dynamical symmetries of U(6), namely: the spherical U(5), the prolate and oblate deformed SU(3) and the $\gamma$-unstable O(6) limits. The potential energy surfaces to the IBM Hamiltonian have been obtained in this paper by using the intrinsic state formalism which introduces the shape variables $\beta$ and $\gamma$. The Gadolinium and Ruthenium isotopic chains have been taken as examples in illustrating the U(5) − SU(3) and U(5) − O(6) shape-phase transitions, respectively.

Furthermore, microscopic description of octupole shape-phase transitions in light actinide and rare-earth nuclei are presented by Nomura *et al.*,[10] where a systematic analysis of low-lying quadrupole and octupole collective states is presented based on the microscopic energy density functional framework. By mapping the deformation constrained self-consistent axially symmetric mean-field energy surfaces onto the equivalent Hamiltonian of the sdf-IBM, that is, onto the energy expectation value in the bosoncondensate state, the Hamiltonian parameters are determined. Their study is based on the global relativistic energy density functional DD-PC1. The resulting IBM Hamiltonian is used to calculate excitation spectra and transition rates for the positive and negative-parity collective states in four isotopic chain characteristics for two regions of octupole deformation and collectivity: Th, Ra, Sm and Ba. Consistent with the empirical trend, the microscopic calculations, which are based on the systematics of $\beta_2, \beta_3$ energy maps, the resulting low-lying negative-parity bands and transition rates show evidence of a shape transition between stable octupole deformation and octupole vibrations characteristic for $\beta_3$-soft potentials.

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that we have degenerated $\beta$ and $\gamma$ bands, and therefore they must have the same representation states of the same spin of these two bands.

Furthermore, we used the RMSD[26] to calculate the deviation between the energy levels of the IBM and the DG formula from the experimental results. It is shown from Table 5 that best agreement is obtained for the ground state and the $\gamma$-band when we used the DG formula, while good agreement is occurred for the $\beta$-band by using the IBM model.

Also, in Fig. 1(b) which illustrates the potential energy surface $V(\beta, \gamma)$ for the even–even deformed $^{228-232}$Th isotopes as functions of the deformation parameters $\beta$ and $\gamma$, we found the minimum values (−2.99 MeV) for the prolate shape of the $^{228}$Th at $\beta = 1.3$ and $\gamma = 0$, (−2.40 MeV) for the prolate shape at $\beta = 1.3$ and $\gamma = 0$ for $^{230}$Th and (−2.86 MeV) for the prolate shape at $\beta = 1.3$ and $\gamma = 0$ for 232Th In addition in Fig. 1(a) note that the deformation parameters $\beta < 0$ for oblate and $\beta > 0$ for prolate shapes[11,30] so for $\gamma = 60°$, it is oblate type and when $\gamma = 0$, it is prolate type. We conclude from this study on PES that the prolate deformation is deeper than the oblate

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# Expected Coming Work

1. The phenomena of the duple backbendings of superdeformed bands
2. new general models to solve the problems in the SD, ND nuclei
3. new general models to solve the problems in the IB,S of SD, ND nuclei
4. Study two new types of deformation.
   - Hyperdeformed nuclei (HD)
   - Triaxial superdeformed nuclei (TSD)
5. The phenomena of staggering $\Delta I=4$
6. Suggested A model can deal with the properties & other phenomena of coexistence nuclei

الحمد لله رب العالمين

# Thank You