THE ROTATION OF THE FISSIONING COMPOUND NUCLEUS IN REACTIONS INDUCED BY POLARIZED NEUTRONS

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1.Introduction.

In our previous papers [1-3] we developed the theory which explained the earlier observed (see e.g. [4]) T-odd correlation of the type:

$$\vec{\sigma}_n[\vec{k}_{LF}\cdot\vec{k}_\alpha]$$

in ternary fission induced by polarized neutrons. Here $\vec{\sigma}_n$ is the neutron spin, while $\vec{k}_{LF}$ and $\vec{k}_\alpha$ are the momenta of the light fragment and the ternary particle (usually alpha) emitted in ternary fission. This correlation, which was called TRI-correlation, was described by the differential cross-section of the type:

$$\frac{d^2\sigma}{d\Omega_{LF}d\Omega_\alpha}=B_0+D\cdot\vec{\sigma}_n[\vec{k}_{LF}\cdot\vec{k}_\alpha] \qquad (1)$$

The experimental geometry was chosen in such a way that the directions of the unit vectors $\vec{\sigma}_n$ and $\vec{k}_{LF}$ were parallel to the y and z axes, while the vector $\vec{k}_\alpha$ varied in the (x,y) plane. The effect measured was defined as:

$$D=\frac{\sigma_+-\sigma_-}{\sigma_++\sigma_-}\quad , \qquad (2)$$

where σ_+ and σ_- stand for the differential cross-sections with the neutron beam positive and negative helicities. The magnitude of the effect for the ^{233}U target was about $-3\cdot10^{-3}$. It is important to point that this magnitude and the sign of the effect were practically independent of the angle θ between the vectors $\vec{k}_{LF}$ and $\vec{k}_\alpha$ in a wide range of angles around $\theta\approx90^0$.

However, the recent measurements [5] for the ^{235}U target demonstrated the existence of the new effect, which was called ROT. Contrary to the TRI, this effect does not change with the inversion of either $\vec{k}_{LF}$ or $\vec{k}_\alpha$ but changes its sign in the vicinity of $\theta\approx90^0$. This effect is rather well reproduced [6] in the classical trajectory calculations of the alpha-particle emission in the rotating Coulomb field of the fission fragments with the angular momentum of about few ħ, equal to the total spin J of the polarized fissioning nucleus ^{236}U which appears after the absorption of the polarized neutron by the target nucleus. The rotation of alpha particles slightly lags behind the rotation of the outgoing fragments' motion axis (Coriolis effect). This leads to the shift of the whole alpha angular distribution with respect to this axis. The inversion of the neutron helicity causes the inversion of the compound-nucleus polarization direction and, therefore to the inversion of the system's rotation direction. This leads to the shifts of the alpha-particle angular distributions in the opposite directions. The difference of the angular distributions for the opposite helicities causes the effect observed. It was possible to reproduce the experimental effect with the angular shift about 0.1^0.
It is well known that the classical trajectory calculations of the alpha angular distributions in ternary fission without the rotation of the decaying system reproduce the experimental data fairly well because the motion of alphas in the Coulomb fields of the fragments is quasi-classical. However in ref. [6] the rotation of the system was taken into account in the trajectory calculations for the first time. Its success is quite surprising because the results of these

calculations seem to contradict the uncertainty relation for the system's angular momentum and the angle θ of its rotation in the plane perpendicular to it:

$$\Delta J \cdot \Delta\theta \sim \hbar \qquad (3)$$

In the case considered the nuclear angular momentum was of the order of few ħ units. Therefore one should expect the uncertainty of the rotation angle to be $\Delta\theta \sim 1$ radians. As pointed above, the trajectory calculations gave the value $0.2^0 \approx 0.003$ radian in full contradiction to (3). Besides this, the expression used in [6] for the angular velocity of the rotating system was quite contradictory. These contradictions will be considered below.

2. Contradictions in the initial conditions of the trajectory calculations

The effective angular velocity of the fissioning system's rotation was estimated in [6] as:

$$\vec{\Omega} = \vec{P}(J)\frac{R}{\Im}, \qquad (4)$$

where $\Im$ is the moment of inertia of the fissioning system and *P(J)* is the compound-nucleus polarization, resulting from the absorption of the neutron with polarization p_n:

$$P(J) = \begin{cases} \dfrac{2I+3}{3(2I+1)} p_n \equiv \dfrac{J+1}{3J} p_n & for \quad J = I + 1/2 \\ -\dfrac{1}{3} p_n & for \quad J = I - 1/2 \end{cases} \qquad (5)$$

Ex. (5) does not take into account the axial symmetry of the deformed fissioning nucleus which therefore possess the additional quantum number K of its spin projection on the symmetry axis. At low excitation energies close to the fission barrier this nucleus fissions through only one-two transition states with fixed K values.

This axially symmetrical deformation was taken into account in ref. [6] by introducing into eq.(4) of the quantity R, which was named in [6] " the angular momentum of the nuclear collective rotation" and related to the values of the total nuclear spin *J* and its projection *K* on the nuclear symmetry axis as:

$$R = \hbar\sqrt{J(J+1) - K^2} \qquad (6)$$

It was assumed that this vector $\vec{R}$ was directed along the vector of the neutron beam polarization so that the rotation of the symmetry axis takes place in the plane perpendicular to the neutron polarization vector $\vec{p}_{\rm n}$ directed along the y axis. The direction of the light fragment emission (i.e. the direction of the nuclear symmetry axis) coincides then with z axis. This shows that the introduction of vector $\vec{R}$ is questionable even in the classical approach. Indeed with our (and experimental) choice of the axes $R = J_y$ while $K = J_z$. If $K = 0$ then the whole nuclear spin J is caused by the collective rotation. Thus $R = J\hbar$ in contradiction to (6). If $K \neq 0$ then the direction of the collective rotation vector $\vec{R}$ (and, therefore the direction of the angular velocity vector $\vec{\Omega}$) can not coincide with the direction of the fissioning nucleus polarization vector because:

$$\vec{J} = \vec{R} + \vec{K} \qquad (7)$$

in contradiction to (4) and (6).

In the quantum approach the inconsistencies of Eqs. (4)-(6) are even more evident because the vector components J_y and J_z do not commute (the corresponding eigenvalues can not be measured simultaneously). One might only introduce the new operator $\hat{R}^2$ of the angular momentum squared $\hat{R}^2$ defined as:

$$\hat{R}^2 = \hat{J}_x^2 + \hat{J}_y^2 = J^2 - J_z^2 \qquad (8)$$

Exactly this was done by Bohr and Mottelson [7] to whom the authors of [6] refer. The eigenvalue of this operator is

$$R(R+1) = J(J+1) - K^2 \qquad (9)$$

in contradiction to (6). Exactly this eigenvalue enters the well-known expression for the collective energy:

$$E_{rot} = \frac{\hbar^2}{2\Im_\perp}(J(J+1) - K^2), \qquad (10)$$

which is given in [7]. By comparing this formula with the expression for the rotation energy of the rigid rotator with the symmetry axis along the z axis and the moments of inertia $\Im_x = \Im_y = \Im_\perp$:

$$E_{rot} = \frac{1}{2}\Im_\perp(\Omega_x^2 + \Omega_y^2) \ , \qquad (11)$$

one might obtain the expression for the square of the rotation velocity in the plane (x,y):

$$\Omega_\perp{}^2 = (\Omega_x^2 + \Omega_y^2) = \frac{\hbar^2 R(R+1)}{\Im_\perp{}^2} \qquad (12)$$

This allows to obtain the expression:

$$\Omega_\perp = \sqrt{\Omega_x^2 + \Omega_y^2} = \frac{\hbar}{\Im_\perp}\sqrt{R(R+1)} = \frac{\hbar}{\Im_\perp}\sqrt{J(J+1) - K^2} \qquad (13)$$

which resembles the Eqs. (4)-(6). However the angular momentum of the collective rotation R is defined not by Eq. (6), but rather by the solution of the quadratic equation (9). The direction of this momentum in the (x,y) plane (and, therefore the direction of the rotation velocity vector) remains undefined and, in complete agreement with quantum mechanics, can not coincide with the y axis, as supposed in [6].

Thus the expressions (4)—(6) for the angular velocity which were used in ref.[6] are invalid both in classical and in quantum approaches. Eq. (5) for the polarization of the fissioning system does not take into account the specific features of the low-energy fission causing the existence of the K quantum number. The attempt to correct this drawback by the introduction of the quantity (6) is physically meaningless and leads to the contradictions both with classical and quantum mechanics.

3. **The correct quantum approach to the rotation of the polarized fissioning nucleus**

We had seen that quantum mechanics does not allow to define the eigenvalues of the two angular momentum projections $J_z=K$ and J_y at the same time, while our physical problem demands this. Strictly speaking, in quantum mechanics we can define only the eigenvalues of the square of the spin operator and of its projection on one given axis, but can not speak about the direction of the spin vector in space. However, by averaging over the ensemble of particles we can simultaneously define all the three components $P_i = \frac{<J_i>}{J}$ of the polarization vector with unlimited accuracy (for instance, the rotation angle of the neutron polarization was measured in [8], which was equal to 10^{-6} radians). The ensemble -averaged quantities $<J_i>$ are usually obtained with the aid of the density matrix $\rho^J_{MM'}$. Actually the average value <*A*> of any spin operator $\hat{A}$ for the system with the fixed spin J and its projection M can be defined as:

$$< A >= \sum_{MM'} \rho^{J}_{MM'} A_{M'M} = Sp(\rho A), \qquad (14)$$

where the matrix element $$A_{M'M} =< \Psi_{JM'} | \hat{A} | \Psi_{JM} > \qquad (15)$$

is defined with the aid of the system's wave function Ψ_{JM}, and

$Sp\rho = 1$

The Eq. (5) can be obtained in this way for the compound nucleus resonance with the fixed values of J and M, which appeared after the absorption of the polarized neutron. Mind that, as is well known, neutron resonances in deformed nuclei possess no definite value of the K quantum number, as a result of the complete K-mixing caused by the dynamical enhancement of the Coriolis interaction [9,10]. Therefore for isolated resonances, induced by polarized neutrons their decay into various neutron and gamma channels is governed by the factor (5). However for sufficiently low neutron energies the saddle- point in the fission channel selects only one-two values of K corresponding to the lowest collective excitations. The probability of this selection is defined by the quantity

$$| b^{J_s}_{sK_s} |^2 =| a^{J_s}_{sK_s} |^2 | c^{J_s}_{sK_s} |^2 \qquad (16)$$

where $| a^{J_s}_{sK_s} |^2$ is the probability to find the component K_s in the s-resonance wave – function. Due to the complete K-mixing $a^{J_s}_{sK_s}$ has random sign and the average value

$$< (a^{J_s}_{sK_s})^2 >= (2J_s + 1)^{-1} \ . \qquad (17)$$

$| c^{J_s}_{sK_s} |^2$ is the transmission probability of this K-value from the resonance *s* via the transition state with J_s, K_s to the scission point. One can assume that $| c^{J_s}_{sK_s} |^2 \approx 1$ for one fixed value of K_s and is approximately zero for all the other values.

Thus we obtain the polarized fissioning system with the fixed values of J, M and K , caused by the absorption of the polarized neutron. With our choice of coordinates the nuclear symmetry axis coincides with z, while the neutron beam is polarized along y-axis. Now we can find the averaged values of all the 3 components of the nuclear spin for a given value of K using the above equations (14), (15) and the known wave function Ψ^{J}_{MK} of the deformed fissioning nucleus (see e.g. [7]):

$$\Psi^{J}_{MK} = \sqrt{\frac{2J+1}{16\pi^2}} \left[D^{J}_{MK}(\omega)\Phi_K(q) + (-1)^{I+K} D^{J}_{M,-K}(\omega)\Phi_{\overline{K}}(q) \right] \qquad (18)$$

Using now the definition of the average spin projections on the *i=(x, y, z)* axes:

$$< J_i(K) >= \sum_{MM'} \rho^{J}_{MM'} < \Psi^{J}_{MK} | \hat{J}_i | \Psi^{J}_{MK} > \qquad (19)$$

and the expression for the density matrix (see [1]):

$$\rho^{J}_{MM'} = \frac{1}{2(2I+1)} \left[\delta_{MM'} + ip_n A(J,J)(C^{JM}_{J1M1} + C^{JM}_{J1M-1} \right] \qquad (20)$$

with

$$A(J,J) = \left(\sqrt{\frac{J}{2(J+1)}} \delta_{J,J_<} - \sqrt{\frac{J+1}{2J}} \delta_{J,J_>} \right) \qquad (21)$$

we obtain:

$$< J_x(K) >=< J_z(K) >= 0 \qquad , \qquad (22)$$

$$< J(K) > = < J_y(K) > = \frac{g_{KJJ}}{2} p_n \hbar \qquad (23)$$

Here

$$g_{K_s J_s J_s} = \frac{J_s(J_s+1) - I(I+1) + 3/4}{J_s} \cdot \frac{J_s(J_s+1) - K_s^2}{(J_s+1)} \qquad (24)$$

and $J_s = \begin{cases} I + 1/2 = J_> \\ I - 1/2 = J_< \end{cases}$, while I is the target spin.

Thus we obtained a result (see (22)-(23), which is quite unexpected from the classical point: Even for K≠0 the average projection $<J_z>$ (as well as $<J_x>$) equals to zero. This is the purely quantum result, caused by the fact that the wave-functions (18) contain the contributions of K and $(-K)$ with equal weights (i.e. the rotation eigenstates are always doubly-degenerate in the sign of K, as seen from (10)). The only non-zero component of the average spin vector is its projection on the y-axis (i.e. on the direction of the neutron beam polarization), which is perpendicular to the nuclear symmetry axis z. Therefore for the fixed K-value the total spin of the deformed fissioning nucleus is polarized along the direction of the neutron beam polarization and its polarization is given by:

$$P(J,K) = \frac{< J(K) >}{J} = \frac{g_{KJJ}}{2J} p_n \qquad (25)$$

Now we can estimate the angular velocity of the rotation around this axis using the expression:

$$\omega_q(J,K) = \frac{< J(K) >}{\Im}, \qquad (26)$$

which gives:

$$\omega_q(J_s,K_s) = \frac{g_{K_s J_s J_s} \cdot \hbar}{2\Im} \mathrm{p}_n = \begin{cases} \dfrac{J_s(J_s+1) - K^2}{J_s} \dfrac{\hbar}{2\Im} \cdot \mathrm{p}_n & for \quad J_s = I + 1/2 \\ \\ -\dfrac{J_s(J_s+1) - K^2}{(J_s+1)} \dfrac{\hbar}{2\Im} \cdot \mathrm{p}_n & for \quad J_s = I - 1/2 \end{cases} \qquad (27)$$

If several transition states contribute to fission (i.e. if several values $|c_{sK_s}^{J_s}|^2$ in (16) are non-zero), then:

$$\boldsymbol{\omega}_q^{eff} = \sum_{K_s} |b_{sK_s}^{J_s}|^2 \, \boldsymbol{\omega}_q(J_s,K_s) \qquad (28)$$

If all the K-values contribute equally (i.e. if all the coefficients $|b_{sK_s}^{J_s}|^2 = |a_{sK_s}^{J_s} c_{sK_s}^{J_s}|^2$ equal their average value $(2J_s+1)^{-1}$), then the polarization $P(J,K)$ of eq.(25) is averaged over all the K-values and

$$\frac{1}{(2J+1)} \sum_K P(J,K) = P(J) \quad , \qquad (29)$$

where $P(J)$ is defined by (5). This is exactly what applies to the non-fission channels of the neutron resonance decay. One can say that the use of Eq.(5) means averaging over all the possible directions of the deformed nucleus symmetry axis. However, if the coefficient $|c_{sK_s}^{J_s}|^2$ equals to 1 for only one K-value (as it often happens in fission channel), then one should use (25), (27) instead of (5), (4).

4. On the absolute values and signs of the TRI and ROT effects

The main physical difference between our approach and the one used in [6] lies in the fact that the rotation angular velocity vector is defined not by the angular momentum R whose direction can not be defined, but rather by the polarization vector $\vec{P}(J,K)$ of the fissioning nucleus whose all the three components are defined by Eqs. (22)—(25). Exactly this classical character of the polarization vector removes all the contradictions connected with the uncertainty relation (3) and allows to measure simultaneously the ensemble-averaged vector $<\vec{J}(K)>$ and the system's rotation angle in the plain perpendicular to it. As mentioned above, the rotation of the polarization vector can be measured with the precision higher than 10^{-6} radians. No wonder that we can measure the rotation angle of the polarized system with the precision higher than 0.1^0. Comparing our Eq. (27) for the angular velocity $\Omega_q(J,K)$ with the empirical Eqs. (4)—(6) used in ref. [6] we see that the ratio

$$\frac{\Omega_q(J,K)}{\Omega}=\frac{3}{2}\frac{\sqrt{J(J+1)-K^2}}{J+1} \qquad (30)$$

varies for J=3,4 in the interval from 0.5 to 1.3. Thus one might expect that in the case of the strong isolated resonance when the interference effects can be neglected, the results of the classical calculations of [6] might be correct to within this factor even if the K-values of the transition states are unknown or when the results of their definition by the different methods differ considerably, as in the case of ^{236}U fission [11].

Much more significant might be the purely quantum effects of the resonance interference which can not be taken into account in the classical approach. The unified quantum description of TRI and ROT effects [3,12] gives the following expression for the polarization dependent part of the ternary particle angular distribution:

$$\frac{d^2\sigma^{Cor}}{d\Omega_\alpha d\varepsilon}=\frac{p_n\pi}{2(2I+1)k_n^2}\sum_{ss'J_sJ_{s'}cK_s}|h_s^{J_s}||h_{s'}^{J_{s'}}|b_{sK_s}^{J_s}b_{s'K_{s'}}^{J_{s'}}\Gamma_{cK_s}\sqrt{(2J_s+1)(2J_{s'}+1)}\times$$

$$\times g_{K_sJ_sJ_{s'}}|A_c^0|\{\vec{\sigma}_n[\vec{\mathrm{k}}_{LF}\cdot\vec{\mathrm{k}}_\alpha]F_{odd}+\vec{\sigma}_n[\vec{\mathrm{k}}_{LF}\cdot\vec{\mathrm{k}}_\alpha](\vec{\mathrm{k}}_{LF}\cdot\vec{\mathrm{k}}_\alpha)F_{even}\} \quad , \qquad (31)$$

Here the factor $h_s^{J_s}=\dfrac{\sqrt{\Gamma_{sn}^{J_s}}}{E-E_s^{J_s}+i\Gamma_s^{J_s}/2}=|h_s^{J_s}|\exp\{i\delta_{sJ_s}\}$ (32)

shows the resonance energy-dependence of the cross-section and the fission width through the transition state (J_s, K_s) into the channel c:

$$\Gamma_{sJ_s,cK_s}=|b_{sK_s}^{J_s}|^2\Gamma_{cK_s} \qquad (33)$$

is defined by the above coefficients $b_{sK_s}^{J_s}=a_{sK_s}^{J_s}c_{sK_s}^{J_s}$ of Eq. (16).

The amplitudes $A^0(\theta,\varepsilon)$ of alpha – particles' angular and energy (ε) distribution for the ternary fission of the unpolarized nucleus is:

$$A_c{}^0(\theta,\varepsilon)=\sum_l|d_{cl}(\varepsilon)|\exp\{i\delta_{cl}(\varepsilon)\}Y_{l0}(\Omega_\alpha)=|A_c^0(\theta,\varepsilon)|\exp\{i\delta_c^0\},$$

while the functions F_{odd} and F_{even} define the contributions to (31) from the odd and even values of the alpha's angular momentum and therefore the values of TRI and ROT effects:

$$F_{odd} = | d_{c1}^{Cor}(\varepsilon) | a_1 \sin(\delta_{sJ_s s'J_{s'}} + \delta_{c1}^{Cor}(\varepsilon) - \delta_c^0(\theta,\varepsilon)) + + | d_{c3}^{Cor}(\varepsilon) | a_3 \sin(\delta_{sJ_s s'J_{s'}} + \delta_{c3}^{Cor}(\varepsilon) - \delta_c^0)(\mathrm{k}_{LF} \cdot \mathrm{k}_\alpha)^2 + ... \quad (34)$$

$$F_{even} = | d_{c2}^{Cor}(\varepsilon) | b_1 \sin(\delta_{sJ_s s'J_{s'}} + \delta_{c2}^{Cor}(\varepsilon) - \delta_c^0(\theta,\varepsilon)) + + | d_{c4}^{Cor}(\varepsilon) | b_2 \sin(\delta_{sJ_s s'J_{s'}} + \delta_{c4}^{Cor}(\varepsilon) - \delta_c^0)(\mathrm{k}_{LF} \cdot \mathrm{k}_\alpha)^2 + ... \quad (35)$$

One can see that Eq. (31) contains contributions not only from the terms with s=s' (which correspond to the isolated resonances) but also from the interference terms with s≠s'. The signs of these interference terms are random due to the random signs of the coefficients $b_{sK_s}^{J_s} = a_{sK_s}^{J_s} c_{sK_s}^{J_s}$, while the magnitudes of the "non-diagonal" quantities $g_{K_s J_s J_{s'}}$ differ from those of the "diagonal" ones (24), which define the angular velocities of the system's rotation for isolated resonances. Unfortunately the analysis carried out in [11] shows that the approximation of isolated resonances is invalid in the case of ^{236}U fission. The interference terms' influence is especially strong for the neutron energies between the resonances—exactly in the region where the experiments [4,5] were done.

Another important consequence of the resonance interference is the appearance of the additional phase shifts:

$$\delta_{sJ_s s'J_{s'}} = \delta_{sJ_s} - \delta_{s'J_{s'}} \quad (36)$$

in the $\sin$ arguments of the functions F_{odd} and F_{even} which define the relative magnitudes and signs of the TRI and ROT effects. The experimentally observed difference in the relative values and signs of the effects for two U isotopes should be explained by the difference of the quantities (36).

5. Summary

We have shown that the classical trajectory calculations [6] (especially if they are done with the correct quantum initial conditions (27), (28)) might describe the experimental ROT effect with good accuracy in the case of strong isolated resonances. We have also derived the formula for the polarization of the deformed compound-nucleus with the fixed values of the quantum numbers J and K. This new expression should be used in the description of the fission reaction induced by the low-energy polarized neutrons.

In the case of the strongly overlapping neutron resonances the influence of the quantum interference effects might be quite strong. Therefore the classical trajectory calculations might reproduce only the order of magnitude of the effect and could not define the correct relative sign and magnitude of the TRI and ROT effects.

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