USE OF THE MONTE CARLO METHOD FOR MODELING OF THE PREEQUILIBRIUM MULTIPARTICLE NUCLEON SPECTRA

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Abstract

The calculation scheme of the preequilibrium multiparticle nucleon emission is realized with the Monte Carlo method. In calculations the emission of ten neutrons and ten protons is taken into account as a function of composite system energy. Nucleons are emitted at the stage of system transition into statistical equilibrium state. In the framework of one history of the statistical test method (Monte Carlo method) the competition of nucleon emission process (neutron or proton) and complication of particle-hole configuration is played. If the choice has fallen on the nucleon emission, the energy of the given nucleon is chosen by means of random number. Accumulation of nucleon spectrum is produced taking into account the order of the particle emission. The given history is being continued for the following residual nucleus. The number of particles decreases by unity, and number of holes does not vary, because the nucleon above the Fermi energy is emitted. The energy of the following composite system is reduced in appropriate way. Consideration of the branching processes is finished in two cases:

1. If the nucleus energy is not sufficient for the nucleon emission.
2. If the statistical equilibrium state is achieved.

Testing the proposed method (MCP) is performed as compared to the calculation results with the classical exciton model for the first nucleons.

I. Introduction

In spite of obvious successes in the nucleon spectra description, the using of the exciton model of preequilibrium decay [1] for population calculations of residual nuclei is limited to relatively narrow range of incident particle energies. Actually, at a one nucleon emission only one residual nucleus is formed, the populations of all subsequent residual nuclei are taken into account in effective way in one's populations.

Nucleons are emitted mainly from initial configurations, i.e. long before the statistical equilibrium is achieved. Hence, at enough excitations of residual composite system which has not achieved equilibrium, several more preequilibrium particles can be emitted. From this point of view the emitted particle spectrum is the integral characteristic, in which all emitted nucleons and, hence, residual nuclei are summed.

Distributions of residual nuclei can be obtained only taking into account the preequilibrium emission of several sequentially emitted particles.

II. Multiparticle emission

The idea of multiparticle preequilibrium emission in the framework of exciton model is not new [2], but, as far as we know, was not realized yet.

In the proposed scheme of calculation (MCP) of the intermediate and high energy nucleon-induced reaction cross-sections at the preequilibrium stage which starts from various initial excitations and configurations, residual nuclei have to transit to equilibrium, emitting nucleons.

1. At the first stage of calculations, according to the initial energy of excitation and the initial quasi-particle configuration (ph) of the first nucleus the list of nuclei which can be formed as a result of nucleon emission, and their highest excitation energies Emax is defined.

The example of the list, including nuclei with $E_{max} > 0$, for ^{90}Zr(p,xn) reaction at E_p=20 MeV is given in Table 1. For this reaction at relatively low energy of incident protons as a result of consistent nucleon emission can be populated up to seven residual nuclei and sequentially emitted three nucleons.

Table 1. The list of residual nuclei for reaction ^{90}Zr (p,xn) at Ep=20 MeV. K – number of emitted nucleons, E_{max} - the highest nucleus energy, B – nucleon binding energy in the given nucleus.

№	K_p	K_n	Z	A	E_{max}, MeV	B_n, MeV	B_p, MeV
1	0	0	41	91	25.16	12.05	5.16
2	0	1	41	90	13.11	10.15	5.08
3	0	2	41	89	2.96	12.71	4.24
4	1	0	40	90	20.00	11.97	8.36
5	1	1	40	89	8.03	9.31	7.86
6	2	0	39	89	11.64	11.47	7.07
7	2	1	39	88	0.17	9.36	6.71
8	3	0	38	88	4.57	11.11	10.62

2. At the second stage of calculations for all possible nuclei and their excitation energies from 0 up to Emax the probabilities of nucleon emission W (n, E, ε) and the probabilities of quasi-particle transitions λ(n, E) are calculated.

In Griffin model [1] quantities under study are calculated according to follow relations:

$$W_\nu(n,E,\varepsilon)=\frac{2s+1}{\pi^2\hbar^3}\mu_\nu\varepsilon\sigma_\nu(\varepsilon)\frac{\omega(p-1,h,E-B_\nu-\varepsilon)}{\omega(p,h,E)}, \qquad (1)$$

$$\lambda(n,E)=\frac{2\pi}{\hbar}\left|\overline{M}\right|^2\varpi(n-\Delta n,E), \qquad (2)$$

where the particle-hole level density of the excited nucleus is more often calculated [3] by formula:

$$\omega(p,h,E)=g\frac{[gE-A(p,h)]^{n-1}}{p!h!(p+h-1)!}, \quad A(p,h)=\frac{1}{4}\left(p^2+h^2+p+h\right), \qquad (3)$$

where g denotes the single-particle level density.

Matrix element of interaction is parametrized in Ref. 4 as:

$$\left|\overline{M}\right|^2=K\cdot A^{-3}E^{-1} \qquad (4).$$

The scale of excitation energies is divided into some of equal intervals for all nuclei. The lists of particle-hole configurations are formed so that the number of particles in the first configuration differed from the previous nucleus by the number of emitted nucleons. The further stages are performed the given number of times (histories).

3. The system of the master equations is solved.

According to [1] the system of the master equations is:

$$\begin{aligned}\frac{dP(n,t)}{dt}&=P(n-2,t)\cdot\lambda_+(n-2,E)+P(n+2,t)\cdot\lambda_-(n+2,E)\\&-P(n,t)\cdot\left[\lambda_+(n,E)+\lambda_-(n,E)+\sum_\nu L_\nu(n,E)\right]\end{aligned}, \qquad (5)$$

where λ_+, λ_- is the probability of nucleus transition to more complex or more simple state, respectively, P denotes the population of n=p+h configuration at time t, E symbolizes the excitation energy of composite system, and L_ν is the particle (ν) emission probability defined as:

$$L_\nu = \int_0^{E-B_\nu} W_\nu(n,E,\varepsilon)d\varepsilon\,. \qquad (6)$$

Statistical falling out of events (nucleon emission or quasi-particle transition) is produced according to ratio of their probabilities which are given by:

$$P_1 = \frac{\lambda_+(n,E)+\lambda_-(n,E)+\lambda_0(n,E)}{\lambda_+(n,E)+\lambda_-(n,E)+\lambda_0(n,E)+\sum_\nu L_\nu(n,E)}$$

(7) – the probability of quasi-particle transition;

$$P_2 = \frac{L_n(n,E)}{\lambda_+(n,E)+\lambda_-(n,E)+\lambda_0(n,E)+\sum_\nu L_\nu(n,E)}$$

(8) – the probability of neutron emission.

The random number $x \in (0;1)$ is used to define the type of process. If x < P1, quasi-particle transition occurs, if P1 < x < P1+P2, then neutron emission occurs, and in the third case, if x > P1+P2, proton emission takes place. At the given stage of studies the emission of more complex particles is not considered. After quasi-particle transition the solving of master-equation system is being continued and the composite system continues motion to equilibrium.

4. After nucleon emission the energy of nucleon is defined by solving the equation:

$$\xi = \int_0^{\varepsilon} W(n,E,\varepsilon)d\varepsilon \qquad (9)$$

where ξ is a random number, and ε denotes the emitted nucleon energy. Accumulation of nucleon spectrum is produced.

The nucleon emission leads to reduction of excitation energy of composite system and quantities Z, A by $(\varepsilon+B_\nu)$ and Z_ν, A_ν respectively, as well as to decrease of quasi-particle number by unity. The obtained values are set as initial quantities for the following calculation. Then, transition to the third stage is performed, i.e. the system of master-equations for new initial conditions is solving.

5. The given history is finished, if the statistical equilibrium state is achieved or if the residual nucleus energy is not sufficient for the nucleon emission. Accumulation of nucleus population is produced at the fixed excitation energy.

Thus, at sufficient amount of histories the spectra of preequilibrium particles and the populations of the residual nuclei being in the equilibrium state are obtained.

III. Comparison with results of analytical calculations

The first stage of the proposed model testing is the comparison with the calculation data based on the usual exciton model. The purpose of this stage is to check not only the correctness of computer modeling, but also, mainly, the mathematical representation of nucleon emission process at the stage of equilibrium establishment.

The main quantity determining the preequilibrium nucleon spectrum is the product of the quasi-particle state population at the given time and the probability of nucleon emission at the given energy. Comparison of the indicated quantities is difficult due to only representation complexity of this multidimensional picture. Therefore, in Fig.1 the comparison of quantities summed over energy for the given time and for the dedicated quasi-particle state (i.e., contribution to preequilibrium part) is given. One can see from Fig.1, the good agreement of results is achieved for quantities of preequilibrium part more than 10^{-5}. In practice, there is no need to achieve the agreement of smaller parts of preequilibrium emission because of the negligible contribution to results and the considerable expenses of computing time. This is confirmed by data of Fig. 2, where the good agreement of the first nucleon spectra is obtained.

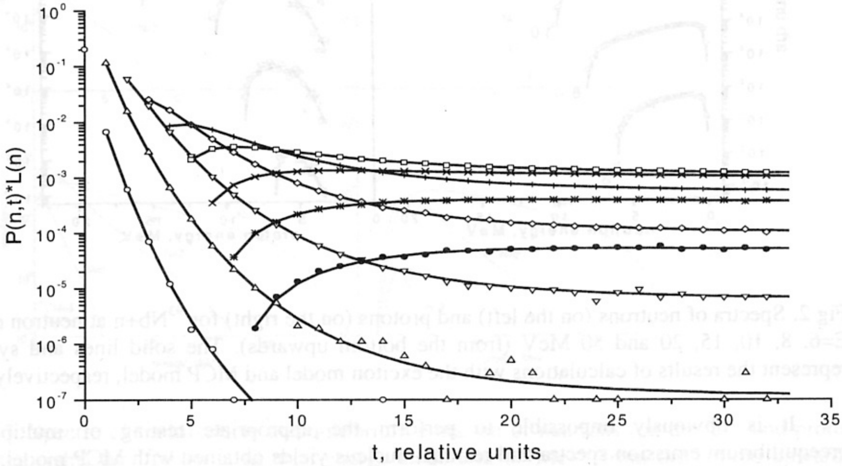

Fig 1. Product of the quasi-particle state population and the probability of nucleon emission from various states of system being equilibrated versus time. The solid lines and symbols represent the results of calculations with the exciton model and the results of modeling with the Monte-Carlo method, respectively.

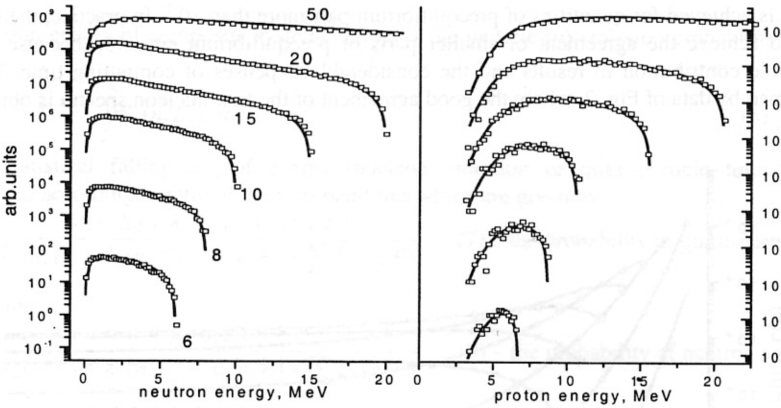

Fig 2. Spectra of neutrons (on the left) and protons (on the right) for ^{93}Nb+n at neutron energy E=6, 8, 10, 15, 20 and 50 MeV (from the bottom upwards). The solid lines and symbols represent the results of calculations with the exciton model and MCP model, respectively.

It is obviously impossible to perform the appropriate testing of multiparticle preequilibrium emission spectra and residual nucleus yields obtained with MCP model. Their reliability can be estimated only qualitatively. The data analysis of Fig.3 shows that the calculated spectra of the subsequent particles are reasonable good. Thus, at incident neutron energy of 25 MeV the secondary particle spectra are lower than the first nucleon spectra due to lack of energy, remaining in the nucleus after emission of the first particles having hard enough spectrum. It is obvious, that the proton spectrum is cut off by Coulomb barrier in the soft part.

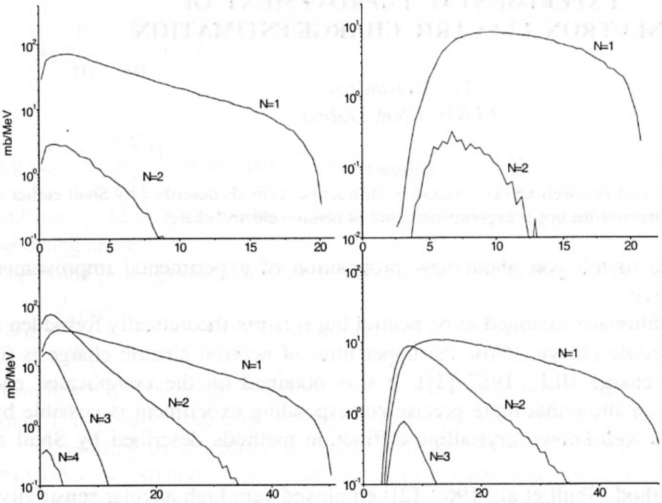

Fig. 3. Spectra of multiparticle preequilibrium emission of neutrons (on the left) and protons (on the right) for ^{93}Nb+n at neutron energy E=20 and 50 MeV (from the bottom upwards). The curves show results of calculations with MCP model, numerals at curves is number of particle.

For 50 MeV neutron energy the second and even the third particles are quite competitive with the first particles, and their spectra are obviously softly and have accordingly lower highest energy. "Softening" of spectra is ascribed to the decrease of excitation energy of residual composite systems and the increase of effective initial number of particles and holes for preequilibrium emission of the subsequent particles. The general view of the spectra for the no first nucleons in Fig.3 is similar to shape of spectra obtained with statistical model of nuclear reactions. This is natural taking into account the fact that the preequilibrium emission occurs frequently from states near equilibrium.

IV. Conclusion

The calculation scheme of the preequilibrium multiparticle nucleon emission is realized with the Monte Carlo method, which allows to compute the spectra of multiparticle emission during an establishment of statistical equilibrium.

Testing the proposed method (MCP model) is performed as compared to the calculation results with usual exciton model for a one preequilibrium particle (neutron and/or a proton).

References

[1] Griffin T.T.// Phys. Rev. Letters, 1966, v.17, p.478.
[2] Fu C.Y.// Nucl. Sci. Eng.,86, 1984, p.344.
[3] Ericson T.,// Adv. in Phys., 9, 1060, p.425.
[4] Kalbach-Clain C.,// Nucl. Phys. 1973, v.A210, p.590.