# THICK FILTER NEUTRON TRANSMISSION DATA

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The results of neutron transmission measurements in dependence on the sample thickness n – exp(-nσ) as well as the reaction cross sections measured on the filtered beams - $\sigma_\alpha\exp(-n\sigma)$ (cross sections with self-indication) in neutron resonance region are very important from practical point of view for the problems of neutron transport in reactor media. So, the space distribution of resonance neutrons in the block in moderating medium with some approximations is determined by the following relation [1]:

$$F(\vec{r},E)=\Sigma_p\int_V d\vec{r}'\,K(\vec{r}-\vec{r}',E)+\int_S d\vec{r}_s'\,K(\vec{r}-\vec{r}_s',E)\left(\frac{\vec{r}-\vec{r}_s'}{|\vec{r}-\vec{r}_s'|},\vec{n}\right) \tag{1}$$

where

$$K(\vec{r}-\vec{r}',E)=\frac{\exp\left[-\sigma(E)\rho|\vec{r}-\vec{r}'|\right]}{4\pi|\vec{r}-\vec{r}'|^2} \tag{2}$$

is so-called kernel of the Peierls equation [2], and in more general case this is the Green function of integral transport equation [3,1].

In multigroup schemes for calculation of neutron space-energy distribution in the media the fluxes $\langle F(\vec{r})\rangle_g$ and reaction rates $\langle\sigma_\alpha F(\vec{r})\rangle_g$ averaged over energy intervals g (groups) are determined. The averaged over resonances in the group transmissions and average cross sections with self-indication:

$$\langle T(n)\rangle_g=\langle\exp[-n\sigma]\rangle_g,\quad \langle T_\alpha(n)\rangle_g=\langle\sigma_\alpha\exp[-n\sigma]\rangle_g,$$
$$n=\rho|\vec{r}-\vec{r}'|, \tag{3}$$

where ρ is the density of nuclei, is the input information about elementary processes, as it is illustrated by our example (1).

In principle for the resolved resonance region these functionals can be determined directly by using the data e.g. from ENDF library. The strong dependence of the average transmissions at big n on the cross section values in resonance minima however imply the requirement for validation of the evaluated data files with the possible correction of these according to the results of direct transmission measurements for relatively thick samples with the beam attenuation of 100 – 1000 times.

The situation in the region of unresolved resonances levels, where the direct information about the cross sections resonance structure in the averaging interval (group) is not available, is much more complicate. Indirectly the effect of resonance structure appear as a disagreement of thick samples transmission data <exp(-nσ)> with calculated exp(-n<σ>), Fig. 1. The physical interpretation of these data is based on the use of statistical models of resonance cross-section structure in the unresolved region (resonance ladders). Moreover the group averaged model cross sections should be equal with the evaluated average cross sections and the modeled <T(n)> and $<T_\alpha(n)>$ should be functions of the average resonance parameters in the same group.

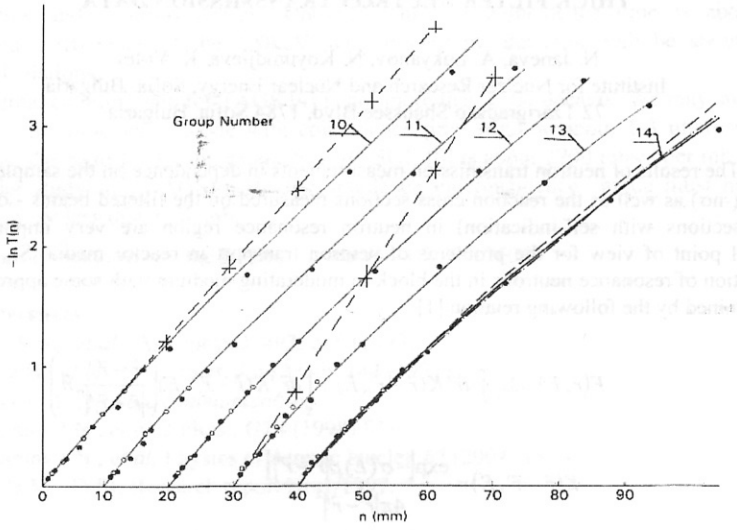


Fig. 1 Transmission function for $^{238}U$ for energy groups 10 through 14 of ABN system: • - present work, and ∘ - averaged data [4]. The curves are shifted on the thickness axis; thickness 10 mm corresponds to 0.0477 nuclei/b.

It has been noted that the functionals (3) strongly depend on cross section value in the resonance minima. This suggests that for modeling the resonance ladder the multilevel presentations of collision matrix accounting the resonance interference effects should be used. In R-matrix scheme using Reich-Moore approximation for exclusion of radiative channels the cross section resonance structure is determined by the corresponding structure of the elements of R-matrix (or R-function for non fissile nuclei) –

$$R(\varepsilon) = s\sum_{\lambda}\frac{\xi_{\lambda}}{\varepsilon_{\lambda}-\varepsilon-iy}, \tag{4}$$

where

$$s=(\pi\overline{\Gamma}_n)/(2\overline{D}),\ y=(\pi\overline{\Gamma}_\gamma)/(2\overline{D}),\ \varepsilon_\lambda=\pi E_\lambda/\overline{D},\ \varepsilon=\pi E/\overline{D},\ \xi_\lambda=\Gamma_{\lambda n}/\overline{\Gamma}_n.$$

The parameters s and y are the known in general strength functions. For $\xi_\lambda$ and $\varepsilon_\lambda$ however in the unresolved resonance region are proposed only the usual for random numbers statistical distribution functions – Porter-Thomas for $\xi_\lambda$ and Wigner $Q(\varepsilon_\lambda)$ for $\varepsilon_\lambda$. In principle arbitrary cross section functionals can be expressed by Re R and Im R. So, for determination of their average values directly should be used the function of joint statistical distribution of R real and imaginary parts. The characteristic function of such distribution is determined as follows

$$F(u,u') = \left\langle e^{i\left(Ru-R^{*}u'\right)}\right\rangle = \ldots \tag{5}$$

$$F(\gamma,\beta,y) = \prod_{\lambda}\int Q(\varepsilon_\lambda)d\varepsilon_\lambda\left[\frac{\varepsilon_\lambda^2+y^2}{\varepsilon_\lambda^2+y^2+2\gamma y-2i\beta\varepsilon_\lambda}\right]^{1/2}, \tag{6}$$

where $\gamma=s(u+u')$ $\beta=s(u-u')$ [5].

Let us form the model resonance ladder by N sequential levels in the sum (4) with arbitrary, for now, parameters $\xi_\lambda$ and $\varepsilon_\lambda$, supposing that the contribution of the remaining levels in R repeats this ladder periodically, then the model R-function can be presented in the following form:

$$R_N(\varepsilon) = i\frac{s}{N}\sum_{\lambda=0}^{N-1}\xi_\lambda ctg\left[(\varepsilon_\lambda - \varepsilon - iy)/N\right] \tag{7}$$

The function $F_N$ is analogous to the characteristic one F (5) and using our model R-function (7) can be determined as

$$F_N(\gamma,\beta,y) = \frac{1}{\pi N}\int_0^{\pi N}\exp\left[-\frac{1}{N}\sum_{\lambda=0}^{N-1}\xi_\lambda\frac{\gamma\, sh(2y/N)+i\beta\sin 2(\varepsilon-\varepsilon_\lambda)/N}{ch(2y/N)-\cos 2(\varepsilon-\varepsilon_\lambda)/N}\right]d\varepsilon \tag{8}$$

The conformity condition for the functions $F_N$ and F in wide intervals of parameters $\gamma$ and $\beta$ changing gives at the end the optimal perimeter set of $\xi_\lambda$, $\varepsilon_\lambda$, and the level number N as well, of our model resonance ladder $R_N(\varepsilon)$ (7) [5].

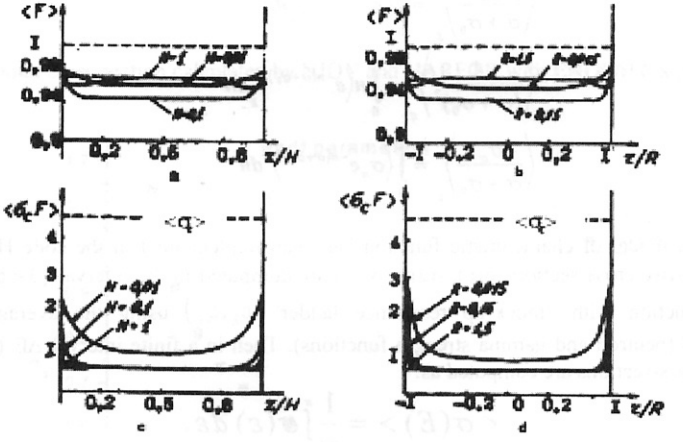

Fig. 2 The space distribution of the neutron flux (a,b) and absorption for the energy group (0.465 – 1. keV) and $^{238}U$ for layer (a,c) and sphere (b,d), H – layer thickness , R – sphere radius.

The use of resonance ladder optimized from statistical approach point of view in the unresolved region permits to calculate the group averaged functionals $\langle T(n)\rangle$ and $\langle T_\alpha(n)\rangle$ the same way as it is in the resolved levels region. Calculations of the space distributions of group beams in the blocks of various geometry (1) and corresponding distributions of the resonance absorption (Fig. 2) can serve as an example for the application of our method.

Universality of the ladder choice following criteria of characteristic function (5) identity allows to calculate also other group averaged cross section functionals that can be used in practical applications. The "resonance self-shielding factors" (9) are most popular here for reactor media that are necessary for evaluation of the corresponding group cross-sections

$$f^g(\sigma_0) = \frac{1}{\langle \sigma_0 \rangle}\left[\frac{\langle 1/(\sigma+\sigma_0)\rangle_g}{\langle 1/(\sigma+\sigma_0)^2\rangle_g} - \sigma_0\right], \qquad (9)$$
$$f_\alpha^g(\sigma_0) = \frac{\langle \sigma_\alpha/(\sigma+\sigma_0)\rangle_g}{\langle \sigma_\alpha\rangle\langle 1/(\sigma+\sigma_0)^2\rangle_g}$$

where $\sigma_0$ is the cross section of the nuclei of non resonant moderator added to the resonant absorber in the homogenized mixture [6,4]. It is easy to see, that the group averaged cross section functionals in (9) are expressed by the transmission functions $<T(n)>_g$ and $<T_\alpha(n)>_g$ as follows

$$\left\langle \frac{1}{\sigma+\sigma_0}\right\rangle_g = \int_0^\infty \left\langle e^{-n(\sigma+\sigma_0)}\right\rangle_g dn$$
$$\left\langle \frac{1}{(\sigma+\sigma_0)^2}\right\rangle_g = \int_0^\infty n\left\langle e^{-n(\sigma+\sigma_0)}\right\rangle_g dn \qquad (10)$$
$$\left\langle \frac{\sigma_\alpha}{\sigma+\sigma_0}\right\rangle_g = \int_0^\infty \left\langle \sigma_\alpha e^{-n(\sigma+\sigma_0)}\right\rangle_g dn$$

The model of characteristic function has been implemented in the code HARFOR [7]. The point-wise cross sections $\sigma(\varepsilon)$, $\sigma_\gamma(\varepsilon)$, $\sigma_\alpha(\varepsilon)$ are computed in the interval $(0 < \varepsilon < \pi)$ via the $R_N(\varepsilon)$ -function with "mock-up resonance ladder" $(\varepsilon_\lambda, \xi_\lambda)$ using the average resonance parameters (neutron and gamma strength functions). Then in a finite interval $\Delta E$ (group g) the average cross-sections are computed as:

$$<\sigma(E)> = \frac{1}{\pi}\int_0^\pi \sigma(\varepsilon)\, d\varepsilon, \qquad (11)$$

$\overline{E}$ (eV) is the midpoint of averaging interval $\Delta E$. All cross sections functional are calculated in the interval $\Delta E$ through the cross sections $\sigma(\varepsilon)$, $\sigma_\gamma(\varepsilon)$, $\sigma_\alpha(\varepsilon)$ in the same way.

The code HARFOR was tested against the code PURR(NJOY) [8]. We used HARFOR and PURR(NJOY) module to produce infinitely diluted cross sections $\sigma(\infty)$ and effective self-shielded cross-sections $\sigma_\gamma(\sigma_0, T^0)$ for $^{232}$Th in the unresolved resonance region. NJOY is a widely used processing code that produces point-wise and multi-group cross sections from ENDF/B data for practical applications. NJOY uses single level Breit-Wigner formalism with the correction of neutron widths fluctuation for cross sections calculation. The two different formalisms applied in the codes HARFOR and NJOY they calculate equally cross sections $\langle\sigma\rangle$ and $\langle\sigma_\gamma\rangle$ in (4-50) keV Fig. 3, Fig. 4.

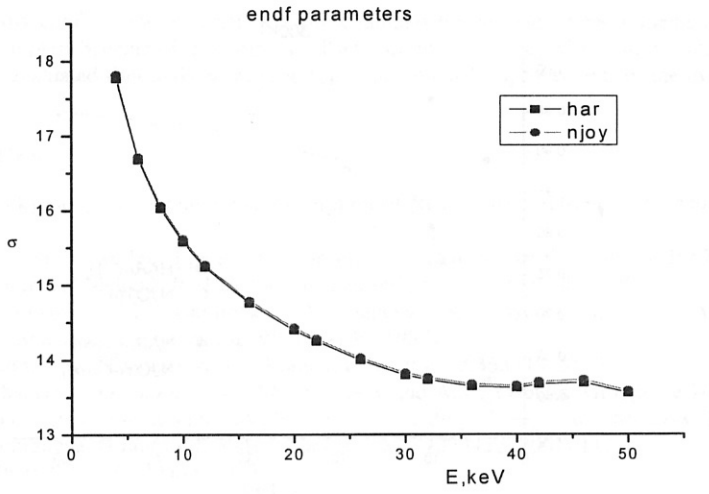


Fig.3 $^{232}$Th total cross section calculated by NJOY and HARFOR with ENDF/B 6.8 parameters.

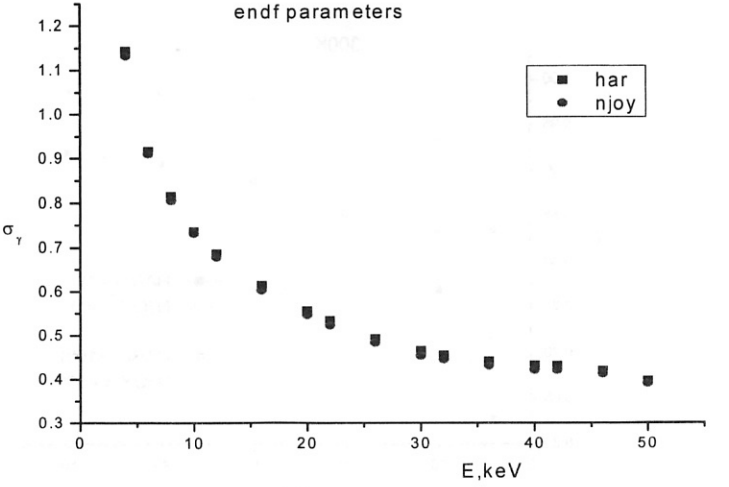


Fig. 4. $^{232}$Th cross section for neutron capture calculated by NJOY and HARFOR

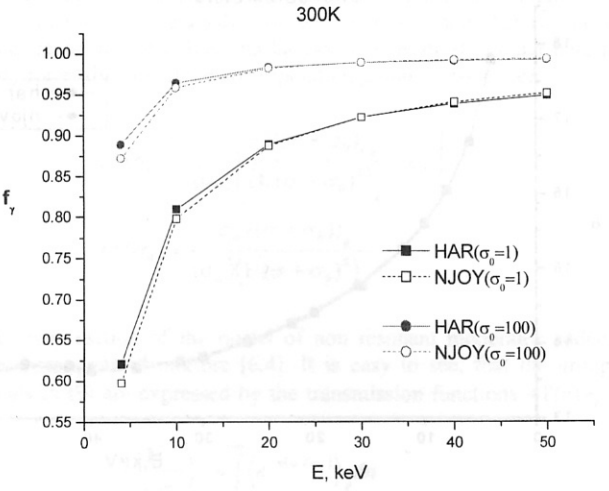


Fig. 5. $^{232}$Th self-shielding factors for neutron capture calculated by NJOY and HARFOR with ENDF/B 6.8 parameters for $\sigma_0$=1 barn (down) and 100 barns (upper).

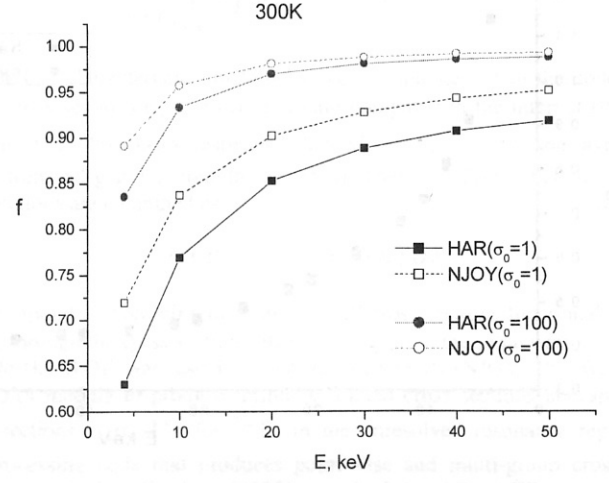


Fig. 6. $^{232}$Th self-shielding factors for total cross-section calculated by NJOY and HARFOR with ENDF/B 6.8 parameters fort $\sigma_0$=1 barn (down) and 100 barns (upper).

This work confirms the importance of thick samples transmission measurements that can be treated as simplest benchmark experiments. These are the only integral measurements easily reproduced by evaluated files and can be used for validation and improvements of the evaluated libraries.

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