ANALYTICAL CALCULATION OF THE NEUTRONS SPECTRUM FOR DIRECT MEASUREMENT OF N-N SCATTERING AT PULSED REACTOR YAGUAR

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Abstract

Analytical calculation of a single neutron detector counts per YAGUAR reactor pulse is presented and comparison with coincidence scheme is given.

1 Introduction

There is a project to measure directly n-n collision for checking charge symmetry of nuclear forces [1]. It is accepted that the best neutron source to perform such measurements is the Russian pulsed YAGUAR reactor. Some preliminary measurements and numerical simulations for expected experimental geometry had been performed [2]. We want to show here an analytical approach to calculations. First we obtain analytical momentum spectrum of scattered neutrons, then the time of flight spectrum of neutrons detected by a single counter. After that we consider coincidence scheme where we have two detectors, and calculate time of flight spectrum for one detector and delay time spectrum for the second one. We considered coincidence scheme because from the very beginning of discussions about the project, and all the time during preparation of the experiment, many people continue to express the opinion that the coincidence scheme has an advantage comparing to the single detector measurement. They claim that loss of intensity, which they usually estimated at the level of 20%, will be surpassed by much higher suppression of background. We show here analytically that in the coincidence scheme effect is so much suppressed, that the question about the background level becomes irrelevant.

2 Estimation of the effect

The scheme of the experiment is presented in Fig. 1 borrowed from [1]. The YAGUAR reactor 1 gives a pulse of length $t_p = 0.68$ ms, during which a huge amount of neutrons with flux density $\Phi = 0.77 \times 10^{18}$ n/cm^{2}s is released. After a moderator at room temperature T neutrons in the thermal Maxwellian spectrum arrive at the volume 2 ($V = 1.13$ cm^3), where they collide with each other and some of them after collision fly along the neutron guide 3 with collimators 4, and arrive at the detector 5, where they are registered with $\sim$ 100% efficiency. The collimators 4 determine the solid angle $\Delta\Omega = 0.64 \times 10^{-4}$, at which the volume V is visible by the detector. The estimated number of neutrons that can be registered at a single pulse is equal to

$$N_e = 2n^2 V t_p v_T |b|^2 d\Omega, \tag{1}$$

where factor 2 takes into account that the detector can register scattered neutron or neutron-scatterer. The square of the scattering amplitude $|b|^2$ is defined as: $|b|^2 = |b_0|^2/4$,

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where b_0 is the singlet scattering amplitude, which is accepted to be 18 fm, and factor 1/4 is statistical weight of the singlet scattering. Therefore $|b|^2 = 8.1 \times 10^{-25}$ cm^2. The speed v_T corresponds to the thermal speed $v_T = 2200$ m/s, and the factor $v_T|b|^2$ determines number of collisions in the neutron gas per unit time. The factor n^2 is the square of the neutron density: $n = \Phi/v_T = 3 \times 10^{12}$ cm^{-3}. After substitution of all the parameters into (1) we find $N_e \approx 170$ neutrons per pulse. However it is the estimation number. To find real number counted by the single detector, N_s, it is necessary to calculate the scattering process. Calculation shows that $N_s = FN_e$, where factor F is of the order unity. Monte Carlo calculations in [1] give $F = 0.83$. Analytical calculations presented below give $F = 0.705$. The number of neutrons per pulse counted at coincidence, if the neutrons trap 6 is replaced by another detector, can be estimated as

$$N_{ec} = N_s d\Omega\tau/t_T, \tag{2}$$

where τ is the width of the coincidence window, $t_T = L/v_T$ is the average length of measurement time after the reactor pulse, and $L \approx 12$ m is the average distance between collision volume and the detectors. In the experimental scheme of Fig. 1 the time t_T is of the order 5 ms. If we accept $\tau \approx t_p = 0.5$ ms, then the ratio τ/t_T is 0.1. The factor $d\Omega$ is included in (2), because only neutrons in this solid angle will be registered by the second detector. The total factor, which suppresses the estimated number of neutrons registered per single pulse in coincidence scheme, is of the order 10^{-5}, therefore the estimated number of counts in coincidence scheme will be 10^{-3}, so the experiment becomes non feasible, and the level of the background, which is determined by neutron scattering on the residual gas atoms present at even very good vacuum conditions, becomes irrelevant. The analytical calculations, presented below, show that the real number of counted neutrons in coincidence scheme contains even additional small factor $F_c = 0.15$.

3 The analytical calculation of neutron scattering in the thermal neutron gas

Our calculations will be based on the standard scattering theory of neutron scattering in the atomic gas. Our main feature is that we shall make calculations directly in the laboratory reference frame without transition to the center of mass system. First we remind all the definitions of the standard scattering theory and then present analytical calculations of all the required integrals.

3.1 The standard scattering theory

The standard scattering theory starts with the Fermi golden rule, according to which one can write down the probability of the neutron scattering per unit time on an arbitrary system as

$$dw(\boldsymbol{k}_i \to \boldsymbol{k}_f, \lambda_i \to \lambda_f) = \frac{2\pi}{\hbar} |\langle \lambda_f, \boldsymbol{k}_f | U | \lambda_i, \boldsymbol{k}_i \rangle|^2 \, \delta(E_{fk} + E_{f\lambda} - E_{ik} - E_{i\lambda}) \rho(E_{fk}), \tag{3}$$

where $|\boldsymbol{k}_i >$, $|\lambda_i >$ are initial, $|\boldsymbol{k}_f >$, $|\lambda_f >$ are final states of the neutron and system with energies E_{ik}, $E_{i\lambda}$, E_{fk}, $E_{f\lambda}$ respectively, U is the neutron-system interaction potential,

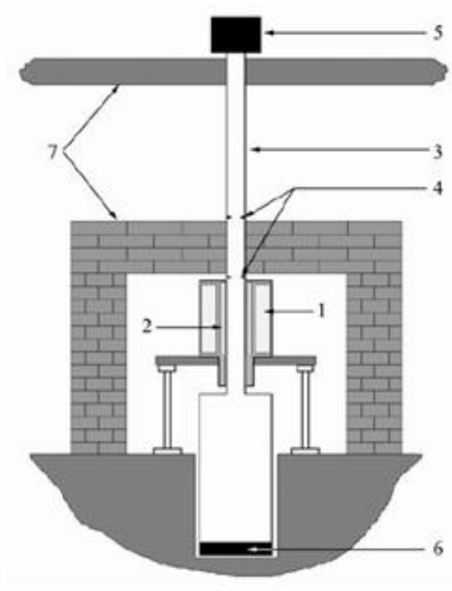

Figure 1: Scheme of the experiment on direct measurement of n-n scattering [1]. 1 — reactor core; 2 — volume of collisions; 3 — neutron guide; 4 — collimators; 5 — detector; 6 — neutrons trap.

which in the neutron atom scattering is accepted in the form of the Fermi pseudo potential

$$U = \frac{\hbar^2}{2m} 4\pi b \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2). \tag{4}$$

Here $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ are positions of the neutron and the system, $\rho(E_{fk})$ is the density of the neutron final states

$$\rho(E_k) = \left(\frac{L}{2\pi}\right)^3 d^3k, \tag{5}$$

$E_k = \hbar^2 k^2/2m$, m is the neutron mass, and L is the size of some arbitrary space cell.

We suppose that the system is an atom with mass $M = m$, and momentum $\boldsymbol{p}$. The initial and final states of the neutron and atom are described with similar wave functions

$$|\boldsymbol{k}_{i,f} >= \frac{1}{L^{3/2}} \exp(i\boldsymbol{k}_{i,f}\boldsymbol{r}), \qquad |\lambda_{i,f} >\equiv |\boldsymbol{p}_{i,f} >= \frac{1}{L^{3/2}} \exp(i\boldsymbol{p}_{i,f}\boldsymbol{r}), \tag{6}$$

where $\boldsymbol{k}_{i,f}$ and $\boldsymbol{p}_{i,f}$ are initial and final neutron and atom momenta respectively.

The flux density of the single incident neutron is

$$j_i = \hbar k_i / mL^3. \tag{7}$$

The scattering cross section at the given initial and final states is the ratio

$$d\sigma(\boldsymbol{k}_i \to \boldsymbol{k}_f, \boldsymbol{p}_i \to \boldsymbol{p}_f) = \frac{1}{j_i} dw(\boldsymbol{k}_i \to \boldsymbol{k}_f, \boldsymbol{p}_i \to \boldsymbol{p}_f). \tag{8}$$

At the next step we need to sum this cross section over final states of the system and average over initial states. In our case summation over the system final states is the integration over density of the atomic final states

$$\rho(E_{pf}) = \left(\frac{L}{2\pi}\right)^3 d^3p_f. \tag{9}$$

This integration gives the cross section for the given initial states as

$$d\sigma(\boldsymbol{k}_i \to \boldsymbol{k}_f, \boldsymbol{p}_i)$$

$$= 2\frac{2\pi m}{\hbar^2 k_i}\frac{L^9 d^3k_f}{(2\pi)^6}\int d^3p_f \left|\langle \boldsymbol{p}_f, \boldsymbol{k}_f|U|\boldsymbol{p}_i, \boldsymbol{k}_i\rangle\right|^2 \delta(E_{fk} + E_{fp} - E_{ik} - E_{ip}), \tag{10}$$

where $E_p = \hbar^2 p^2/2M$, $E_k = \hbar^2 k^2/2m$, and the additional factor 2 means that the atom and neutron are the same particles, therefore we can detect with the same probability the scattered neutron in the phase element d^3k_f or an atom in the element d^3p_f.

For our experiment we need not a cross section, but the number of the neutrons $dN(\boldsymbol{k}_i, \boldsymbol{p}_i, \boldsymbol{k}_f)$ scattered in the element d^3k_f. This number is determined by the number of collisions of neutrons with atoms, so the number of scattered neutrons is equal to

$$dN(\boldsymbol{k}_i, \boldsymbol{p}_i, \boldsymbol{k}_f) = dn_a(p_i)dn_n(k_i)V dt_p v d\sigma(p_i, \boldsymbol{k}_i \to \boldsymbol{k}_f), \tag{11}$$

where $dn_a(p_i)$, $dn_n(k_i)$ are the number densities of atoms and neutrons with initial momenta $\boldsymbol{p}_i$ and $\boldsymbol{k}_i$ respectively, $v = \hbar|\boldsymbol{p}_i - \boldsymbol{k}_i|/m$ is the relative neutron-atom velocity, and V, dt_p are volume and time, where collisions create detectable neutrons.

Since our atoms and neutrons have the same Maxwellian distribution with the temperature T, the densities $dn_a(p_i)$ and $dn_n(k_i)$ are

$$dn_a(\boldsymbol{q}) = dn_n(\boldsymbol{q}) = n\frac{d^3q}{(2\pi T)^{3/2}}\exp\left(-\frac{q^2}{2T}\right), \tag{12}$$

where n is the average neutrons density, the letter T denotes reduced temperature $T = mk_B[T]/\hbar^2$, and $[T]$ is the temperature in Kelvin degrees. To find the total number of neutrons $dN(\boldsymbol{k}_f)$ scattered into element d^3k_f of the final momentum space we must integrate (11) over $dn_a(p_i)dn_n(k_i)$, after which we get

$$dN(\boldsymbol{k}_f) = 2n^2 V dt_p \frac{1}{(2\pi T)^3}\frac{L^9 d^3k_f}{(2\pi)^6}\frac{2\pi}{\hbar}\frac{2m}{\hbar^2}\int d^3k_i \int d^3p_i \frac{|\boldsymbol{p}_i - \boldsymbol{k}_i|}{k_i}$$

$$\times \exp\left(-\frac{p_i^2 + k_i^2}{2T}\right)\int d^3p_f \left|\langle \boldsymbol{p}_f, \boldsymbol{k}_f|V|\boldsymbol{p}_i, \boldsymbol{k}_i\rangle\right|^2 \delta(k_f^2 + p_f^2 - k_i^2 - p_i^2). \tag{13}$$

The matrix element of the potential (4) is

$$\langle \boldsymbol{p}_f, \boldsymbol{k}_f|V|\boldsymbol{p}_i, \boldsymbol{k}_i\rangle = 4\pi b\frac{\hbar^2}{2m}\frac{(2\pi)^3}{L^6}\delta(\boldsymbol{p}_i + \boldsymbol{k}_i - \boldsymbol{p}_f - \boldsymbol{k}_f), \tag{14}$$

and its square is

$$\left|\langle \boldsymbol{p}_f, \boldsymbol{k}_f|V|\boldsymbol{p}_i, \boldsymbol{k}_i\rangle\right|^2 = |4\pi b|^2\left(\frac{\hbar^2}{2m}\right)^2\frac{(2\pi)^3}{L^9}\delta(\boldsymbol{p}_i + \boldsymbol{k}_i - \boldsymbol{p}_f - \boldsymbol{k}_f). \tag{15}$$

After substitution of (15) into (13) we can extract $|b|^2$ from the square of the matrix element, $d\Omega$ from d^3k_f and introduce the thermal speed $v_T = \hbar\sqrt{2T}/m$. As a result we obtain

$$dN(\boldsymbol{k}_f) = N_e g(\boldsymbol{k}_f)\frac{dk_f}{\sqrt{2T}}, \tag{16}$$

where N_e is given in (1), and $g(\boldsymbol{k}_f)$ is

$$g(\boldsymbol{k}_f) = \frac{2}{\pi^3}\frac{k_f^2}{(2T)^3}\int d^3k_i\int d^3p_i\frac{|\boldsymbol{p}_i-\boldsymbol{k}_i|}{k_i}$$

$$\times\int d^3p_f\exp\left(-\frac{p_f^2+k_f^2}{2T}\right)\delta(\boldsymbol{p}_i+\boldsymbol{k}_i-\boldsymbol{p}_f-\boldsymbol{k}_f)\delta(k_f^2+p_f^2-k_i^2-p_i^2). \tag{17}$$

Integration over d^3p_i gives

$$g(\boldsymbol{k}_f) = \frac{2}{\pi^3}\frac{k_f^2}{(2T)^3}\int d^3p_f\exp\left(-\frac{p_f^2+k_f^2}{2T}\right)\int d^3k_i\frac{|\boldsymbol{P}-2\boldsymbol{k}_i|}{k_i}\delta(k_f^2+p_f^2-k_i^2-(\boldsymbol{P}-\boldsymbol{k}_i)^2), \tag{18}$$

where $\boldsymbol{P}=\boldsymbol{p}_f+\boldsymbol{k}_f$ is the total momentum of two particles.

With all these definitions in hands we can directly calculate the spectrum of scattered neutrons

3.2 Analytical calculation of the integrals

First we calculate the integral

$$Q(\boldsymbol{k}_f,\boldsymbol{p}_f) = \int d^3k_i\frac{|\boldsymbol{P}-2\boldsymbol{k}_i|}{k_i}\delta(k_f^2+p_f^2-k_i^2-(\boldsymbol{P}-\boldsymbol{k}_i)^2)$$

$$= 2\int\frac{d^3k_i}{k_i}|2\boldsymbol{k}_i-\boldsymbol{P}|\delta((\boldsymbol{k}_f-\boldsymbol{p}_f)^2+(2\boldsymbol{k}_i-\boldsymbol{P})^2). \tag{19}$$

After change of variables $2\boldsymbol{k}_i-\boldsymbol{P}=\boldsymbol{u}$ we obtain

$$Q(\boldsymbol{k}_f,\boldsymbol{p}_f) = \frac{1}{2}\int u\frac{d^3u}{|\boldsymbol{u}+\boldsymbol{P}|}\delta(u^2-q^2), \tag{20}$$

where $q^2=(\boldsymbol{k}_f-\boldsymbol{p}_f)^2$.

After representation $ud^3u=(u^2du^2/2)d\varphi d\cos\theta$, where polar axis is chosen along the vector $\boldsymbol{P}$, we can integrate over $d\varphi$ and $d(u^2)$. As a result we get

$$Q(\boldsymbol{k}_f,\boldsymbol{p}_f) = \int_{-1}^{1}\frac{\pi q^2 d\cos\theta}{2\sqrt{q^2+2Pq\cos\theta+P^2}}. \tag{21}$$

Integration over $d\cos\theta$ gives

$$Q(\boldsymbol{k}_f,\boldsymbol{p}_f) = \frac{\pi q}{2P}(q+P-|q-P|). \tag{22}$$

The last factor is equal to $2q$, if $q<P$, and it is equal to $2P$, if $q>P$. Which one of these inequalities is satisfied depends on the angle θ_f between vectors $\boldsymbol{k}_f$ and $\boldsymbol{p}_f$. Inequality $q<P$ is satisfied, when $\cos\theta_f>0$, and inequality $q>P$ is satisfied, when $\cos\theta_f<0$. Therefore Eq. (22) is representable in the form

$$Q(\boldsymbol{k}_f,\boldsymbol{p}_f) = \pi q\left(\Theta(\cos\theta_f<0)+\Theta(\cos\theta_f>0)\frac{q}{P}\right), \tag{23}$$

where $\Theta(x)$ is the step function equal to unity, when inequality in its argument is satisfied, and to zero in the opposite case.

3.3 The spectrum of neutrons, counted by a single detector

Substitution of (23) into (18) gives

$$g(\boldsymbol{k}_f) = \int d^3p_f w(\boldsymbol{k}_f, \boldsymbol{p}_f), \tag{24}$$

where

$$w(\boldsymbol{k}_f, \boldsymbol{p}_f) = \frac{2}{\pi^3}\frac{k_f^2}{(2T)^3}\exp\left(-\frac{p_f^2+k_f^2}{2T}\right)Q(\boldsymbol{k}_f, \boldsymbol{p}_f). \tag{25}$$

To obtain spectrum of neutrons counted by a single detector we represent $d^3p_f = p_f^2 dp_f d\Omega_f$, and integrate $Q(\boldsymbol{k}, \boldsymbol{p})$ over $d\Omega_f$. As a result we obtain (in the following we omit subscripts f of variables)

$$I(k,p) = \int Q(\boldsymbol{k}, \boldsymbol{p}) d\Omega = 2\pi^2\left(\int_{-1}^{0} d\cos\theta|\boldsymbol{k}-\boldsymbol{p}| + \int_0^1 d\cos\theta\frac{(\boldsymbol{k}-\boldsymbol{p})^2}{|\boldsymbol{k}+\boldsymbol{p}|}\right)$$

$$= 2\pi^2\left(\int_0^1 d\cos\theta|\boldsymbol{k}+\boldsymbol{p}| + \int_0^1 d\cos\theta\left[\frac{2(k^2+p^2)}{|\boldsymbol{k}+\boldsymbol{p}|} - |\boldsymbol{k}+\boldsymbol{p}|\right]\right)$$

$$= \frac{(2\pi)^2}{pk}(p^2+k^2)(p+k-\sqrt{p^2+k^2}). \tag{26}$$

Substitution of (26) into (25) and change of variables $x = p/k$, $y = k/\sqrt{2T}$ gives

$$g(\boldsymbol{k}_f) \equiv f(y) = \frac{2}{\pi}\exp(-y^2)y^2 J(y), \tag{27}$$

where

$$J(y) = 2y^4\int_0^\infty 2x dx \exp(-x^2y^2)(1+x^2)\left[(x+1)-\sqrt{x^2+1}\right]. \tag{28}$$

Integration by parts gives

$$J(y) = 2y^2\int_0^\infty dx \exp(-x^2y^2)(1+2x+3x^2-3x\sqrt{x^2+1}) = y\sqrt{\pi} + J_1(y), \tag{29}$$

$$J_1(y) = 2y^2\int_0^\infty x dx \exp(-x^2y^2)(2+3x-3\sqrt{x^2+1}) = -1+3\frac{\sqrt{\pi}}{2y}\{1-e^{y^2}[1-\Phi(y)]\}, \tag{30}$$

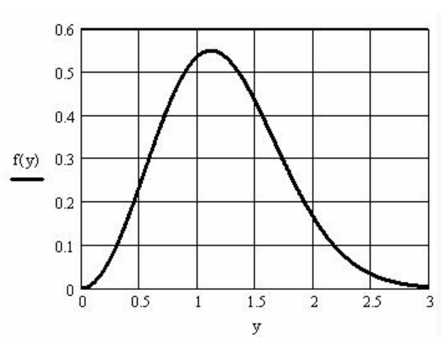

Figure 2: Spectrum $y^2F(y)\exp(-y^2)$ of the neutrons detected by a single detector in dimensionless units $y = v/v_T$.

where $\Phi(y) = \int_0^y \exp(-x^2)2dx/\sqrt{\pi}$. Substitution of (30) into (29) gives

$$J(y) = y\sqrt{\pi} - 1 + 3\frac{\sqrt{\pi}}{2y}\{1 - e^{y^2}[1 - \Phi(y)]\}. \tag{31}$$

The momentum spectrum $f(y)$ from Eq. (27) with account of (31) is shown in Fig. 2. Numerical integration of this function gives $F = \int_0^\infty f(y)dy = 0.705$.

3.4 Time of flight spectrum of a single detector

In the experiment the time of flight (TOF) spectrum is measured. To transform (27) into TOF spectrum we multiply it by unity

$$1 = dt\delta(t - L/v_T y), \tag{32}$$

where L is the distance between scattering volume and the detector, and integrate over dy. After that we obtain

$$\dot{N}_s(y(t)) = f(L/v_T t)\frac{L}{v_T t^2} = f(y)\frac{y}{t}. \tag{33}$$

4 Registration by two detectors in coincidence

Let's consider the case, when neutrons are registered in coincidence by two detectors on the opposite sides of the collision volume. It means that the angle between $\boldsymbol{k}_f$ and $\boldsymbol{p}_f$ is approximately 180°. Since we register both neutrons, we should not integrate (23) over d^3p_f. Instead we should accept $\boldsymbol{k}_f\boldsymbol{p}_f < 0$, $\boldsymbol{p}_f \approx -\boldsymbol{k}_f$, and $d^3p_f = p_f^2 dp_f d\Omega$ with the same $d\Omega$ as in d^3k_f. Taking into account Eq. (16), (24) and (25) we can represent the number of neutrons counted by two detectors as

$$dN(\boldsymbol{k}_f, \boldsymbol{p}_f) = N_e d\Omega_{pf}\frac{2}{\pi^2}\frac{k_f^2 dk_f}{(2T)^{7/2}} q p_f^2 dp_f \exp\left(-\frac{p_f^2 + k_f^2}{2T}\right). \tag{34}$$

After transformation to dimensionless variables $y = k_f/\sqrt{2T}$ and $z = p_f/\sqrt{2T}$ we get

$$dN(\boldsymbol{k}_f, \boldsymbol{p}_f) = N_e d\Omega_p G(y, z)dydz, \tag{35}$$

where $G(y, z) = \exp(-y^2 - z^2)2y^2z^2(y + z)/\pi^2$, and we replaced q by $k_f + p_f$.

To get TOF spectrum in one detector and coincidence count in the second one with coincidence window τ we must multiply (35) by the unit $1 = dt\delta(t - L/v_T y)dt'\delta(t' - L/v_T z + t)$ and integrate over $dydz$. As a result we obtain

$$\dot{N}_c \equiv dN(\boldsymbol{k}_f, \boldsymbol{p}_f)/dt = N_0 d\Omega_p G\left(\frac{L}{v_T t}, \frac{L}{v_T(t + t')}\right)\frac{dt' L^2}{v_T^2 t^2 (t + t')^2}. \tag{36}$$

After integration over dt' in the range of the coincidence window τ we can put $z \approx y$, and finally get

$$\dot{N}_c \approx N_0 d\Omega_p \frac{4y^7}{\pi^2}\exp(-2y^2)\frac{\tau}{t^2}. \tag{37}$$

For comparison of TOF spectrum of two and single detectors it is useful to find ratio of (37) to (33). This ratio is

$$W = \frac{\dot{N}_c}{\dot{N}_s} = d\Omega_p \frac{\tau}{t} R(y), \tag{38}$$

where $R(y) = \frac{4y^6}{\pi^2 f(y)} \exp(-2y^2) 4y^6/\pi^2 f(y)$ is shown in Fig. 3. Its integral $\int dy R(y)$ is equal to 0.15. So we can tell that the ratio is approximately $W \approx 0.1 d\Omega\tau/t$, as is said in section 2.

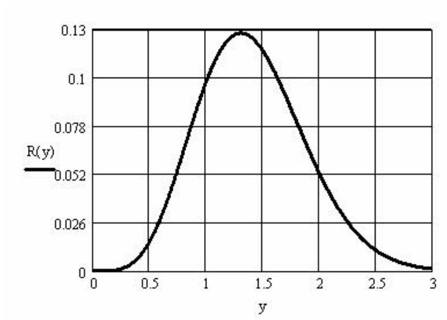

Figure 3: Dependence of $R(y)$ on $y = k/\sqrt{2mT}$.

5 Conclusion

We have shown that the effect of n-n scattering experiment and spectrum of detected neutrons in a single detector can be calculated analytically with the standard scattering theory without transformation to center of mass system. Analytically calculated factor $F = 0.705$ is close to that $F = 0.83$, calculated by Monte Carlo method. The difference can be attributed to slightly different spectra of neutrons in the collision volume. In Monte Carlo calculations spectrum contained Maxwellian part and epithermal tail, while for analytical calculations we used only Maxwellian part. We did not calculated background which is related to scattering of neutrons on gas molecules, but we claim that it also can be calculated analytically. One of the main conclusions of this paper is that coincidence scheme for this type of experiment is absolutely impractical, because the effect becomes so low, that the level of the background is irrelevant.

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