SOME PROBLEMS IN DETERMINING LEVEL DENSITY AND RADIATIVE STRENGTH FUNCTIONS IN LIGHT AND NEAR-MAGIC NUCLEI

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Abstract

The values of some functional dependencies of level density and radiative strength functions that reproduce the experimental intensities of the two-step gamma-cascades to the ground and first excited states of ^{28}Al have been determined. It was shown that the assumption about independence of the dipole cascade transitions radiative strength functions on energy of decaying level leads to rather essential error in observation of both level density and radiative strength functions.

Introduction

High level density in most nuclei does not allow determination of their parameters (spin, parity and so on) and, consequently, these data cannot be involved in calculation. In these cases, there are usually used theoretical notions on the level density ρ and radiative strength functions k:

$$k = f/A^{2/3} = \Gamma_{\lambda l}/(E_\gamma^3 \times A^{2/3} \times D_\lambda). \tag{1}$$

This definition of k for used gamma-transition energy E_γ and spacing D_λ between the decaying levels λ provides week dependence on nuclear mass A and allows direct comparison of k for the nuclei with sufficiently different masses. In this case it is implied that the partial width $\Gamma_{\lambda l}$ of the transitions between the levels λ and l is averaged over some set $m = \rho \times \Delta E$ of its random values from a given energy interval ΔE. But such approach is considered as inapplicable for light and near-magic nuclei with low level density and clearly expressed influence of nuclear structure on the gamma-transition spectrum. It seems useful to determine experimentally the energy region where the notions of level density and radiative strength functions can be used. Only this argument stipulated our effort to determined the mentioned parameters for the ^{28}Al compound nucleus [1]. It should be noted, that up to now the reliable data on the level density and radiative strength functions can be derived mainly from the experimental intensities of the two-step gamma transitions [2]:

$$I_{\gamma\gamma} = \sum_{\lambda,f}\sum_{l} \frac{\Gamma_{\lambda l}}{\Gamma_\lambda}\frac{\Gamma_{lf}}{\Gamma_l} = \sum_{\lambda,f} \frac{\Gamma_{\lambda l}}{<\Gamma_{\lambda l}> m_{\lambda l}} n_{\lambda l} \frac{\Gamma_{lf}}{<\Gamma_{lf}> m_{lf}}, \tag{2}$$

proceeding between the compound state (neutron resonance) and a group of low-lying levels of the nucleus under study that were determined according to algorithm [3] for all the possible energy intervals ΔE of their primary gamma-transitions E_1. Analogous data derived from the spectra of evaporated nucleons or gamma-ray spectra following decay of levels E_{ex} excited in nuclear reactions have too large systematic errors. They are conditioned by the use of theoretical models for calculating penetration coefficients of nuclear surface for evaporated nucleons or by very large (at least several hundreds) transfer coefficients of experimental systematic errors into the determined ρ and k values or by principal incorrectness [4,5] of the data processing procedure. In the first case, one cannot expect required accuracy in determination of level density and its energy dependence below, at least, neutron binding energy. In the second case it is necessary to get realistic estimations of systematic uncertainties and to reduce them to minimum possible magnitude using all the possibilities of experiment, mathematic and mathematical statistics.

1 Specific in determination of ρ and k from experimental spectra

Equation (2) contains more parameters than the number of selected intervals but the range of possible variations for these parameters is always limited and rather narrow if:

(a) experimental spectra are decomposed into two components containing solely primary or solely secondary gamma-transitions according to method [3] based on some grounds;

(b) the ratio $\Gamma_{\lambda l}$ and Γ_{lf} of partial widths for the primary and secondary transitions of equal energy and multipolarity is set.

Minimization of discrepancy between the experimental and determined within iterative process [2] ρ and k requires also to use the density of neutron resonances, number of low-lying levels and total radiative width of s-resonances from [6]. Process [2] quickly enough converges to some average value of the cascade gamma-decay parameters (in limits of principally irremovable scatter) if experimental distribution (2) does not have considerable local bumps. Acceptable values of the parameter χ^2 for ^{28}Al are achieved sometimes only after 50000-100000 iterations. In any case, no correlation between varying input parameters ρ and k and deviation of their best values from the average was found.

As it is seen from Fig. 1, the mentioned above condition is not fulfilled in case of ^{28}Al and there are observed serious structure effects. Moreover, these effects weakly enough manifest themselves in instrumental spectrum and very strongly in dependence of cascade intensity on energy of the primary transition. This testifies to necessity to use method [3] for extraction of ρ and k from the experiment and shows uncertainty of the results of analysis like that described in [7].

Usually model reproduction of the cascade gamma-decay process of high-lying levels ($E_{ex} \approx B_n$) is performed within hypothesis of independence of radiative strength functions on the energy of decaying level. The values of ρ and $f = kA^{2/3}$ [8] were determined from the data on the reactions ^{28}Si(^{3}He,^{3}He,γ)^{28}Si and ^{28}Si(^{3}He,$\alpha\gamma$)^{27}Si using this assumption. This assumption cannot be used for the excitation energy, $E_{ex} < 0.5B_n$ even for nuclei with high level density what follows [9] from the experimental and calculated

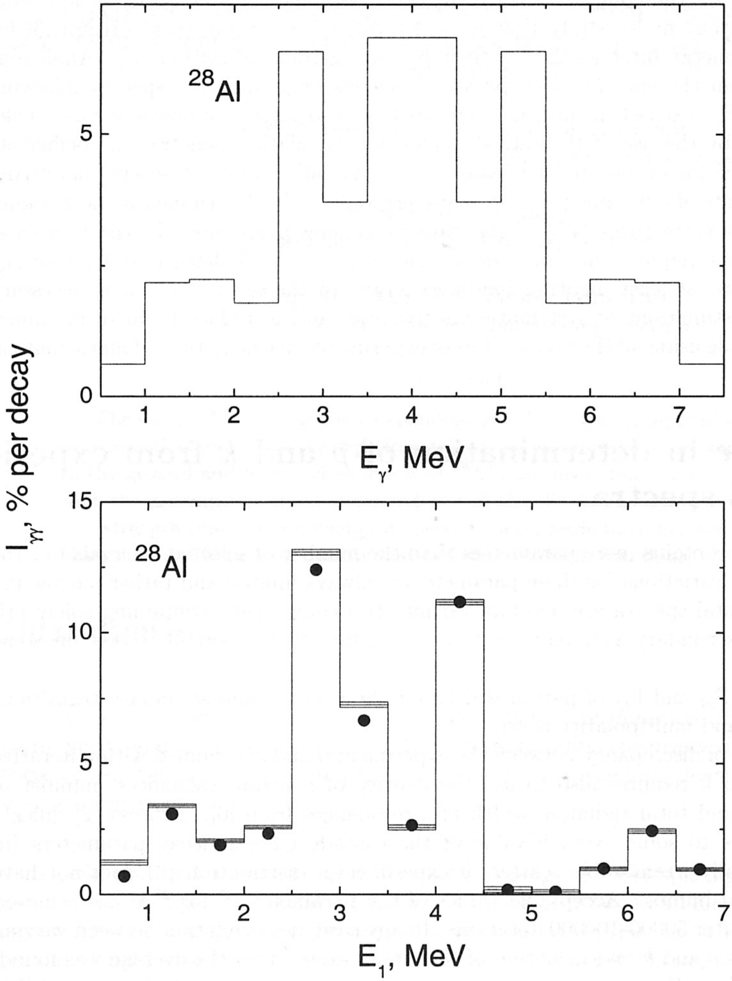

Fig. 1. *Top: intensities of two-step cascades in the function of energy of primary, o second of gamma-transitions. Bottom: histogram is the intensity of two-step cascades int ^{28}Al, summed up [1] on interval of 0.5 MeV of their primary gamma-transition energy Points - the same for the cascades, the resolved in the form of pairs peaks.*

population of these levels. The direct, although not exhaustive notion about the function $k = F(E_\gamma, E_{ex})$ can be obtained from the experimental data on the two-step gamma-cascades only if there is observed not less than 95-99% of their intensity.

However, at present such statistics of information is unattainable. Therefore, it is of practical interest even the data on the sign of deviation of ρ with respect to the experimental value when the standard approach $k = F(E_\gamma, E_{ex})/F(E_\gamma, B_n)$ =const is used in analysis [2].

1.1 Spectrum of possible functions ρ, k and their most probable values

Fig. 1 demonstrates intensity spectrum obtained from experimental spectra by redistribution of cascade intensities resolved as the pair of peaks in so that their intensity is summed in corresponding energy intervals of the primary transitions (and subtracted from energy interval of the secondary transitions). Low level density in ^{28}Al results in absence of continuous distribution of a number low intensity cascades. Combination of the method [10] to determine quanta ordering in cascade with the data [11,12] from the evaluated decay scheme provides extremely small systematic errors in both total cascade intensity and its energy dependence. In further analysis we neglect them. Sufficiently larger errors in the observed ρ are caused by the use of the assumption $F(E_\gamma, B_n)/F(E_\gamma, E_{ex}) =$ const.

But their magnitude can be estimated only in indirect way. Fig. 2 shows the results of determination of the cascade gamma-decay parameters for three variants of analysis:

(a) the level density is fixed and set by the non-interacting Fermi-gas model;

(b) the level density and radiative strength functions are varied parameters of the process [2];

(c) the function ρ below $E_{ex} \approx 7$ MeV is set by the simplest interpolation of the cascade intermediate levels observed in [1].

Variants (a) and (c) do not provide precise reproduction of $I_{\gamma\gamma}$ in principle. This caused with anomalous high experimental values of k for the primary $E1$ transitions with the energy $E_1 \leq 3.5$ MeV and very small for the energy exceeding 4.5 MeV (multipolarity of the primary and secondary transitions in this nucleus is equal due to corresponding parity of corresponding levels). This can be compensated by extremely high and absolutely unreal level densities at $E_{ex} > 4$ MeV, as it follows from the variant (b). But observed in this case radiative strength functions of the cascade primary transitions are several order of magnitude less than the values of k derived from known [10] primary transition intensities according to eq.(1). The Porter-Thomas fluctuations of such scale are to small in order to explain the data presented in Fig. 2. It should be noted here that all the known levels with negative parity from the energy interval $3.7 < E_{ex} < 5.7$ MeV are populated by intense primary transitions and this effect cannot be explained by random fluctuations. Accounting for the fact that more than 90% of the experimental spectra area [1] of the nucleus under consideration is concentrated as resolved peaks, one can assume that the main portion of the desired level density and radiative strength functions is caused just by individual cascades. The last variant of method [2] takes into account possible inequality of level density with different parities. Therefore, impossibility of

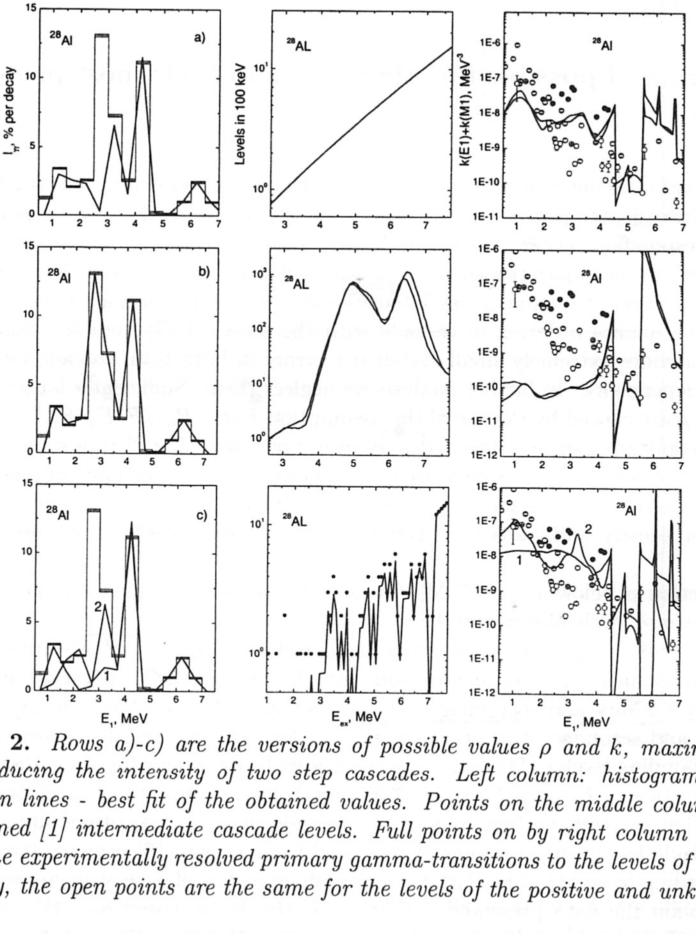

Fig. 2. *Rows a)-c) are the versions of possible values ρ and k, maximally accurately reproducing the intensity of two step cascades. Left column: histogram is experiment, broken lines - best fit of the obtained values. Points on the middle column - number of obtained [1] intermediate cascade levels. Full points on by right column show the k(E1) for the experimentally resolved primary gamma-transitions to the levels of negative [11,12] parity, the open points are the same for the levels of the positive and unknown parity.*

precise reproduction of $I_{\gamma\gamma}^{exp}$ using level densities (a) and (c) have to connect only with change in energy dependence of k when changing the energy of decaying level.

Unfortunately, practically there is no possibility to estimate the value of this effect within the method [9] for ^{28}Al owing to large difference in energy dependence of radiative strength functions for primary and secondary transitions and, correspondingly, very strong change in the determined ρ values. In heavier nuclei, corrections of function $k = F(E_\gamma, B_n)$ required to derive $F(E_\gamma, E_{ex})$ change level density sufficiently weaker.

Possibility of the practically precise reproduction of experimental distribution $I_{\gamma\gamma}$ by values ρ and k with the large systematic errors on the condition: $\sum(dI/d\rho \times \delta\rho) = -\sum(dI/dk \times \delta k)$ it leads to the appearance in the procedure [2] of the false solutions. Mathematically this is equivalent to the presence in function of the maximum likelihood of false maximums. Mathematical statistics provides for this possibility and recommends for their identification the using of a maximum spectrum of the initial values of the parameters of iterative process.

With the multiple repetition of random process [2] the probability of equality $\sum(dI/d\rho\times\delta\rho) = -\sum(dI/dk\times\delta k)$ it diminishes with an increase in the sum $\sum(|dI/d\rho\times\delta\rho|+|dI/dk\times\delta k|)$. Therefore the average value of the obtained parameters will be most probable. Of course, this statement is true only if relation (2) between the measured functions and desired parameters corresponds to reality. Its discrepancy can be discovered only in the analysis of additional experimental information. For example, there can be use of theoretical models that limit the region of possible values of required parameters. The results shown in Fig. 2 clearly illustrate this situation because in the frameworks of the level density exceeding of this parameter over the predictions of the non-interacting Fermi-gas model is impossible. Similar problem arises also when ρ and k are extracted from the total gamma-ray spectra depopulating the levels excited in reaction like (^{3}He,^{3}Heγ). Possible scale of the problem in this case can significantly exceed that for two-step cascades because intensity of the two-step cascades directly depends on the absolute value of level density. But the intensity of primary gamma-transitions, extracted from the nuclear reactions gamma-spectra is not depend on the absolute value ρ.

2 Full gamma-spectrum of the capture of the thermal neurons

Comparison between the experimental [13] and calculated total gamma-spectra following thermal neutron capture provides complementary information on confidence level and values of possible systematical errors of the observed ρ and k. Results of this analysis are shown in Fig. 3. Theoretical and experimental spectra do not agree in all presented variants of calculation. This means, first of all, that the desired parameters in each variant a priory contain larger or smaller systematical errors. At every decay, the considered nucleus gives of quite certain energy $B_n = 7.725$ MeV independently on number and energy of emitted quanta. This fact was used in [13] for normalization of the spectra and its use in the experiment like [8] would provide serious decrease in systematic errors of the determined there ρ and k values. In analysis [3], however, this circumstance is, most probably, negative: large variations of calculation parameters are reduced in essentially less errors of calculated spectra. This means that relatively small discrepancy between

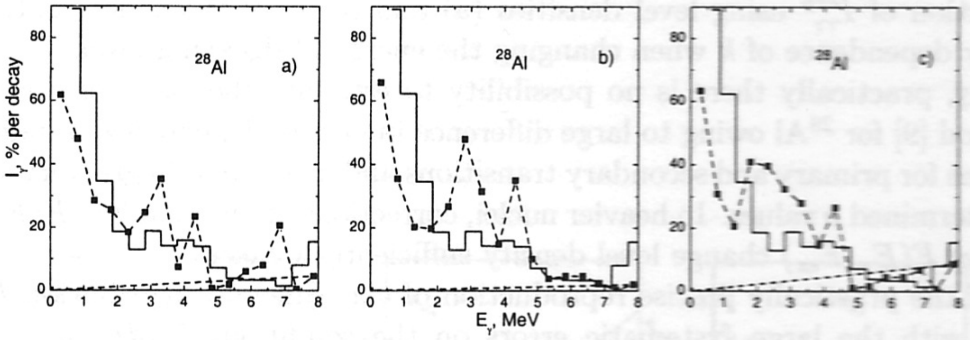

Fig. 3. . *The histogram represents the experimental gamma-quantum spectrum following thermal neutron capture in aluminum. Points are its calculated values for the density of levels and radiative strength functions, represented in fig. 2a-2c.*

calculation and experiment can be related with large errors of ρ and k.

In all variants, calculated intensity of low-energy $(E_\gamma, < 2$ MeV) gamma-quanta is noticeably less than that experimental. This can be caused either by systematic errors in data [13] or by large values of radiative strength functions of the secondary gamma-transitions in mentioned region of their energy. Maximum exceeding of theoretical intensity in variant (b) results, most probably from excessive value of level density in Fig. 2b. The least discrepancy of theoretical and experimental intensities above 7 MeV can be achieved only if the total calculated intensity of all primary transitions of smaller energy has minimal value (most probably due to the least systematic errors of level density in this set of considered ρ and k.

3 Conclusion

Data analysis about the two-step cascades, obtained during capture of thermal neutrons into ^{27}Al, revealed the strong divergence of the form of the energy dependence of radiative strength functions for the gamma-transitions of one and the same energy and multipolarity during decaying of their initial and intermediate level respectively. Effect has considerably larger value, than this is observed in the more heavy nuclei [9]. General trend of energy dependence of the sums of radiative strength functions providing the best reproduction of the experimental intensity of the two-step cascades corresponds to the data obtained from the absolute intensities of the primary gamma-transitions. It demonstrates increase in strength functions of probably low-energy $M1$ transitions depopulating the compound state. This conclusion is correct if the density of the cascade intermediate levels unessentially differs from the simplest extrapolations of exponential type.

Tomandl I. Two-step cascades following thermal neutron capture in ^{27}Al, Fizika B (Zagreb), 2003, **12(4)**, p 299.

2. Vasilieva E.V., Sukhovoj A.M., Khitrov V.A., Direct Experimental Estimate of Parameters That Determine the Cascade Gamma Decay of Compound States of Heavy Nuclei, Phys. At. Nucl. 2001, **64(4)**, 153.
   Khitrov V. A., Sukhovoj A. M., New technique for a simultaneous estimation of the level density and radiative strength functions of dipole transitions at $E_ex < B_n - 0.5$ MeV // INDC(CCP)-435, Vienna, 2002, P. 21 http://arXiv.org/abs/nucl-ex/0110017.
3. Boneva S. T., Khitrov V.A., Sukhovoj A.M., Excitation study of high-lying states of differently shaped heavy nuclie by the method of two-step cascades Nucl. Phys. 1995, **A589**, 293.
4. Sukhovoj A.M., Khitrov V.A., Li Chol, On correctness of some processing operations for two-step cascade intensities data from the $(n_{th}, 2\gamma)$ reaction, JINR E3-2004-100, Dubna, 2004.
   http://arXiv.org/abs/nucl-ex/0409016.
5. Khitrov V.A., Sukhovoj A.M., Pham Dinh Khang, Vuong Huu Tan, Nguyen Xuan Hai, On the role of some sources of systematic errors in determination of level density and radiative strength functions from the gamma-spectra of nuclear reactions, In: XI International Seminar on Interaction of Neutrons with Nuclei, Dubna, 22-25 May 2003, E3-2004-9, Dubna, 2004, p. 107.
   http://arXiv.org/abs/nucl-ex/nucl-ex/0305006.
6. Neutron Cross Section, vol. 1, part A, edited by Mughabhab S. F., Divideenam M., Holden N. E., Academic Press 1981.
7. Bečvář F. *et al.*, Phys. Rev. C **52**, (1995) 1278.
   Voinov A. *et al.*, Nucl. Instrum. Methods Phys. Res. A **497**, 350 (2003).
8. Guttormsen M. at all, J. Phys. G: Nucl. Part. Phys. **29** (2002) 263.
9. Bondarenko V.A., Honzatko J., Khitrov V.A., Li Chol, Loginov Yu.E., Malyutenkova S.Eh., Sukhovoj A.M., Tomandl I., Cascade population of levels and probable phase transition in vicinity of the excitation energy 0.5Bn of heavy nucleus In: XII International Seminar on Interaction of Neutrons with Nuclei, Dubna, May 2004, E3-2004-169, p. 38.
   http://arXiv.org/abs/nucl-ex/0406030
   http://arXiv.org/abs/nucl-ex/0410015
10. Popov Yu.P., Sukhovoj A.M., Khitrov V.A., Yazvitsky Yu.S., Izv. AN SSSR, Ser. Fiz. **48** (1984) 1830.
11. http://www.nndc.bnl.gov/nndc/ensdf.
12. http://www-nds.iaea.org/pgaa/egaf.html.
13. Groshev L.V. et al. Atlac thermal neutron capture gamma-rays spectra, Moscow, 1958.