

# Prompt Fission Neutron Spectra of $^{233}\text{U}(\text{n},\text{F})$

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# Collaboration on PFNS of 2000-2011

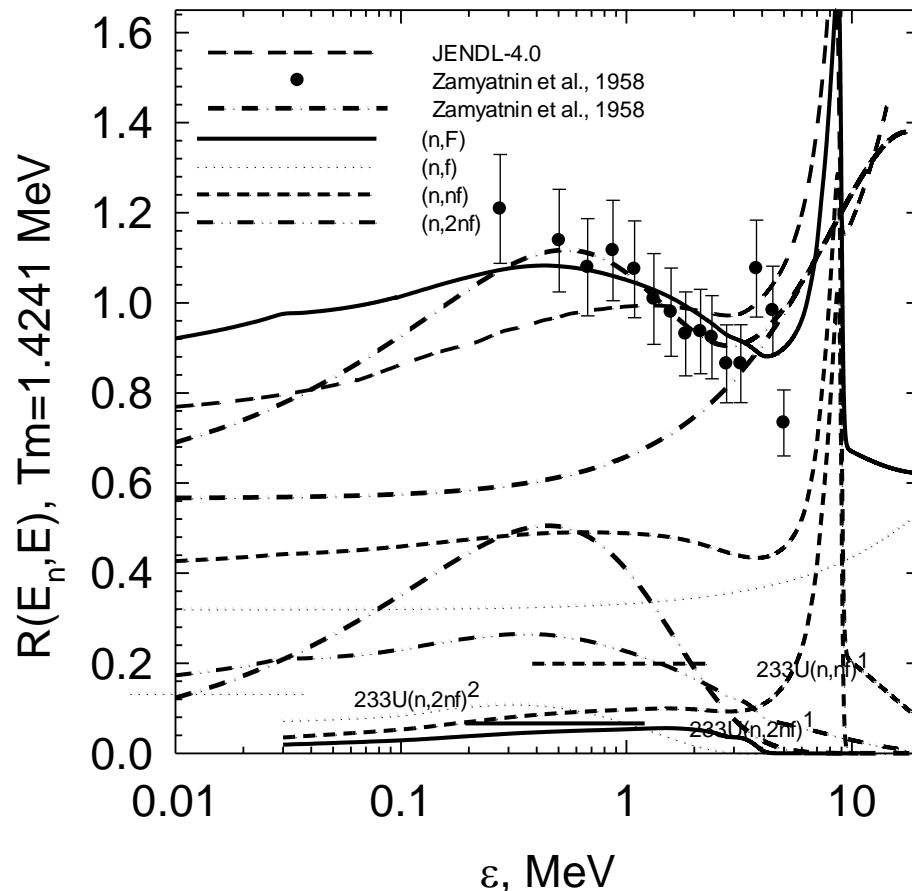
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# EXPERIMENTS on

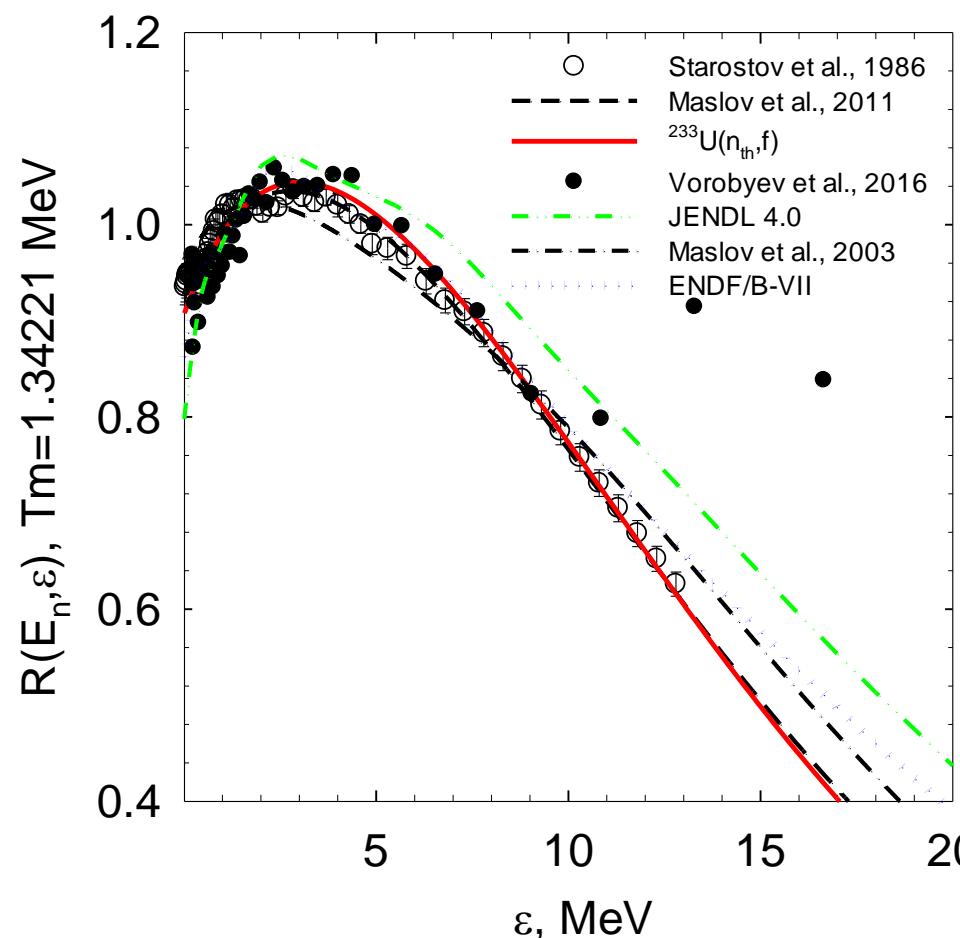
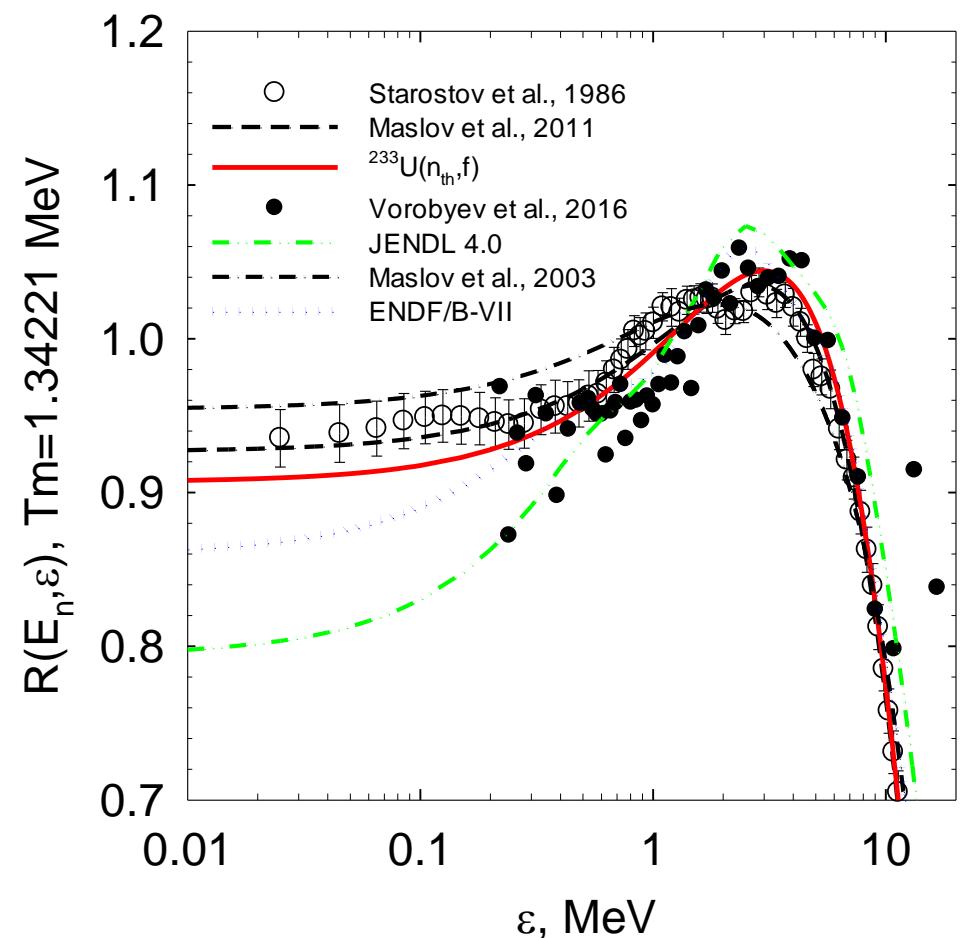
- PFNS of  $^{235}\text{U}(n,F)$   $^{238}\text{U}(n,F)$   $^{239}\text{Pu}(n,F)$   
LANSCE, 2019-2023
- Asymmetry PFNS in  $^{239}\text{Pu}(n,F)$   $^{235}\text{U}(n,F)$  PFNS  
Kelly e. a., Phys. Rev. Lett., 2019, v. 122, p. 072503
- Asymmetry in neutron emission spectra  $E_n=14 \text{ MeV}$   
 **$^{235}\text{U}+n$ ;  $^{239}\text{Pu}+n$ ;  $^{238}\text{U}+n$**  – first observed by  
Kammerdiener J.L., UCRL-51232, 1972.
- Asymmetry in neutron emission spectra of  $^{232}\text{Th}+n$   
and  **$^{238}\text{U}+n$  at  $E_n=6, 12, 14, 18 \text{ MeV}$**

For  $^{233}\text{U}(\text{n},\text{F})$  PFNS  
next to nothing

# $^{233}\text{U}(\text{n},\text{F})$ PFNS $E_{\text{n}}=14$ MeV



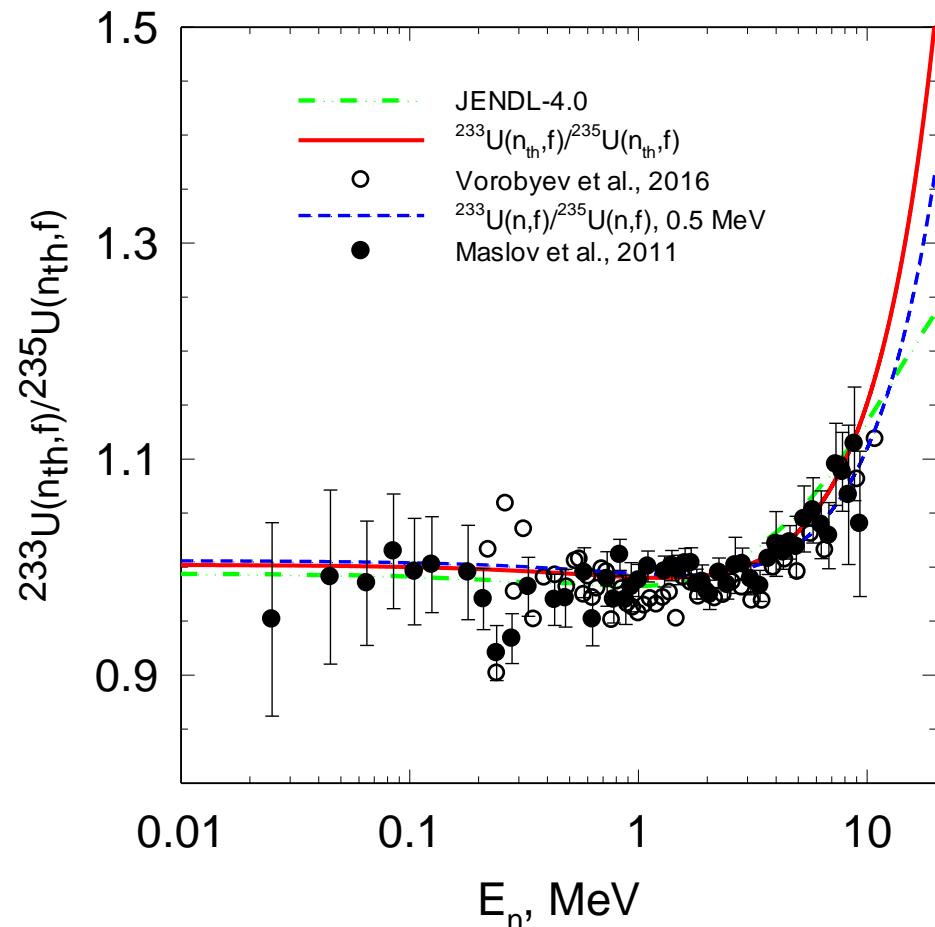
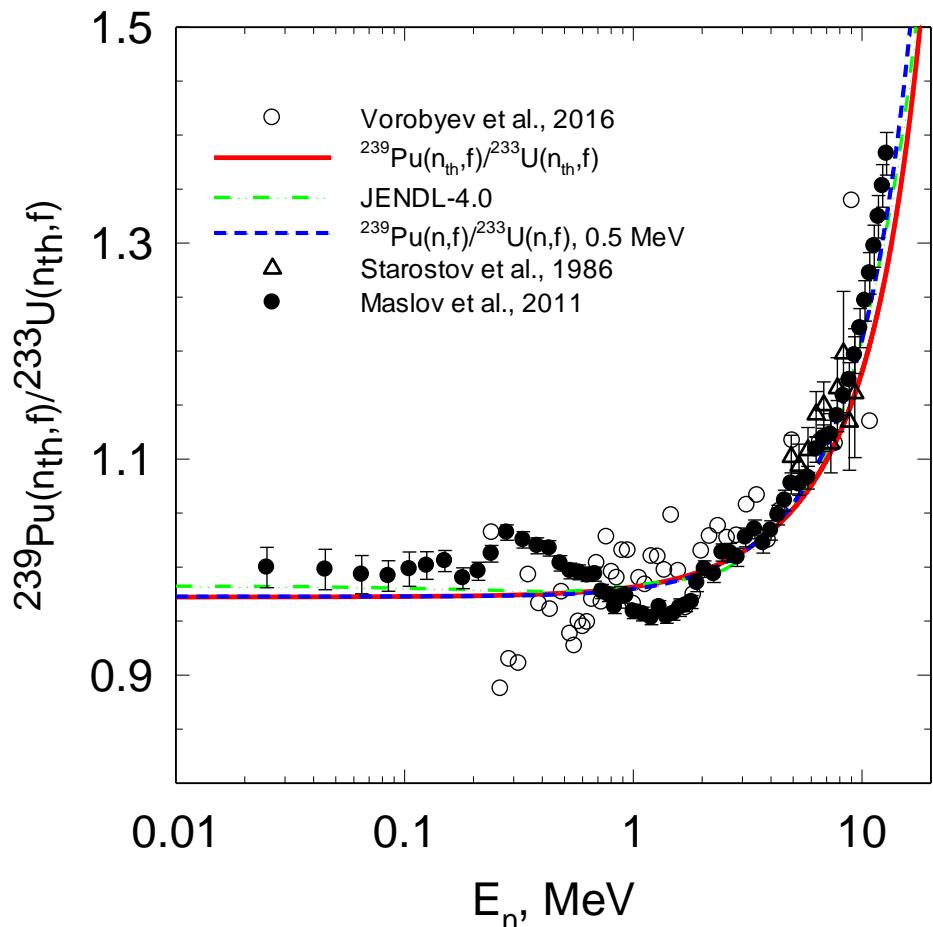
$E_n=0.02536$  eV     $^{233}\text{U}(n,\text{F})$  PFNS     $E_n=0.02536$  eV



En=0.5 MeV  $^{239}\text{Pu}/^{233}\text{U}$

PFNS ratio

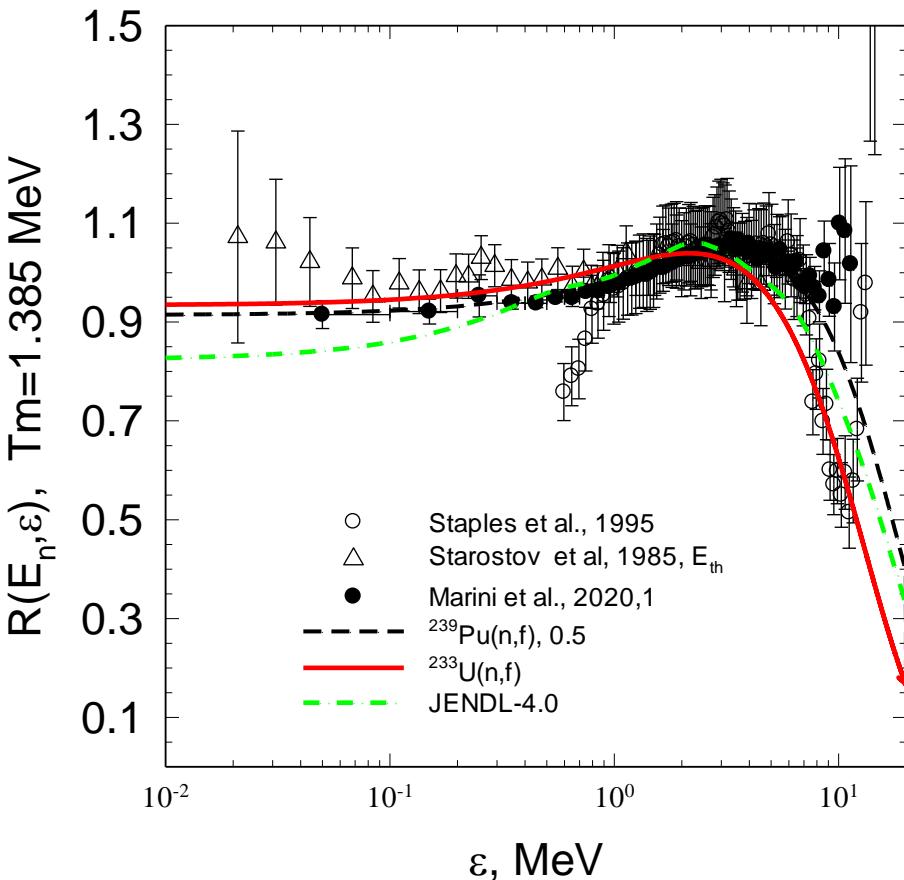
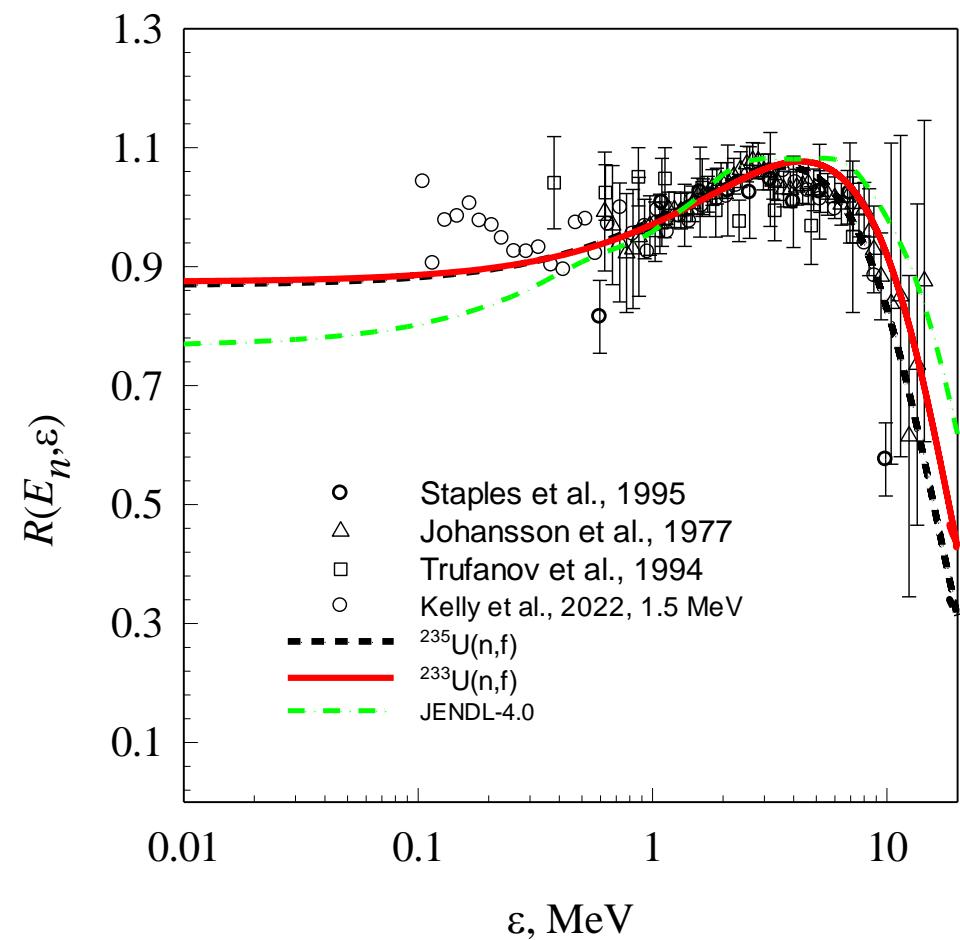
$^{233}\text{U}/^{235}\text{U}$  En=0.5 MeV



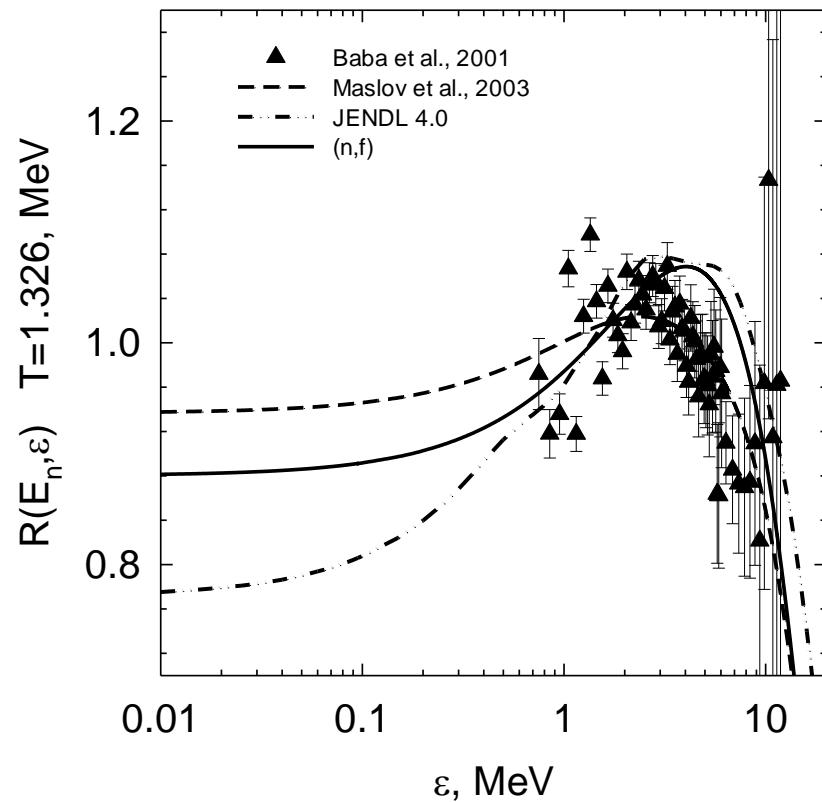
En=0.5 MeV, 1.5 MeV 233U(n,F)  
235U(n,F)

PFNS

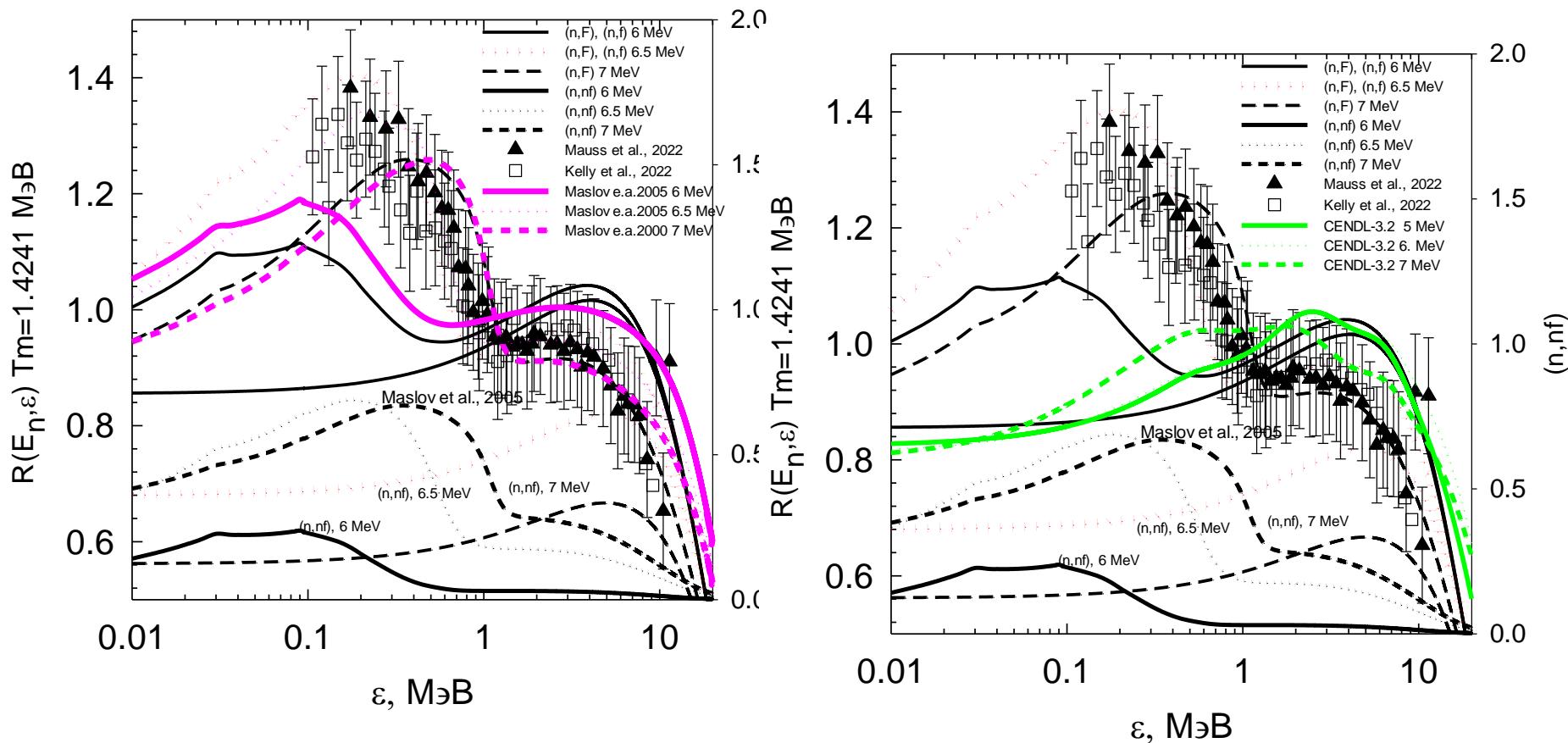
En=0.02536 eV 233U(n,F)  
1.5 MeV 239Pu(n,F)



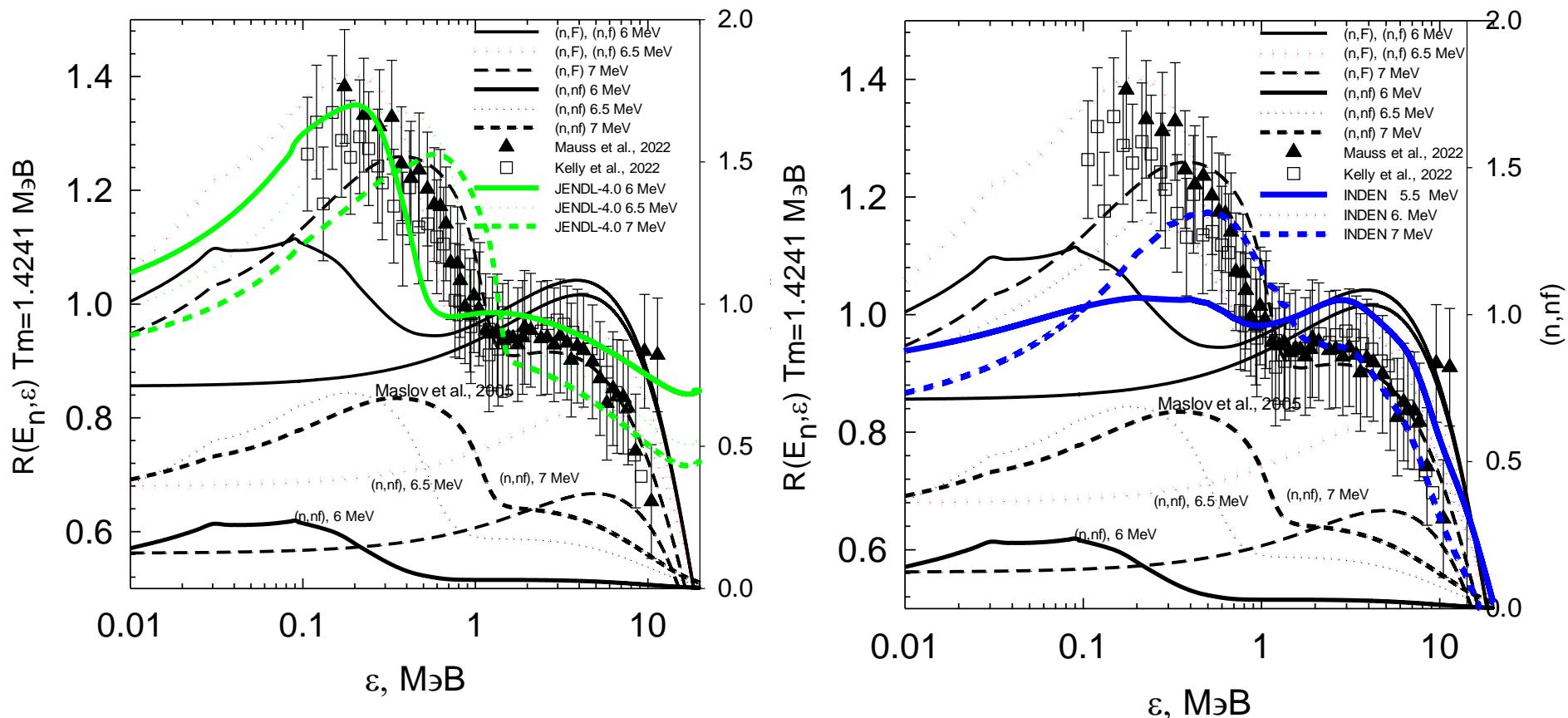
# $^{233}\text{U}(\text{n},\text{F})$ PFNS    $E_{\text{n}}=0.5 \text{ MeV}$



Maslov V.M. Physics of Atomic Nuclei, 2023, Vol. 86, pp. 627–669.  
 En=6.5 MeV 235U(n,F) PFNS En=6.5 MeV 235U(n,F)



Maslov V.M. Phys. At. Nucl. 2023, Vol. 86, pp. 627–669.  
 En=6.5 MeV 235U(n,F) PFNS En=6.5 MeV 235U(n,F)



$$\sigma_{nF}(E_n) = \sigma_{nf}(E_n) + \sum_{x=1}^X \sigma_{n,xnf}(E_n)$$

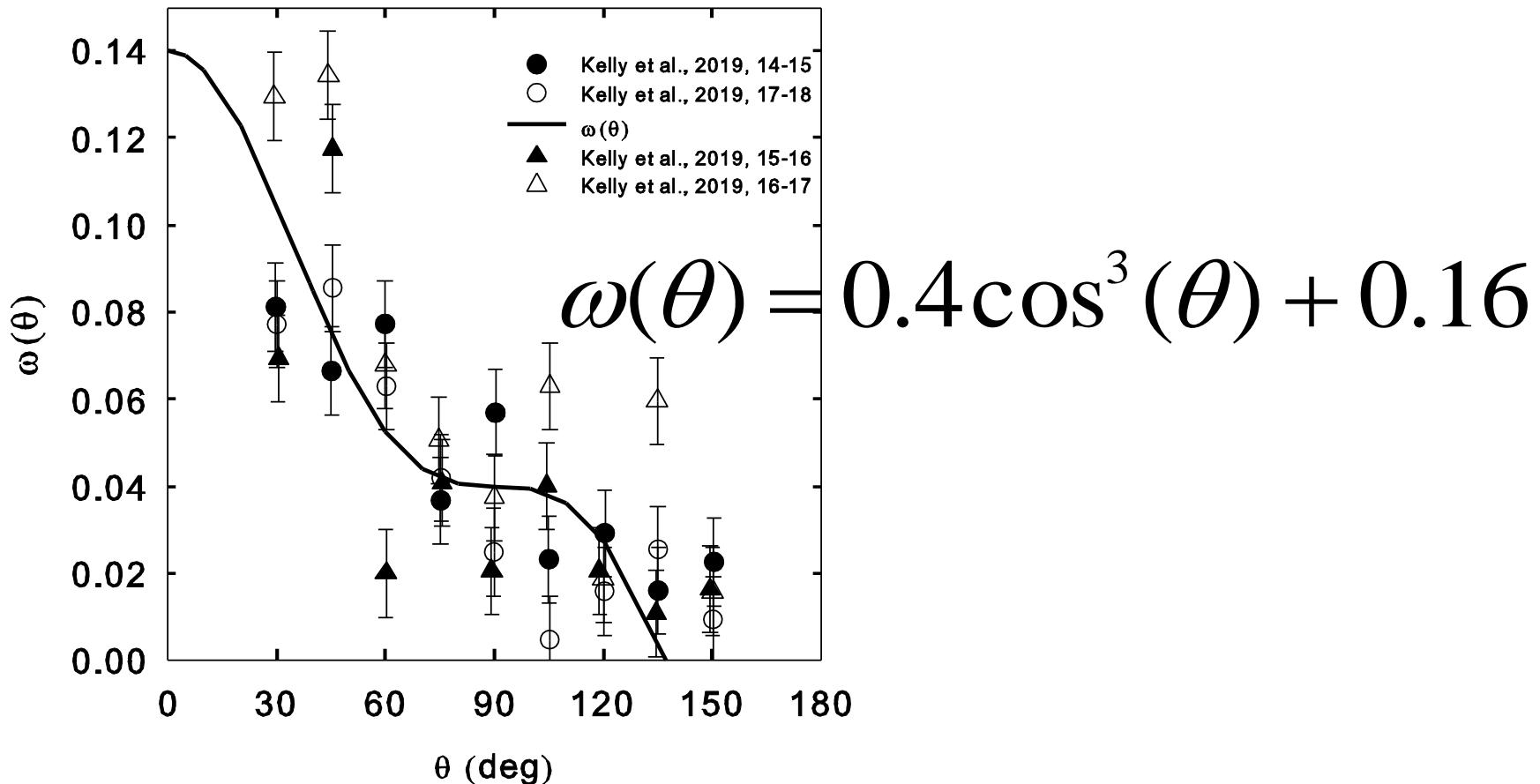
$$\sigma_{n,xnf}(E_n) = \sum_{J\pi} \int_0^{U_x} W_{A+1-x}^{J\pi}(U) P_{f(A+1-x)}^{J\pi}(U) dU$$

$$\frac{d\sigma_{nnx}^1(\varepsilon, E_n)}{d\varepsilon} \approx \frac{d\tilde{\sigma}_{nnx}^1(\varepsilon, E_n)}{d\varepsilon} + \sqrt{\frac{\varepsilon}{E_n}} \frac{\langle \omega(\theta) \rangle_\theta}{E_n - \varepsilon}$$

$$\langle \omega(\theta) \rangle_g \approx \omega(\theta \approx 90^\circ)$$

$$\omega(\theta) = 0.4 \cos^3(\theta) + 0.16$$

# Angular distribution of $^{239}\text{Pu}(n,\text{F})$ pre-fission neutrons



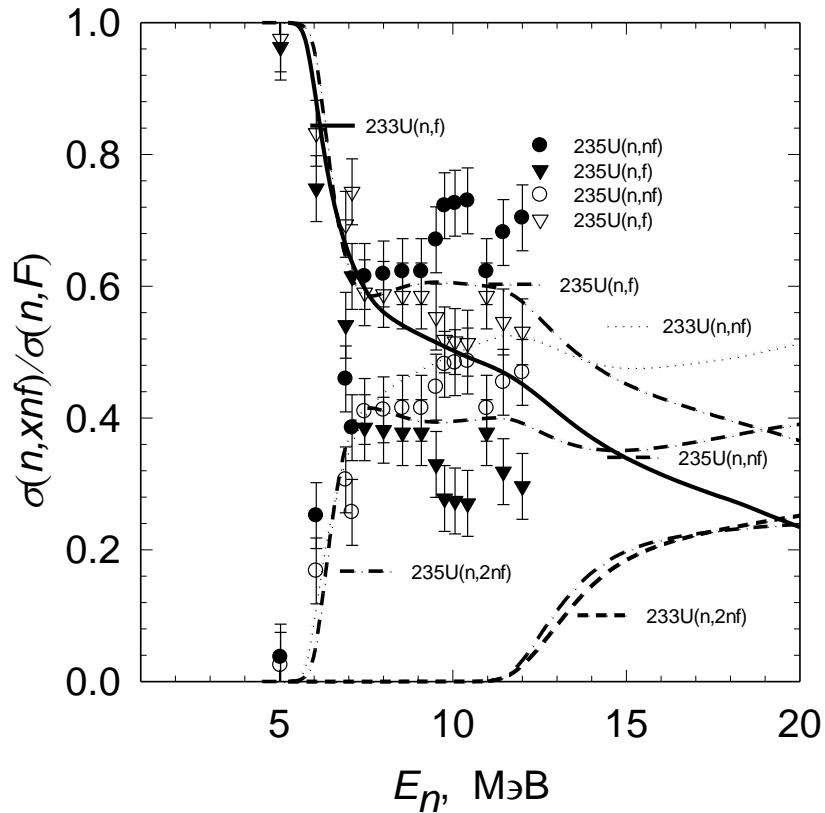
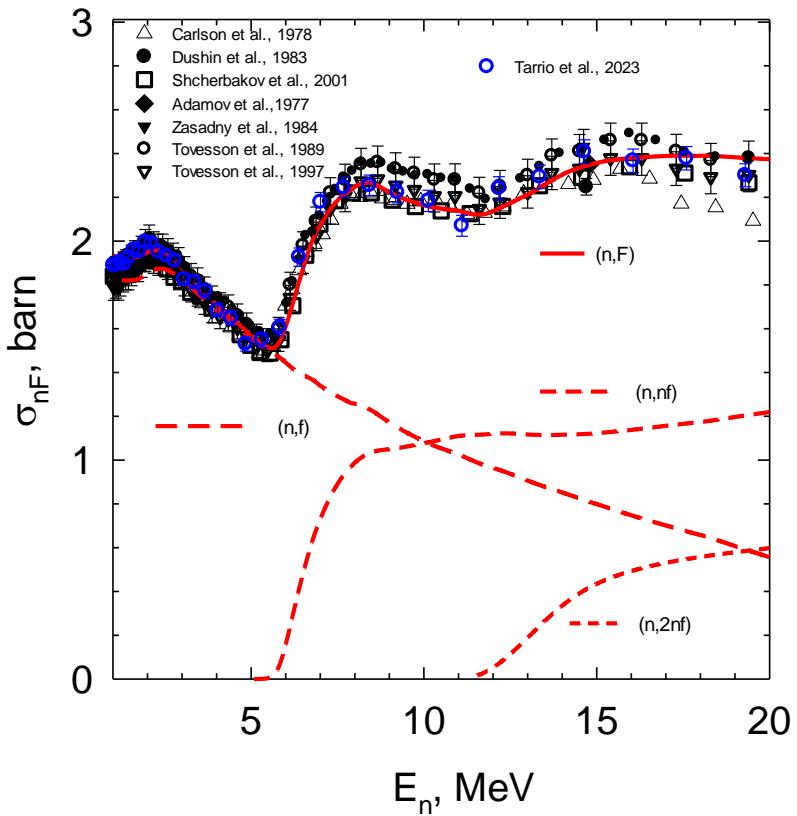
**Angular anisotropy (AA) of secondary neutrons is evidenced in neutron emission spectra (NES)  $^{232}\text{Th}$ ,  $^{235}\text{U}$ ,  $^{238}\text{U}$ ,  $^{239}\text{Pu}$ , due to**

1.  $(n,n)$
2. **g.s. band levels  $J^\pi = 0^+, 2^+, 4^+, 6^+, 8^+$  (e-e)**
3.  $\gamma$ -bands with  $K^\pi=0^+, 2^+$ , octupole bands  $K^\pi=0^-$  (e-e)
4.  $(n,n\gamma)$

NES AA is due to PE/semi-direct  $(n,nX)^1$

**Angular anisotropy of PFNS due to tiny part of  $(n,nX)^1$  neutrons in exclusive pre-fission neutrons in  $(n,xnf)^{1,\dots,x}$ .**

# $\sigma(n,F)$ , $\sigma(n,xnf)$ $\sigma(n,xnf)/\sigma(n,F)$



# PFNS as a superposition

$$\begin{aligned}
S(\varepsilon, E_n, \theta) &= \tilde{S}_{A+1}(\varepsilon, E_n, \theta) + \tilde{S}_A(\varepsilon, E_n, \theta) + \tilde{S}_{A-1}(\varepsilon, E_n, \theta) + \tilde{S}_{A-2}(\varepsilon, E_n, \theta) = \\
&\nu_p^{-1}(E_n, \theta) \cdot \{ \nu_{p1}(E_n) \cdot \beta_1(E_n, \theta) S_{A+1}(\varepsilon, E_n, \theta) + \nu_{p2}(E_n - \langle E_{nnf}(\theta) \rangle) \beta_2(E_n, \theta) S_A(\varepsilon, E_n, \theta) + \\
&+ \beta_2(E_n, \theta) \frac{d^2 \sigma_{nnf}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} + \nu_{p3}(E_n - B_n^A - \langle E_{n2nf}^1(\theta) \rangle - \langle E_{n2nf}^2(\theta) \rangle) \beta_3(E_n, \theta) S_{A-1}(\varepsilon, E_n, \theta) + \beta_3(E_n, \theta) \cdot \\
&\left[ \frac{d^2 \sigma_{n2nf}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} + \frac{d^2 \sigma_{n2nf}^2(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \right] + \nu_{p4}(E_n - B_n^A - B_n^{A-1} - \langle E_{n3nf}^1(\theta) \rangle - \langle E_{n3nf}^2(\theta) \rangle - \langle E_{n3nf}^3(\theta) \rangle) \cdot \\
&\beta_4(E_n, \theta) S_{A-2}(\varepsilon, E_n, \theta) + \beta_4(E_n, \theta) \left[ \frac{d^2 \sigma_{n3nf}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} + \frac{d^2 \sigma_{n3nf}^2(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} + \frac{d^2 \sigma_{n3nf}^3(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \right] \}.
\end{aligned}$$

$$\nu_p(E_n) = \nu_{post} + \nu_{pre} = \sum_{x=1}^X \nu_{px}(E_{nx}) + \sum_{x=1}^X (x-1) \cdot \beta_x(E_n)$$

Exclusive neutron spectra of  $(n,xnf)^{1,\dots,x}$ ,  $(n,n\gamma)$  and  $(n,xn)^{1,\dots,x}$  are calculated within Hauser-Feshbach formalism alongside with  $(n,F)$  and  $(n,xn)$  reaction cross sections, angular dependence of first  $(n,nX)^1$  emission  $\omega(\theta)$  being included.

$$U_x = E_n + B_n - \sum_{x, 1 \leq k \leq x} (< E_{nxnf}^k(\theta) > + B_{nx})$$

$$E_{nx} = E_r - E_{fx}^{pre} + E_n + B_n - \sum_{x, 1 \leq k \leq x} (\langle E_{nxnf}^k(\theta) \rangle + B_{nx})$$

$$E_F^{pre}(E_n) = \sum_{x=1}^X E_{fx}^{pre}(E_n) \sigma_{n,xnf} / \sigma_{n,F}$$

$$E_F^{post} \approx E_F^{pre} \left( 1 - \nu_{post} / (A + 1 - \nu_{pre}) \right)$$

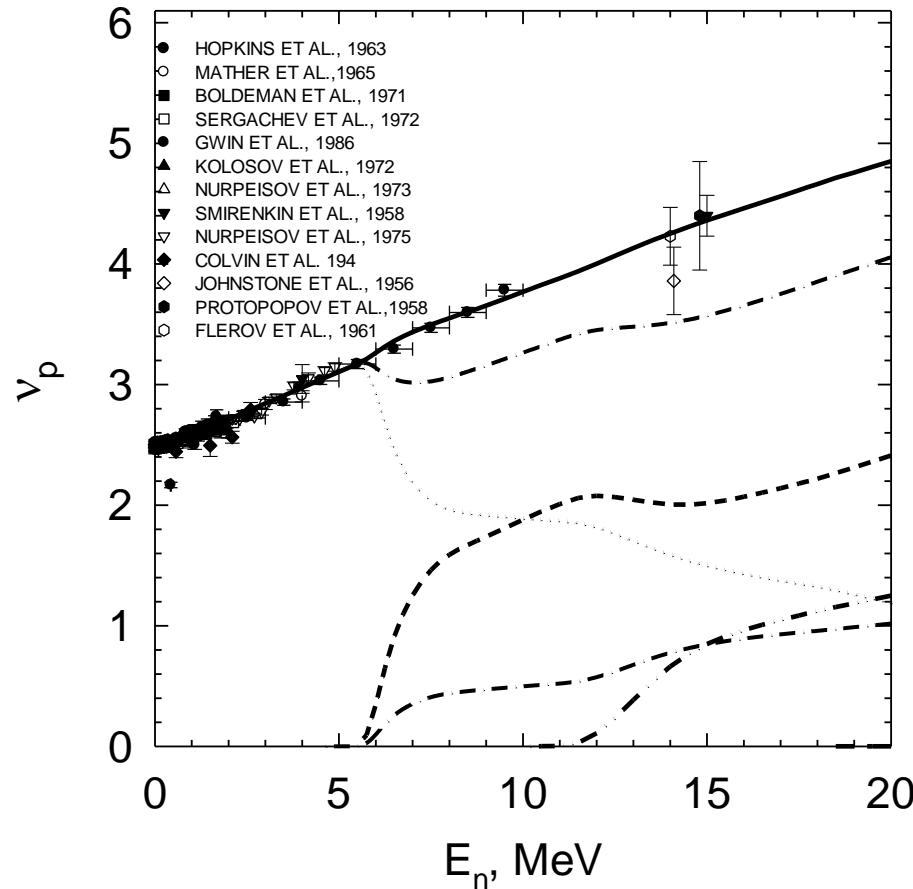
$$\nu_p(E_n) = \nu_{post} + \nu_{pre} = \sum_{x=1}^X \nu_{px}(E_{nx}) + \sum_{x=1}^X (x-1) \cdot \beta_x(E_n)$$

$$\frac{d^2\sigma_{nnx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \approx \frac{d^2\tilde{\sigma}_{nnx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} + \sqrt{\frac{\varepsilon}{E_n}} \frac{\omega(\theta)}{E_n - \varepsilon}$$

$$\langle \omega(\theta) \rangle_g \approx \omega(\theta \approx 90^\circ)$$

Maslov V.M. Anisotropy of Prompt Fission Neutron Spectra of  $^{239}\text{Pu}(n, F)$  and  $^{235}\text{U}(n, F)$ , Physics of Particles and Nuclei Letters, 2023, Vol. 20, No. 6, pp. 1373–1384.

# $^{233}\text{U}(n,\text{F})$ , $^{233}\text{U}(n,xnf)$ pre- and post-fission neutrons multiplicity



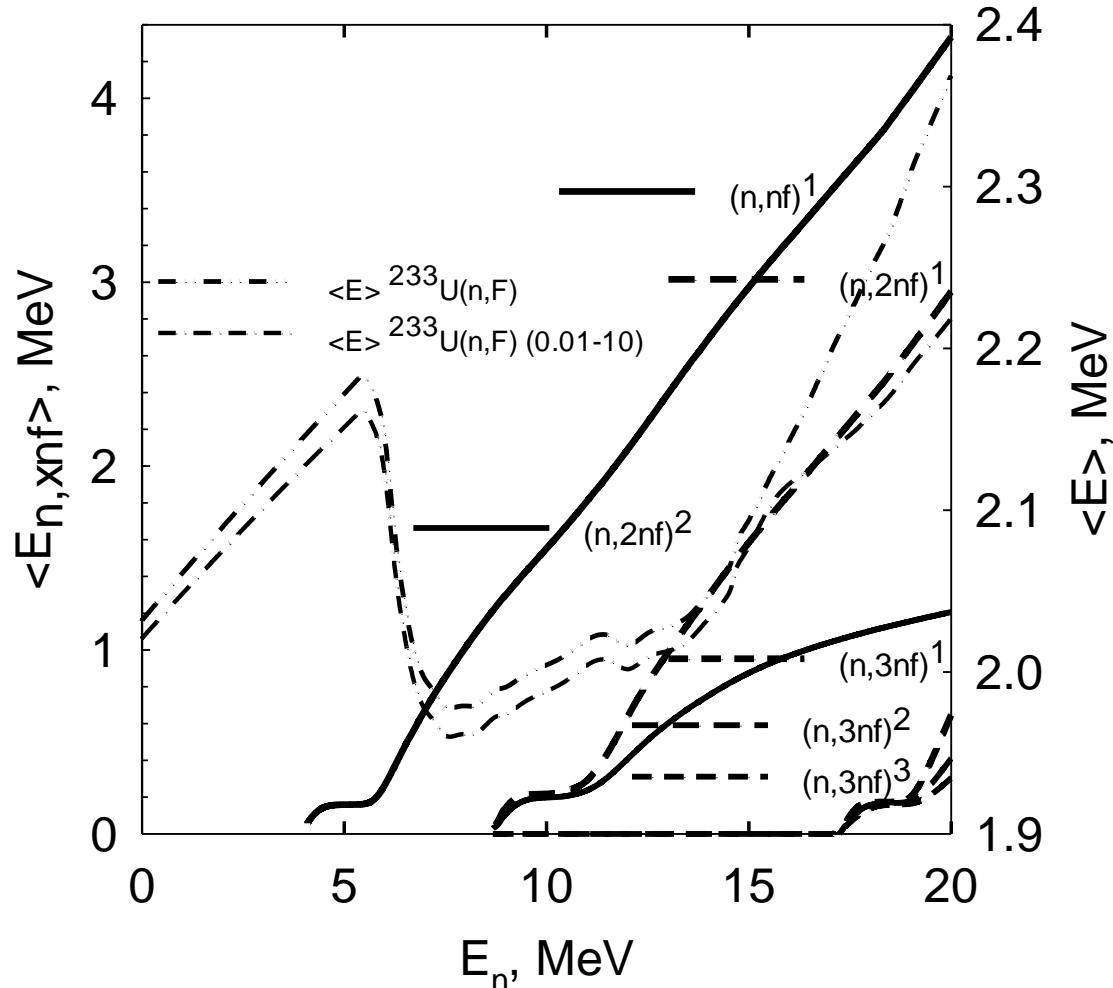
$$\frac{d\tilde{\sigma}_{nnx}^1(\varepsilon, E_n)}{d\varepsilon} = \sum_{J,\pi} W^A(E_n - \varepsilon, \theta, J^\pi)$$

$$\frac{d^2\sigma_{nnf}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} = \frac{d^2\sigma_{nnx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \frac{\Gamma_f^A(E_n - \varepsilon, \theta)}{\Gamma^A(E_n - \varepsilon, \theta)}.$$

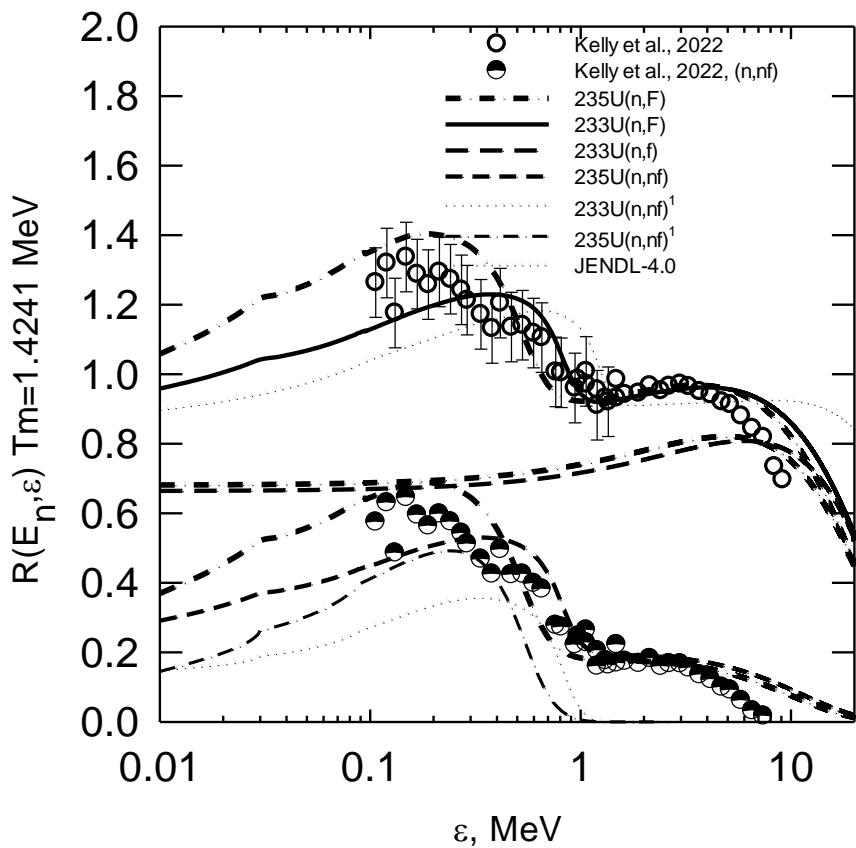
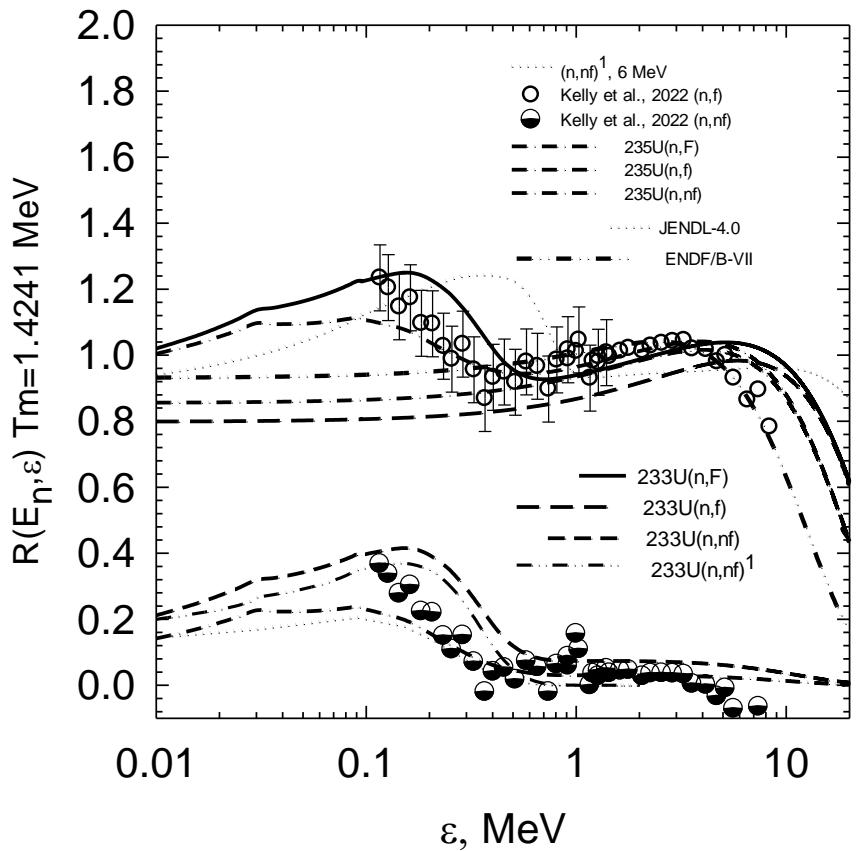
$$\frac{d^2\sigma_{n2nx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} = \frac{d^2\sigma_{nnx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \frac{\Gamma_n^A(E_n - \varepsilon, \theta)}{\Gamma^A(E_n - \varepsilon, \theta)}$$

$$\frac{d^2\sigma_{n2nf}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} = \int_0^{E_n - B_n^A} \frac{d^2\sigma_{n2nx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \frac{\Gamma_f^{A-1}(E_n - B_n^A - \varepsilon - \varepsilon_1)}{\Gamma^{A-1}(E_n - B_n^A - \varepsilon - \varepsilon_1)} d\varepsilon_1$$

# $^{233}\text{U}(\text{n},\text{xnf})$ pre-fission neutrons $\langle E_{\text{n,xnf}} \rangle$ , $\langle E \rangle$ of PFNS

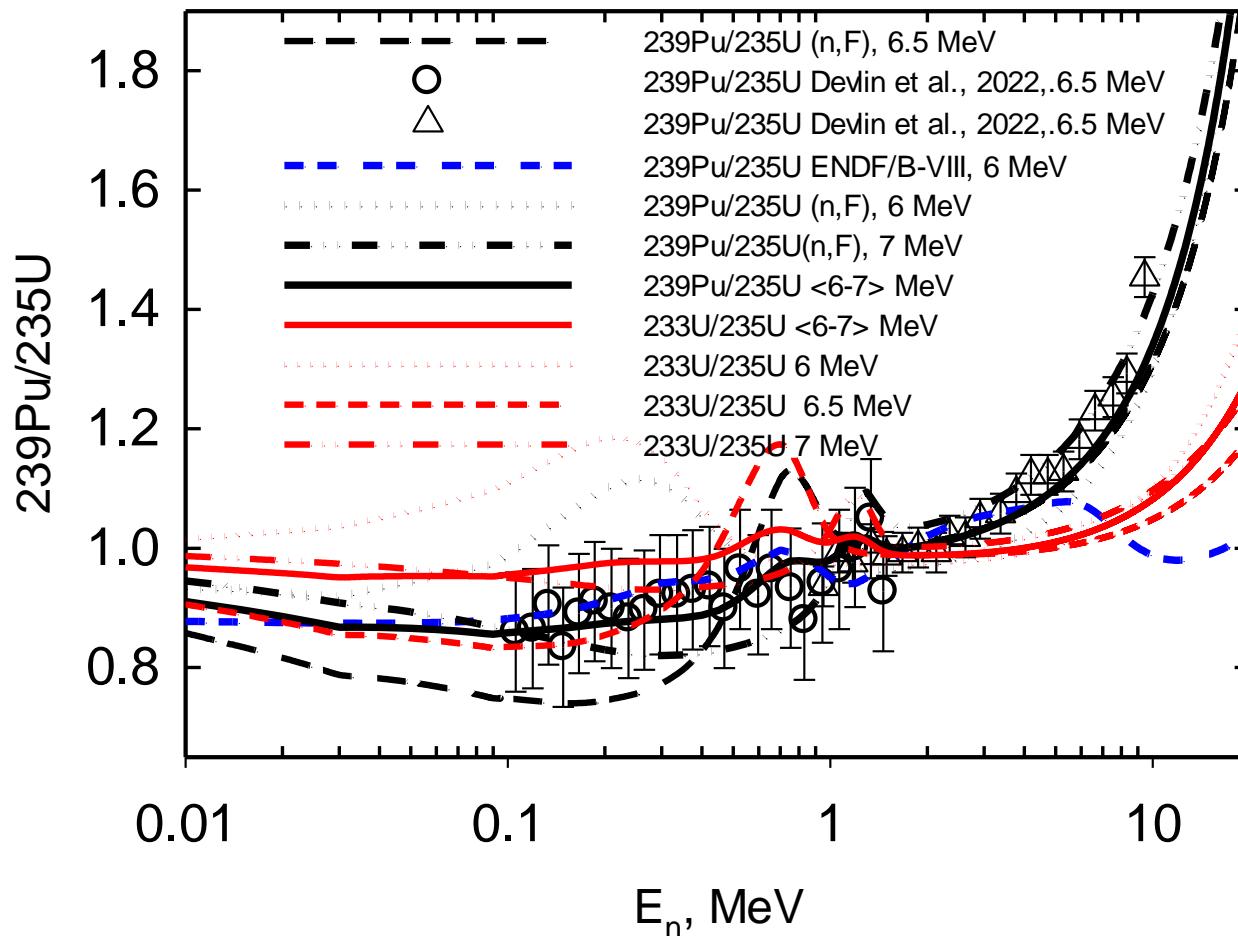


En=6 MeV 233U(n,F)&235U(n,F) PFNS En=6.5 MeV

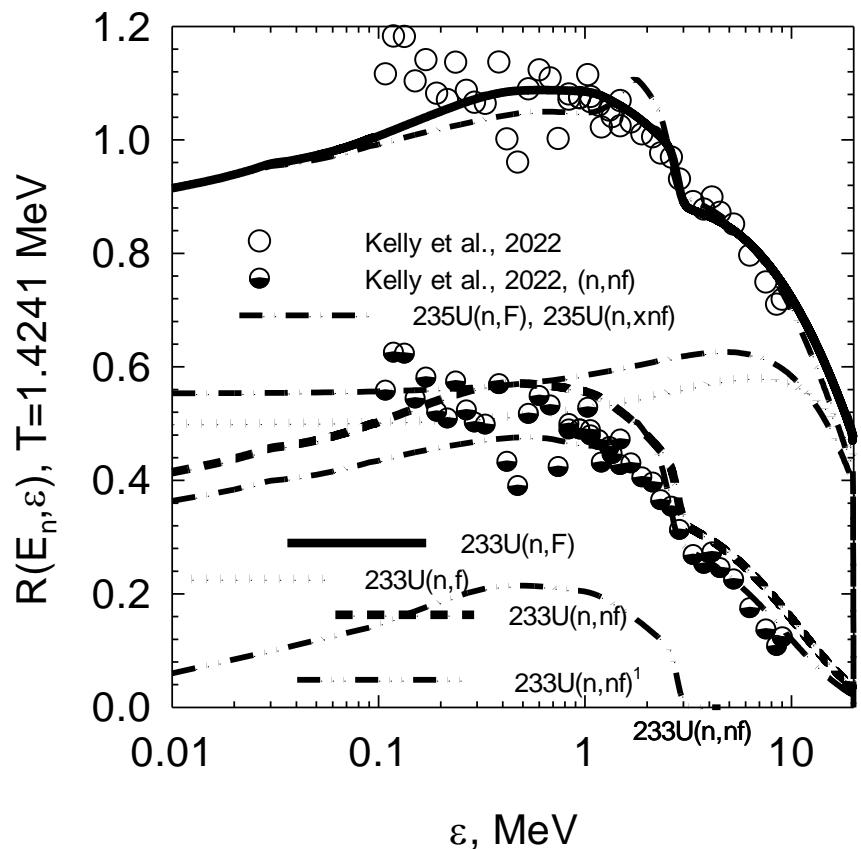
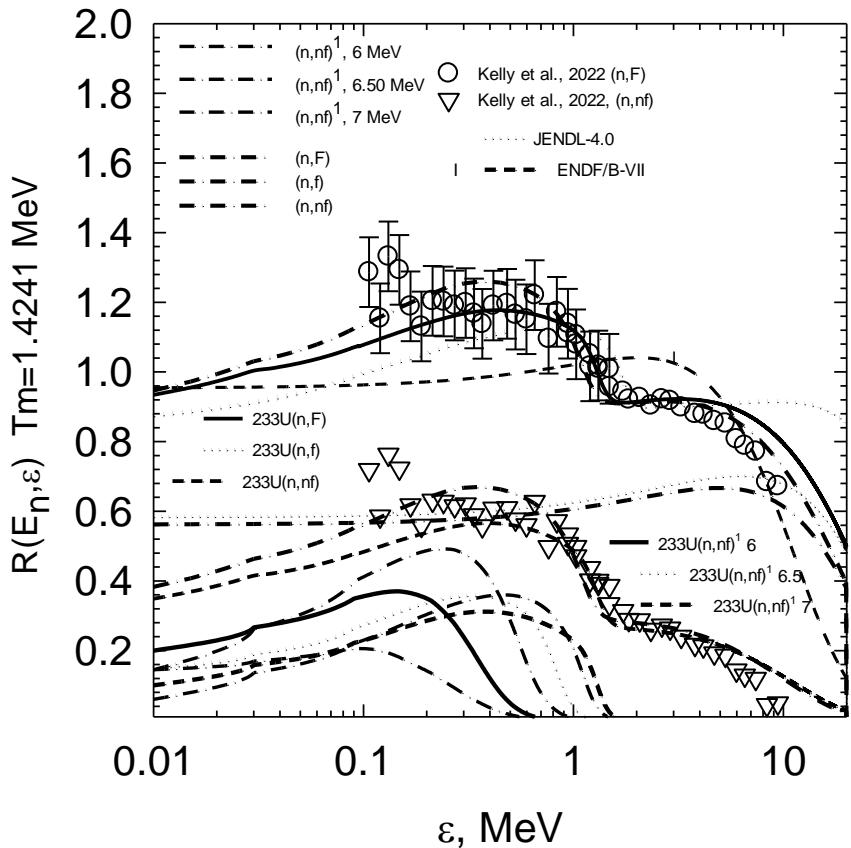


$E_n=6\text{-}7 \text{ MeV}$   $^{239}\text{Pu}/^{235}\text{U}$  PFNS ratio  $^{233}\text{U}/^{235}\text{U}$

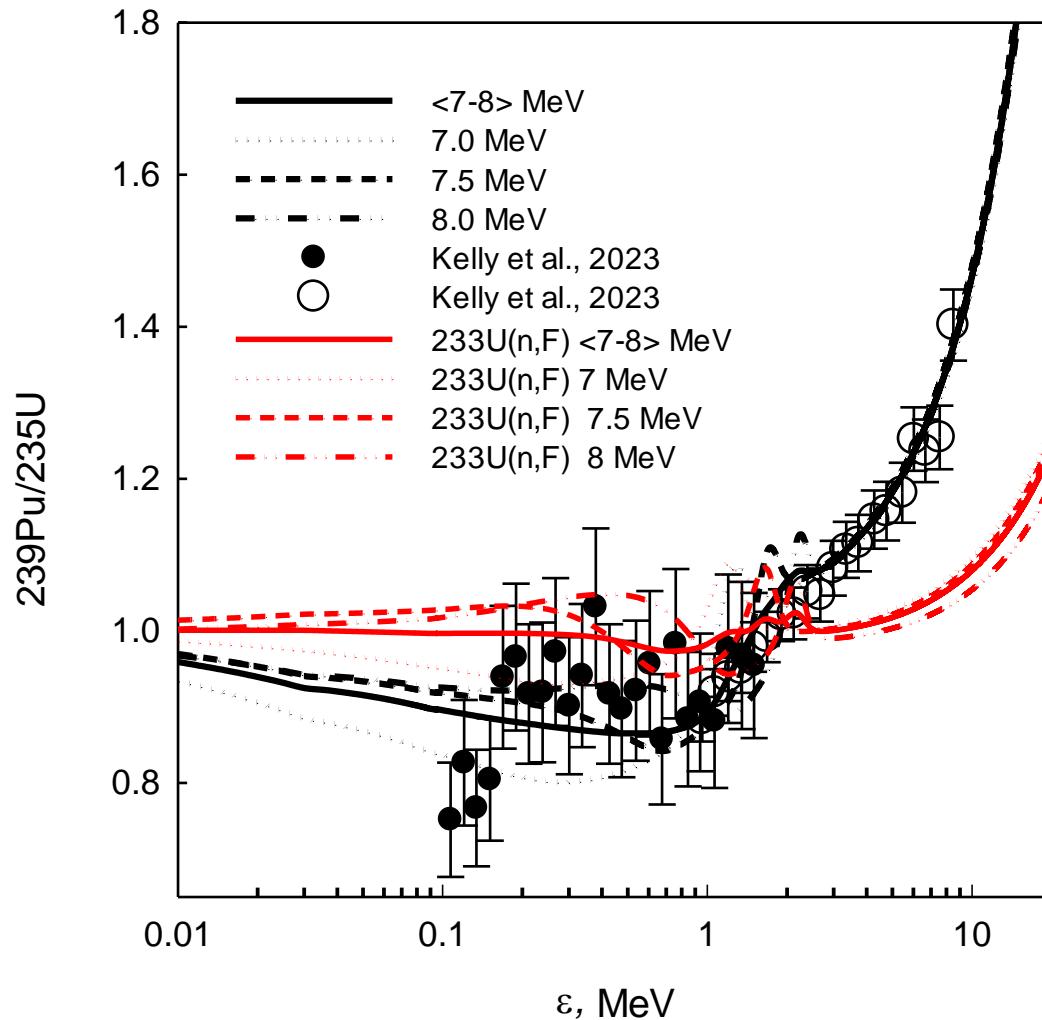
PFNS ratio, 6.5 MeV



$E_n = 7 \text{ MeV}$     $^{233}\text{U}(n,\text{F}) \& {}^{235}\text{U}(n,\text{F})$  PFNS       $E_n = 8.5 \text{ MeV}$

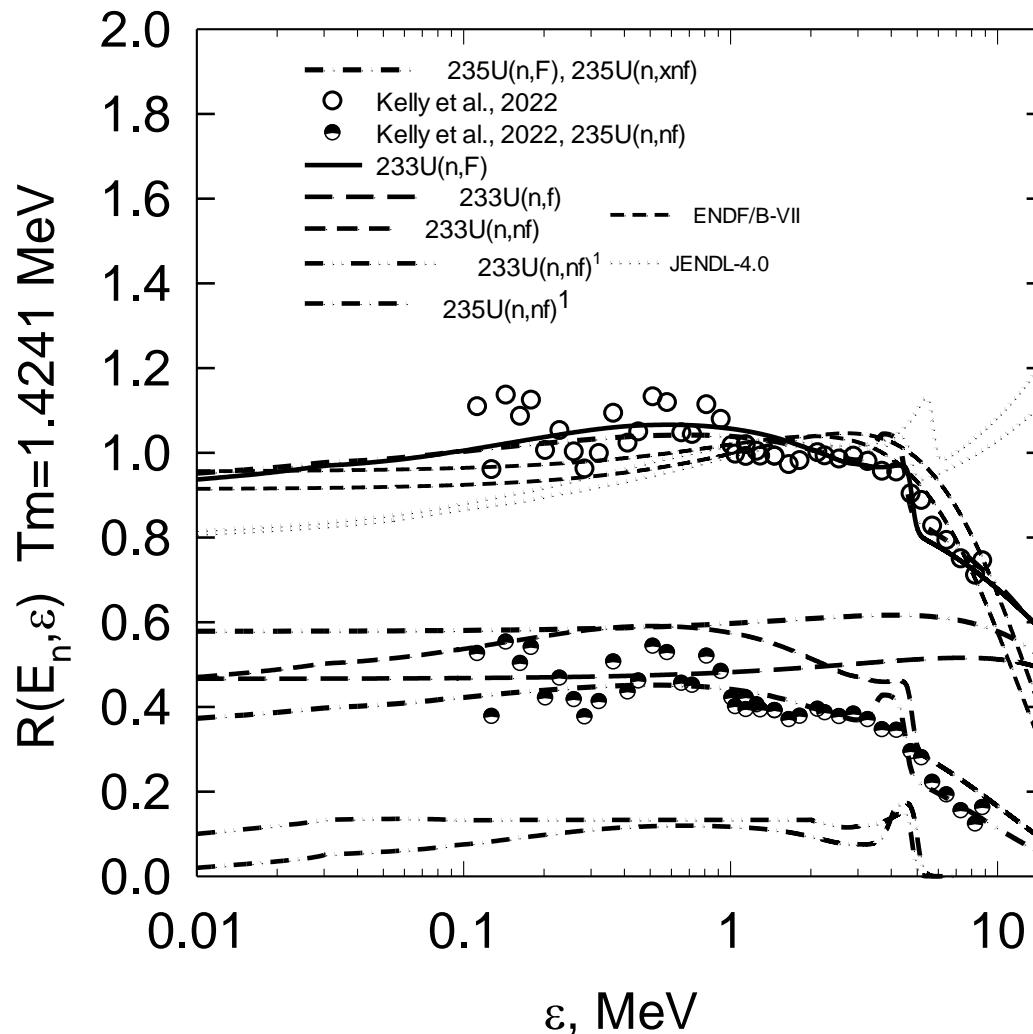


$E_n=7\text{-}8 \text{ MeV}$   $^{239}\text{Pu}/^{235}\text{U}$  PFNS ratio  $^{233}\text{U}/^{235}\text{U}$

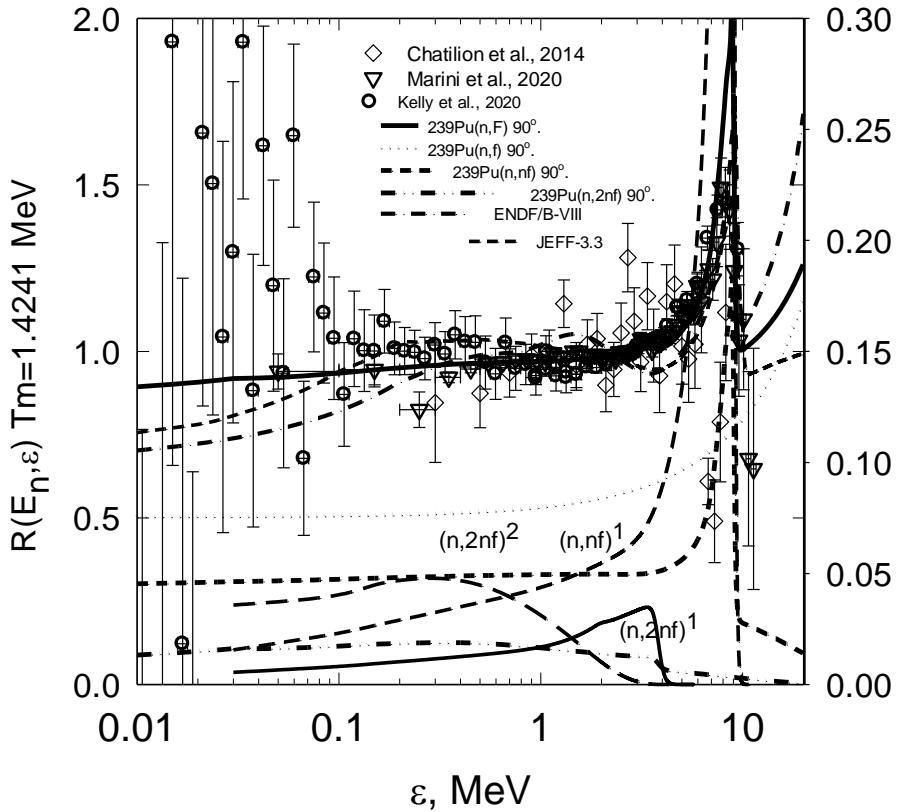


# 233U(n,F)&235U(n,F) PFNS

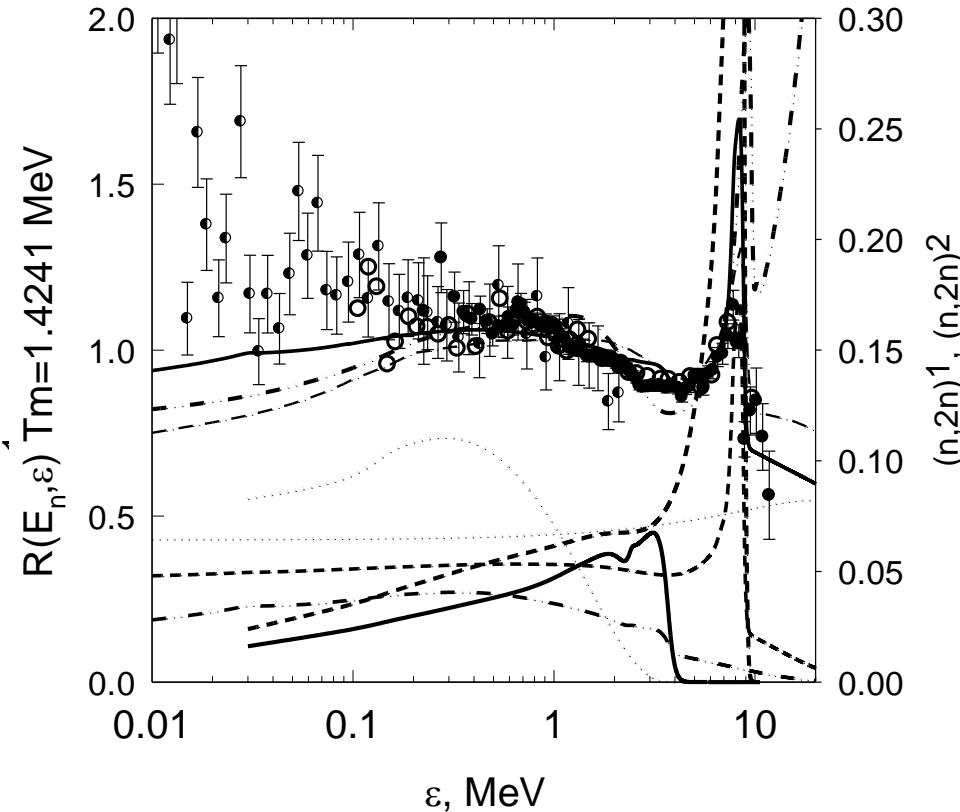
$E_n = 10.5 \text{ MeV}$



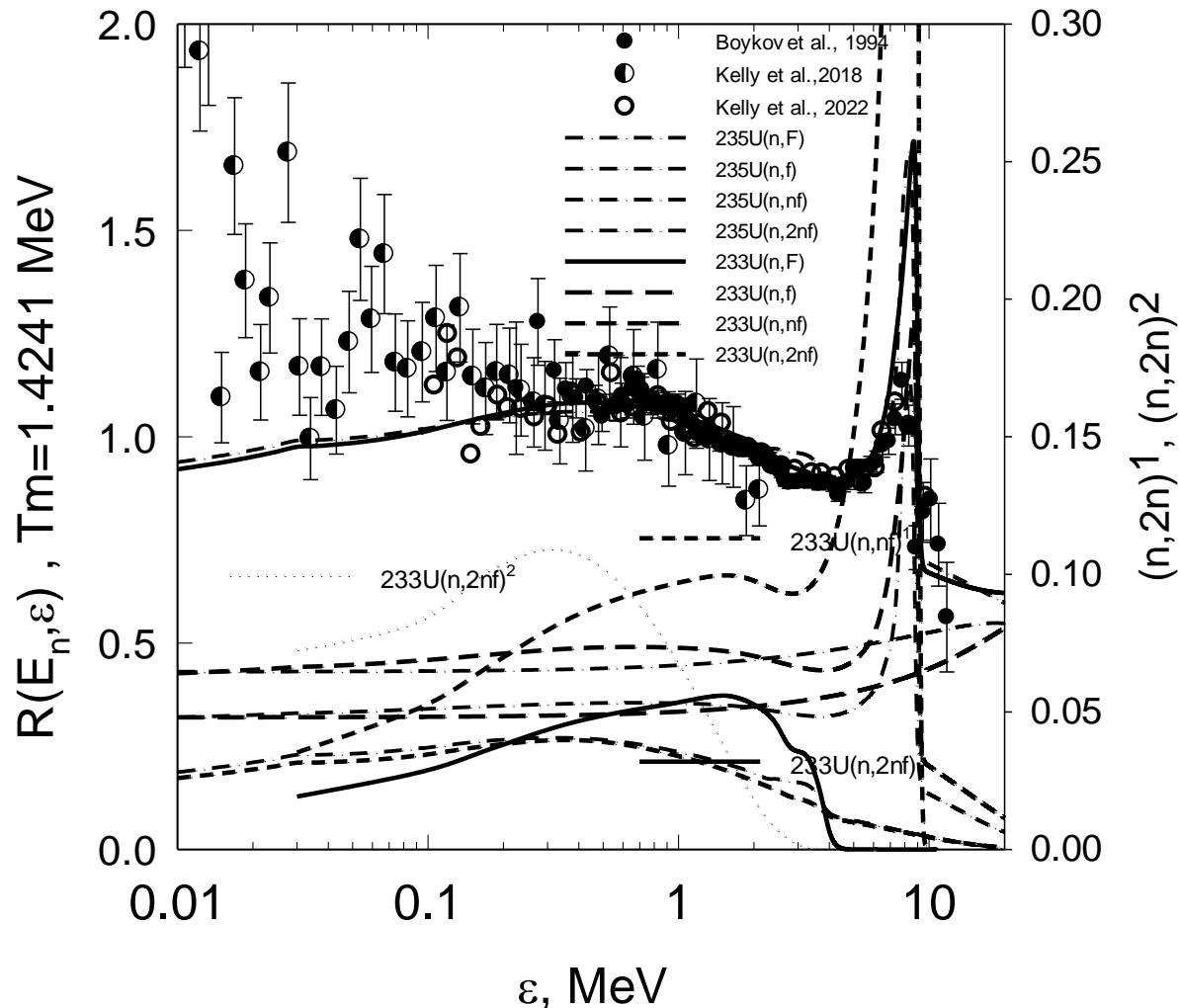
## $^{239}\text{Pu}(n,\text{F})$ PFNS 14.7 MeV



## $^{235}\text{U}(n,\text{F})$ , PFNS 14.7 MeV

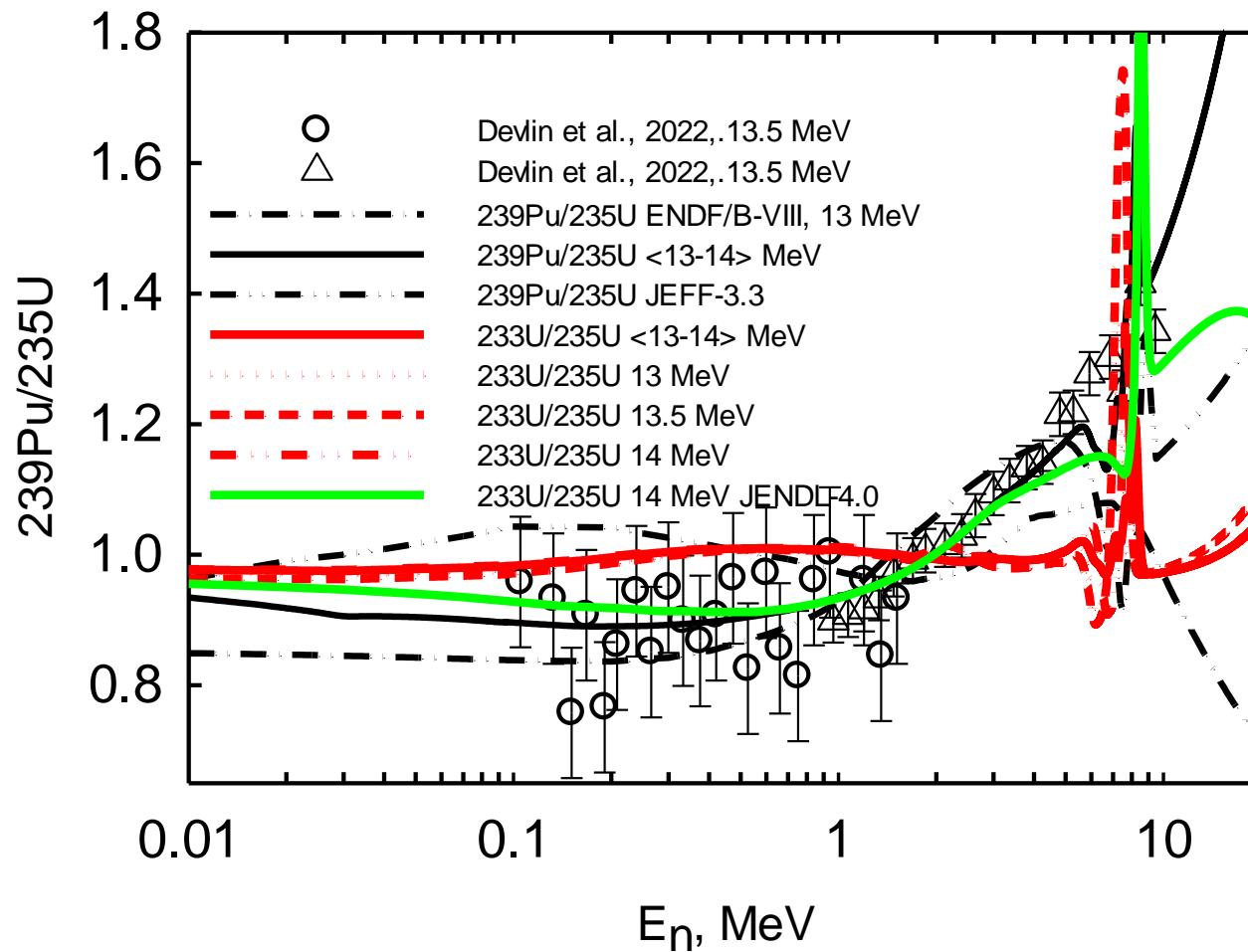


# 233U(n,F)&235U(n,F) PFNS      En=14.5 MeV

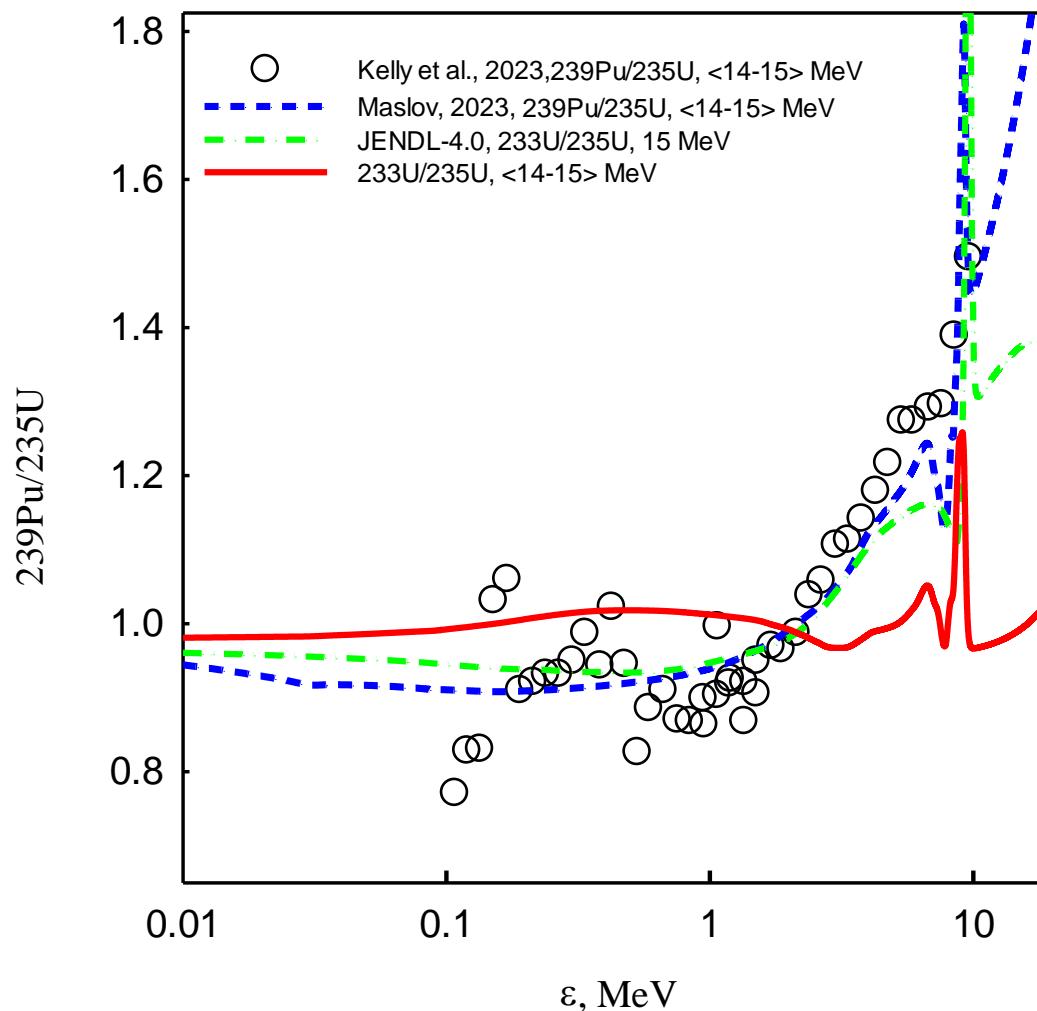


$E_n = 13-14$  MeV  $^{239}\text{Pu}/^{235}\text{U}$  PFNS ratio  $^{233}\text{U}/^{235}\text{U}$

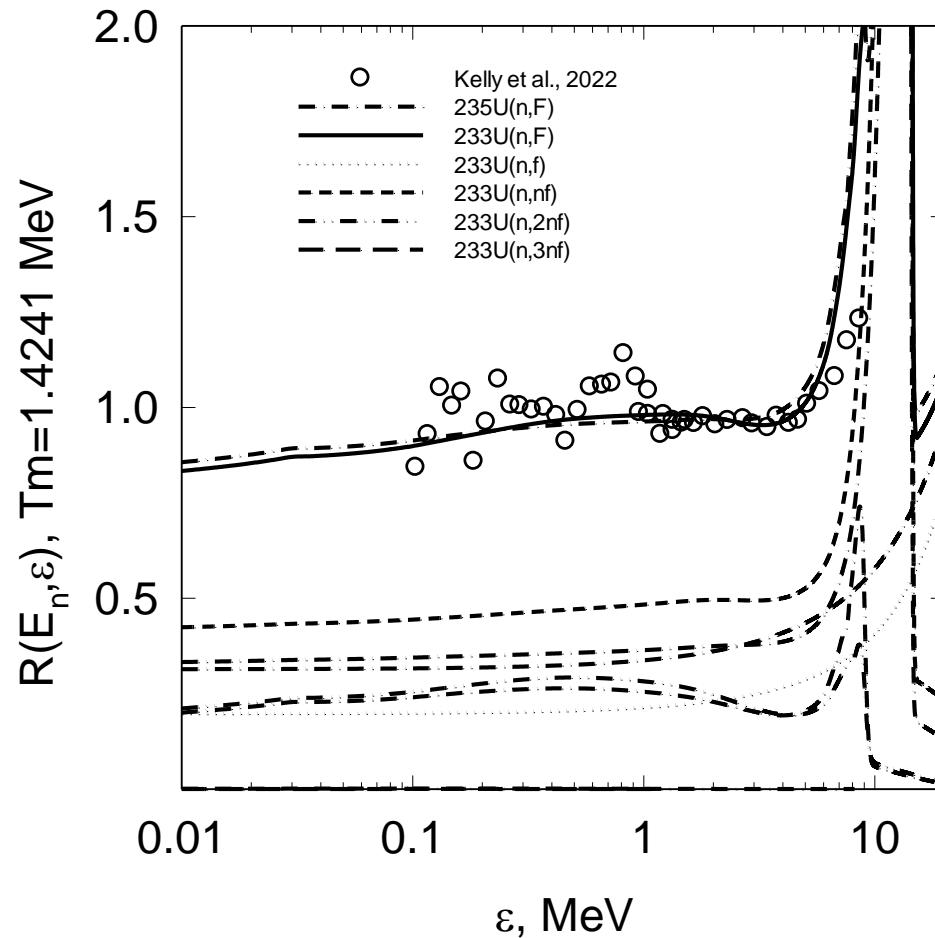
PFNS ratio, 13.5 MeV



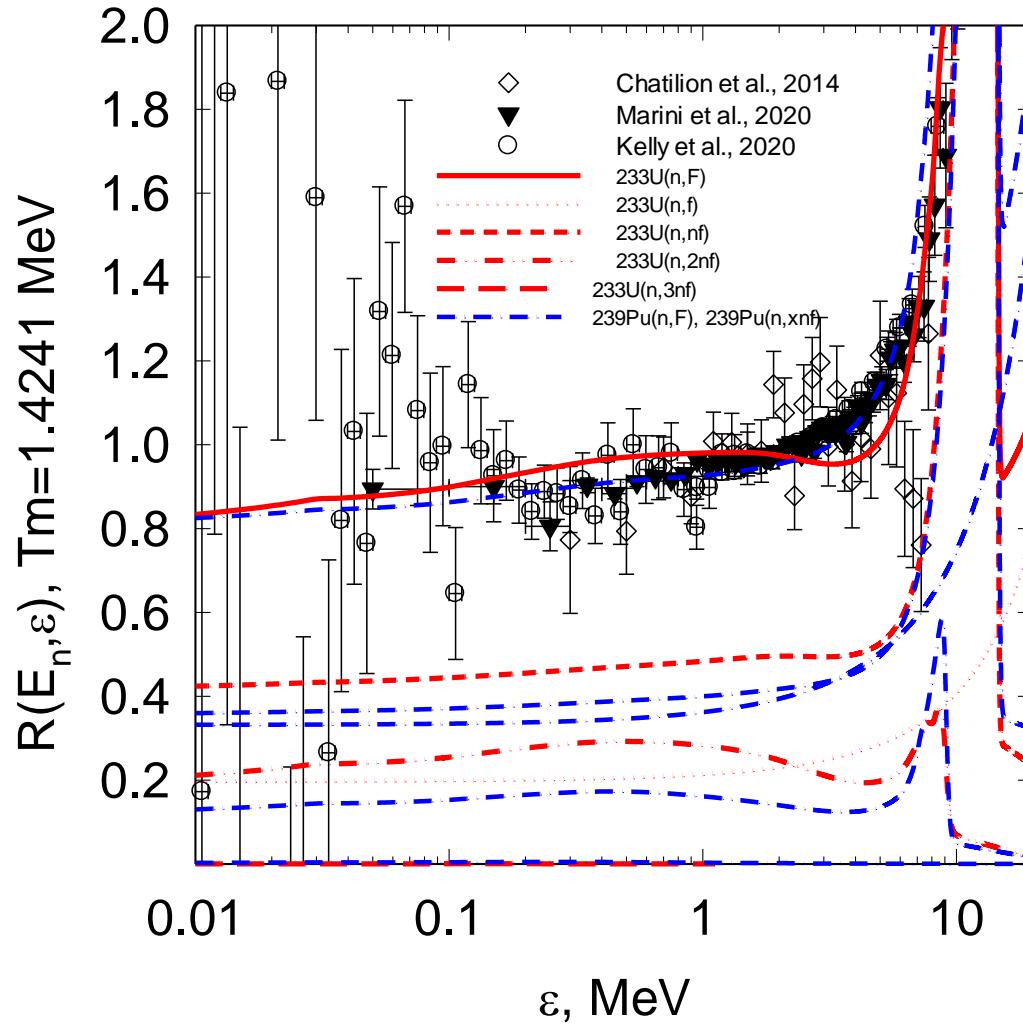
$E_n=14-15$  MeV  $^{239}\text{Pu}/^{235}\text{U}$  PFNS ratio  $^{233}\text{U}/^{235}\text{U}$



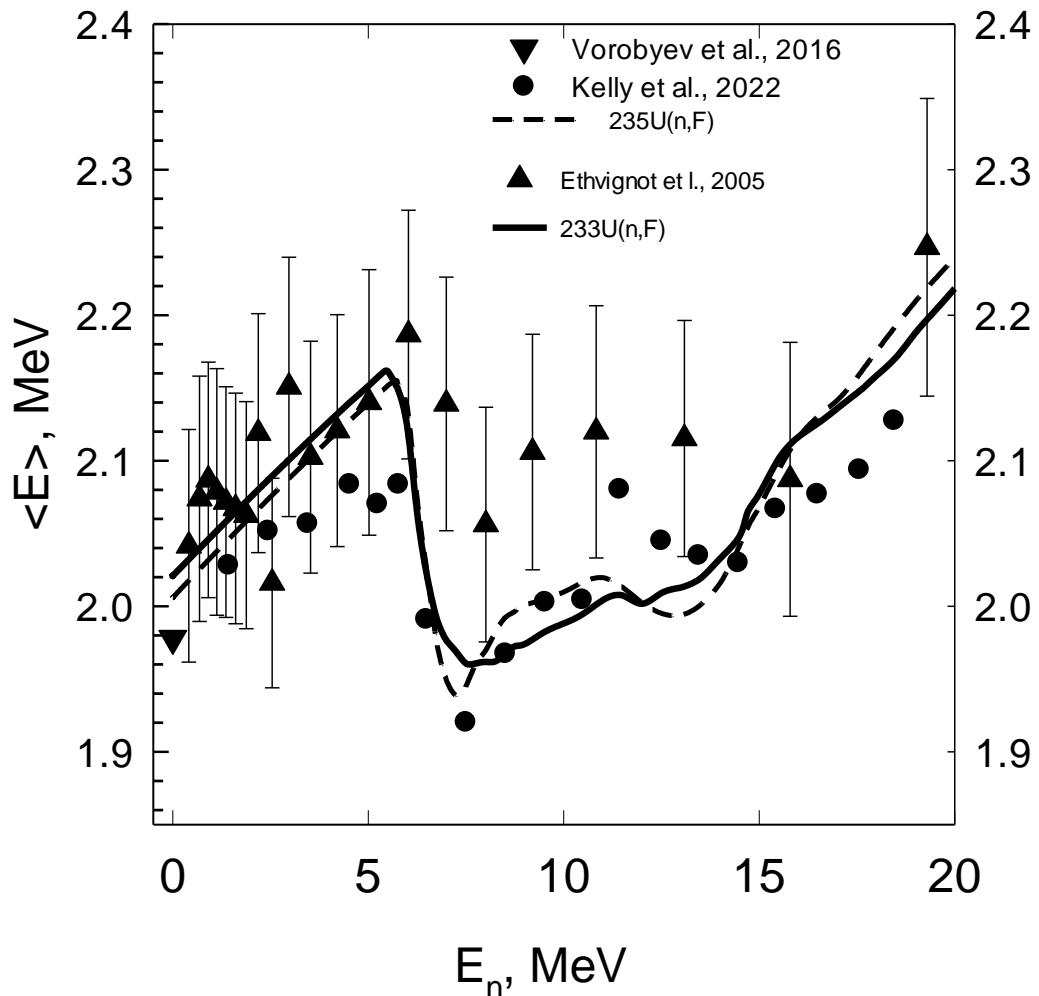
233U(n,F)&235U(n,F) PFNS      En=19-20 MeV



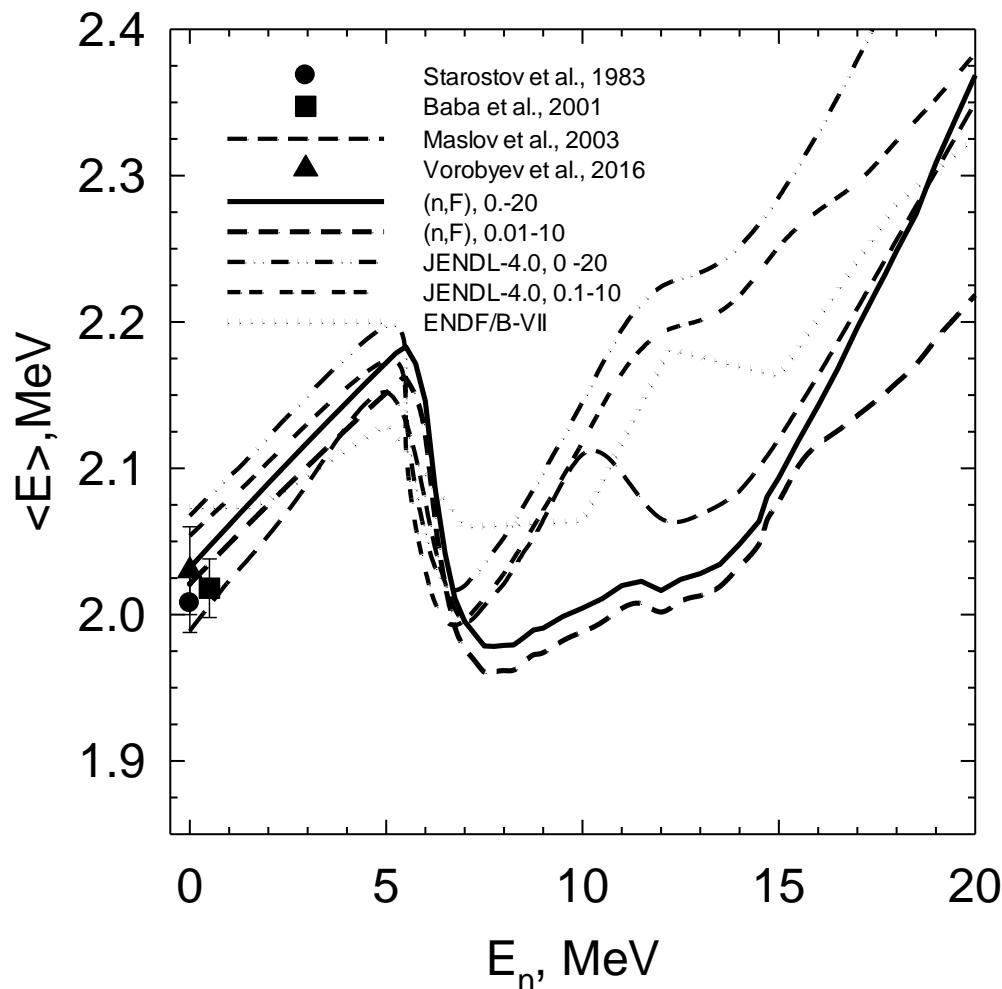
# $^{233}\text{U}(n,\text{F}) \& ^{239}\text{Pu}(n,\text{F})$ PFNS      $E_n=19-20 \text{ MeV}$



# $\langle E \rangle$ PFNS $^{233}\text{U}(n,F)$



## $\langle E \rangle$ PFNS $^{233}\text{U}(n,F)$



$$U_x = E_n + B_n - \sum_{x, 1 \leq k \leq x} (\langle E_{nxnf}^k(\theta) \rangle + B_{nx})$$

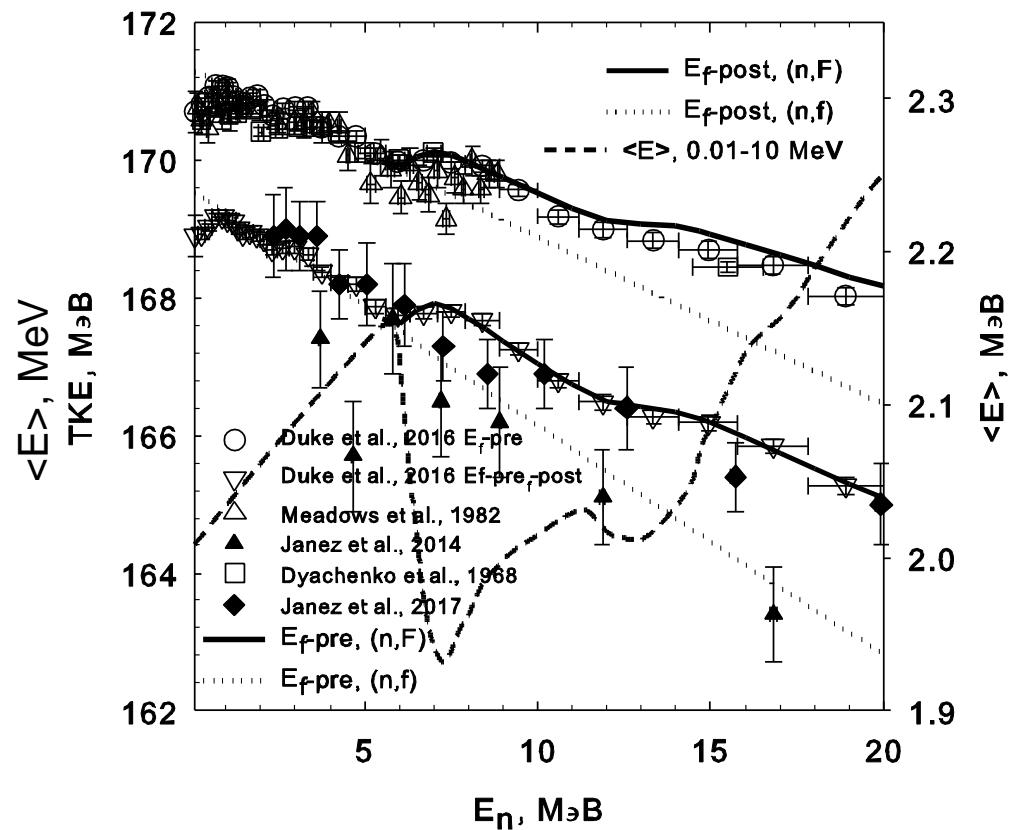
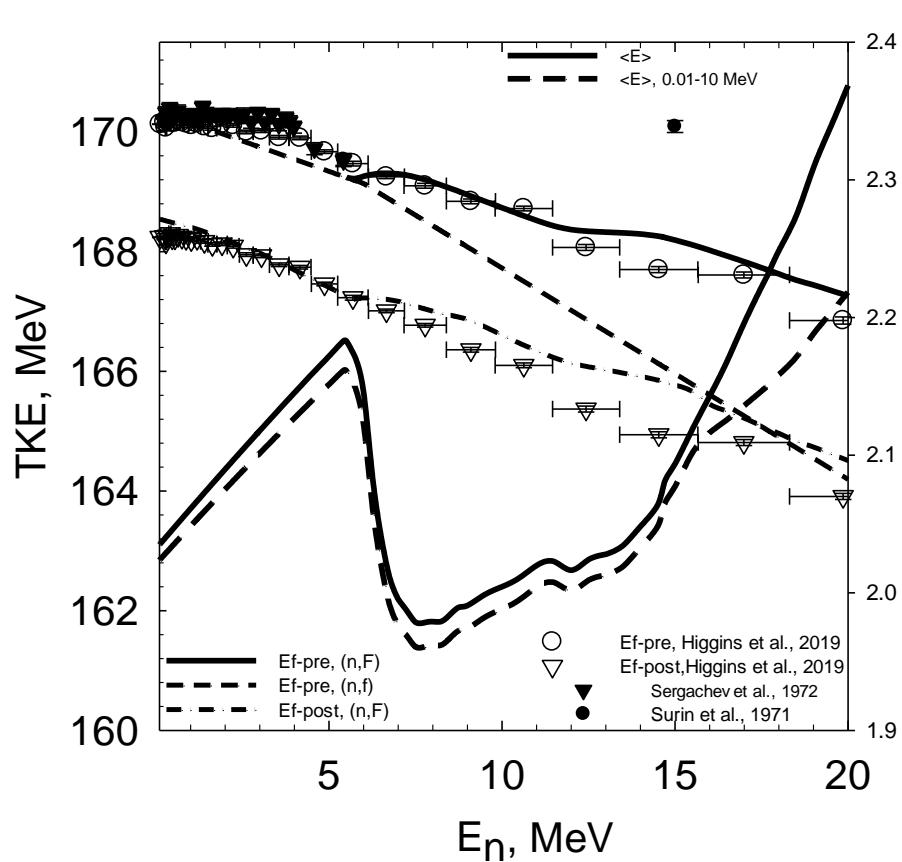
$$E_{nx} = E_r - E_{fx}^{pre} + E_n + B_n - \sum_{x, 1 \leq k \leq x} (\langle E_{nxnf}^k(\theta) \rangle + B_{nx})$$

$$E_F^{pre}(E_n) = \sum_{x=1}^X E_{fx}^{pre}(E_n) \sigma_{n,xnf} / \sigma_{n,F}$$

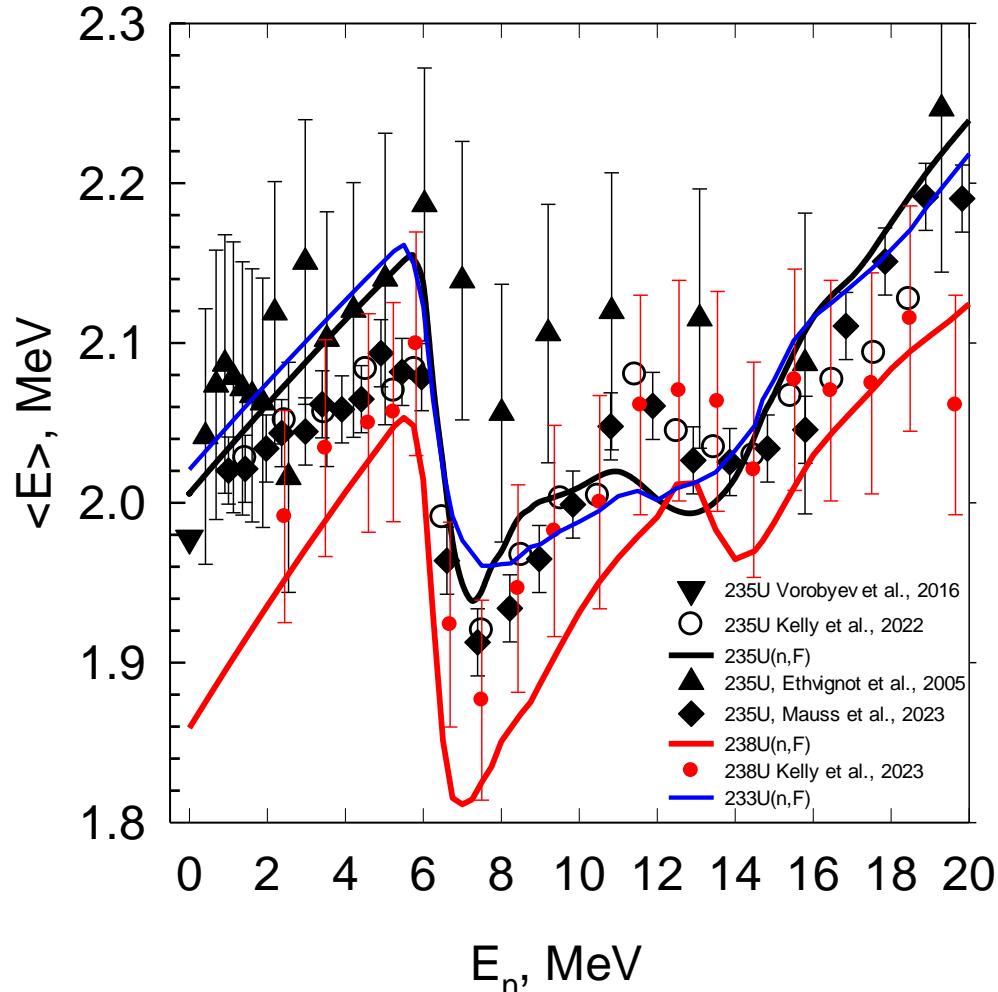
$$E_F^{post} \approx E_F^{pre} \left( 1 - \nu_{post} / (A + 1 - \nu_{pre}) \right)$$

$$\nu_p(E_n) = \nu_{post} + \nu_{pre} = \sum_{x=1}^X \nu_{px}(E_{nx}) + \sum_{x=1}^X (x-1) \cdot \beta_x(E_n)$$

# TKE& $\langle E \rangle$ PFNS 233,235U(n,F)



# $\langle E \rangle$ PFNS $^{233}\text{U}(n,F)$ , $^{235}\text{U}(n,F)$ , $^{238}\text{U}(n,F)$



# Conclusions

**Angular** dependence of first  $(n,nX)^1$  emission  $\omega(\theta)$   $\langle E \rangle$  of  $(n,nf)^1$  neutrons depends on emission angle  $\theta$   
**Fission** cross section, prompt neutron number and total kinetic energy depend on  $\theta$  as well

**Exclusive** neutron spectra  $(n,xnf)^{1,\dots,x}$  at  $\theta \sim 90^\circ$  are consistent with  $^{235}\text{U}(n,F)(^{235}\text{U}(n,xn))$  and  $^{239}\text{Pu}(n,F)(^{239}\text{Pu}(n,xn))$  css within  $E_n \sim 0.01 - 20$  MeV  
**Exclusive** neutron spectra of  $(n,xnf)^{1,\dots,x}$ ,  $(n,ny)$  and  $(n,xn)^{1,\dots,x}$  – by Hauser-Feshbach formalism

# Conclusions

**Approximation** obtained for  $\omega(\theta)$  fits the measured  $^{235}\text{U}+n$  &  $^{235}\text{U}+n$  NES at 14 MeV.

**The correlation** of angular dependence of  $(n,xnf)^1$  neutron emission with emissive fission  $(n,xnf)$  and angular anisotropy of  $^{235}\text{U}+n$  &  $^{239}\text{Pu}+n$  NES is established.

**On that background**

**The PFNS shapes and energies  $\langle E \rangle$  and TKE for  $^{233}\text{U}(n,F)$  &  $^{233}\text{U}(n,xnf)$  provided**

# Conclusions

In  $^{239}\text{Pu}(n,xnf)^{1,\dots,x}$  and  $^{235}\text{U}(n,xnf)^{1,\dots,x}$  PFNS demonstrate different responses to forward and backward  $(n,xnf)^1$  neutrons emission with respect to the incident neutron momentum

In  $^{233}\text{U}(n,xnf)^{1,\dots,x}$  and  $^{233}\text{U}(n,F)$  PFNS stronger response to forward and backward  $(n,xnf)^1$  neutrons emission ?

# Conclusions

- Maslov V.M. Anisotropy of Prompt Fission Neutron Spectra of  $^{239}\text{Pu}(n, F)$  and  $^{235}\text{U}(n, F)$ , Physics of Particles and Nuclei Letters, 2023, Vol. 20, No. 6, pp. 1373–1384.

<https://pepan.jinr.ru/index.php/PepanLetters/Issue/20/6>; <https://rdcu.be/dsLEI>;

- Maslov V.M. Anisotropy of Prompt Fission Neutron Spectra of  $^{233}\text{U}(n, F)$ , Physics of Particles and Nuclei Letters, 2024, in press.