Neutron spectrum unfolding method based on shifted Legendre polynomials, its application to the IREN facility

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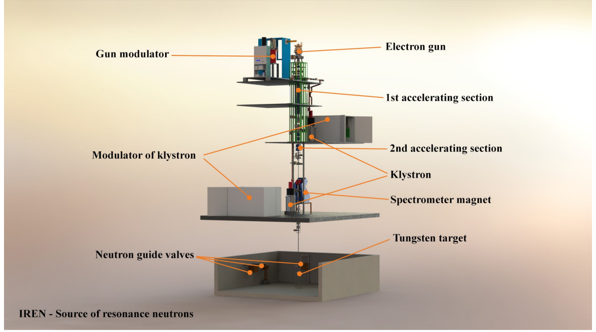
Introduction

- Individual dosimetric occupational monitoring is based on the results of dosimetric monitoring of workplaces, which consists of determining the energy distribution of neutron flux density when monitoring continuous radiation fields, as well as the time a worker spends in these conditions.
- Radiation fields behind the protective shields of the JINR nuclear physics facilities (particle accelerator, nuclear reactors) are formed mainly by *neutrons* of a wide energy spectrum.
- Radiation monitoring at accelerators cannot be carried out using standard dosimeters and neutron radiometers alone, since their operating range is limited by a maximum neutron energy of about 10 MeV.
- Bonner Multi-sphere spectrometer is a common used tool for Radiation monitoring at accelerators.



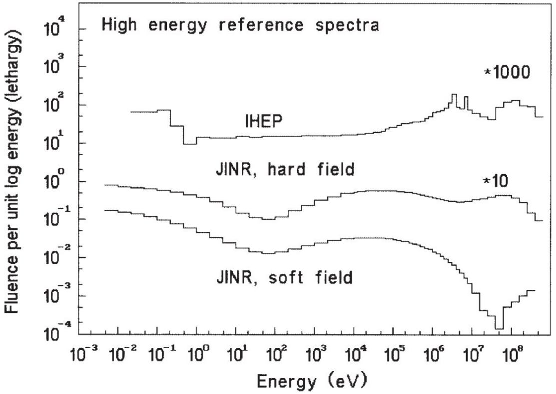
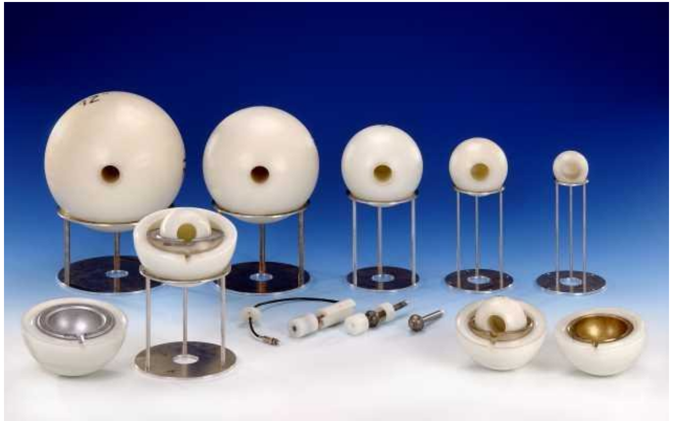
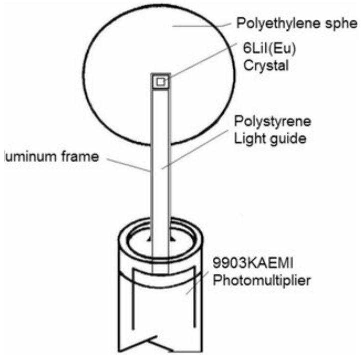


FIG. 4.21. High energy reference spectra (JINR and IHEP).

Bonner Multi-sphere Spectrometer

- A multi-sphere Bonner spectrometer is used to measure the neutron flux density.
- The measurement method is based on the moderation of fast neutrons in polyethylene moderator balls of various diameters.
- Various detectors are used to detect thermal neutrons, e.g., inorganic scintillators such as ^{6}LiI.





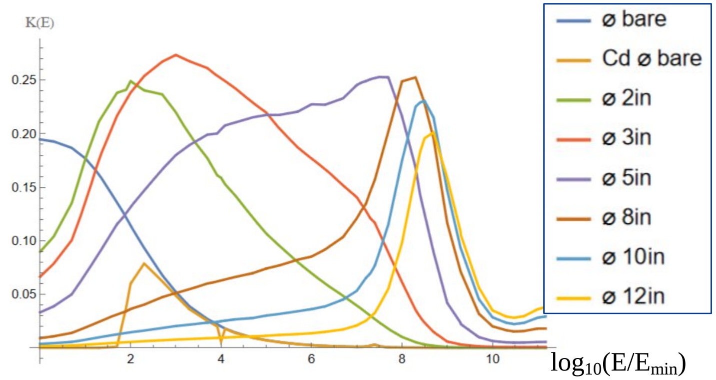
Unfolding the neutron spectra

Fredholm integral equations of the 1st kind:

$$Q_j = \int_{E_{min}}^{E_{max}} K_j(E) \cdot \varphi(E)\, dE, \quad j = 1,\ldots,M,$$

- where Q_j is the Bonner spectrometer reading for the j-th sphere,
- $\varphi(E)$ - neutron spectrum,
- $K_j(E)$ is the kernel of the j-th equation, which is a response function of the detector to neutrons of various energies,
- M is the number of spheres used to measure the spectrum.
- The integration limits E_{min} and E_{max} are specified by the domain of definition of the neutron spectrum E and the set of detectors used for measurements.

This is an **ill-posed inverse** problem.



$K_j(E)$, response function of JINR Bonner spectrometer

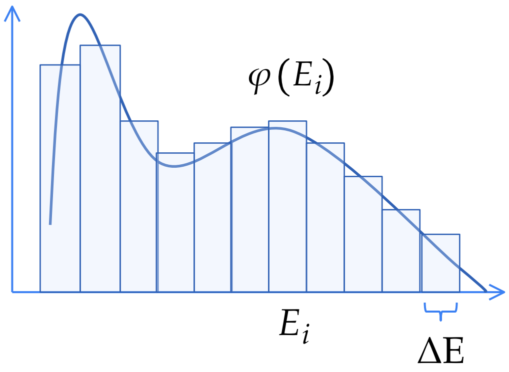
Martinkovic J., Timoshenko G. N. P16-2005-105 Calculation of Multisphere Neutron Spectrometer Response Functions in Energy Range up to 20 MeV, JINR preprint, 2005

Unfolding methods

- Reconstruction of the spectrum via a numerical solution of a system of Fredholm integral equations of the 1st kind with partitioning $\varphi(E)$ over a discrete grid and applying the Tikhonov regularization:

$$Q_j = \int_{E_{\min}}^{E_{\max}} K_j(E)\cdot\varphi(E)\,dE \approx \sum_{i=1}^{N} R_{ji}\Phi_i \Delta E, \quad j = 1,\ldots,M,$$

where $\Phi_i \equiv \varphi(E_i)$ is a vector that is a discrete analogue of a continuous quantity of $\varphi(E)$ being the neutron spectrum (E_i, $i = 1,\ldots,N$); R_{ji} is the matrix obtained from the kernel of the integral equations.



- Representation of the spectrum in the form of a linear combination of L trial functions, when the expansion coefficients C_i are found by the formula:

$$\varphi(E) = \sum_{i=1}^{L} C_i \cdot F_i(E) \rightarrow \quad Q_j = \sum_{i=1}^{L} A_{ji}\cdot C_i, \quad A_{ji} = \int_{E_{\min}}^{E_{\max}} K_j(E)\cdot F_i(E)\,dE.$$

- other methods (*Brooks F. D., Klein H. Neutron spectrometry – historical review and present status, Nuclear Instruments and Methods in Physics Research, A 476, 2002, p.111*).

Previously developed software

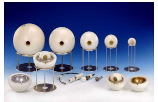
Dosimetry in JINR

The need for relevant and modern calculation codes

Statistical regularization method

JINR “RECONST”

BSS



1960 1970 1980 1990 2000 2010 2020 ?

Tikhonov Regularization Method

FRUIT MAXED GRAVEL
SAND-II FERDOR SPEC-4
NSDUAZ BUNKI LOHUI
BUMS MITOM BESPOKE
SPECTRA-UF MIEKE UNFANA
LOUHI …

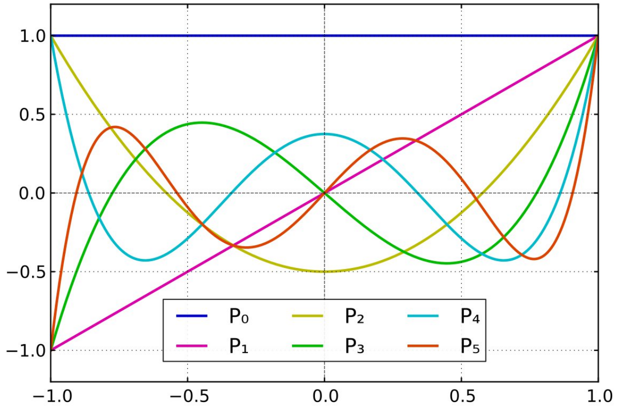
Development of computing computer capabilities and new algorithms spectrum recovery

Method of functional expansion of flux density using shifted Legendre polynomials with the Tikhonov regularization

We propose a method for decomposing the neutron flux density using shifted Legendre polynomials defined on the interval $[0, l]$:

$$P_n^*(x) = P_n\left(\frac{2x}{l} - 1\right),$$

where $P_n(x) = \frac{1}{2^n n!} \cdot \frac{d^n}{dx^n}\left(x^2 - 1\right)^n$ is the Legendre polynomial of n-th order defined on the interval $[-1,1]$.



Method of functional expansion of flux density using Legendre polynomials with Tikhonov regularization

$$Q_j = \int_{E_{\min}}^{E_{\max}} K_j(E)\cdot\varphi(E)\,dE, \quad j = 1,\dots,M,$$

Wide energy range from **10^{-8} to 10^{3} MeV**, proceed to integration over lethargy $u(E) = \lg(E/E_{\min})$:

$$Q_j = \ln 10 \cdot \int_0^{l_E} K_j(u)\cdot\varphi(u)\,E(u)\,du, \quad j = 1,\dots,M, \; l_E = \lg(E_{\max}/E_{\min}).$$

In our method, the problem of reconstructing the spectrum is reduced to finding expansion coefficients C_i using shifted Legendre polynomials:

$$\Phi(u) \equiv \varphi(u)\,E(u) = \sum_{i=1}^{N} C_i\, P_{i-1}\left(2u/l_E - 1\right), \quad u \in [0, l_E].$$

From the system of integral equations we obtain a system of linear algebraic equations, presented in the matrix form as

$$\boldsymbol{AC} = \boldsymbol{Q}$$

where the matrix elements **A** are defined as $A_{ji} = \ln 10 \cdot \int_0^{L_E} K_j(u)\cdot P_{i-1}(2u/l_E - 1)\,du.$

Tikhonov regularization with mth derivative

The condition of the minimum of the stabilizing functional with mth derivative:

$$M^{\alpha}[C] = \|\mathbf{AC}-\mathbf{Q}\|^2 + \alpha \times \int_0^{l_E} \left\{ \Phi^2(u) + [\Phi'(u)]^2 + \ldots + \left[\Phi^{(m)}(u)\right]^2 \right\} du = \sum_{j=1}^{M} \left[\sum_{i=1}^{N} A_{ji} C_i - Q_j\right]^2 + \alpha \times Z,$$

$$Z = \sum_{i,k=1}^{N} C_i C_k \int_0^{l_E} \left[P_{i-1}(2u/l_E - 1) P_{k-1}(2u/l_E - 1) + P'_{i-1}(2u/l_E - 1) P'_{k-1}(2u/l_E - 1) + \ldots + P_{i-1}^{(m)}(2u/l_E - 1) P_{k-1}^{(m)}(2u/l_E - 1) \right] du.$$

Solving regularized system of linear algebraic equations relative to the new expansion coefficients $\boldsymbol{C}^{\alpha}$ of the neutron spectrum and regularization parameter $\alpha > 0$:

$$\left(\boldsymbol{A}^T \boldsymbol{A} + \alpha \boldsymbol{B}\right) \boldsymbol{C}^{\alpha} = \boldsymbol{A}^T \boldsymbol{Q}$$

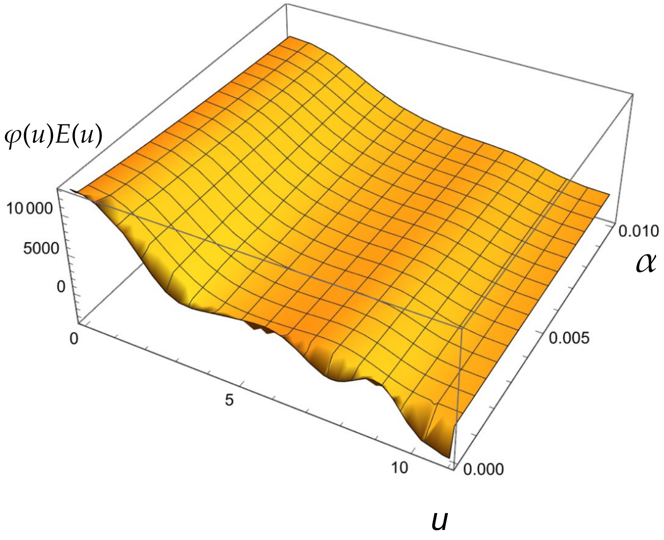
$$B_{ik} = \frac{2 l_E}{2i-1} \delta_{ik} + \sum_{n=1}^{m} 4^{1-n} \left(\frac{2}{l_E}\right)^{2n-1} \times \sum_{j=1}^{[N/2]-1} (-1)^{j-1} (n-j)!\,(n+j-1)!\, \left\{ \begin{matrix} \binom{i+n-j}{n-j} \binom{i}{n-j} \binom{k+n+j-1}{n+j-1} \binom{k}{n+j-1} \delta_{k-i,2(j-1)} + \\ + \binom{k+n-j}{n-j} \binom{k}{n-j} \binom{i+n+j-1}{n+j-1} \binom{i}{n+j-1} \delta_{i-k,2j} \end{matrix} \right\}$$

The choice of the regularization parameter (α)

The choice of the regularization parameter α is carried out in accordance with the *generalized residual principle*

$$\|\mathbf{A}\mathbf{C}^{\alpha}-\mathbf{Q}\|^{2}-\delta^{2}-\mu^{2}(\mathbf{Q},\mathbf{A})=0,$$

- δ is the error in measurements of the neutron spectrum $\varphi(E)$
- μ ($\mathbf{Q}$,$\mathbf{A}$) is a measure of incompatibility (physical restrictions on the spectrum form)



Ensuring the non-negativity of the $\varphi(E)$ function, the regularization parameter $\alpha \geq \alpha^*$, where α^* is the root of equation

$$\min_{u\in[0,l_E]} \sum_{i=1}^{N} C_i(\alpha^*)\cdot P_{i-1}(2u/l_E-1)=0\,.$$

Dose assessment

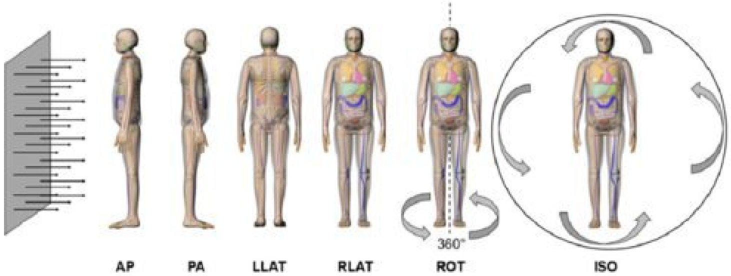
The Bonner multisphere spectrometer is used to measure neutron spectra in stationary fields to assess exposure of personnel.

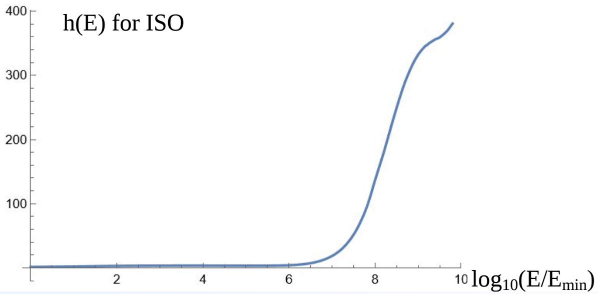
$$\Phi^{\alpha}(u) \equiv \varphi^{\alpha}(u)\,E(u) = \sum_{i=1}^{N} C_i^{\alpha} \cdot P_{i-1}(2u/l_E - 1),$$

$$\dot{H}^{\alpha} = \int_{E_{\min}}^{E_{\max}} h(E) \cdot \varphi^{\alpha}(E)\,dE = \ln 10 \times \int_{0}^{l_E} h(u) \cdot \Phi^{\alpha}(u)\,du,$$

where $h(E)$ [pSv·cm^2] is the *dose conversion coefficient* for mono energetic particles in various irradiation geometries (up to 20 MeV - NRB 99/2009; up to 10 GeV - ICRP116), for different irradiation types: *AP*, *PA*, *LLAT*, *RLAT*, *ROT*, *ISO*.

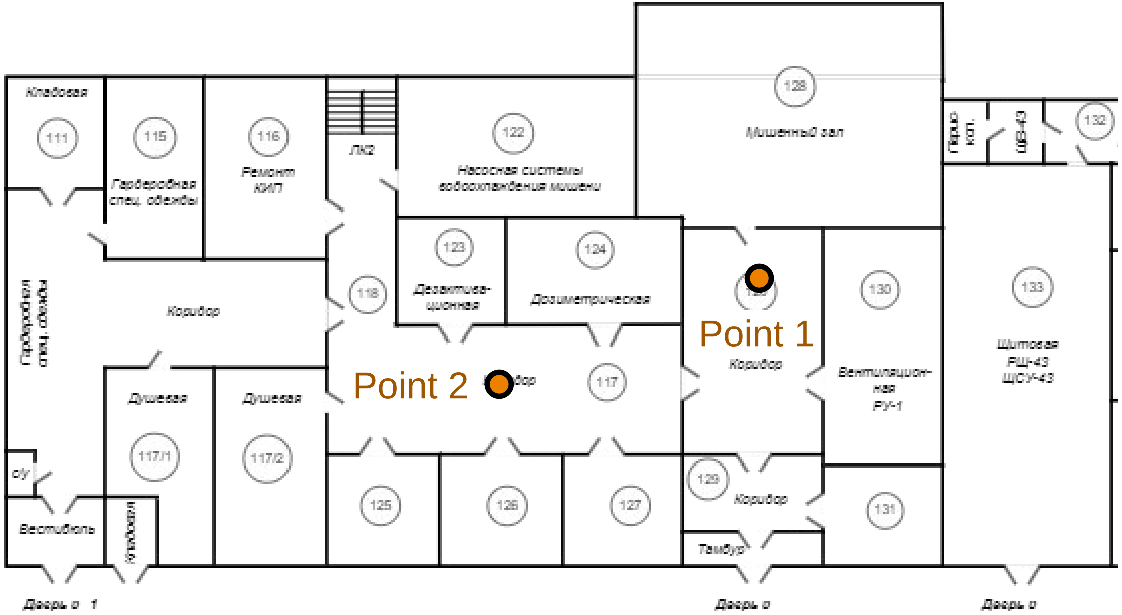
$\dot{H}^{\alpha}$ - Dose rate ($\dot{E}_{eff\_AP}$, $\dot{E}_{eff\_ISO}$, $\dot{H}^*(10)$, $\dot{H}_p(10,0^{\circ})$)

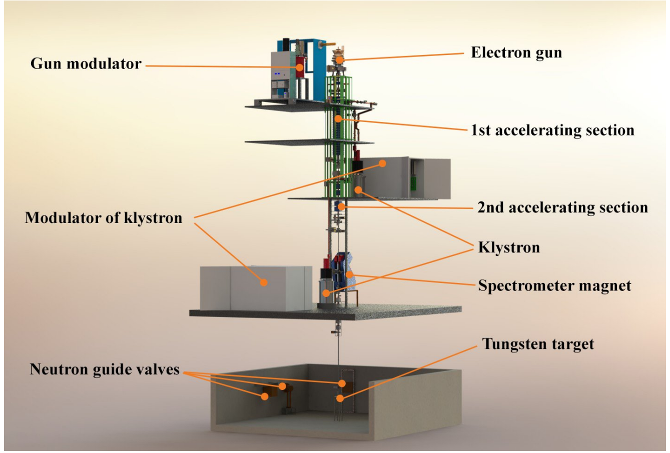




IREN facility (Intense Resonance Neutron Source)

General schematic of the IREN facility accelerator and target halls.

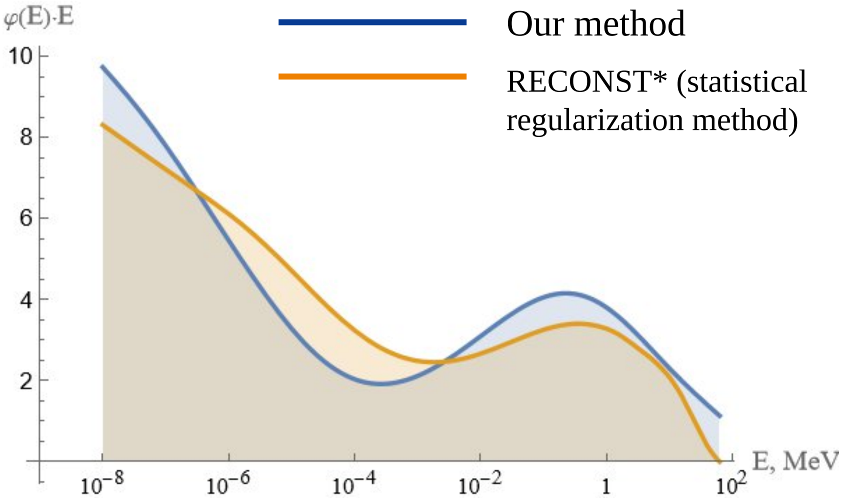


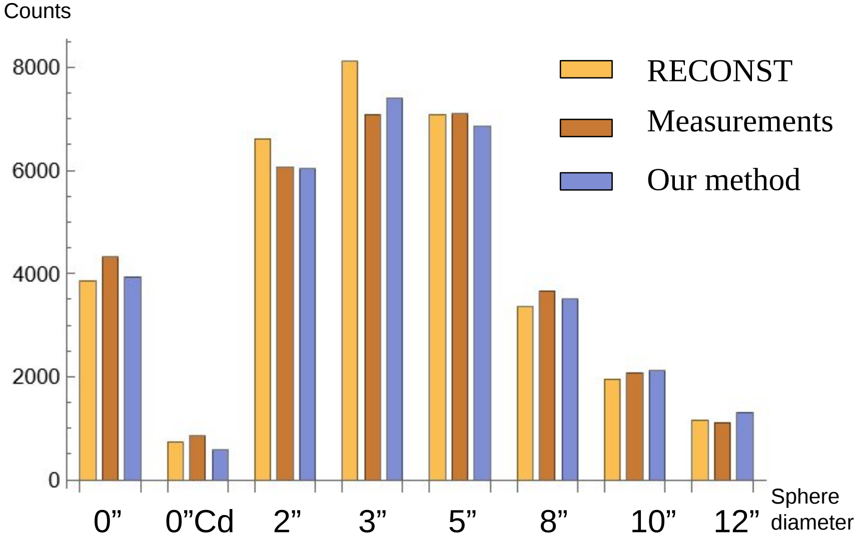


https://flnp.jinr.int/en-us/main/facilities/iren

IREN facility, point 1

- Input data: measurement results for 8 spheres: bare, Cd bare, 2, 3, 5, 8, 10, 12 inches.
- $\delta(\mathbf{Q}) = 5\%$
- $N = 15$ (number of Legendre polynomials)
- 1st and 2nd derivatives in the regularization, $\Phi'(u)$ and $\Phi''(u)$
- $\alpha \approx 4.3 \cdot 10^{-3}$

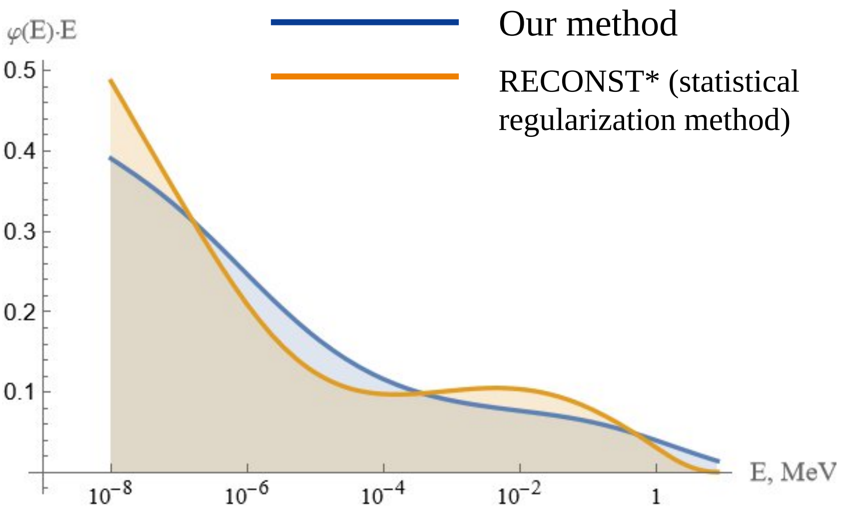




*Krylov A.R., Timoshenko, G.N., Aleinikov, V.E. Measurement of neutron spectra in fields of hard scattered radiation in the energy range from 10^8 to hundreds of MeV, (R-16-91-177), 1991. Dubna: JINR.

IREN facility, point 2

- Input data: measurement results for 8 spheres: bare, Cd bare, 2, 3, 5, 8, 10, 12 inches.
- $\delta(\mathbf{Q}) = 5\%$
- $N = 15$ (number of Legendre polynomials)
- 1st and 2nd derivatives in the regularization, $\Phi'(u)$ and $\Phi''(u)$
- $\alpha \approx 7.8 \cdot 10^{-3}$



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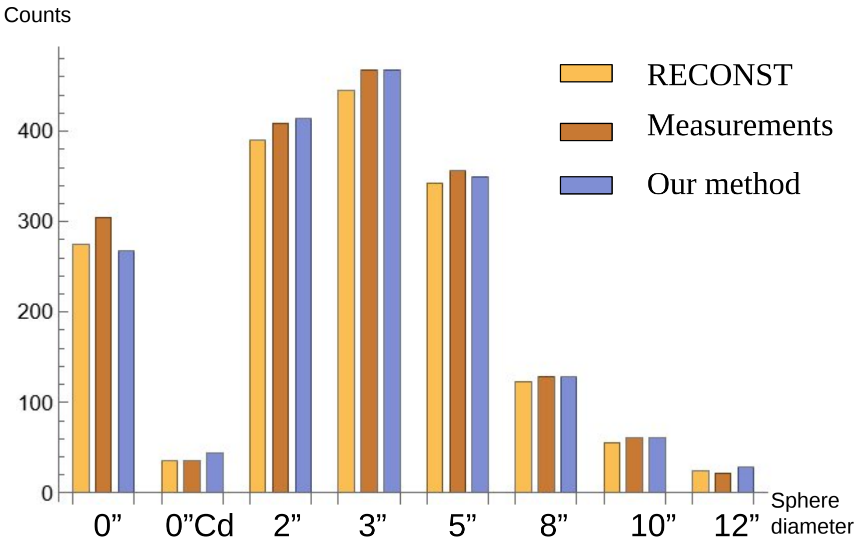


Dose assessment

Measurement point	$\dot{E}_{eff}$ ISO, pSv/s	$\dot{E}_{eff}$ AP, pSv/s	$\dot{H}^{*}(10)$, pSv/s	$\dot{H}_{p}(10,0^{\circ})$, pSv/s
IREN facility, point 1	$3.5 \cdot 10^3$	$6.3 \cdot 10^3$	$7.9 \cdot 10^3$	$7.6 \cdot 10^3$
IREN facility, point 2	$3.1 \cdot 10^1$	$6.3 \cdot 10^1$	$9.4 \cdot 10^1$	$10.0 \cdot 10^1$
IREN facility, point 1 RECONST	$2.9 \cdot 10^3$	$5.3 \cdot 10^3$	$6.6 \cdot 10^3$	$6.7 \cdot 10^3$
IREN facility, point 2 RECONST	$2.9 \cdot 10^1$	$5.5 \cdot 10^1$	$8.8 \cdot 10^1$	$8.5 \cdot 10^1$

Dose conversion coefficient h(E) from Petoussi-Henss N, Bolch WE, Eckerman KF, Endo A, Hertel N, Hunt J, Pelliccioni M, Schlattl H, Zankl M. International Commission on Radiological Protection; International Commission on Radiation Units and Measurements. ICRP Publication 116. Conversion coefficients for radiological protection quantities for external radiation exposures. Ann ICRP. 2010. 40(2-5), 1-257. doi: 10.1016/j.icrp.2011.10.001.

Advantages of the method

1. The solution gives the coefficients of expansion C_i by Legendre polynomials, rather than a direct selection of the values of the function $\varphi(E)\,E$, a lower probability of missing peaks.
2. Analytical solution for the stabilizing functional.
3. Unfolding the spectrum taking into account a specified error in the source data.

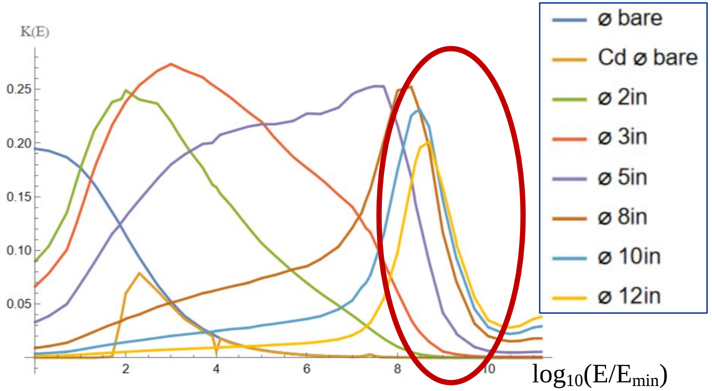
Constrains

1. The method is suitable for reconstructing spectra in stationary fields.
2. Set of Bonner spectrometer spheres limits the energy range of unfolded spectrum.
3. The reliability of the reconstructed neutron spectra significantly depends on the quality of the response functions.

Future development of the method

1. Taking into account the influence of Bonner detector spheres on the reconstructed spectrum by introducing *weighting factors* of contributions for each sphere in the stabilizing functional.

2. Expansion the working energy range of the method. Calculation of response functions in Monte Carlo simulation software (Geant4) for spheres sensitive to high energies. Choice of the diameter and material of the spheres.



Conclusions

1. A method has been developed for reconstructing the energy spectra of neutron flux density by decomposing the spectrum into Legendre polynomials using Tikhonov regularization.
2. The developed method made it possible to reconstruct the neutron spectra for two locations at the IREN based on actual measurements.
3. Effective dose rate for ISO and AP, Ambient dose rate equivalent and Personal dose rate equivalent for AP were accessed for the reconstructed spectra.
4. Comparison of the reconstructed spectra and dose rates with the results of the statistical regularization method showed good agreement.

Thank you!

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1. Chizhov K, Beskrovnaya L, Chizhov A, “Neutron spectra unfolding from Bonner spectrometer readings by the regularization method using the Legendre polynomials”, *Physics of Elementary Particles and Atomic Nuclei*, 3, 2024 (accepted for publication)
2. Chizhov K, Chizhov A. Dose assessment of personnel neutron irradiation on high-energy accelerators using a multi-sphere Bonner spectrometer. *Mathematical Modeling* 7 (2), 63-64, 2023.