

# **Prompt Fission Neutron** **Spectra of $^{240}\text{Pu}(n,F)$**

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# Collaboration on PFNS of 2000-2011

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# EXPERIMENTS on

- Asymmetry PFNS in  $^{239}\text{Pu}(n,F)$   $^{235}\text{U}(n,F)$  PFNS Kelly e. a., Phys. Rev. Lett., **2019**, v. 122, p. 072503
- **Asymmetry in neutron emission spectra  $E_n=14$  MeV  $^{235}\text{U}+n$ ;  $^{239}\text{Pu}+n$ ;  $^{238}\text{U}+n$**  – first observed by Kammerdiener J.L., UCRL-51232, 1972.
- Asymmetry in neutron emission spectra of  $^{232}\text{Th}+n$  and  $^{238}\text{U}+n$  at  $E_n=6, 12, 14, 18$  MeV (measured)
- Asymmetry of pre-fission neutron emission in  $^{238}\text{U}(n,F)$  vis  $^{232}\text{Th}(n,F)$  (predicted)

For  $^{240,242}\text{Pu}$

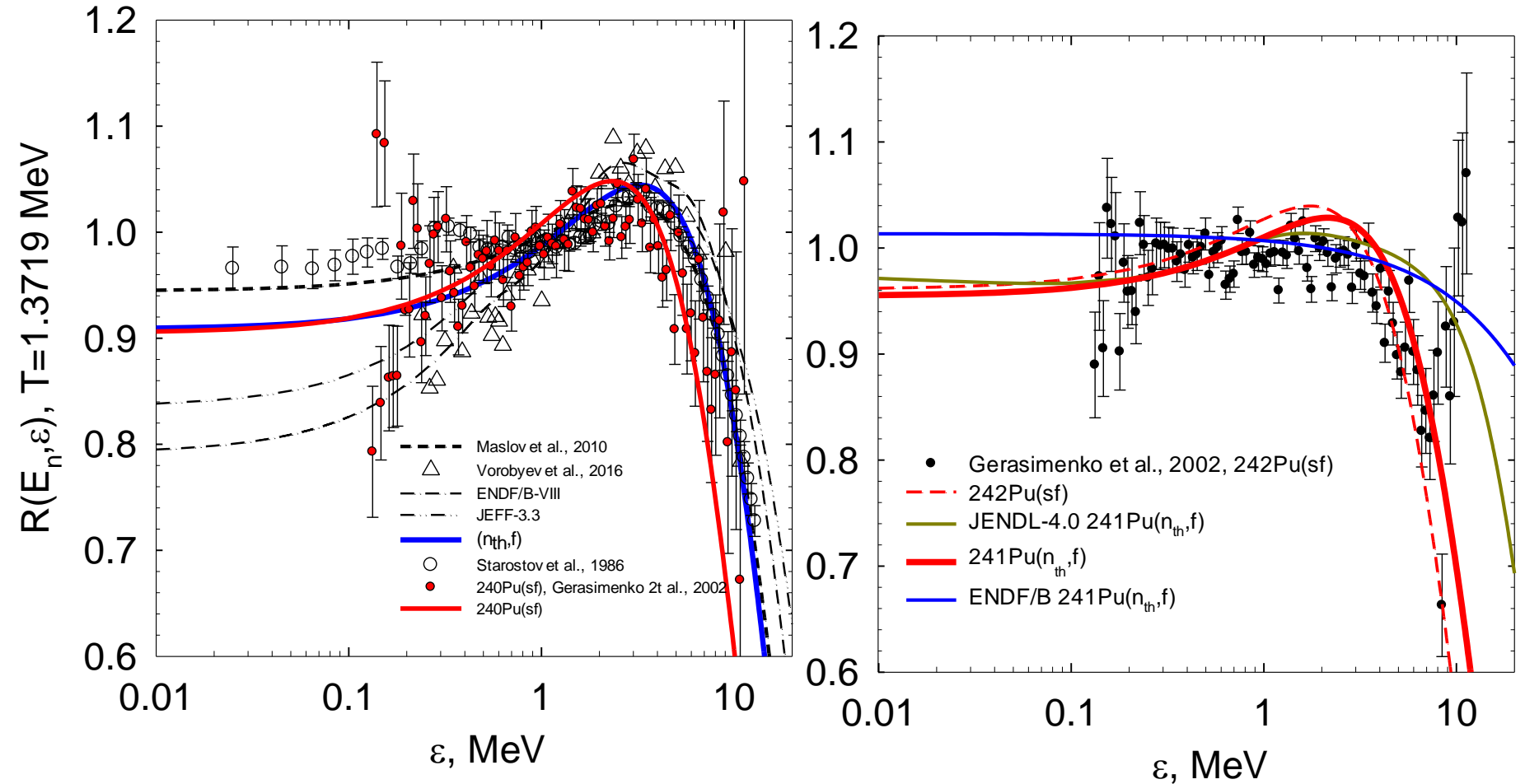
$^{240}\text{Pu}(\text{sf}), ^{242}\text{Pu}(\text{sf})$  --SFNS only

$^{240}\text{Pu}(\text{sf})$     $^{241}\text{Pu}(\text{sf})$     $^{242}\text{Pu}(\text{sf})$

$^{239}\text{Pu}(\text{n}_{\text{th}},\text{f})$     $^{240}\text{Pu}(\text{n}_{\text{th}},\text{f})$     $^{241}\text{Pu}(\text{n}_{\text{th}},\text{f})$

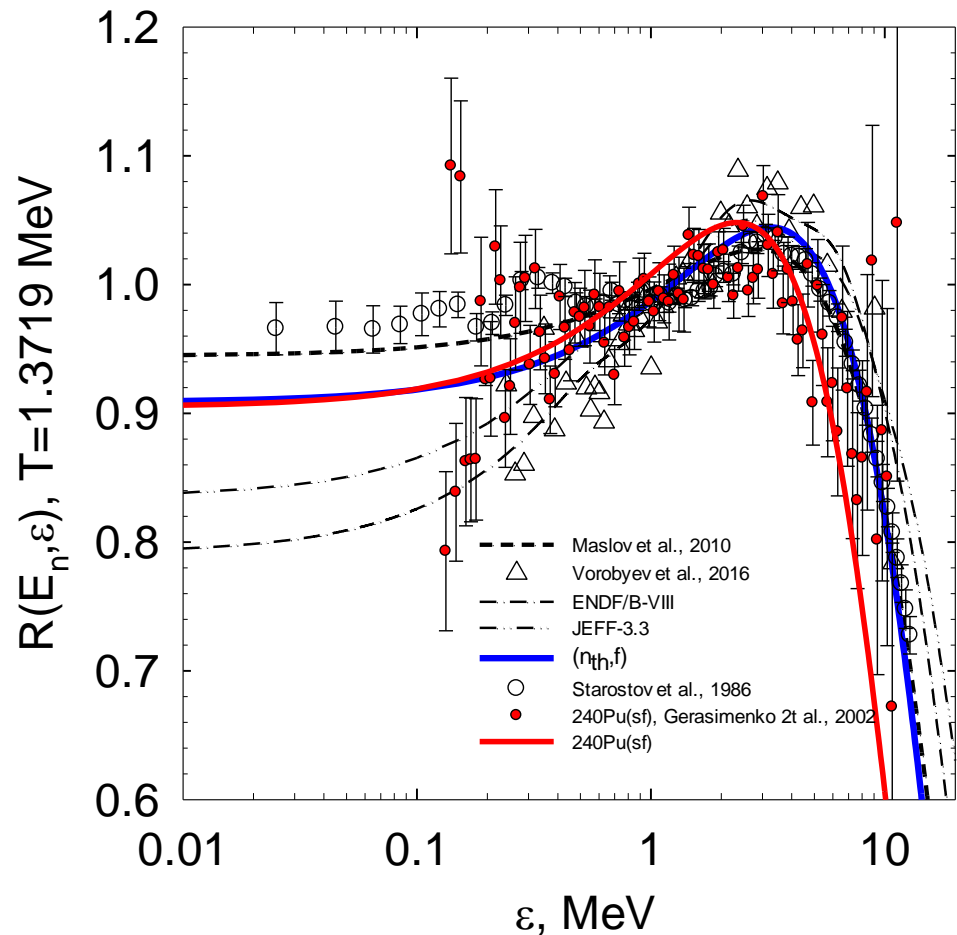
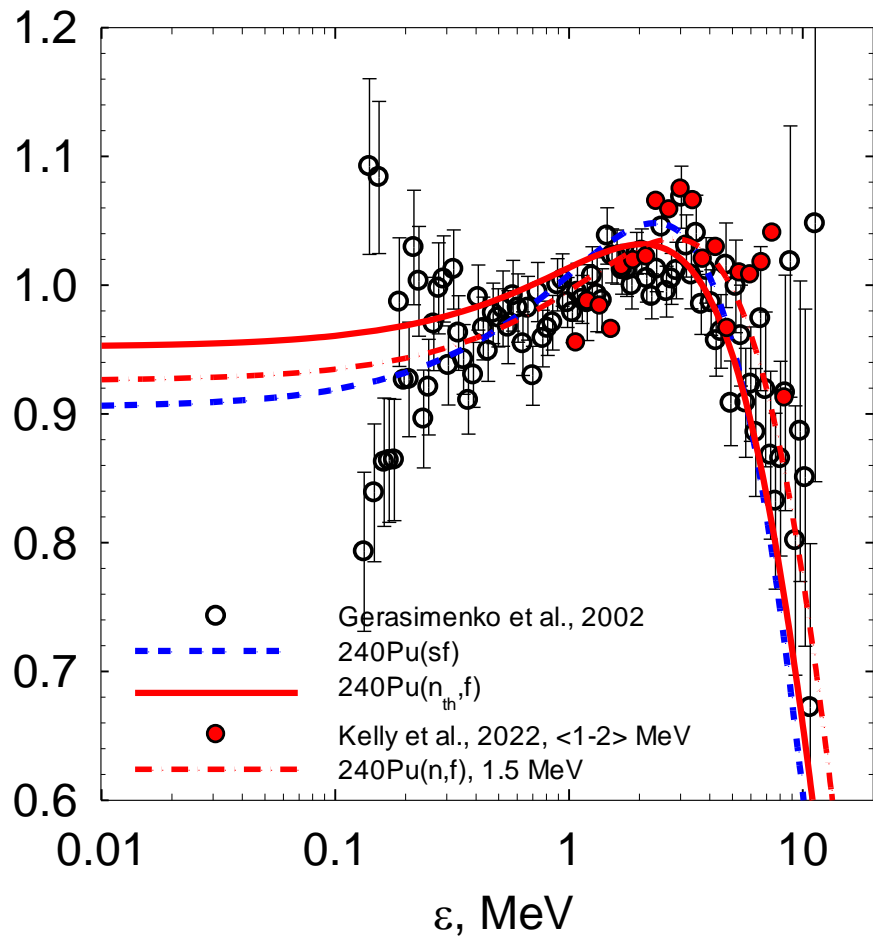
$^{239}\text{Pu}(n_{\text{th}},f)$   $^{240}\text{Pu}(sf)$

$^{241}\text{Pu}(n_{\text{th}},f)$   $^{242}\text{Pu}(sf)$

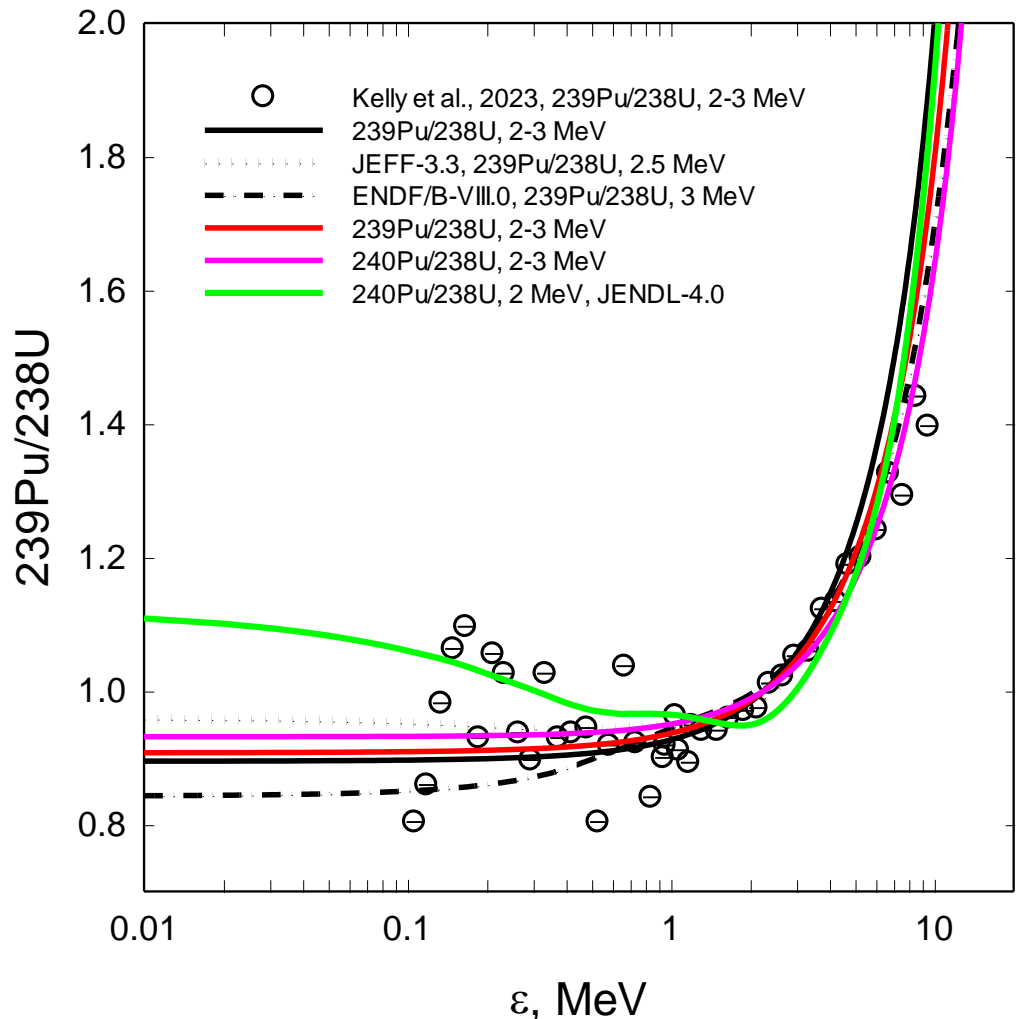


$^{240}\text{Pu}(n_{\text{th}},f)$   $^{240}\text{Pu}(sf)$

$^{239}\text{Pu}(n_{\text{th}},f)$   $^{240}\text{Pu}(sf)$



$^{239}\text{Pu}(n,f)/^{238}\text{U}(n,f)$   $E_n=2-3\text{MeV}$   $^{240}\text{Pu}(n,f)/^{238}\text{U}(n,f)$



$$\sigma_{nF}(E_n) = \sigma_{nf}(E_n) + \sum_{x=1}^X \sigma_{n,xnf}(E_n)$$

$$\sigma_{n,xnf}(E_n) = \sum_{J\pi}^J \int_0^{U_x} W_{A+1-x}^{J\pi}(U) P_{f(A+1-x)}^{J\pi}(U) dU$$

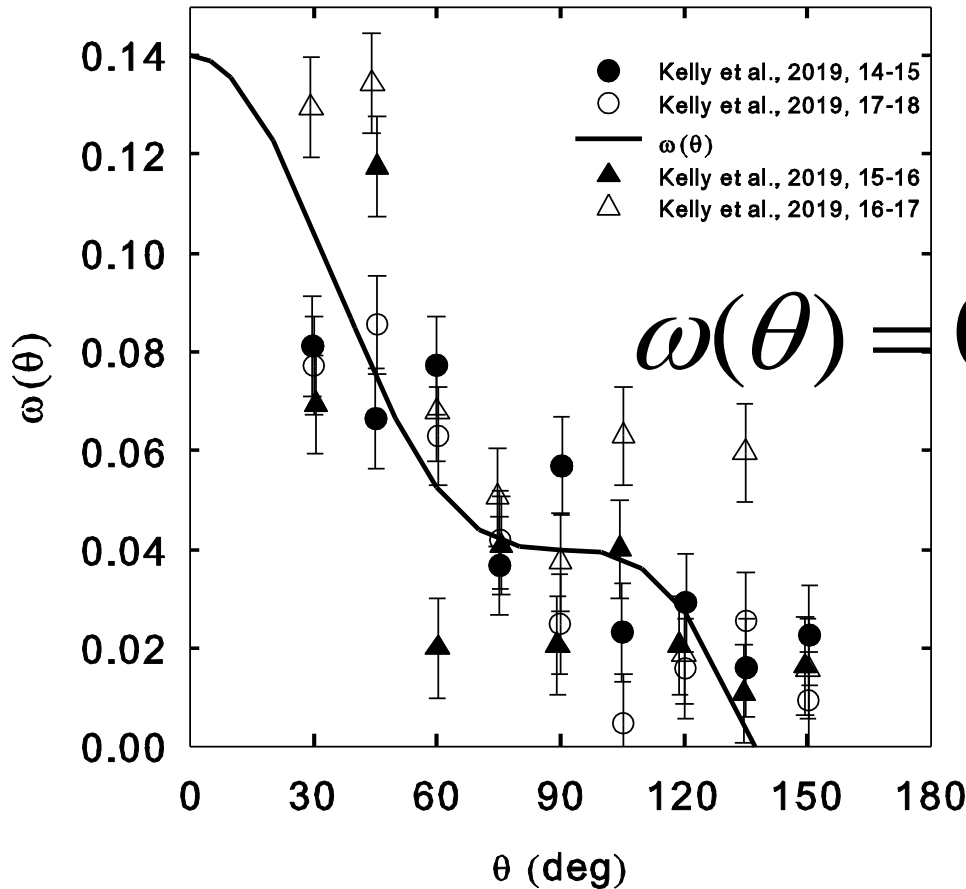
$$\frac{d\sigma_{nnx}^1(\varepsilon, E_n)}{d\varepsilon} \approx \frac{d\tilde{\sigma}_{nnx}^1(\varepsilon, E_n)}{d\varepsilon} + \sqrt{\frac{\varepsilon}{E_n} \frac{\langle \omega(\theta) \rangle_\theta}{E_n - \varepsilon}}$$

$$\langle \omega(\theta) \rangle_g \approx \omega(\theta \approx 90^\circ)$$

$$\omega(\theta) = 0.4 \cos^3(\theta) + 0.16$$



# Angular distribution of $^{239}\text{Pu}(n,F)$ pre-fission neutrons



$$\omega(\theta) = 0.4 \cos^3(\theta) + 0.16$$

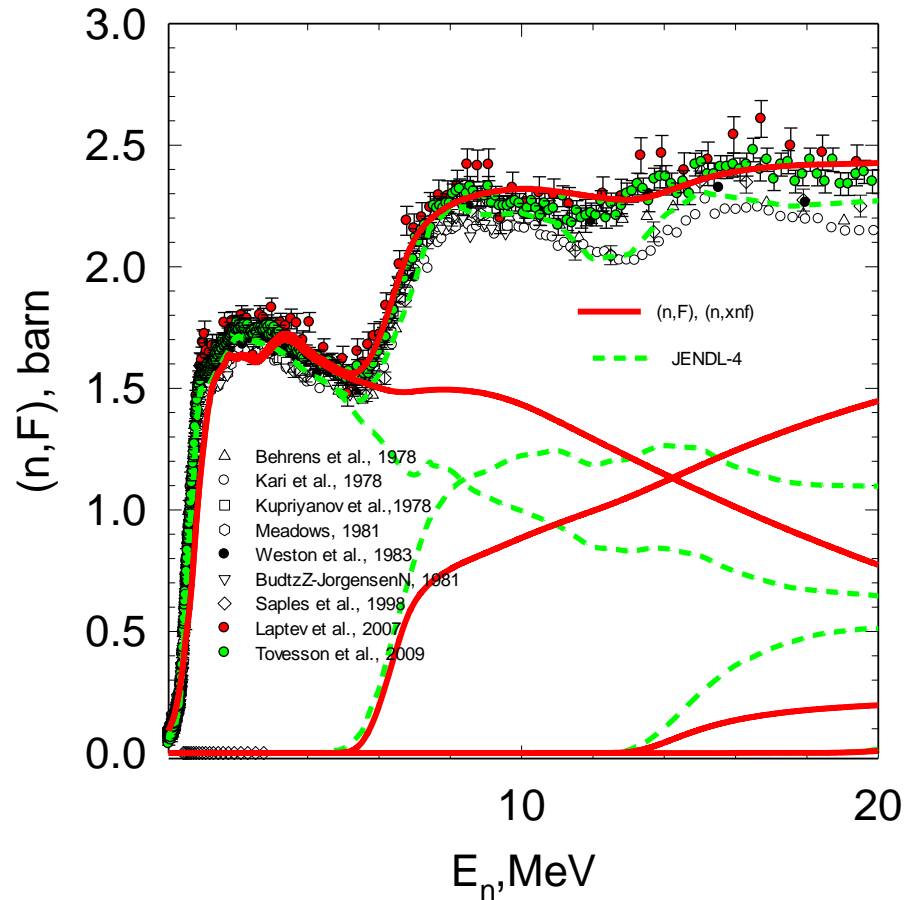
**Angular anisotropy (AA) of secondary neutrons** is evidenced in neutron emission spectra (NES) due to

1. (n,n)
2. **g.s. band levels**  $J^\pi = 0^+, 2^+, 4^+, 6^+, 8^+$  (e-e)
3.  $\gamma$ -bands with  $K^\pi = 0^+, 2^+$ , octupole bands  $K^\pi = 0^-$  (e-e)
4. (n,n $\gamma$ )

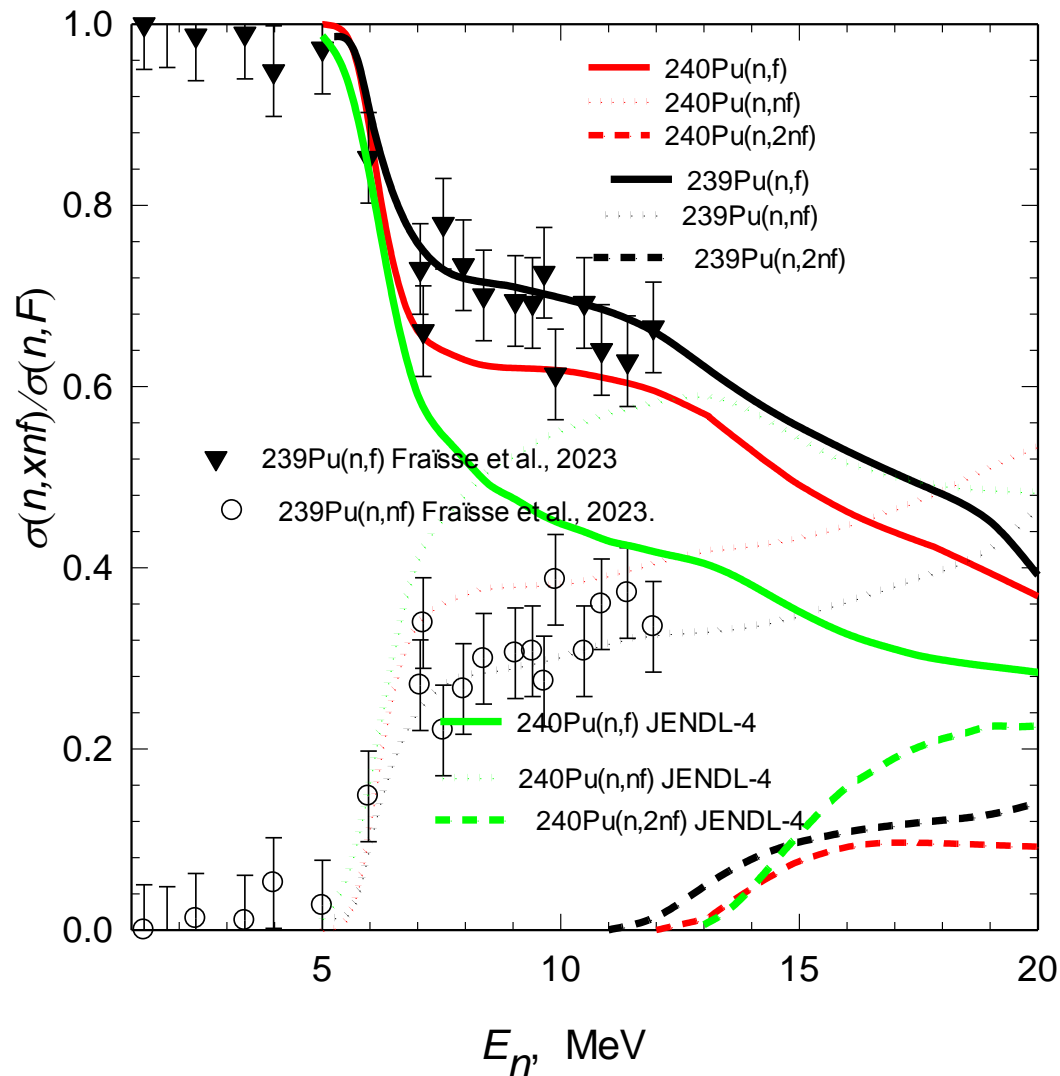
NES **AA** is due to PE/semi-direct  $(n,nX)^1$

**Angular anisotropy of PFNS due to** tiny part of  $(n,nX)^1$  neutrons in exclusive pre-fission neutrons in  $(n,xnf)^{1,\dots,x}$ .

# $^{240}\text{Pu}(n,F)$ , $^{240}\text{Pu}(n,xnf)$



# $^{240}\text{Pu}(n,xf)/^{240}\text{Pu}(n,F)$



# PFNS as a superposition of (n,f) &(n,xf)

$$\begin{aligned}
 S(\varepsilon, E_n, \theta) &= \tilde{S}_{A+1}(\varepsilon, E_n, \theta) + \tilde{S}_A(\varepsilon, E_n, \theta) + \tilde{S}_{A-1}(\varepsilon, E_n, \theta) + \tilde{S}_{A-2}(\varepsilon, E_n, \theta) = \\
 &v_p^{-1}(E_n, \theta) \cdot \left\{ v_{p1}(E_n) \cdot \beta_1(E_n, \theta) S_{A+1}(\varepsilon, E_n, \theta) + v_{p2}(E_n - \langle E_{nnf}(\theta) \rangle) \beta_2(E_n, \theta) S_A(\varepsilon, E_n, \theta) + \right. \\
 &+ \beta_2(E_n, \theta) \frac{d^2 \sigma_{nnf}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\varepsilon} + v_{p3}(E_n - B_n^A - \langle E_{n2nf}^1(\theta) \rangle - \langle E_{n2nf}^2(\theta) \rangle) \beta_3(E_n, \theta) S_{A-1}(\varepsilon, E_n, \theta) + \beta_3(E_n, \theta) \cdot \\
 &\left[ \frac{d^2 \sigma_{n2nf}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} + \frac{d^2 \sigma_{n2nf}^2(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \right] + v_{p4}(E_n - B_n^A - B_n^{A-1} - \langle E_{n3nf}^1(\theta) \rangle - \langle E_{n3nf}^2(\theta) \rangle - \langle E_{n3nf}^3(\theta) \rangle) \cdot \\
 &\left. \beta_4(E_n, \theta) S_{A-2}(\varepsilon, E_n, \theta) + \beta_4(E_n, \theta) \left[ \frac{d^2 \sigma_{n3nf}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} + \frac{d^2 \sigma_{n3nf}^2(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} + \frac{d^2 \sigma_{n2nf}^3(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \right] \right\}.
 \end{aligned}$$

Exclusive neutron spectra of  $(n, xn f)^{1, \dots, x}$ ,  $(n, n\gamma)$  and  $(n, xn)^{1, \dots, x}$  are calculated within Hauser-Feshbach formalism alongside with  $(n, F)$  and  $(n, xn)$  reaction cross sections,

Angular dependence of first  $(n, nX)^1$  emission  $^{240}\text{Pu}$  being included via  $\langle \omega(\theta) \rangle_{\theta} \approx \omega(\theta \approx 90^\circ)$

$$U_x = E_n + B_n - \sum_{x, 1 \leq k \leq x} (\langle E_{n, x, n, f}^k(\theta) \rangle + B_{nx})$$

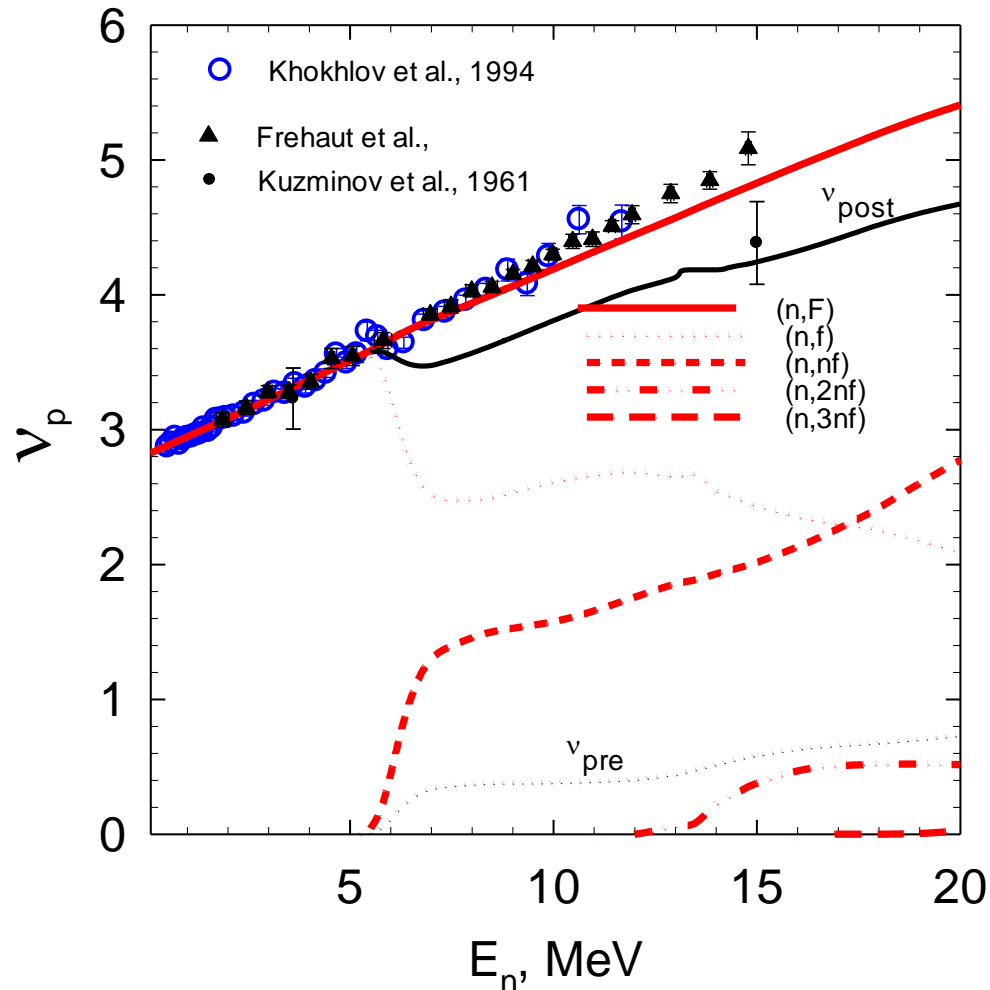
$$E_{nx} = E_r - E_{fx}^{pre} + E_n + B_n - \sum_{x, 1 \leq k \leq x} (\langle E_{n, x, n, f}^k(\theta) \rangle + B_{nx})$$

$$E_F^{pre}(E_n) = \sum_{x=1}^X E_{fx}^{pre}(E_n) \sigma_{n, x, n, f} / \sigma_{n, F}$$

$$E_F^{post} \approx E_F^{pre} \left( 1 - \nu_{post} / (A + 1 - \nu_{pre}) \right)$$

$$\nu_p(E_n) = \nu_{post} + \nu_{pre} = \sum_{x=1}^X \nu_{px}(E_{nx}) + \sum_{x=1}^X (x-1) \cdot \beta_x(E_n)$$

# $^{240}\text{Pu}(n,F)$ , $^{240}\text{Pu}(n,xnf)$ , pre- and post-fission neutrons multiplicity





$$\frac{d^2 \sigma_{nnx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \approx \frac{d^2 \tilde{\sigma}_{nnx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} + \sqrt{\frac{\varepsilon}{E_n}} \frac{\omega(\theta)}{E_n - \varepsilon}$$

$$\langle \omega(\theta) \rangle_g \approx \omega(\theta \approx 90^\circ)$$

$$\sigma_c(E_n) = \sigma_a(E_n)(1 - q - \tilde{q}),$$

$$\tilde{q} = \int_0^{E_n - E_d} \frac{d\tilde{\sigma}_{nnx}^1}{d\varepsilon} d\varepsilon = \int_0^{E_n - E_d} \sqrt{\frac{\varepsilon}{E_n}} \frac{\langle \omega(\theta) \rangle_\theta}{E_n - \varepsilon} d\varepsilon = \langle \omega(\theta) \rangle_\theta \left[ \ln \left( \frac{1 + \sqrt{1 - \delta}}{1 - \sqrt{1 - \delta}} \right) - 2\sqrt{1 - \delta} \right]$$

$\delta = E_d/E_n$  ,  $E_d$ -upper discrete level energy

$$\frac{d\tilde{\sigma}_{nnx}^1(\varepsilon, E_n)}{d\varepsilon} = \sum_{J, \pi} W^A(E_n - \varepsilon, \theta, J^\pi)$$

$$\frac{d^2\sigma_{nnf}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} = \frac{d^2\sigma_{nnx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \frac{\Gamma_f^A(E_n - \varepsilon, \theta)}{\Gamma^A(E_n - \varepsilon, \theta)}$$

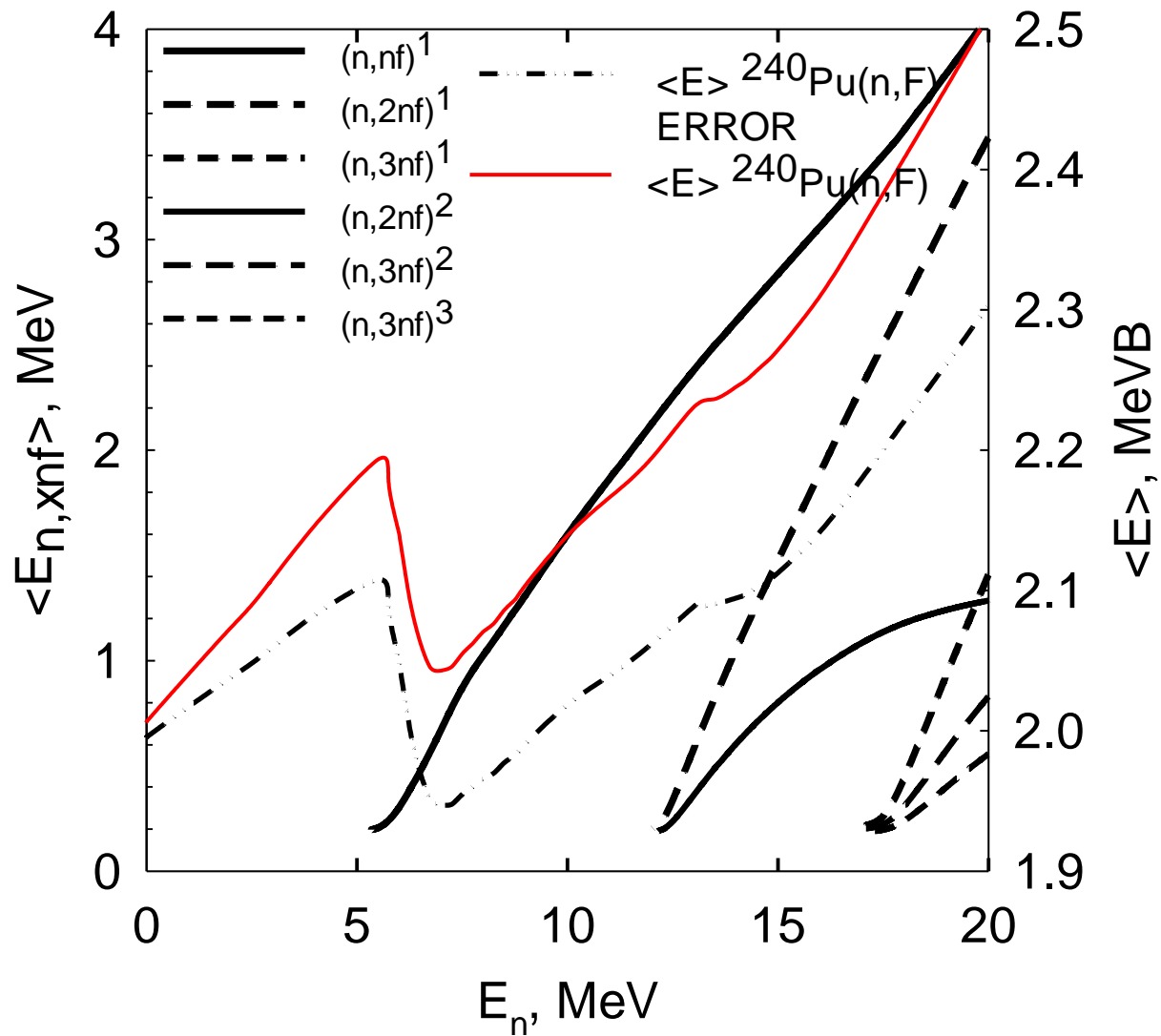
$$\frac{d^2\sigma_{n2nx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} = \frac{d^2\sigma_{nnx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \frac{\Gamma_n^A(E_n - \varepsilon, \theta)}{\Gamma^A(E_n - \varepsilon, \theta)}$$

$$\frac{d^2\sigma_{n2nf}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} = \int_0^{E - B_n^A} \frac{d^2\sigma_{n2nx}^1(\varepsilon, E_n, \theta)}{d\varepsilon d\theta} \frac{\Gamma_f^{A-1}(E_n - B_n^A - \varepsilon - \varepsilon_1)}{\Gamma^{A-1}(E_n - B_n^A - \varepsilon - \varepsilon_1)} d\varepsilon_1$$

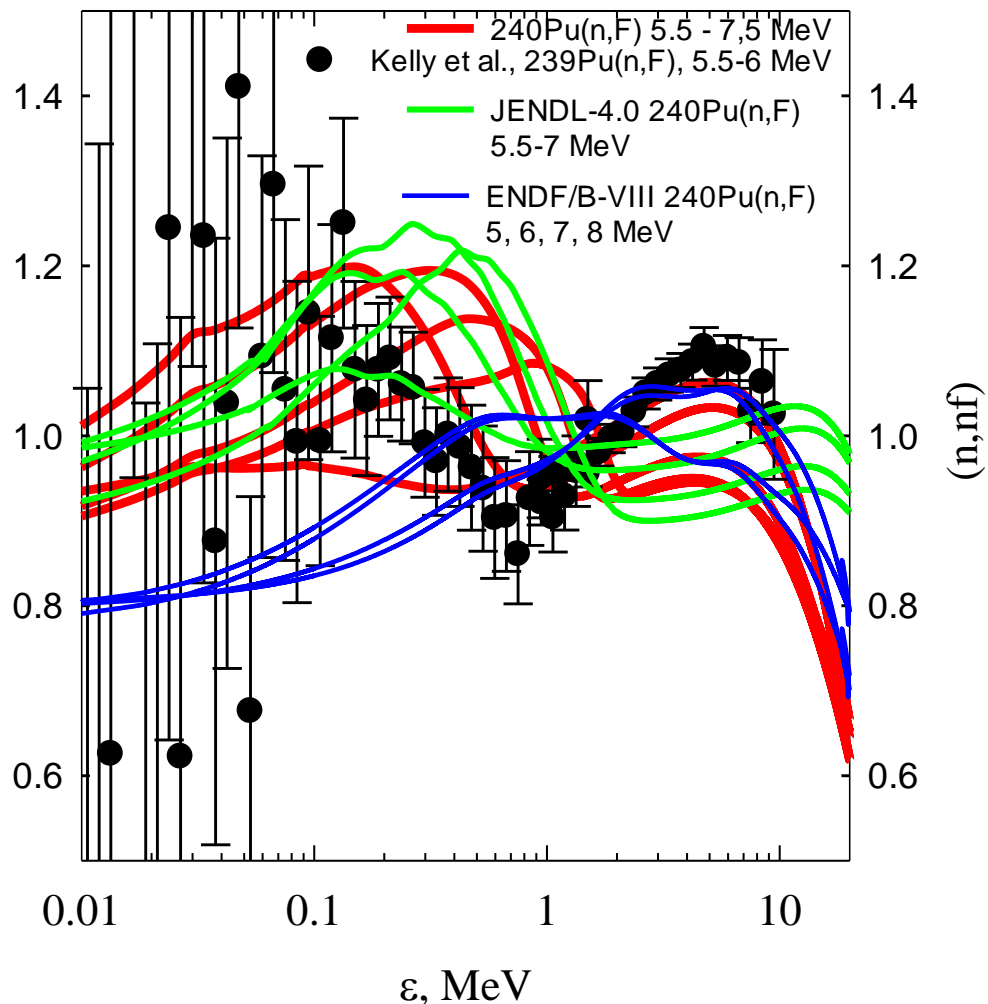
# Exclusive pre-fission neutron contribution to the neutron emission spectra

$$\frac{\sigma_{n,2nf}(E_n, \theta)}{4\pi} \frac{d\sigma_{n,2nf}^{1,2}(\varepsilon, E_n, \theta)}{d\varepsilon}$$

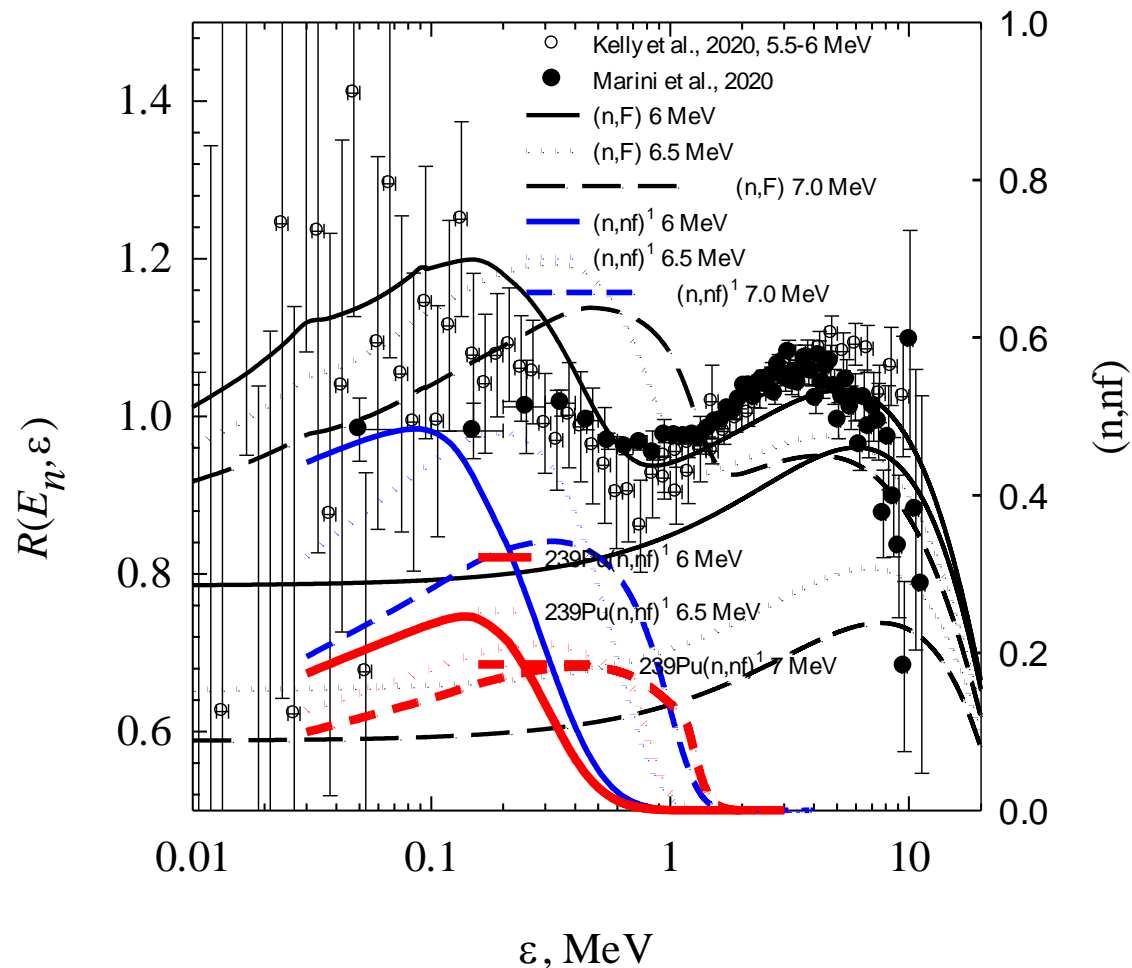
# $\langle E_{n,xf} \rangle$ , $\langle E \rangle$ of PFNS



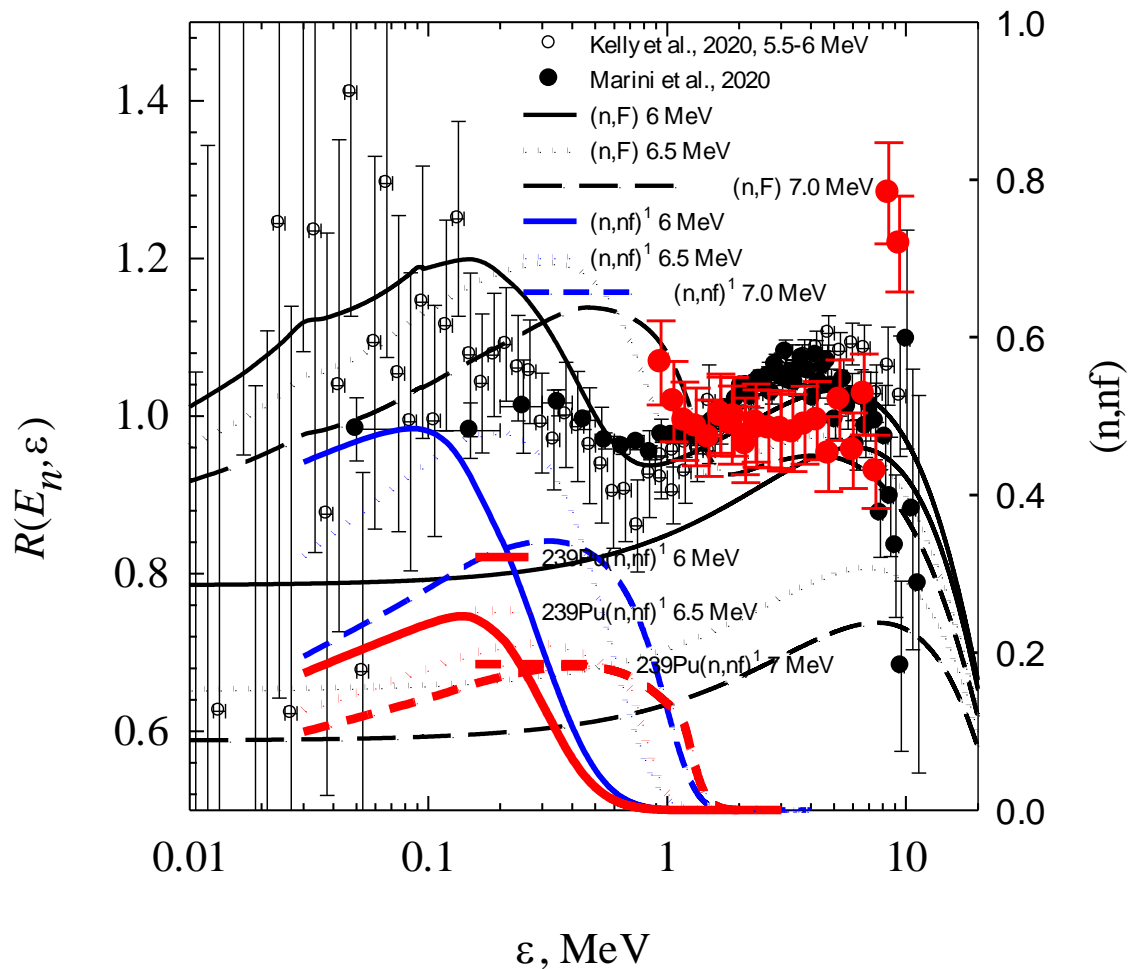
# $^{240}\text{Pu}(n,F)$ 5.5-7 MeV    $^{239}\text{Pu}(n,F)$



# $^{239}\text{Pu}(n,xfn)$ 6-7 MeV $^{240}\text{Pu}(n,xfn)$



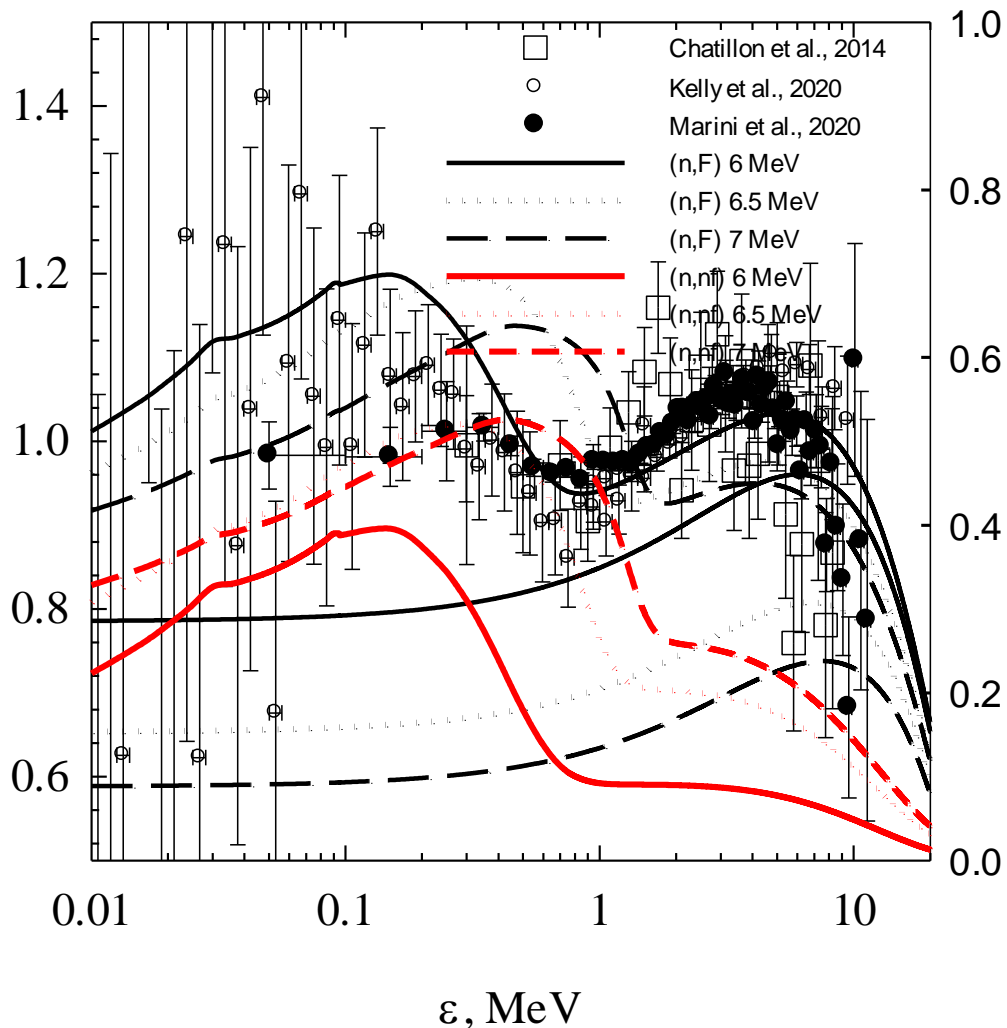
# $^{239}\text{Pu}(n,F)$ 5.5-7 MeV    $^{240}\text{Pu}(n,F)$



# $^{240}\text{Pu}(n,F)$ 6-7 MeV    $^{240}\text{Pu}(n,nf)$

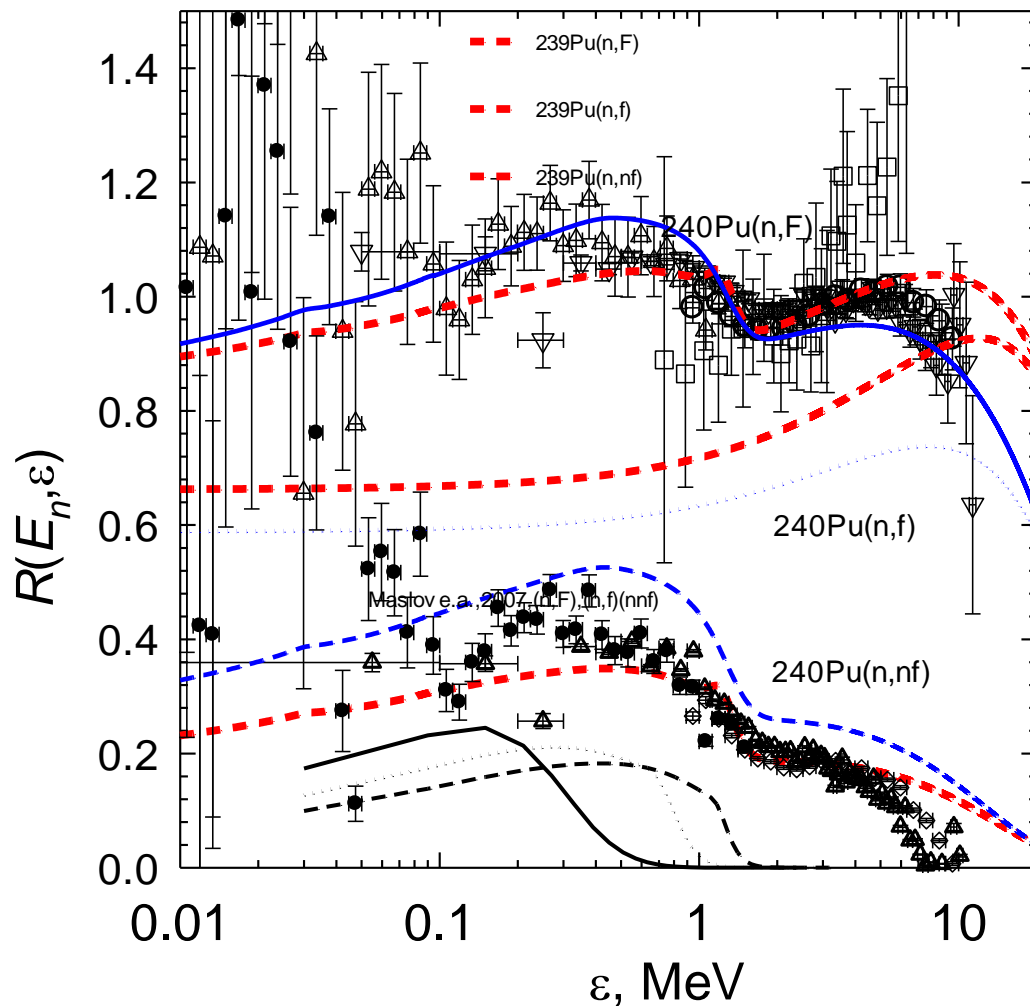
$R(E_n, \varepsilon)$

(n,nf)

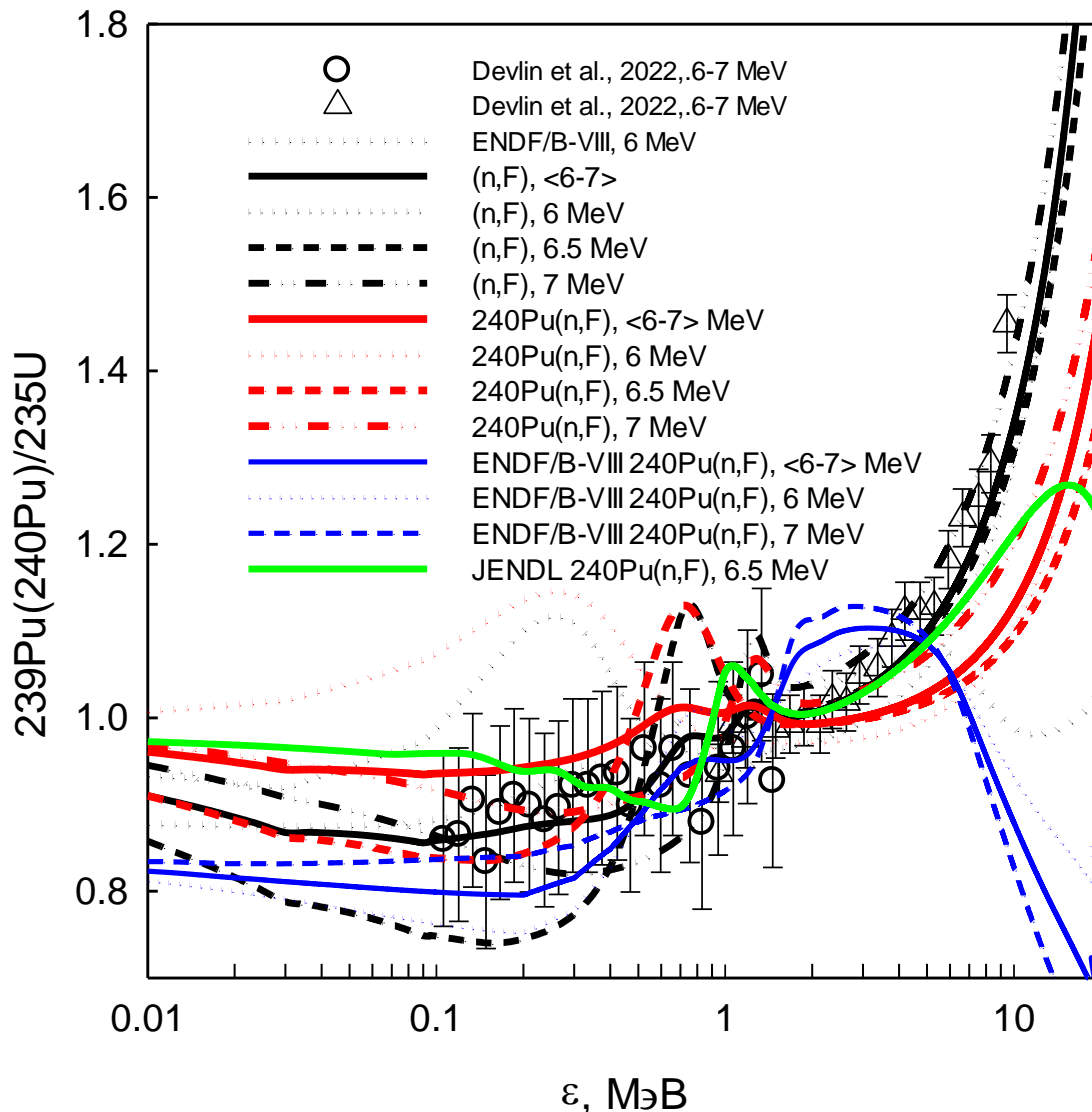




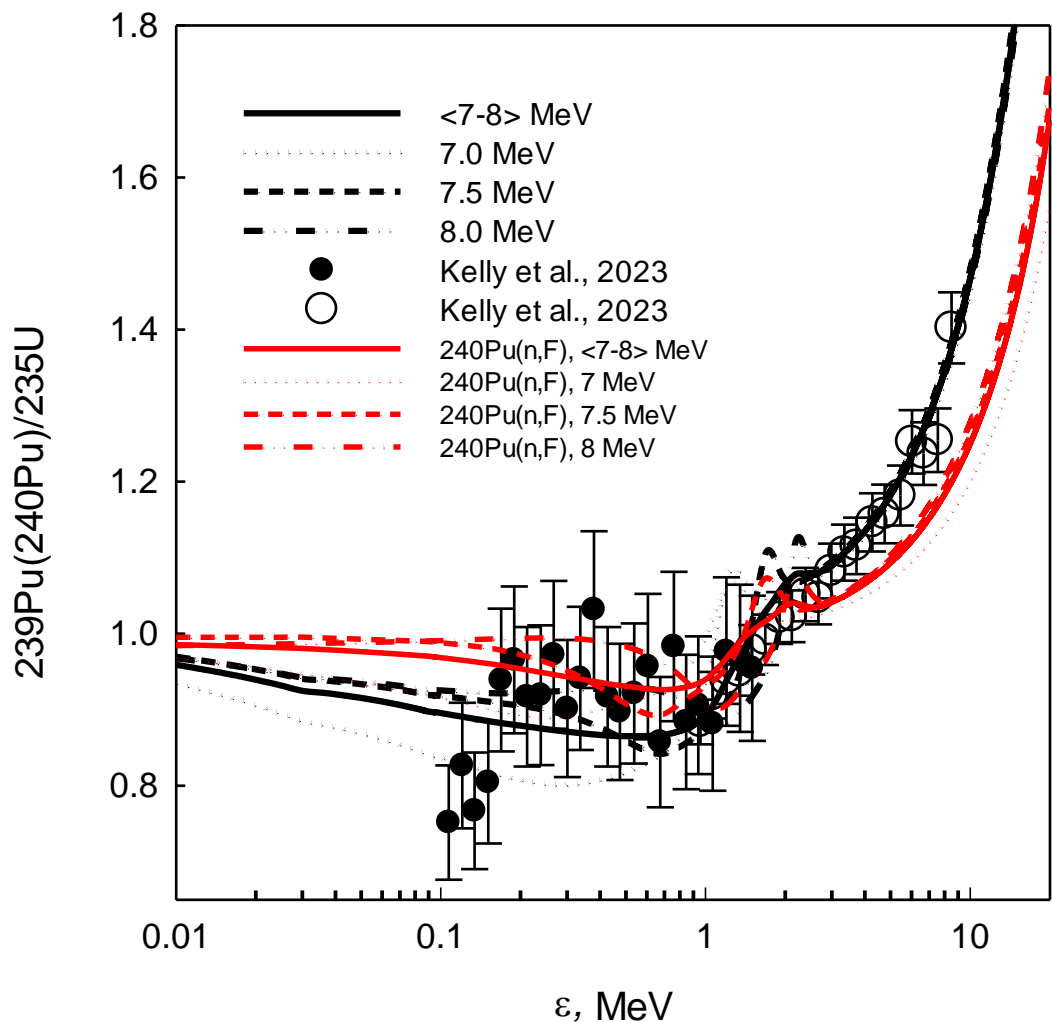
# $^{239}\text{Pu}(n,F)$ $E_n=7$ MeV $^{240}\text{Pu}(n,F)$



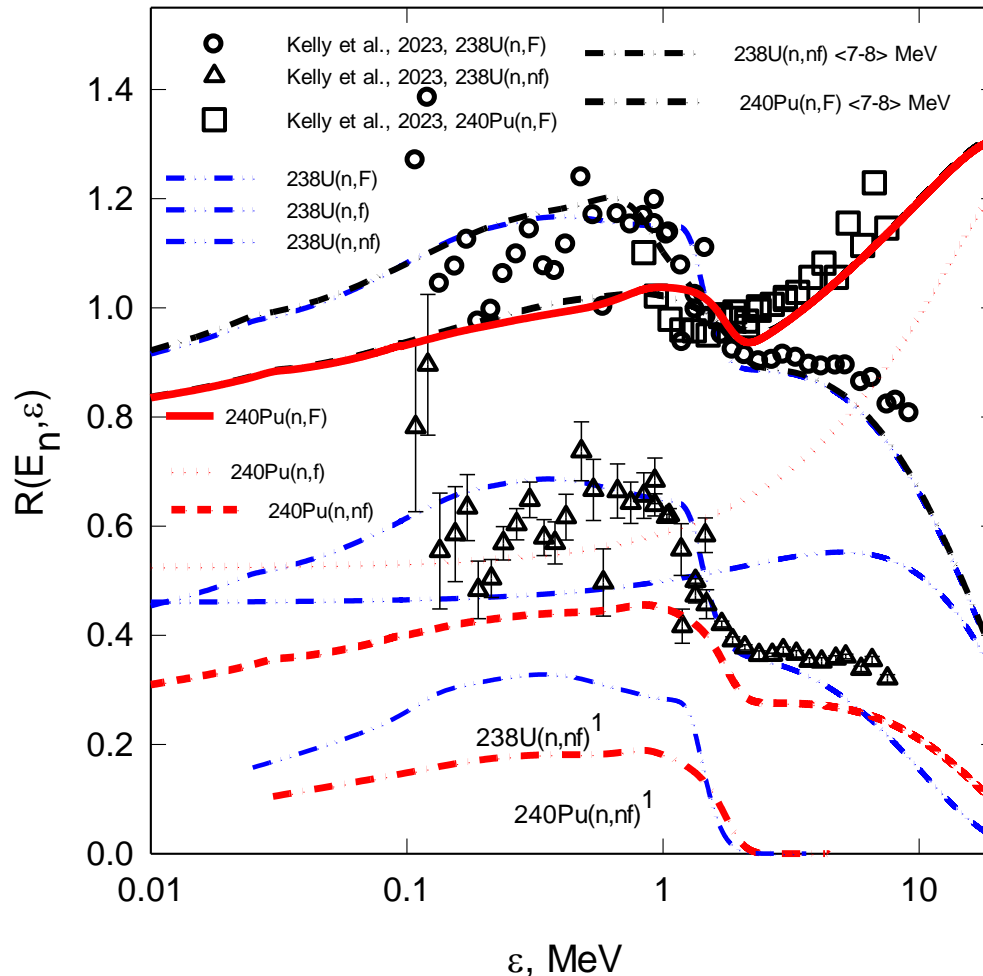
# $^{239}\text{Pu}/^{235}\text{U}$ 6-7 MeV $^{240}\text{Pu}/^{235}\text{U}$



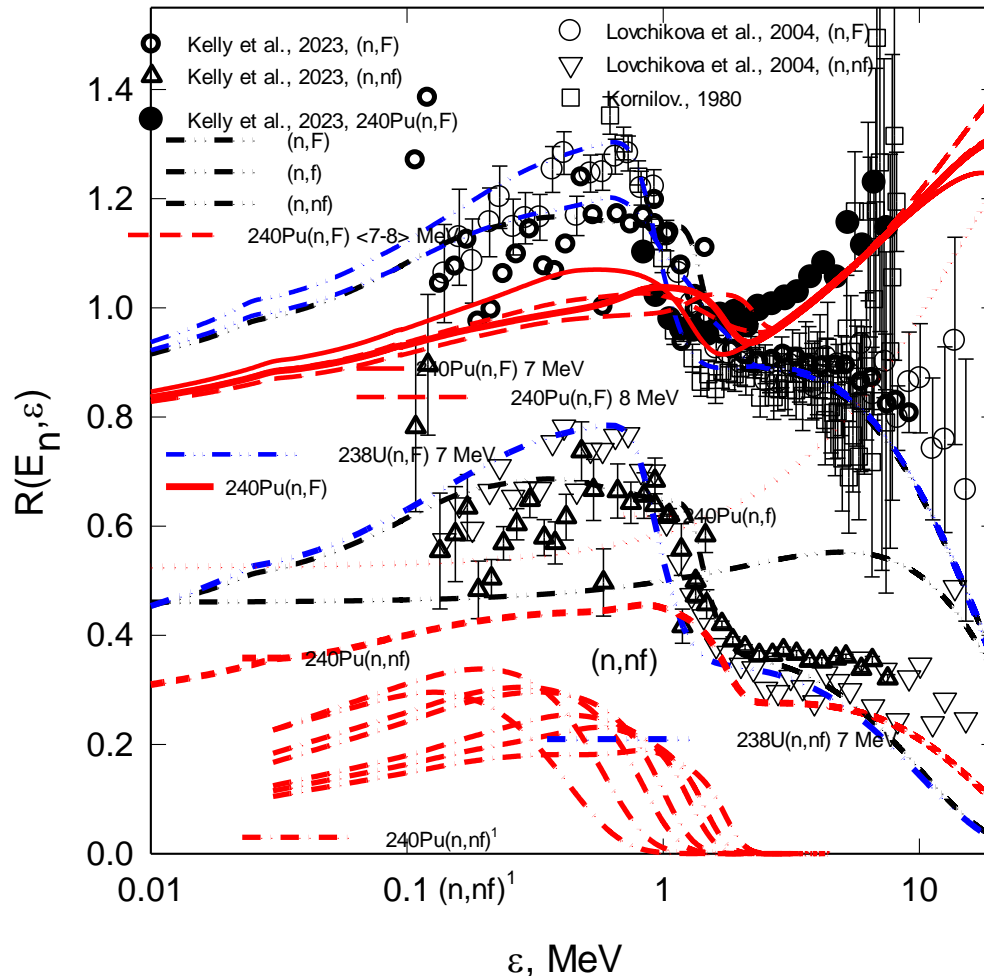
# $^{239}\text{Pu}/^{235}\text{U}$ 7-8 MeV $^{240}\text{Pu}/^{235}\text{U}$



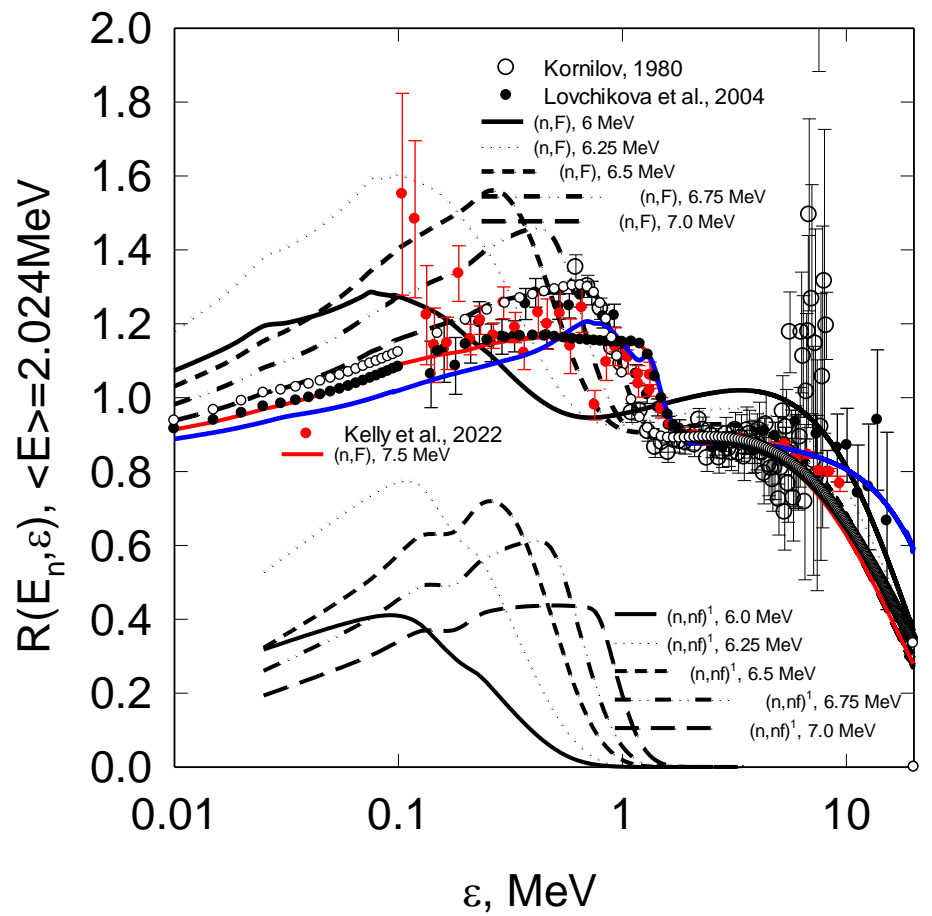
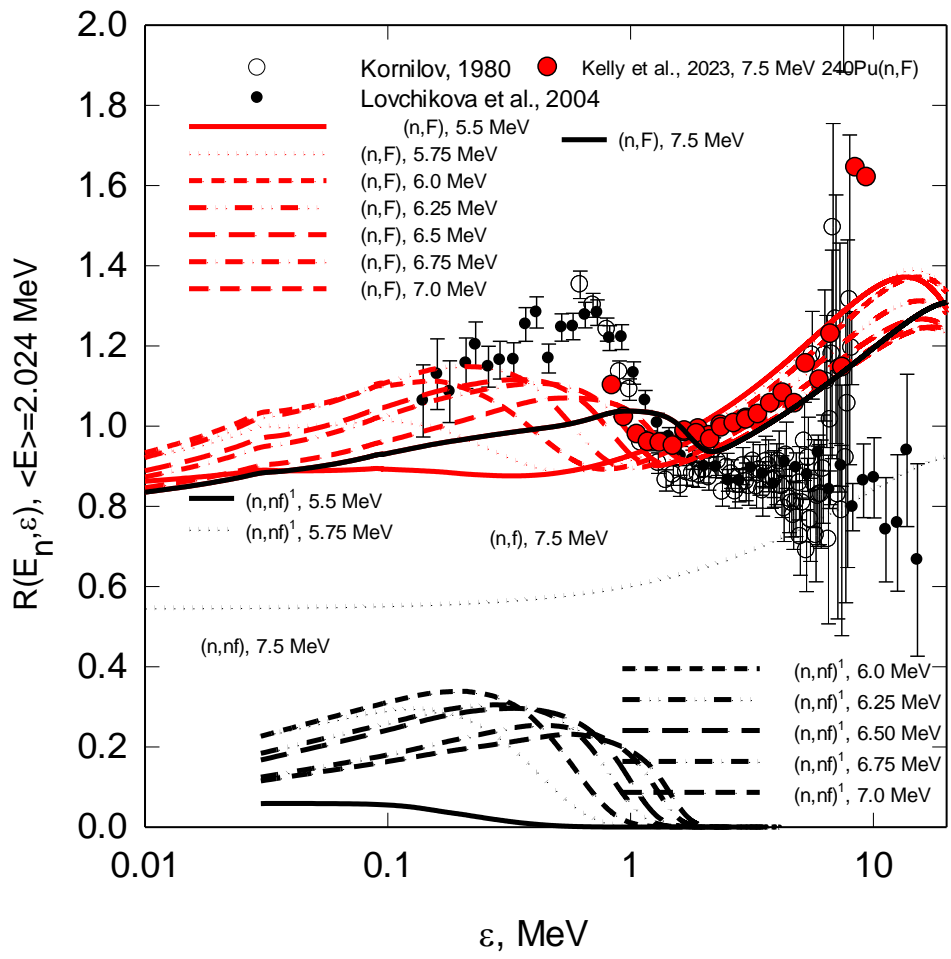
# $^{238}\text{U}(n,F)$ 7-8 MeV    $^{240}\text{Pu}(n,F)$



# $^{238}\text{U}(n,F)$ 7-8 MeV    $^{240}\text{Pu}(n,F)$

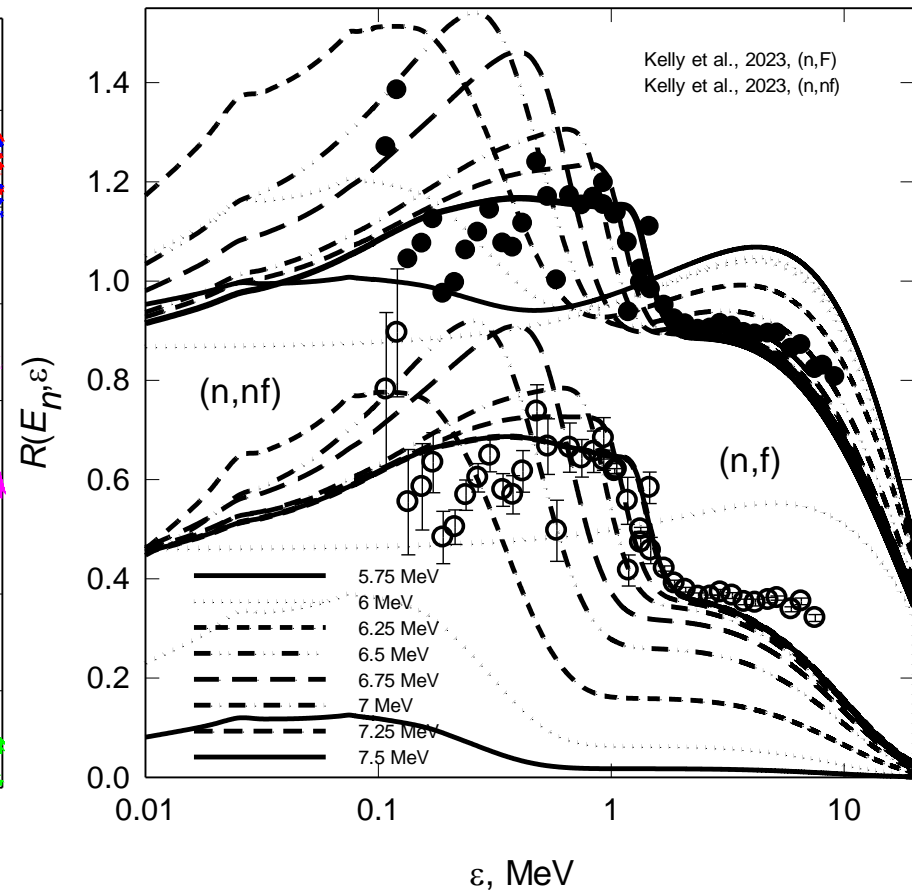
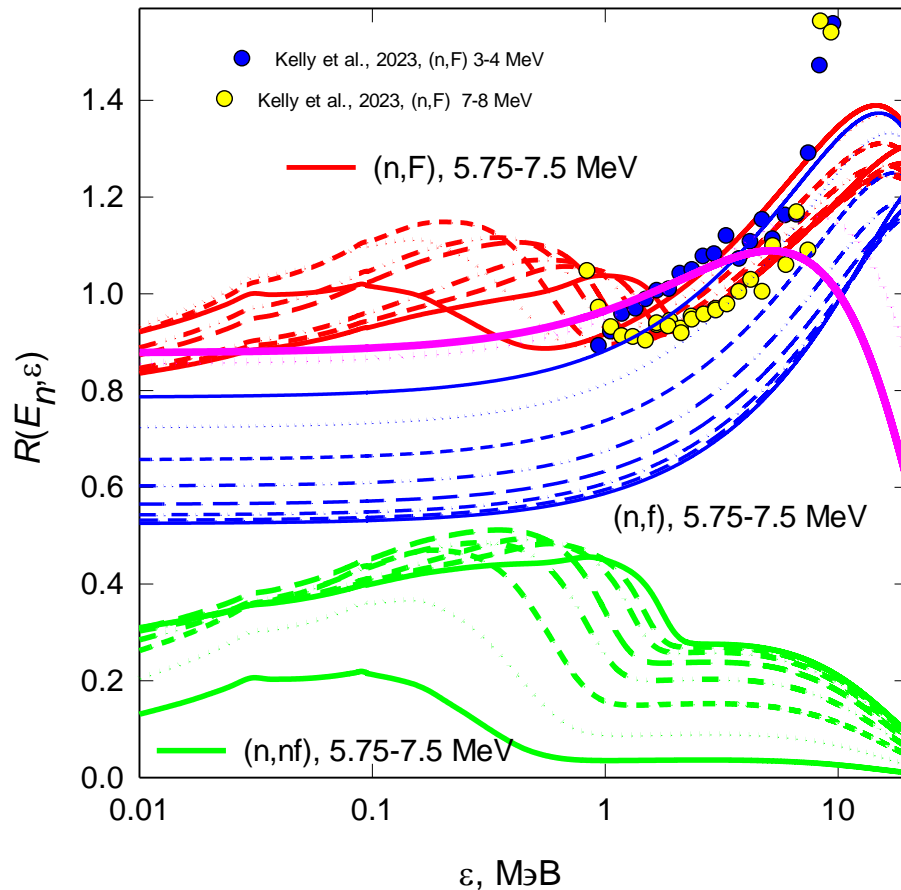


# $^{240}\text{Pu}(n,F)$      6-7 MeV      $^{238}\text{U}(n,F)$

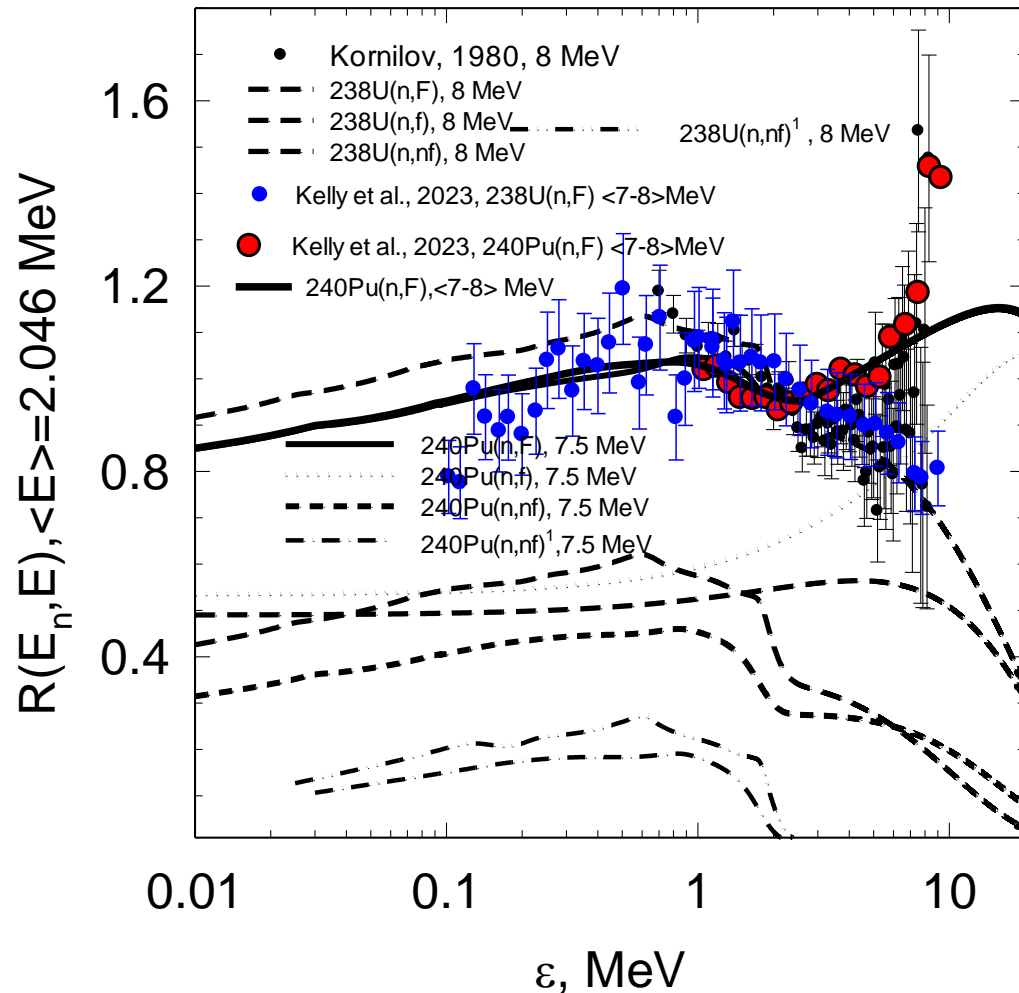


$^{240}\text{Pu}(n,F)$   $E_n=5.75-7.5$  MeV

$^{238}\text{U}(n,F)$

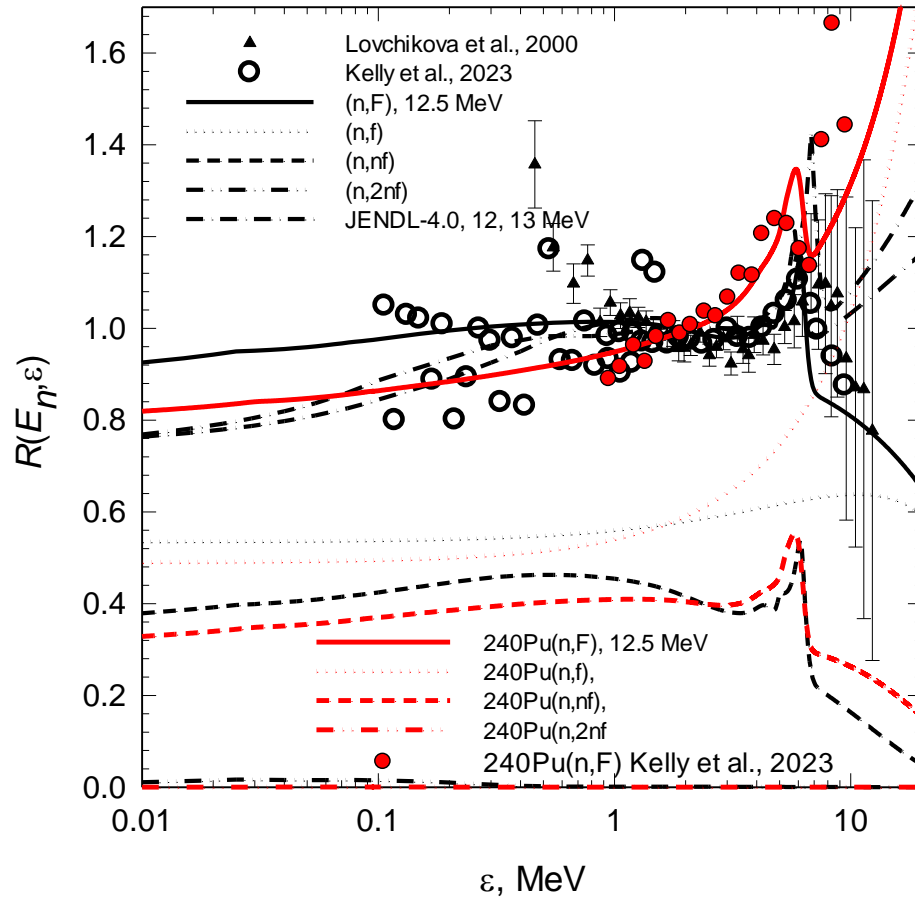


# $^{240}\text{Pu}(n,F)$ 7-8 MeV    $^{238}\text{U}(n,F)$

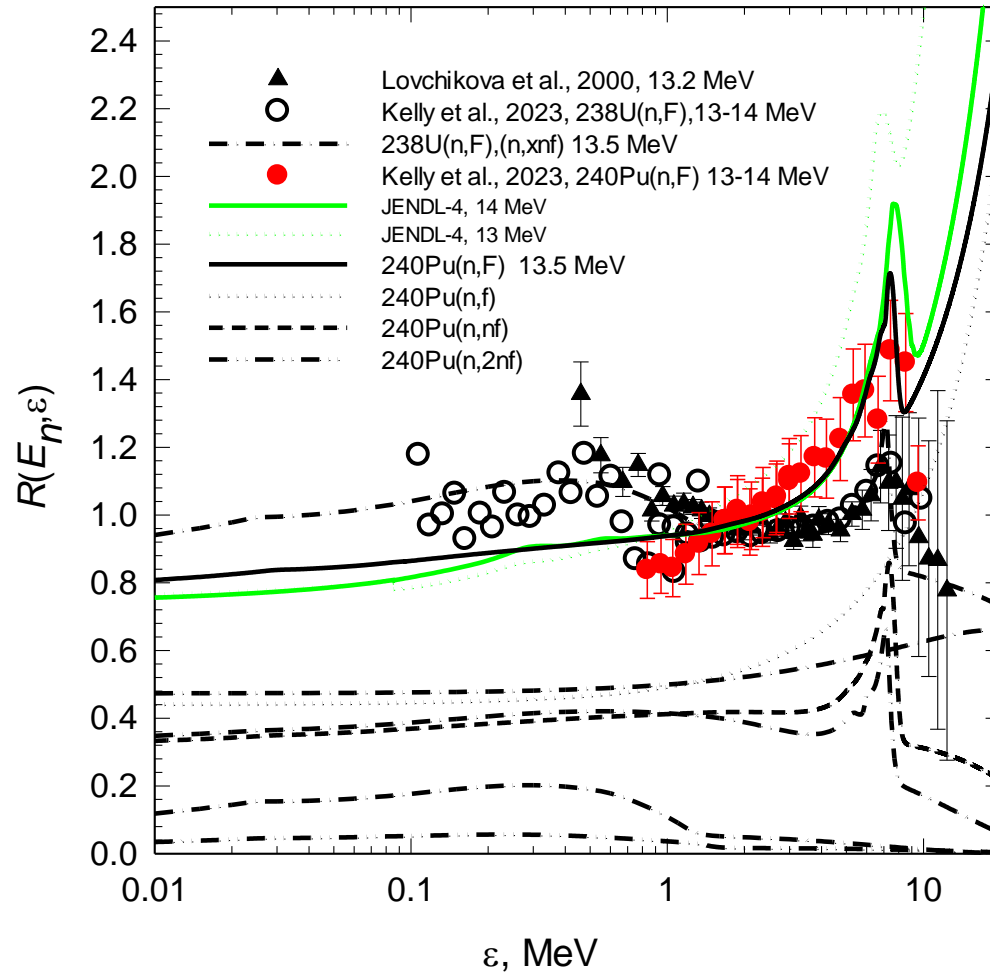




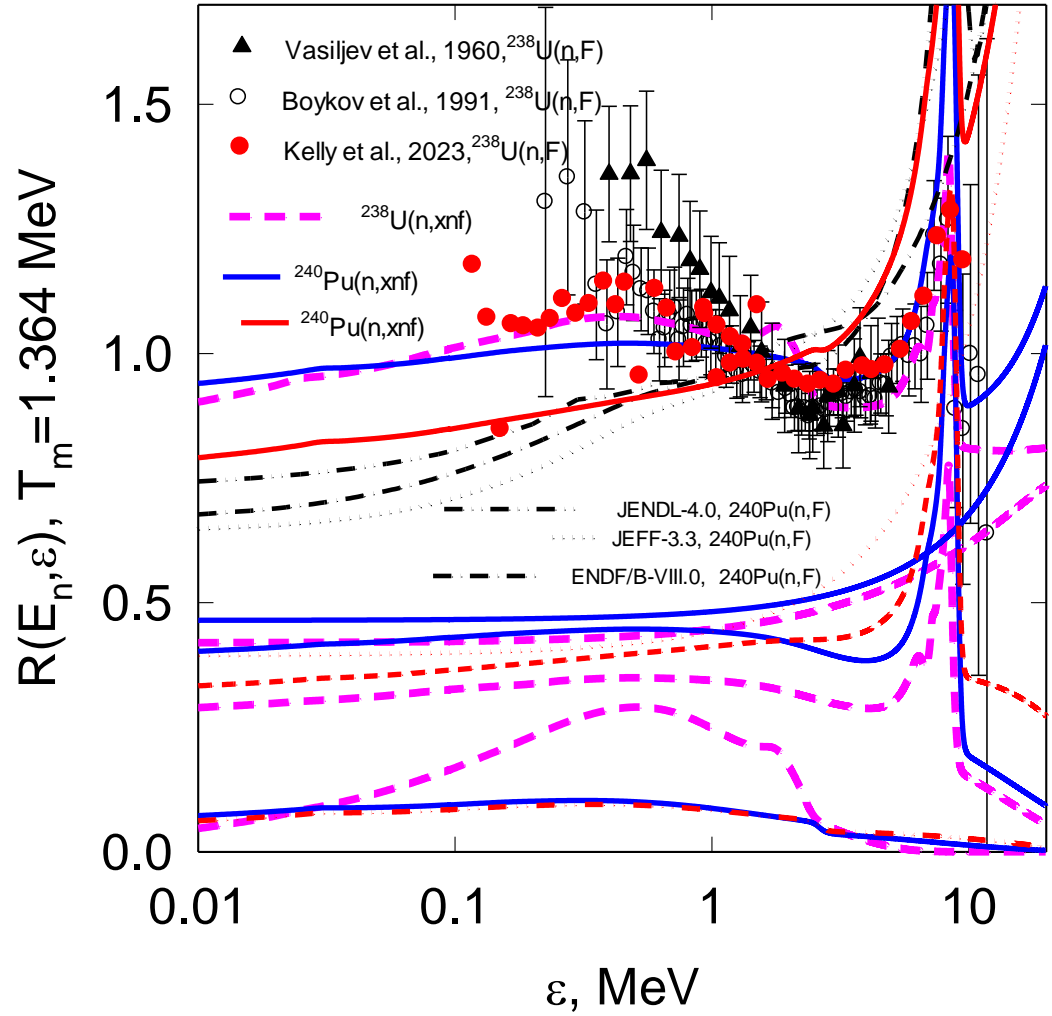
# $^{240}\text{Pu}(n,F)$ 12-13 MeV $^{238}\text{U}(n,F)$



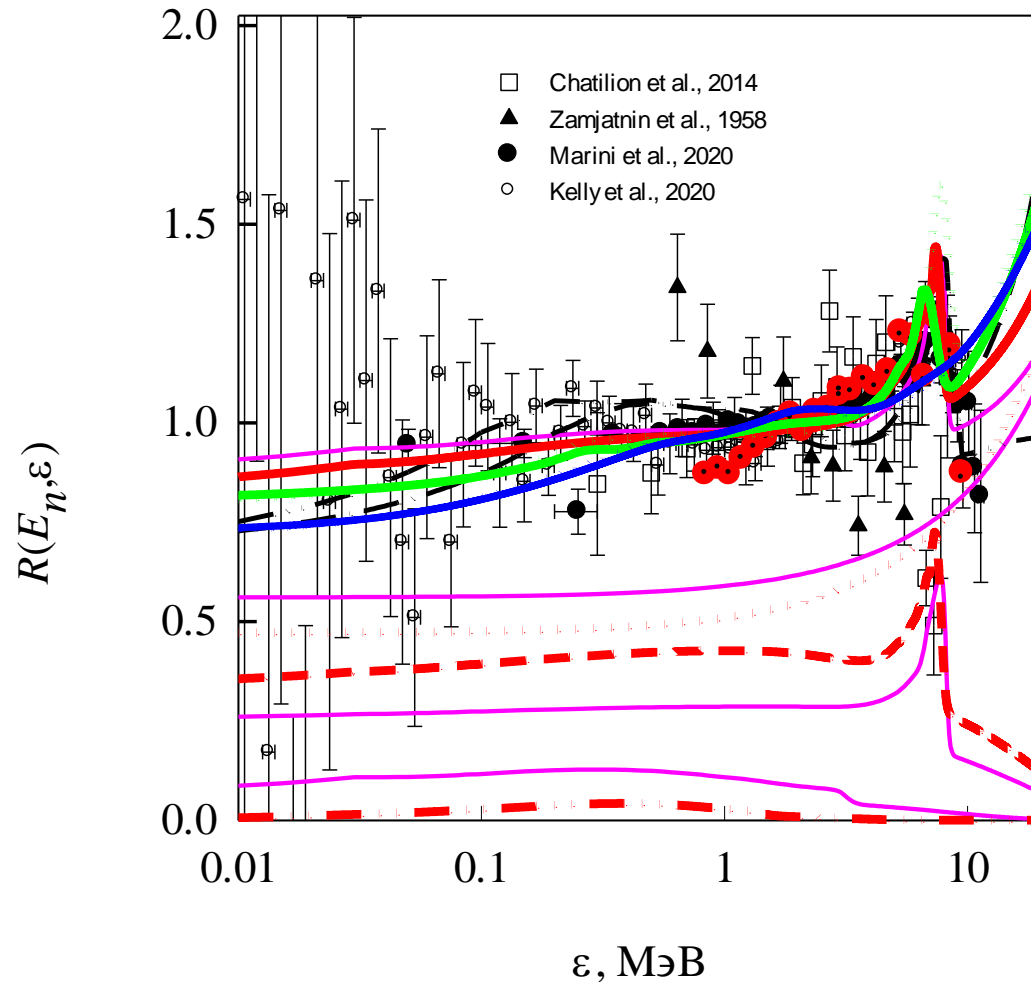
# $^{240}\text{Pu}(n,F)$ 13-14 MeV $^{238}\text{U}(n,F)$



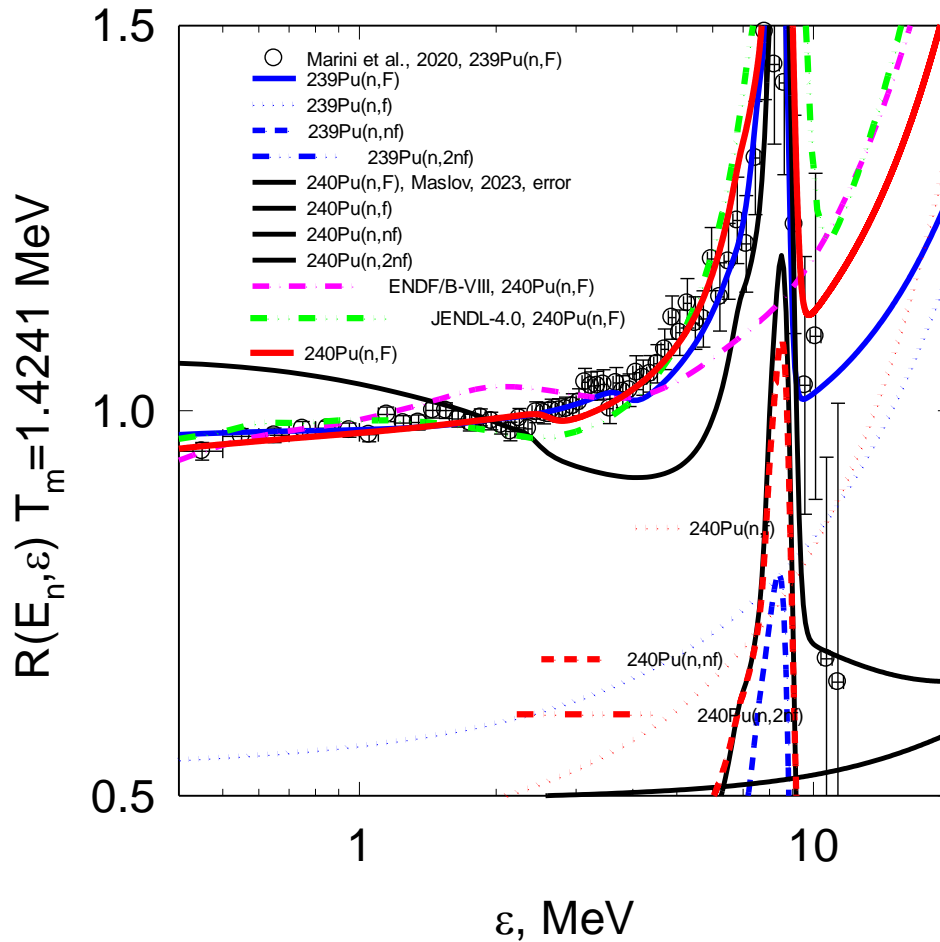
# $^{240}\text{Pu}(n,F)$ 14-15 MeV $^{238}\text{U}(n,F)$



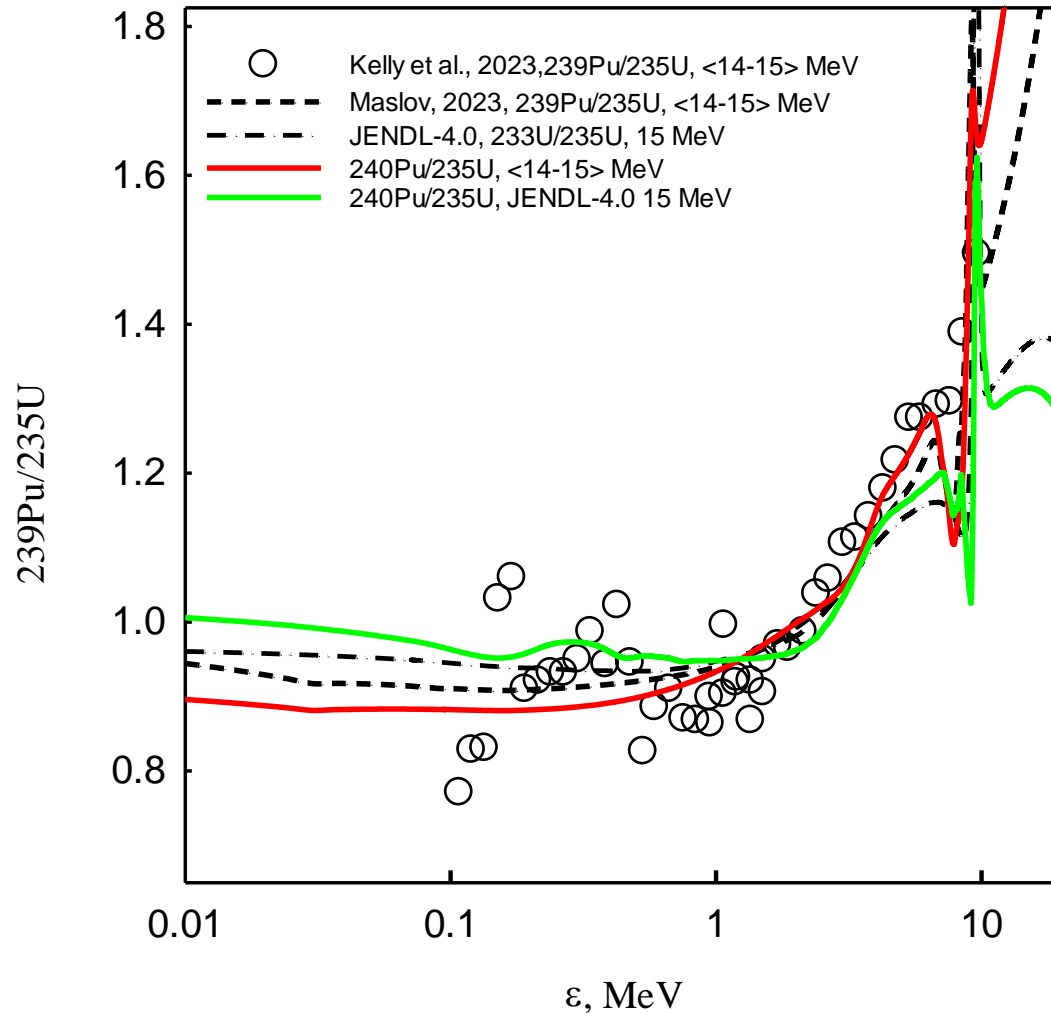
# $^{240}\text{Pu}(n,F)$ 13-14 MeV $^{239}\text{Pu}(n,F)$

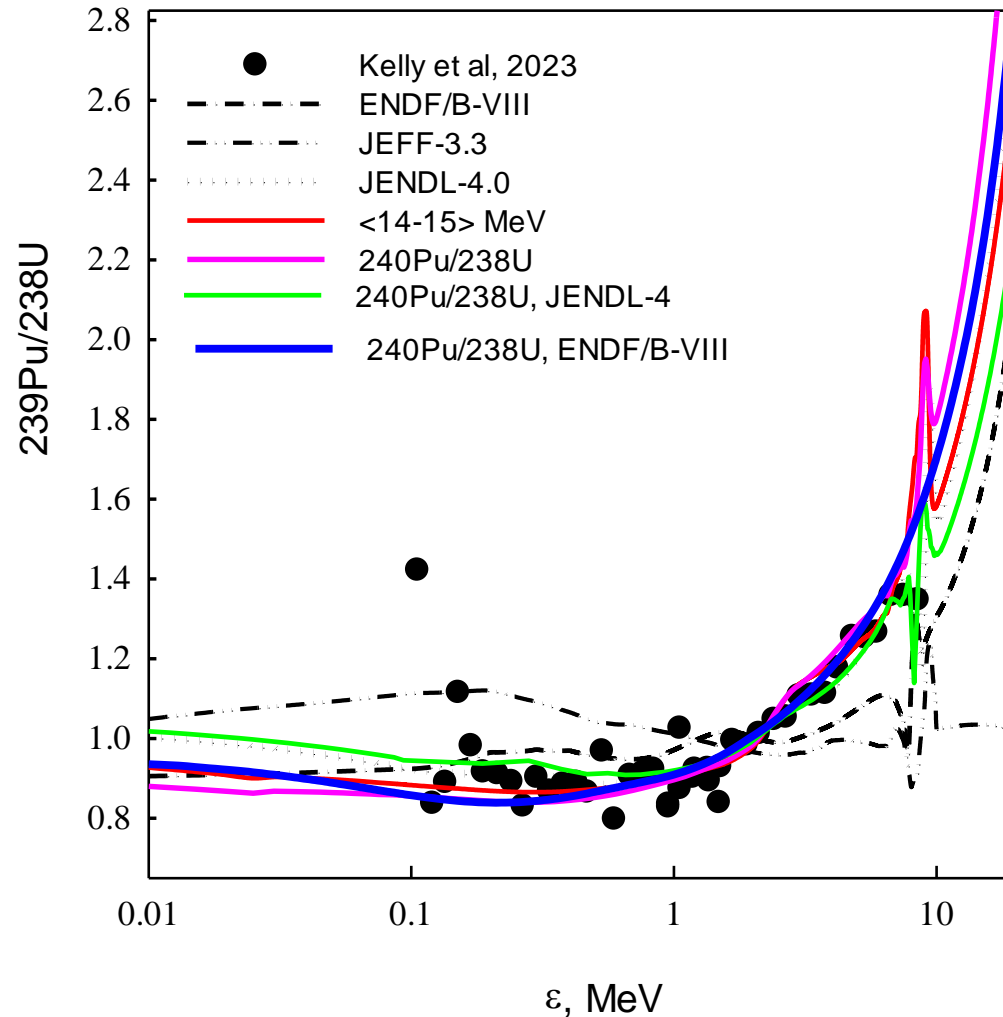


# $^{240}\text{Pu}(n,F)$ 14-15 MeV $^{239}\text{Pu}(n,F)$

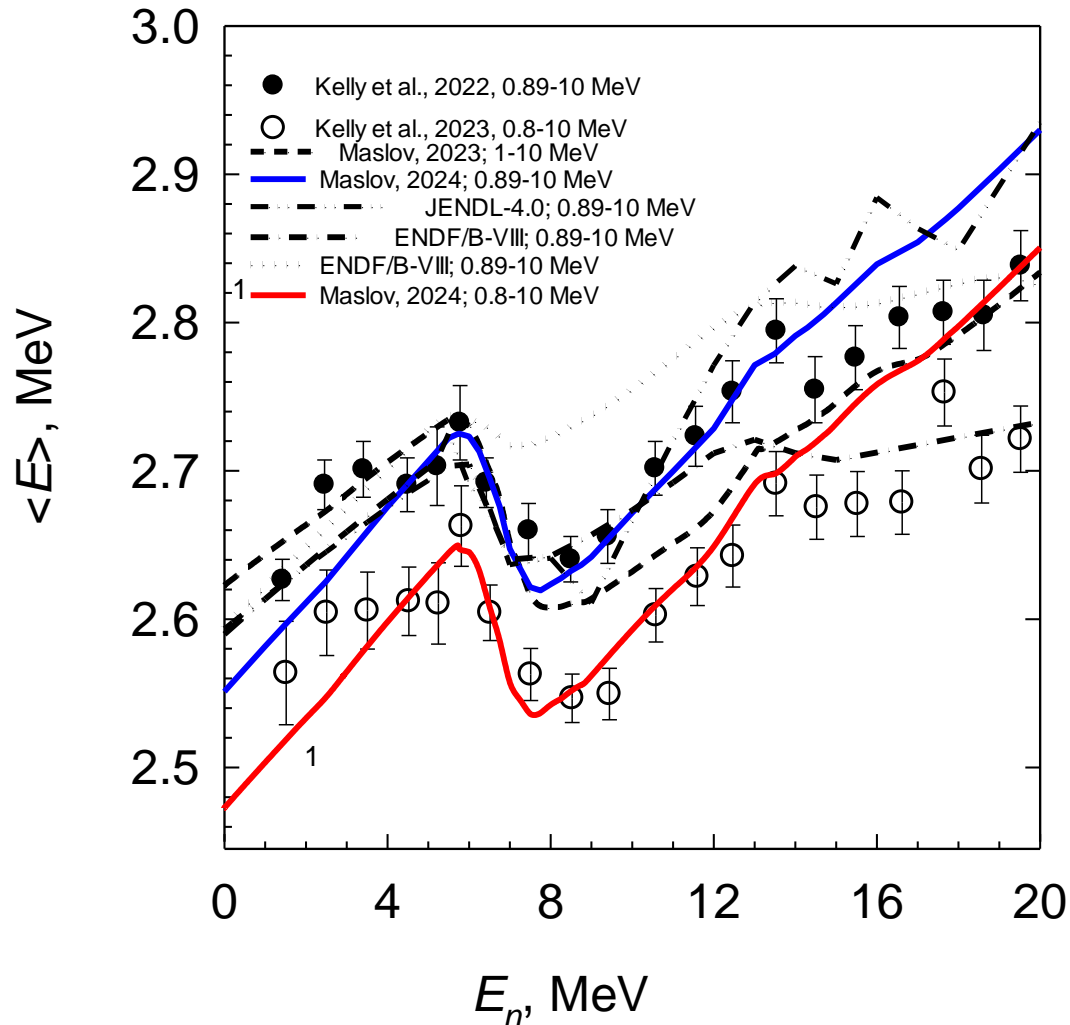


$^{239}\text{Pu}/^{235}\text{U}$      $E_n=14-15$  MeV     $^{240}\text{Pu}/^{235}\text{U}$



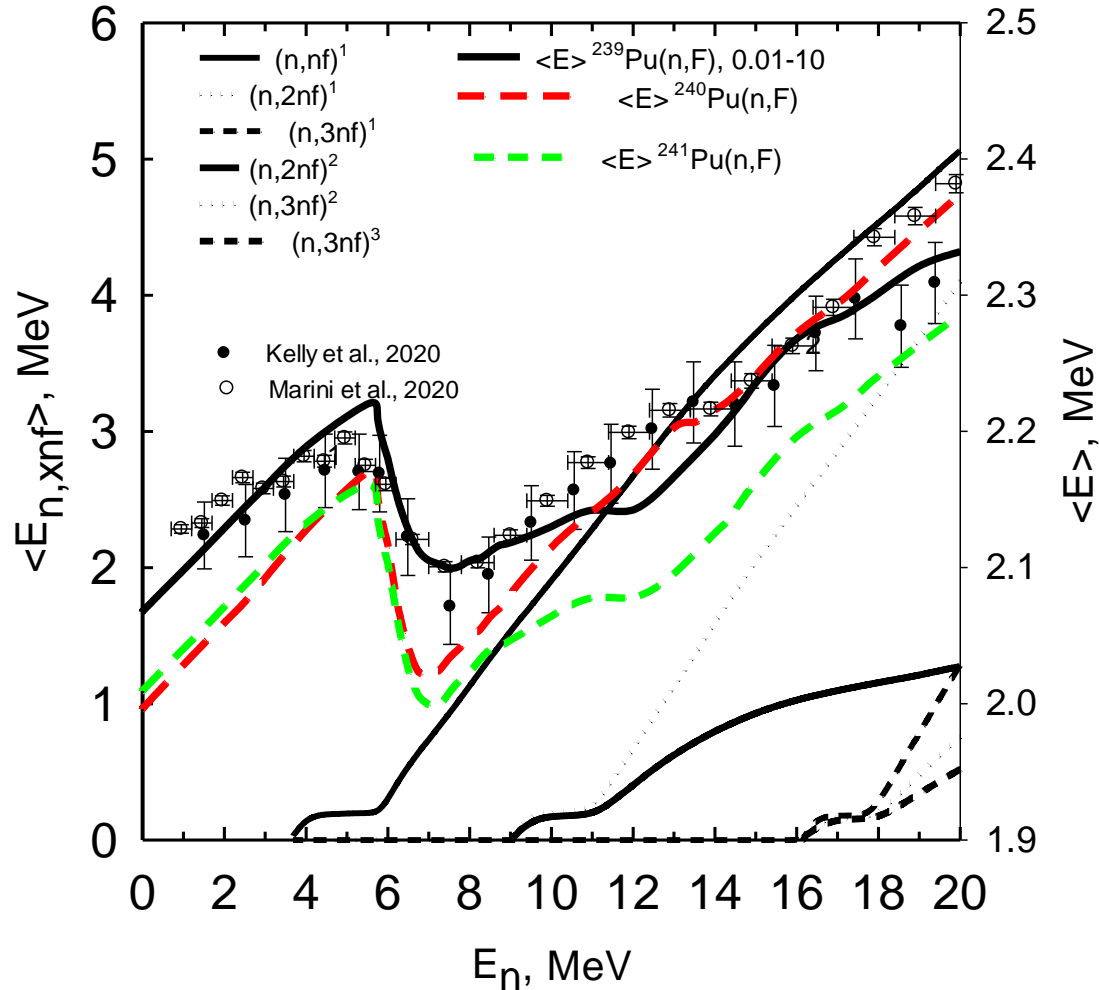
$^{239}\text{Pu}/^{238}\text{U}$  $E_n=14-15\text{ MeV}$  $^{240}\text{Pu}/^{238}\text{U}$ 

# $^{240}\text{Pu}(n,F) \langle E \rangle, \varepsilon \sim 0.8-10 \text{ MeV}$





$^{239}\text{Pu}(n,F)$ ,  $^{240}\text{Pu}(n,F)$ ,  $^{241}\text{Pu}(n,F)$   $\langle E \rangle$ ,  $\varepsilon \sim 0.01-10$  MeV



$$U_x = E_n + B_n - \sum_{x, 1 \leq k \leq x} (\langle E_{nxf}^k(\theta) \rangle + B_{nx})$$

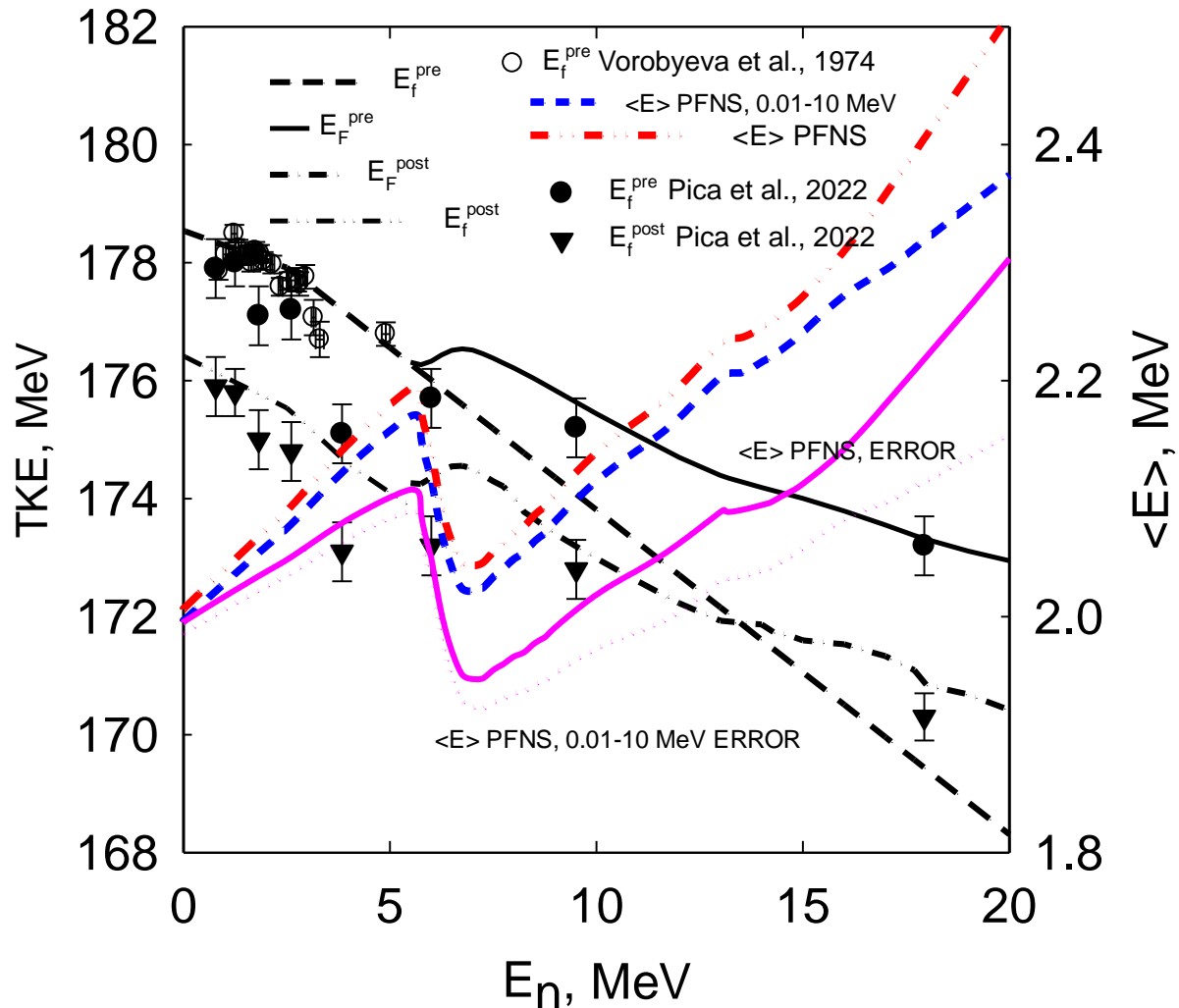
$$E_{nx} = E_r - E_{fx}^{pre} + E_n + B_n - \sum_{x, 1 \leq k \leq x} (\langle E_{nxf}^k(\theta) \rangle + B_{nx})$$

$$E_F^{pre}(E_n) = \sum_{x=1}^X E_{fx}^{pre}(E_n) \sigma_{n,xf} / \sigma_{n,F}$$

$$E_F^{post} \approx E_F^{pre} \left( 1 - \nu_{post} / (A + 1 - \nu_{pre}) \right)$$

$$\nu_p(E_n) = \nu_{post} + \nu_{pre} = \sum_{x=1}^X \nu_{px}(E_{nx}) + \sum_{x=1}^X (x-1) \cdot \beta_x(E_n)$$

# Total Kinetic Energy



# Conclusions

**Angular** dependence of first  $(n, nX)^1$  emission  
 $\langle E \rangle$  of  $(n, nf)^1$  neutrons depends on emission angle  $\theta$

**Fission** cross section, prompt neutron number and  
total kinetic energy depend on  $\theta$  as well

**Exclusive** neutron spectra  $(n, xnf)^{1, \dots, x}$  at  $\theta \sim 90^\circ$  are  
consistent with  $^{240}\text{Pu}(n, F)$ ,  $^{239}\text{Pu}(n, F)$  and  $^{241}\text{Pu}(n, F)$   
css within  $E_n \sim 0.01 - 20$  MeV

**Exclusive** neutron spectra of  $(n, xnf)^{1, \dots, x}$ ,  $(n, n\gamma)$  and  
 $(n, xn)^{1, \dots, x}$  – from Hauser-Feshbach formalism

# Conclusions

**Approximation** obtained for  $\omega(\theta)$  fits the measured  $^{239}\text{Pu}+n$  NES at  $E_n=14$  MeV.

**The correlation** of angular dependence of  $(n,xnf)^1$  neutron emission with emissive fission  $(n,xnf)$  and angular anisotropy of  $^{235}\text{U}+n$  &  $^{239}\text{Pu}+n$  NES is established.

**On that background**

**The PFNS shapes and energies  $\langle E \rangle$  and TKE** for  $^{240}\text{Pu}(n,F)$  &  $^{240}\text{Pu}(n,xnf)$  provided

# Conclusions

In  $^{239}\text{Pu}(n,xf)^{1,\dots,x}$  and  $^{235}\text{U}(n,xf)^{1,\dots,x}$  PFNS

demonstrate different responses to forward and backward  $(n,xf)^1$  neutrons emission with respect to the incident neutron momentum

In  $^{240}\text{Pu}(n,xf)^{1,\dots,x}$  and  $^{240}\text{Pu}(n,F)$  PFNS stronger response to forward and backward  $(n,xf)^1$  might be predicted.