Neutron-Electron Scattering Length Deduced from Neutron Diffraction Experiment on Noble Gas ^{36}Ar

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The earlier proposed method to extract the neutron-electron scattering length b_{ne} from the experimental data on structure factors $S(n,q)$ for noble gases is applied to known experimental data for gaseous isotope ^{36}Ar measured at the densities $n = (0.9 - 2.4)$ nm^{-3} and wave number transfers $q = (0.24 - 10.1)$ Å^{-1}. Our testing analysis of ^{36}Ar data demonstrated that in spite of the small n,e contribution (by 10 times less than for natural Ar) to the neutron scattering by ^{36}Ar atoms one can obtain the b_{ne} value with an accuracy $\sim 30\%$. The necessary accuracy of experimental data normalization is also discussed and the conclusion is made that the proposed method will give more precise b_{ne} value from possible new experiments with Ar, Kr and Xe of natural isotope mixtures.

1.Introduction

The present investigation is related to a very interesting problem of the fundamental importance. There are more than 10 experimental values of the n,e scattering length b_{ne} of the best accuracy (see for example [1]) in the interval between $-1.3 \cdot 10^{-3}$ fm and $-1.6 \cdot 10^{-3}$ fm. The point is that the so-called Foldy scattering length $b_F = \mu_n e^2/(2M_n c^2) = -1.468 \cdot 10^{-3}$ fm caused only by the neutron anomalous magnetic moment μ_n settles down just in the middle of this interval. And so far as b_{ne} is determined through $b_{ne} = (M_n e^2/3\hbar^2) < r_n^2 >$ by the other fundamental constant, the mean square charge radius of the neutron $< r_n^2 >$, the possible equality $b_{ne}=b_F$ would signify the direct bond $< r_n^2 >= \mu_n (3\hbar^2/2M_n^2 c^2)$ between electrical and magnetic values what seems unexpected for nowadays theory.

A solution of the discussed problem being related to the particle physics can be realized by the methods of low-energy physics, which are much cheaper in realization than those of high-energy physics.

2.Method

One of the best results on the n,e scattering length b_{ne} has been obtained [2] by the research of angular dependence of the thermal neutrons scattered by noble gases. Due to possible diffraction effects (see [3]), authors of these experiments used gases of low atom density $n = (0.01 - 0.04)$ nm^{-3}, so the probability of scattering, which could be registered by detectors, varied between 0.02% and 0.15% only.

Much more dense gases can be used for the b_{ne} obtaining in the new method, which has been proposed and tested in the references [4-6], using experimental data from [7] for the structure factors $S(n,q)$ of krypton. This method is based on two facts: 1) the diffraction and

n,e interaction contributions to scattering have very different dependences on the wave number transfer q (oscillating and monotonous); 2) while the diffraction contribution per one atom is proportional to density n or even has a term with n^2, the n,e contribution does not depend on n.

In order to extract the n,e scattering length to a few percents, it is necessary to know the value $S(q)$ with an accuracy $\sim 4\cdot 10^{-4}$, which really can be reached only in relative measurements. The data from [7], which we used for demonstration of our method [4], were obtained by normalization of the scattering by krypton relatively to one by vanadium with an accuracy about $10^{-2}-10^{-3}$ and we achieved the accuracy of b_{ne} ~ 15% [6]. In order to improve the accuracy of relative measurements, for example, by 5 times, it is necessary to measure the ratio of the type Xe to Kr or the natural argon to ^{36}Ar, which have rather different contributions of n,e scattering to the total one. The largest ratio takes place for natural argon to ^{36}Ar.

The nucleus ^{36}Ar has anomalously large scattering cross section $\sigma_s = 77 \div 78$ b and accordingly the very small n,e scattering contribution maximum ~ 0.002. Therefore, results with gases Ar, Kr and Xe of natural isotope mixtures, which have the n,e contribution ~ 0.019, 0.013 and 0.022, respectively, have to be much better.

Particular interest presents a measurement with the natural argon, diffraction on which has to be very close to one on ^{36}Ar gas or they both are the same. In any case, it would be of use to have such a pair of data with practically identical diffraction parts and very different n,e contributions. Actually, the experiment with ^{36}Ar analyzed in this report can be considered as a test of zero-experiment on the extraction of n,e scattering length for the natural Ar. Nevertheless, it seems to us, the analysis of ^{36}Ar data described below presents an interest as one more example of the b_{ne} extraction from the real experimental data [8].

In the paper [8] devoted to deriving the well of the isotropic pair potential, the experiment, in which the neutron diffraction on isotope ^{36}Ar had been measured in the range $q = 0.24-10$ Å^{-1} at n = 0.902, 1.399, 1.900, 2.393 nm^{-3}, was described. Since authors correcting the data for various distorting effects did not take into account the n,e interaction, its small (~10^{-3}) contribution to scattering is contained in their structure factor $S^{\exp}(n,q)$ (469 points for each n with the accuracy ~ 10^{-3}) obtained in [8] and can be extracted.

The neutron scattering intensity corrected for the background, self-shielding and multiple scattering effects per unit of neutron flux and one atom can be expressed in form:

$$I(n,q) = k\sigma_s\{[F_s(E_0,q,A) + \frac{8\pi a_{coh} b_{ne} f(E_0,q)}{\sigma_s}] + \\ + \frac{nC(q)}{1-nC(q)}[\frac{\sigma_{coh}}{\sigma_s} F_s(E_0,q,2A) + \frac{8\pi a_{coh} b_{ne} f(E_0,q)}{\sigma_s}]\}. \quad (1)$$

Here q is the wave number transferred, k is the constant, which takes into account the solid angle and efficiency of neutron detectors, E_0 is the initial neutron energy, A is the atomic weight, a_{coh} is the coherent nuclear scattering length, σ_{coh} is the coherent nuclear scattering cross section, σ_s is the total nuclear scattering cross section, $C(q)$ is the correlation function related to the structure factor $S(n,q)$

$$S(n,q)-1=\frac{nC(q)}{1-nC(q)} \tag{2}$$

and n is the atomic gas density.

The function F_s takes into account the thermal motion of gas atoms [2,9]:

$$F_s(V_0,\theta,A)=\frac{(A+1)^2}{A^2\sqrt{\pi}V_0B_0}\int_0^\infty\frac{V^2}{\sqrt{V_0^2+V-2V_0V\cos\theta}}\times$$

$$\times\exp\left\{-\frac{(V^2-V_0^2\frac{A-1}{A+1}-\frac{2V_0V\cos\theta}{A+1})^2}{4(\frac{A}{A+1})^2B_0^2(V_0^2+V^2-2V_0V\cos\theta)}\right\}dV\,, \tag{3}$$

where $B_0=\sqrt{\frac{2kT}{mA}}=128.9\sqrt{\frac{T}{A}}$ m/s, T is the gas temperature in K, V_0 is the initial neutron velocity, V is the neutron velocity after scattering, θ is the scattering angle in the laboratory frame.

The atom form factor is expressed in the form of [10]

$$f(q(\theta))=\frac{Z}{\sqrt{1+3(q/q_0)^2}}, \tag{4}$$

where the constant q_0 is estimated in Hartry-Fok's approach [10].

We made a comparison of corrections for the thermal motion of gas atoms by formula (3) and by that one used in the investigation [8], and we were convinced that they have the same dependence on q but a little different normalization. Therefore in our further calculations, the original data on $S(n,q)$ from [8] were used as the experimental points $S^{\exp}(n,q)$ already corrected for the thermal motion, and these data were described on the basis of (1) but without taking into account the thermal motion (with replacements F_s for 1) as

$$S^{\exp}(n,q)=\alpha\left\{1+Bf(q)+\frac{nC(q)}{1-nC(q)}[\gamma+Bf(q)]\right\}/S_{theor}(q_{\max}), \tag{5}$$

where $\gamma=\sigma_{coh}/\sigma_s=1$ for even-even nuclei ^{36}Ar, $B=\frac{8\pi a_{coh}b_{ne}}{\sigma_s}$, k and σ_s disappeared due to author's data normalization to high $q_{\max}\cong 10$ Å^{-1} where $S(q,n)\to 1$, a new constant α reflects an error of normalization, and denominator $S_{theor}(q_{\max})$ reflects normalization itself ($S_{theor}(q)$ is numerator without α).

3.Analysis of ^{36}Ar data

The method of obtaining b_{ne} for each q separately (as in [4-6]) was tested too, but due to small number of gas densities (only four densities just as there were fifteen densities from 0.3 to 6.2 nm^{-3} in [5,6]) the error of resulting b_{ne} came out too big. So we used the other approach tested in [6] allowing each density n to analyze independently from others. The formula

$$C(q) = (A_1 - A_2 n)\exp(-A_3 q)\sin(\frac{2\pi q}{A_4 + A_6 / q} + A_5) + A_7 / q, \qquad (6)$$

was chosen in (5) for description of $C(q)$. The data were normalized in [8] so that $S(q_{\max}) \cong 1$ with the accuracy a little better than 10^{-3}. As it is possible to see in our work [6], in order to extract b_{ne} with required accuracy ~ 5% it is necessary to know the normalizing constant with an accuracy $\sim 2 \cdot 10^{-4}$. In the present report we considered two ways of normalization:

1) the constant α was fitted simultaneously with all other parameters;
2) the constant $\alpha = 1$ was fixed for each sample and the data for each gas density were renormalized to wide interval $q \sim (8-10)$ Å^{-1}.

3.1. Normalizing is fitted

Some results of the fitting 4 – 7 parameters of $C(q)$ (6) to the $S^{\exp}(n,q)$ data using the formula (5) for four densities n simultaneously are presented in Table 1, where N is number of the used experimental points.

Table 1

q, Å^{-1}	χ^2 / N	$b_{ne} \cdot 10^3$ fm	α	Number of parameters in $C(n,q)$
2 ÷ 10	1975/1132	0.18±0.17	0.9668±0.0001	4
2 ÷ 10	1952/1175	1.43±0.23	0.9671±0.0006	7
3 ÷ 10	1348/1000	-2.05±0.24	0.9671±0.0001	5
2.5 ÷ 10	1573/1064	1.24±0.18	0.9457±0.0001	4

The Table 1 illustrates that it is not possible to describe with good χ^2 available data $S^{\exp}(n,q)$ for $q > 2$ Å^{-1}. Data processing with $q \geq 3$ essentially improved the fittings. We explain the noted here features by an insufficient for our task accuracy of taking into account, for example, the backgrounds at small angles or by existence of unknown for us processes. In other words, a certain physics of the samples could be here, which is not described by our model (6).

All variants of fitting in the Table 1 were produced by means of the code FUMILI, which is not such universal and fundamental as MINUIT, which estimates parameter errors in multivariable case (as in our one) in view of their correlations and a number of free parameters more reliably.

The simultaneous analysis of experimental data for gas densities n = 1.4, 1.9, 2.4 nm^{-3} (see section 3.2) and $q \geq 3$ Å^{-1} has given result

$$b_{ne} = (-2.15 \pm 0.36) \cdot 10^{-3} \text{ fm} \qquad (7)$$

in the case of fixed α and

$$b_{ne} = (-2.15 \pm 0.49) \cdot 10^{-3} \text{ fm} \qquad (8)$$

at all free parameters, where b_{ne} has the correlation coefficient with α $\rho = -0.966$ and small correlations with the parameters $A_1 - A_5$, which describe the diffraction effects. The error 0.49 in (8) is more adequate because it takes into account an influence of data normalization. In the last case FUMILI fitting gives

$$b_{ne} = (-2.5 \pm 0.5) \cdot 10^{-3} \text{ fm}. \qquad (9)$$

The Fig.1 shows χ^2 correlation contour for MINUIT fitting.

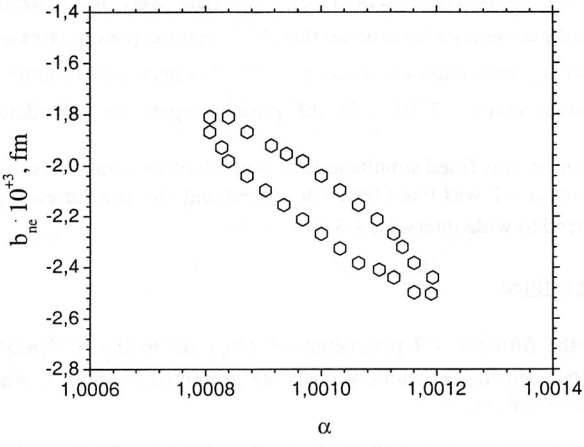

Fig. 1. The result of MINUIT fitting.

3.2. Renormalization of $S^{\exp}(n,q)$

In order to reveal the influence of α precision upon the b_{ne} error in another way than in section 3.1 we replaced α for 1, made the fitting function linear in B (i.e. b_{ne}) using approximations $S^{-1}_{theor}(q_{\max}) \cong 1 - Bf(q_{\max})$, $\gamma = 1$, and have obtained from (5):

$$S^{\exp}(n,q) - 1 = B[f(q) - f(q_{\max})] + \{1 + B[f(q) - f(q_{\max})]\}[S(n,q) - 1]. \qquad (10)$$

Here, in the left part, there are the experimental structure factors from [8] and the right part is their description with the n,e contribution, where $S(n,q) - 1$ corresponds to (2) and (6).

Thus, confiding in all corrections of [8] and using formulas (2), (6), (10) we fitted by FUMILI code five parameters (b_{ne}, A_1, A_3, A_4, A_5) to the experimental values $S^{\exp}(n,q)$. It was found again, that a satisfactory description of the data obtained only at $q > 3$ Å^{-1}. The results of fitting at different n are shown in Table 2. If we use the quantitative rule: $\chi^2 - N$ must be not more than $\sim\sqrt{2N}$, so it is obvious that fits for three high n can be considered as normal ones what is impossible to say about result for the lowest n. It is important to say that the correlation factor for b_{ne} is small, just as they are essential for parameters A_1, A_3, A_4, A_5 ($A_2 = A_6 = A_7 = 0$).

Table 2

n, nm^{-3}	0.902	1.399	1.900	2.393
$b_{ne} \cdot 10^3$,fm	0.42±0.23	-1.01±0.32	-1.72±0.43	-1.36±0.53
χ^2 / N	380/250	294/250	272/250	274/250
q, Å^{-1}	3÷10	3÷10	3÷10	3÷10

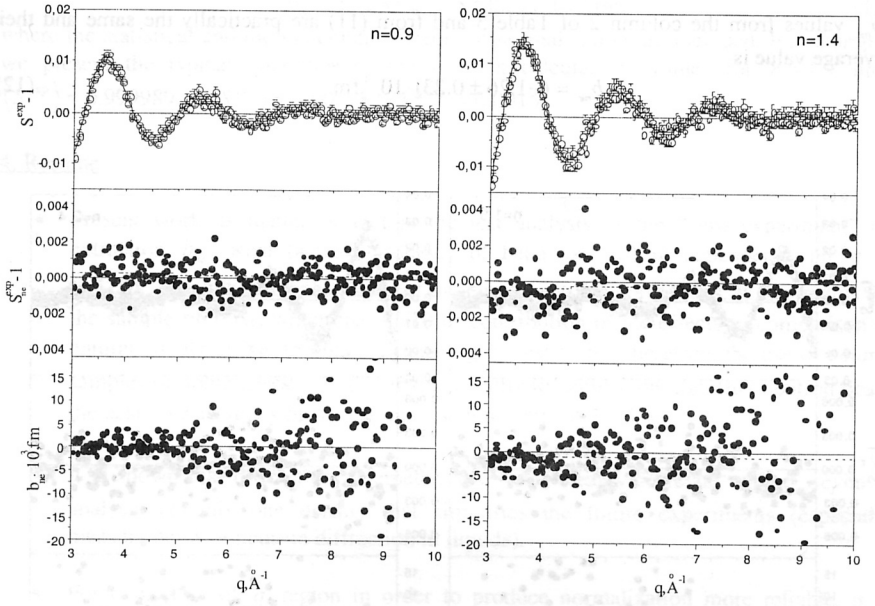

Fig.2. The experimental values for gas densities 0.902 nm^{-3} and 1.399 nm^{-3} and their description with fitted parameters.

The quality of fitting can be seen in Fig.2 and Fig.3 for each gas density n in three pictures. The top pictures give a general representation on the descriptions of experimental points $S^{exp}(q)-1$. The n,e contributions to $S^{exp}(q)-1$ at each q are illustrated in the middle pictures, where the points are the differences between $S^{exp}(q)-1$ and the second term of (10) and the lines are the first term of (10) (both terms are calculated at the fitted parameters). The b_{ne} values calculated from the data of the middle pictures (these data were divided by $8\pi a_{coh}[f(q)-f(q_{max})]/\sigma_s$) are shown in the lower pictures. The scatter of the points $S^{exp}(q)-1$ and b_{ne} is exaggerated since as a matter of fact it belongs to the total statistics of $S^{exp}(n,q)$. At $q>7$ Å^{-1} many of b_{ne} points turned out to be outside the figure field. Disregarding the result with $n=0.902$ nm^{-3} we calculated the weighted average value over three the rest n:

$$b_{ne}=(-1.29\pm 0.23)\cdot 10^{-3} \text{ fm}. \qquad (11)$$

Similar result was obtained also in fitting to the data for three large densities together, where the addition parameter $A_2\neq 0$ was used in (6). This can be seen in Table 3 where the different variants of fitting are given. The results in columns 2 and 5 are obtained at the free parameter A_2, while in columns 3 and 4 they are obtained at the fixed A_2. It is worth to note insignificancy of parameter A_2, what means very small non-linearity of $S(n,q)-1$ in n. The

b_{ne} values from the column 2 of Table 3 and from (11) are practically the same and their average value is

$$b_{ne} = (-1.26 \pm 0.23) \cdot 10^{-3} \text{ fm}. \tag{12}$$

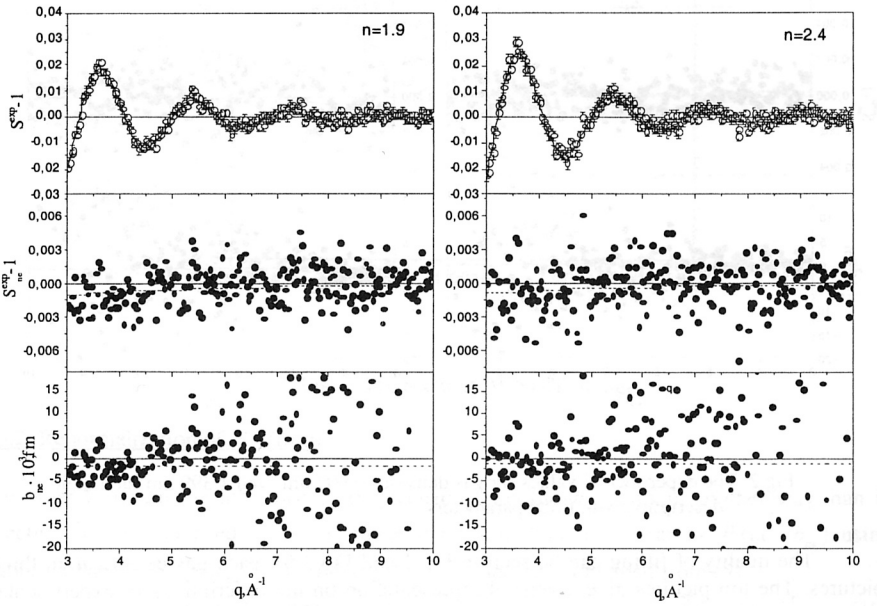

Fig.3. The experimental values for gas densities 1.900 nm^{-3} and 2.393 nm^{-3} and their description with fitted parameters.

And so the precise $\alpha = 1$ approximately halves b_{ne} error (compare (9) and (12)).

Table 3

$b_{ne}\cdot 10^3$,fm	-1.23±0.23	-1.26±0.23	-1.34±0.25	-1.31±0.25	-0.28±0.18	-0.42±0.20
χ^2 / N	852/750	858/750	505/405	500/405	1208/849	826/504
q, Å^{-1}	3÷10	3÷10	3÷7	3÷7	2÷10	2÷7

Of course, it is necessary to add some systematic error to the results (11) and (12). In order to evaluate it we have made the renormalization of data by calculating the average value $< S^{exp} >$ with its error Δ in the interval $q = (8.15 - 9.95)$ Å^{-1} for each n. The following fitting to these "new" data at $q = (3 - 8)$ Å^{-1} and $q_{max} = 9$ Å^{-1} normalized by division of each S^{exp} by $< S^{exp} > -\Delta$, $< S^{exp} >$ and $< S^{exp} > +\Delta$ gave three b_{ne} value for each n. Being averaged over three large n they are (in 10^{-3} fm) -0.77 ± 0.28, -1.33 ± 0.28, -1.90 ± 0.28, what gives the right to write finally

$$b_{ne} = (-1.33 \pm 0.28 \pm 0.57) \cdot 10^{-3}\ \text{fm}, \qquad (13)$$

where the statistical and the systematical errors of normalization are included. For example, we present the typical quantities of the used in calculations values for $n = 1.9$ nm^{-3} $\left\langle S^{\exp} \right\rangle = 0.999989$, $\Delta\left\langle S^{\exp} \right\rangle = 0.000187$, $\Delta S^{\exp} = 0.0015$.

4. Resume

- Present work as matter of fact is the test analysis of the "zero experiment" on extraction b_{ne}. Real (actual) accuracy of behavior $S(q)$ in dependence on q is $\sim 4 \cdot 10^{-4}$, including inexactitudes of the equipment and corrections. It allowed us for the sample of ^{36}Ar, which has ~1/10 of contribution of n,e effects in comparison to natural Ar, Kr or Xe, to obtain a meaning b_{ne} estimate. Therefore, the use of natural samples of noble gases will give the possibility to achieve on high flux neutron beam the desirable accuracy of b_{ne} estimate within limits of 3 – 5 %.

- It is shown that diffraction parameters $(A_1 - A_7)$ weakly correlate with b_{ne} even at analysis of only one density that simplifies the future experiments (especially measurements of neutron diffraction in liquids).

- For expansion of q region in order to produce normalization more reliable, it is advantageous to pass to greater neutron energy.

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