Neutron slowing down as a random walk problem

E.I. Sharapov. Joint Institute for Nuclear Research, 141980, Dubna, Russia

Introduction

Random walk in the mathematical statistics was introduced by Karl Pearson in 1905 [1], as sequences of '**n**' steps (of a some person) taken in random directions of space. The directions and the length of each step have to be chosen from a know probability distributions, and are independent between steps. The problem to solve is to find the probability of "landing" at a required spot **x** in the space after the number **n** of steps. The **n** and **x** are random variables. The random walk formalism assumes the probabilistic description of a physical process.

Neutron slowing down was discovered by the E. Fermi's group in 1934 [2] when they found the increasing radioactivity of their samples with the fast neutron source placed into the paraffin (the protons media) cube. At each collision neutron loses a part of its kinetic energy, or even all the energy, by transferring it to protons. The random variables in question are the neutron energies **Ei** during the neutron flights between subsequent collisions and the number of collisions, '**n**', producing a final required neutron energy En.

[1] K. Pearson 1905. The problem of the Random Walk. Nature 72 no.1865:294, no.1867:342.

[2]. E. Fermi, E. Amaldi, B. Pontecorvo, F. Rasetti, E. Segre. 1934. Azione di sostanze idrogenato sulls radioactivita provocato di neutroni I. Ric. Scient. 5(2):280-283.

Poster overview

- It appears, that the names of Pierre-Simon Laplace, Karl Pearson, Gian-Carlo Wick, Enrico Fermi, and Paul Langevin can be related ('bounded') to the historic problem of random work in the statistical theory and to the problem of fast neutron slowing down in the neutron physics. The 'bound' comes from developing the exact mathematical expressions for the probability density of the sum of **n** independent random variables.
- The most problematic were difficulties of getting the result in the form of an analytical formula for energy distribution of neutrons after a fixed number **n** of impacts with protons.
- The different approaches of the great physicists are reviewed and compared. The neutron impacts with nuclei are treated probabilistically, as in the random walk problem. The final analytical expressions, when available, have been found to be identical.
- The author's simplest approach to deduce analytical formula for the probability density of the sum of **n** logarithmic random variables is shown also, for the pedagogical reasons, for students interested in the neutron physics.

Motivation writing this historical review:

- – The interest to the history of neutron physics since the 30-ies, to the discovery of neutron, its slowing down and induced radioactivity, to the birth, in the struggle of opinions, of new ideas and techniques.
- – The knowledge, from the past pedagogic practice, that the young physicists are often interested only in the modern subjects that may lead fast to their PhD.
- – The desire to come to the conclusion, that the past and presence are always bound and, when so, that may lead to unforeseen successes.

The start of the poster presentation:

Enrico Fermi in the paper [3] of his team wrote: “It is easily shown that an impact of a neutron against a proton reduces, on the average, the neutron energy by a factor of 1/e”. Not all physicists had agree with this estimate. Therefore, the Professor Fermi’s young assistant G.C. Wick published a short Letter to Physical Review [4], where he wrote: “The above passage in a paper by Professor Fermi is considered somewhat obscure. Since a more detailed explanation might be of interest also to others, it was thought advisable to make it generally known”.

[3] E. Amaldi et al. 1935. Artificial Radioactivity produced by neutron bombardment II. Proc. Roy. Soc.. A149, 522-558.

[4] Wick G.C. 1936. On the slowing down of neutrons. Phys. Rev. 49:192-193.

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Gian-Carlo Wick analytical formula

– G.C. Wick explained in his letter to the Phys. Rev. editor that E. Fermi had used, as a random variable , the quantity x, namely the logarithmic energy change (‘decrement’) *x = ln(Eo /En*). Here Eo is the neutron source energy and En is the neutron energy after the subsequent **n** collisions. He stated (but not proofed), that it is possible to obtain the following analytical formula for the probability distribution function:

$$p_n(x) = [x^{n-1}/(n-1)!]\, e^{-x} \qquad (1)$$

– The expression *p(x)dx* means the probability that the logarithmic variable x lies between values *x* and *x + dx.* The setting n=1 confirms the Fermi’s ‘*1/e statement’* for the logarithmic energy change after one neuron-proton collision.

– With this distribution and its known variance, one can obtains with ‘initial’ energy of E_0 = 2 MeV and the ‘final’ energy of E_n = 1 eV,

– *the following values: <n> = 15.5+- 3.8,* -- the average number **n**, (to compare with the ‘arithmetic’ average <<n>> = 20.9),

– $n_{\mathbf{peak}}$ =14.5 -- the most probable number of collisions to reduce neutron energy to the value of 1 eV.

Laplace and Breit-Condon formulas

- One of the founders of the theory probability, the great French scientist Laplace was first to develop the serie [5]

$$f_n(x) = [1/a^n(n-1)!]\{x^{n-1} - nC_1 (x-a)^{n-1} + nC_2 (x-2a)^{n-1} - \ldots\} \qquad (2)$$

for the probability density function fn(x) of the sum x of n random variables, each having the same uniform distribution in the range from 0 to 'a'. In it, nC_i are the Bernoulli coefficients. The formula is exact, but requires calculation of many terms in this sum.

- Breit and Condon [6], working on the neutron slowing down problem with the random variable x = En/Eo , during their derivation of Wick's Eq. (1), came to the Laplace serie independently. They switching to logarithmic variable $u = -ln(x)$ had obtained with it exactly the same analytical formula, as Wick in Eq. (1):

$$f_n(u)du = e^{-u}u^{n-1}du/(n-1)!, \qquad (3)$$

- Next, to work directly with the energy distribution of neutrons, they transformed to the variable x and have obtained:

$$f_n(x)dx = (log1/x)^{n-1}dx/(n-1)! \qquad (4)$$

[5] Laplace P-S. 1820. Theorie Analytique des Probabilities, Troisieme Edition, Paris (1820). In: Oeuvres complete de Laplace, 7:257-263, Paris (1878).

[6] Condon E.U., Breit G. 1936. The energy distribution of neutrons slowed by elastic impacts. Phys. Rev. 49:229-231.

Langevin’s and author’s approaches

Langevin started to work on the moderation of neutrons in media composed by nuclei heavier then hydrogen, following the Joliot-Curie’s suggestion in 1940, when the German occupants imprisoned him in Paris. He successfully concluded the work by the 1941-1942 publications submitted from the town of Troyes, where the Gestapo put him under the surveillance. As did other physicists, Langevin calculated the probability for a fast neutron to reduce its energy to E, E+dE by collisions with any media nuclei after one, two, three, or any number **n** of successive impacts. Starting from an neutron energy ratio C=E/Eo as a random variable, he transformed it to the logarithmic variable: **x**=ln(Eo/E) and obtained, for the probability of **x**, the series solution analogical to the Laplace’s one. This was done after first developing the original approach to represent and calculate probabilities, which he named as the “geometrical graphical method”. The numerical results were presented in several figures. The Langevin’s publications are:

- **Paul Langevin. 1942. Sur les chocs entre neutrons rapides et noyaux de masse quelconque. Ann. Phys, Paris, 17:303-317.**
- **Paul Langevin. 1942. Compt. Rendes Acad. Sci., Paris, v. 214, pages 517, 867, 889.**

The author’s pedagogical derivation of the analytical formula for the probability density of the sum of **n** logarithmic random variables, using the Induction method with initial convolutions, is presented in Appendix of the full text of the E. Sharapov’s poster.

Conclusion

- Today it is known, that even though the transport theory dominates discussions related to neutron slowing down, the arguments that similar results can be obtained with the probabilistic description, continue to appear. C.S. Barnett [7] showed recently that first and higher order moments of distribution function, calculated with the random work formalism, gave the correct results.

As recently as 2018, B. Ganapol et al. [8] used the probabilistic neutron slowing down model as a mathematical model for **the entropy** in physics.

- **The final conclusion is, that the past and the presence are always bound in physics, which may lead to an unforeseen successes.** **Thank you.**

[7] C. S. Barnett (1974). On the randomness of a neutron's kinetic energy as it slows down by elastic collisions in an infinite medium. Nuclear Science and Engineering 5(2):234-242.

[8] B. Ganapol et al. (2018). A mathematical realization of Entropy through neutron slowing down. Entropy, 20(4):233-249.