ANGULAR ANISOTROPY OF NEUTRONS EVAPORATED FROM FISSION FRAGMENTS

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The probability estimates for the emission of scission neutrons (those emitted in the vicinity of the scission point) were done several times. The main method used in these estimates was the comparison of the experimental and theoretical energy and angular distributions of neutrons accompanying fission. The basic assumption of the method was the isotropic emission of neutrons in the c.m. system of each fission fragment.

The present work takes into account the anisotropy of such an emission, which results from the large fragments spins appeared at scission and are perpendicular to the line of the fragment's relative motion. Contrary to the majority of previous works the sequential emission of neutrons was calculated in the framework of the standard evaporation model by Monte-Carlo method.

It is shown that the anisotropy $[W(0^o)/W(90^o)-1]$ in the fragment c.m. system depends only on the average value $<J_i>$ but is insensitive to the form of the initial spin distribution and is about 10% for both heavy and light fragments. In the laboratory system this anisotropy is much smaller than the one arising from the fragments' relative motion and practically should not affect the estimates of the scission neutron's yield.

It is well known the process of nuclear fission of heavy elements (Uranium and Californium) is accompanied by prompt neutron emission. It means these neutrons come out before γ-quanta and their time of emission is less than 10^{-14} s. It is quite within reason to suggest these neutrons can be emitted in process of scission, during a period of fragment's acceleration and from fully accelerated fragments. It is well established that the latter part of prompt neutrons amounts to 80% or 90%. It indicates that neutron evaporation comes mostly from fully accelerated fragments.

Apart from scission neutrons (which are assumed to be emitted isotropically in the laboratory system) the neutrons evaporated from fully accelerated fragments have in this system very different neutron yields at various angles. The experiment, which was made by K. Skarsvag and K. Bergheim /1/ for the reaction $^{235}U(n_{slow}, f)$ (see Fig.1), where the angle was measured between emitted neutron and light fragment velocities, demonstrates the absolute maximum of angular distribution curve in the direction of light fragment motion and the relative maximum in the opposite direction. One can find a minimum of angular distribution at 90^o. These neutron yields are in the ratio of 9:4:1.

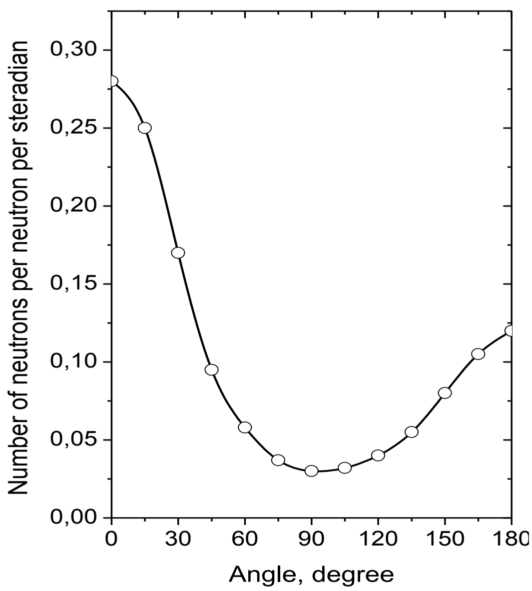

Fig.1. Angular distribution of neutrons in the lab. system for the reaction $^{235}U(n_{slow}, f)$.

Such interrelation between maxima depends on light fragment velocity and multiplicity of evaporated neutrons from light fragment exceed the velocity and multiplicity of heavy one. It is worthy of note that the velocities of both fragments amount to their nearly maximal values in 10^{-20} seconds time (see Fig.2).

For the description of observed data in the laboratory system it is necessary to use some hypothesis about neutron spectra and angular distributions of evaporated neutrons in the fragment's center-of-mass system. Many authors have assumed that the detected neutrons are emitted (in this system) isotropically.

Deviation from this basic assumption is possible due to the angular momentum of the fragment. The large fragment's spins can appear at scission point. It is supposed they prefer to be directed perpendicularly to the line of the fragment's relative motion.

T.Ericson and V.Strutinski /2/ have performed the quasi-classical approach for this task. In such limit the anisotropy A of angular distribution

$$W(\theta) \propto 1 + A \cdot \cos^2(\theta)$$

can be written as:

$$A = \frac{\mu \cdot \omega^2 \cdot R}{2T},$$

where μ – the mass of the emitted particle,

ω – the angular velocity,

R – the nuclear (fragment's) radius ,

T – the nuclear (fragment's) temperature.

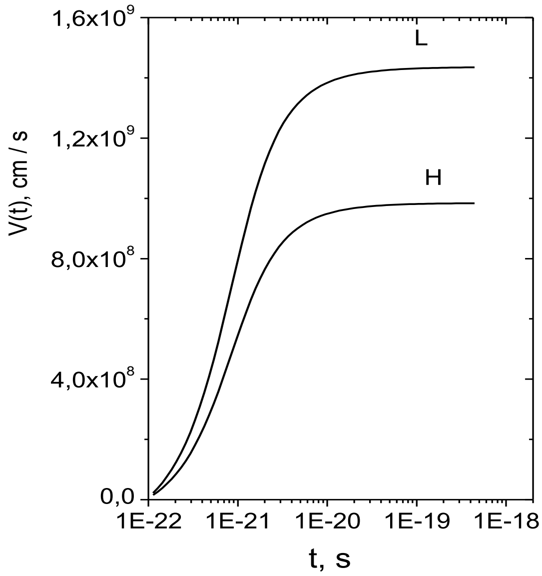

Fig. 2. Velocities of light (L) and heavy (H) fission fragments directly after scission in the reaction $^{235}U(n_{slow}, f)$.

Then Gavron /3/ has offered a statistical model calculation for consideration. In his paper the final angular distribution of evaporated neutrons in the fragment's centre-of-mass system was defined by the sum over spherical harmonics: $W(\theta) = \sum_{lm} P_{lm} \left| Y_{lm}(\theta, \varphi) \right|^2$.

The evaporation cascade is followed by a Monte-Carlo procedure in which the probability P_{lm} of emitting a neutron with given orbital angular momentum l and its projection m is proportional to the sum over all values of final fragment's spin J_f that can couple to initial spin J_i for the given l value:

$$P_{lm} \propto \sum_{J_f} \int_0^{E^*-B_n} \rho_{J_f}(E^* - B_n - \varepsilon) \cdot T_l(\varepsilon) \cdot \left| C^{J_i M_i}_{J_f M_f \, lm} \right|^2 d\varepsilon .$$

The result of such approach depends on the fragment's level density $\rho_{J_f}(E^* - B_n - \varepsilon)$, neutron transmission coefficient $T_l(\varepsilon)$ and vector coupling coefficient $C^{J_i M_i}_{J_f M_f \, lm}$.

Here: E^* – the excitation energy of the fragment;

B_n – the neutron binding energy;

ε – the energy of evaporated neutron.

In his calculation the angular momentum (*J*) and its projection on the fission axis (*M*) were determined at each stage of cascade. The initial spin projection of primal fission fragment is assumed to be equal zero (*M=0*) in consequence of a suggestion that initial fragment's spin is directed perpendicularly to the fission axis.

Gavron has taken the initial spin distribution of the fragment in the "standard" form:

$$\rho_J = (2J+1) \cdot \exp\left(-(J+0.5)^2 / B^2\right).$$

The main presumption in our work was the same, but our calculations were performed not only with the "standard" form of angular distribution, but also with the form:

$$\rho_J = (2J+1) \quad J \le J_{\max}$$
$$\rho_J = 0 \qquad J > J_{\max}.$$

The second form was offered by Kadmensky /4/ as a result of his quantum theory of fission.

The figure 3 demonstrates both kinds of distributions. Parameter "B" for the first curve and maximal value of initial fragment's spin for the second curve were chosen in this way − the average value of angular momentum was the same:

$$1) \quad B = 12 \qquad < J > \approx 9.5$$
$$2) \quad J_{\max} = 14 \quad < J > \approx 9.5.$$

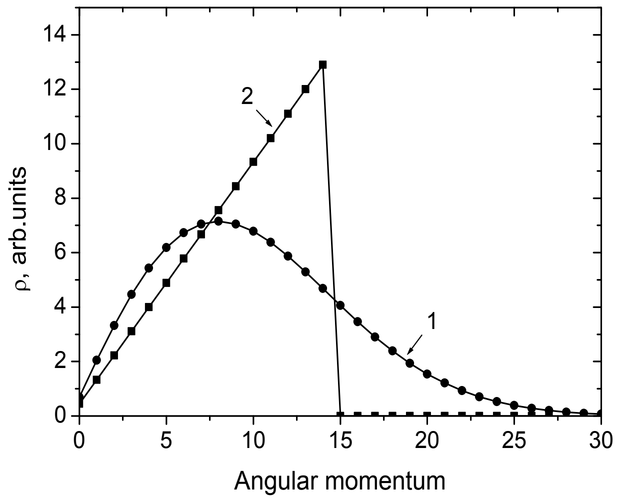

Fig. 3. Initial angular momentum distributions.

The squares under curves are proportional to the number of events. This determines arbitrary units on the ordinate axis for both curves.

The level density depends on both excitation energy and angular momentum of the fragment. To determine kinetic energy of evaporated neutron we took into account that the level density is proportional to the exponential function of a square root of the excitation energy:

$$\rho \propto \exp\left(2\sqrt{aE^*}\right).$$

The spin dependency was used to find an angular momentum of a residual nucleus, which appeared after neutron evaporation.

The excitation energy of fission fragments directly after scission notates by the subscript zero (E_0^*). This subscript increases after neutron evaporation by a number one (i – serial number of evaporated neutron). The excitation energy decreases after emission on neutron binding energy and kinetic energy of evaporated particle ($E_i^* = E_{i-1}^* - B_n - \varepsilon_i$).

Evaporation cascade is finished if the excitation energy is less than the binding energy of a neutron.

To determine the primal excitation energy of fragments we used the averaged experimental values of neutron multiplicity of both light and heavy fragments /5/:

$$\nu_L = 2.05, \quad \nu_H = 1.71$$

and the value of neutron binding energy $B_n = 6\,MeV$. This way:

$$E_L^* = 19.3\,MeV, \quad E_H^* = 13.7\,MeV.$$

The neutron spectra for the first emitted neutron and for neutrons following it are presented in the figure 4. As we can see the second neutron spectrum is softer than the first one. This is because the excitation energy decreases during the process of neutron evaporation.

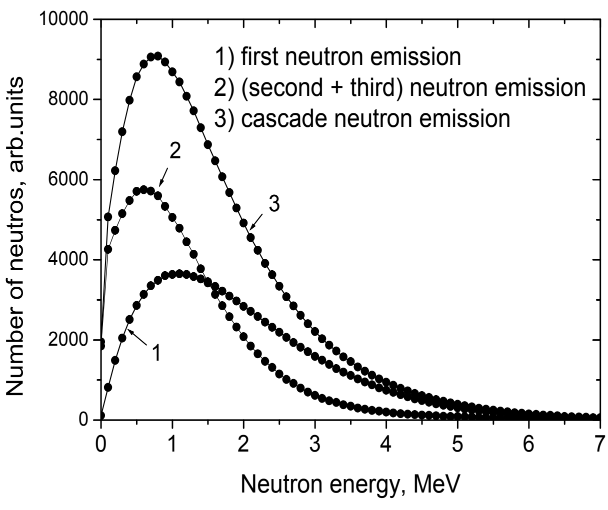

Fig. 4. Neutron energy spectra for light fragment.

The neutron transmission coefficients were calculated by using the well known Blatt-and-Weisskopf form /6/:

$$T_l = \frac{4xXv_l}{X^2 + (2xX + x^2 v'_l) v_l}.$$

Here: $v_l \equiv \frac{1}{G_l^2(R) + F_l^2(R)}$ $\qquad v'_l \equiv \frac{1}{k^2}\left[\left(\frac{dG_l}{dr}\right)^2 + \left(\frac{dF}{dr}\right)^2\right]$

$$F_l(r) = \sqrt{\frac{\pi kr}{2}} J_{l+1/2}(kr) \qquad G_l(r) = \sqrt{\frac{\pi kr}{2}} N_{l+1/2}(kr)$$

$J_p(z)$ – Bessel's function $\qquad$ k – the wave number outside nucleus

$N_p(z)$ – Neuman's function $\qquad$ K – the wave number inside nucleus.

For these calculations the nuclear radius R, it means the fragment's mass number A, and the energy of neutron were used.

Figures 5 and 6 present the pictures of calculated neutron transmission coefficients for light and heavy mass regions of the fission fragments. As we can see for low energy of neutrons the less an orbital momentum of a neutron is the more transmission coefficient value amounts.

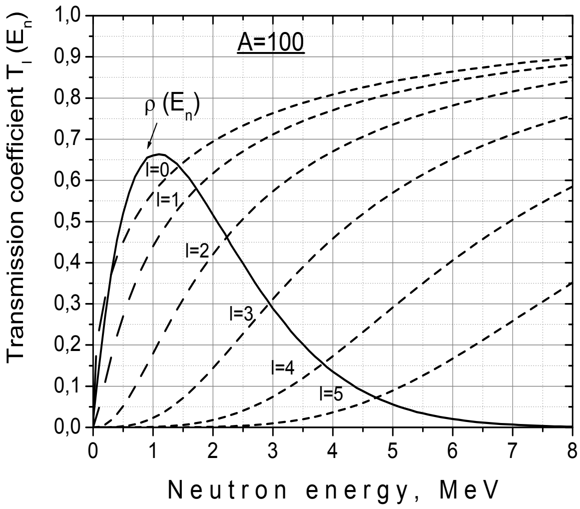

Fig.5. Neutron transmission coefficients and energy spectrum for light fragment region.

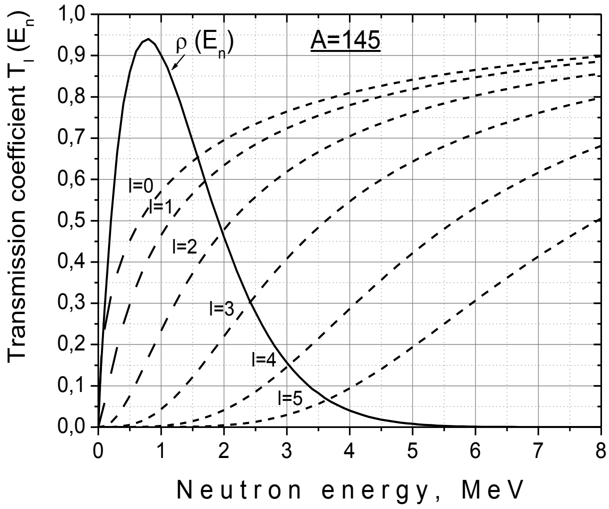

Fig.6. Neutron transmission coefficients and energy spectrum for heavy fragment region.

We can estimate the number of events with given orbital momentum of a neutron if we bring into comparison the transmission coefficient picture and neutron energy spectrum. This way we can cut off the maximal value of neutron orbital momentum by 5 for both light and heavy fragment's mass regions.

The histograms in the figures 7 and 8 were obtained by using a Monte-Carlo calculation and demonstrate the number of events with given orbital momentum of a neutron (l) for the first, second and third neutron emitted from light fragment and for the first and second neutron evaporated by heavy fragment. As we can see the number of neutrons is falling down with the increase of neutron orbital momentum, but this decrease is faster for the second and especially for the third neutrons. It should be mentioned that the number of the third neutron emitted by the light fragment is not negligible although the average multiplicity taken into account equals 2.05 according to Kornilov's data /5/.

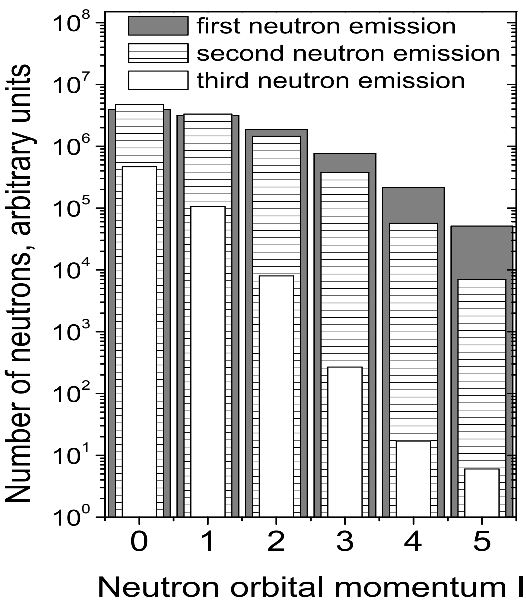

Fig. 7. The number of neutrons with given orbital momentum for light fragment.

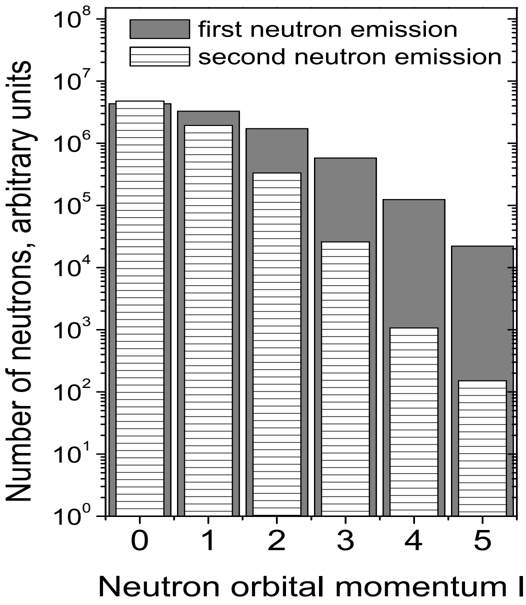

Fig. 8. The number of neutrons with given orbital momentum for heavy fragment.

The figure 9 demonstrates that the calculated anisotropy has strong dependence on the orbital momentum of evaporated neutrons. The more neutron orbital momentum is, the more angular anisotropy in CMS amounts. These curves for the partial anisotropy were received in presumption that the number of events for any orbital momentum of neutrons is the same. But actually, as it was mentioned, the number of events falls down with increasing of neutron's orbital momentum.

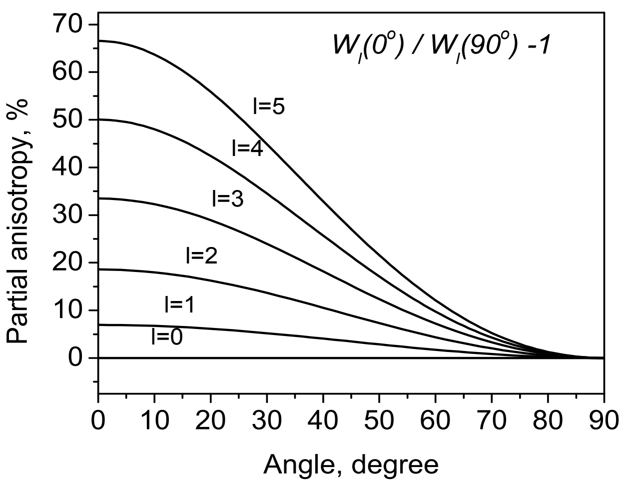

Fig. 9. Dependence of angular anisotropy in CMS on the orbital momentum of evaporated neutrons.

In addition, distributions are used for these calculations depend not only on orbital momentum, but also on magnetic number of a neutron. These distributions are conditioned by the vector coupling coefficients.

The intermediate result of the Monte-Carlo calculations can be presented in the tabular form. Below one can see the examples of such tables calculated for the light fragment of ^{252}Cf. The values in cells are the numbers of events with given orbital momentum of a neutron and its projection for the first and second emitted neutrons. The index number of a line corresponds with the orbital momentum of a neutron. Each table has five lines because we cut-off maximal value of the orbital momentum by *l* equal 5. Columns correspond with magnetic numbers. Non-zero quantities of events in cells and their distributions in each line are determined by Clebsh-

Gordan coefficients. We can say these values in cells are the non-normalized probability P_{lm} coefficients.

Table 1

Monte-Carlo calculations
for neutron emission anisotropy in CM system

Examples of calculated matrices for light fragment
(total number of events 20000000)

Matrix for the first neutron emission

l\m	-5	-4	-3	-2	-1	0	1	2	3	4	5
0						3943478					
1					1017334	1116038	1018705				
2				316613	400702	433940	401021	316112			
3			76890	107306	131195	139910	130871	107342	76851		
4		13708	20065	26270	30871	32795	31358	26358	19889	13443	
5	2144	3272	4570	5688	6511	6907	6541	5715	19889	3158	2041

Matrix for the second neutron emission

l\m	-5	-4	-3	-2	-1	0	1	2	3	4	5
0						4758196					
1					1072972	1168318	1074081				
2				250017	309894	334271	310547	249457			
3			38894	52329	63355	66942	62701	52386	38641		
4		3627	5347	6981	8209	8623	8006	6896	5410	3862	
5	300	441	569	765	893	937	902	801	628	456	271

As we can see, the fraction of events in the 5-th line of the first table is about seven times larger than the fraction of events in the corresponding line of the second table is. This leads to decrease of angular anisotropy for the second neutron in comparison with the first one.

With the help of such tables the anisotropy were calculated for light and heavy fragments. The figure 10 demonstrates magnitudes of anisotropy averaged over all neutrons emitted by fragments. The calculations have statistical character and the result is within the bounds of errors. These results depend on average excitation energy and average initial spin of fission fragment. The curves presented here were calculated with the same parameter B, that is, with the same initial spin of the primal fragment. Actually the average initial momentum of the light fragment may be less than for heavy one /7/. If we take this fact into account, the anisotropy for both light and heavy fragments will be

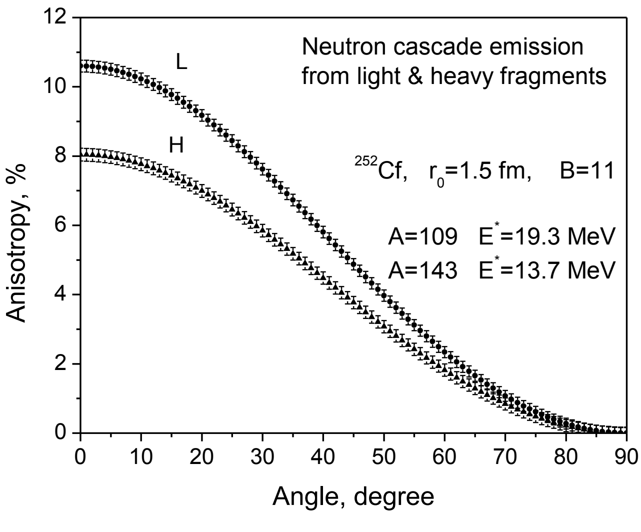

Fig. 10. The average angular anisotropy in CMS for light (L) and heavy (H) fragments of ^{252}Cf.

the same and equals about 8.2%.

It is worthy of note that the angular anisotropy is under the influence of a neutron energy. This is a consequence of a relation between neutron energy and its probability to have a given orbital momentum. Curves in the figure 11 have been calculated for equal numbers of events with fixed neutron energy. In fact angular anisotropy in fragment's center-of-mass system is calculated considering the neutron spectrum. It means the part of neutrons with the energy around 1 MeV predominates.

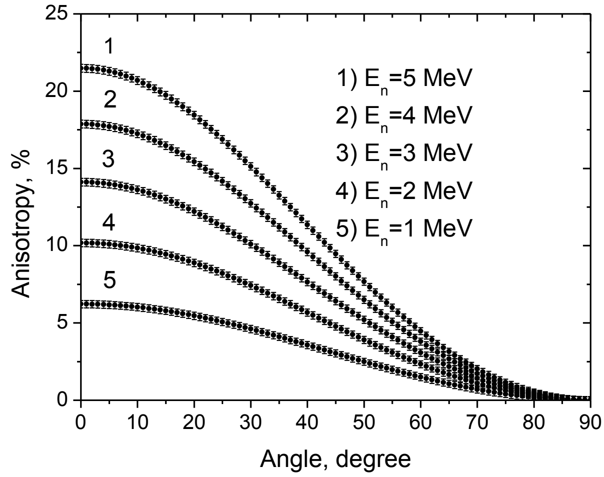

Fig. 11. Energy dependence of angular anisotropy in SMS.

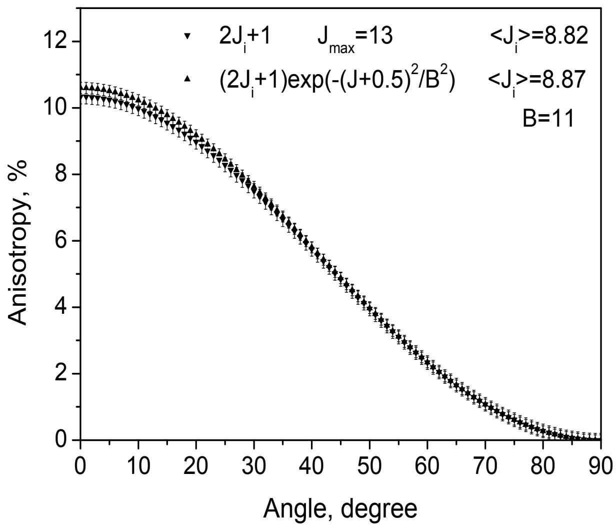

Fig. 12. The comparison between two initial distributions of angular momentum of fission fragments.

We have calculated anisotropy for two forms of initial fragment's spin distributions: with exponential and linear dependences. The latter one was bounded by maximal spin value. It was shown that the anisotropy [W(0^o)/W(90^o)-1] in the fragment's center-of-mass system depends on the average value of initial angular momentum but is insensitive to the form of the initial spin distribution. The curves in the figure 12 correspond with two versions of calculations.

The figure 13 demonstrates the angular distributions of neutron yields in the laboratory system with and without taking into account anisotropy in the fragment's center-of mass system. As we can see the difference between two curves is not large and concentrates in two small regions around fragment's motion directions. The values of neutron yields in the region from 15^o to 165^o are practically the same. If the scission neutrons are assumed to be emitted isotropically in the laboratory system this discrepancy can not be explained by such kind of emission and practically should not affect the estimates of the scission neutrons yield.

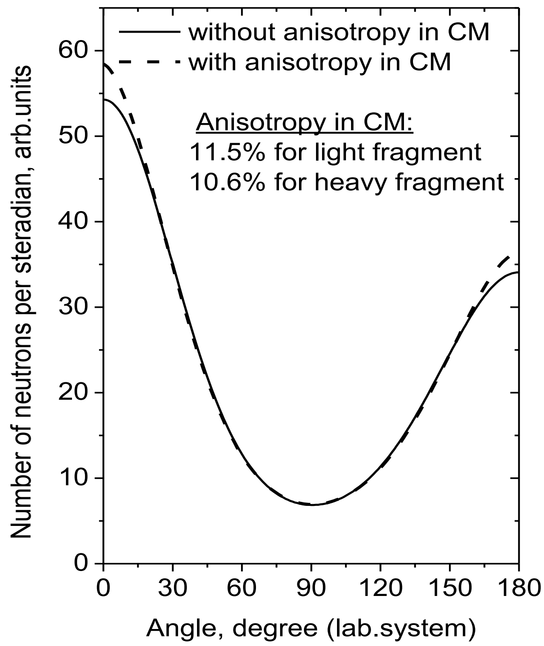

Fig. 13. Angular anisotropy of neutrons from fully accelerated fission fragments in lab. system.

Conclusion:

- It is shown that the anisotropy [W(0^o)/W(90^o)-1] in the fragment's center-of-mass system depends only on the average value of initial angular momentum but is insensitive to the form of the initial spin distribution.
- Angular anisotropy is about 10% for both heavy and light fragments.
- In the laboratory system this anisotropy is much smaller than the one arising from the fragment's relative motion and practically should not affect the estimates of the scission neutrons yield.

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