Constraints on extra-short-range interactions from neutron scattering experiments

V. V. Nesvizhevsky*
Institute Laue Langevin, 6 rue Jules Horowitz, F-38042, Grenoble, France

G. Pignol† and K. V. Protasov‡
Laboratoire de Physique Subatomique et de Cosmologie,
UJF - CNRS/IN2P3 - INPG, 53 Av. des Martyrs, Grenoble, France
(Dated: September 4, 2008)

The available data on neutron scattering were reviewed to constrain a hypothetical new short-range interaction. We show that these constraints are several orders of magnitude better than those usually cited in the range between 1 pm and 5 nm. This distance range occupies an intermediate space between collider searches for strongly coupled heavy bosons and searches for new weak macroscopic forces. We emphasise the reliability of the neutron contraints in so far as they provide several independent strategies. We have identified a promising way to improve them.

PACS numbers: 03.75.Dg, 03.75.Be, 13.40.Gp

I. INTRODUCTION

The existence of other forces in nature, mediated by new bosons, has been extensively discussed in the literature, given their possibility in many of the extensions to the standard model of particle physics [1]. New bosons for example are predicted by most of the Grand Unified Theories embedding the standard model, with a coupling constant of $\approx 10^{-1}$. These strongly coupled bosons would have to be heavier than ≈ 1 TeV if they were not to conflict with present observations; heavier bosons will be searched for at the Large Hadron Collider. Lighter bosons could however have remained unnoticed, provided they interact weakly with matter. Such bosons would mediate a finite range force between two fermions:

$$V(r) = Q_1 Q_2 \, \frac{g^2}{4\pi} \frac{\hbar c}{r} e^{-r/\lambda} \tag{1}$$

where g is the coupling constant, Q_1 and Q_2 the charges of the fermions under the new interaction, and the range of this Yukawa-like potential $\lambda = \frac{\hbar}{Mc}$ is inversely proportional to the boson mass M. In the following we consider the interactions of neutrons with nuclei of atomic number A: the charge of the atom under the new interaction is equal $Q_1 = A$; the neutron charge is equal unity $Q_2 = 1$. A new boson could even be massless, as has been suggested by Lee and Yang [2] well before the birth of the standard model, to explain the conservation of the baryon number. This additional massless boson would mediate a new infinite-range force, and could be seen in searches for violation of the equivalence principle at large distances. The presence of very light bosons ($M \ll 1$ eV) would be shown by deviations from the gravitational inverse square law. Gravity has been probed down to distances of 0.1 mm [3]; new bosons lighter than 2×10^{-3} eV must thus have a coupling constant lower than the gravity strength between nucleons, $g^2 < 10^{-37}$.

Theories with extra large spatial dimensions [4–9] provide strong motivation to search for such forces. If a boson is allowed to travel in large extra-dimensions, with a strong coupling constant in the bulk, it behaves in our 4D world as a very weakly coupled new boson, the coupling being diluted in the extra-dimensions. The light dark matter hypothesis also argues in favour of the existence of new short range interactions [10].

While gravity experiments are most competitive in the distance range > 10 μm, the measurements of the Casimir or Van der Waals forces (for a review, see e.g. [11]) give the best constraints in the nanometer range (10 nm $< \lambda <$ 10 μm), and antiprotonic atoms constrain the domain below 1 pm [12, 13], it has been suggested that experiments with neutrons could be competitive in the intermediate range [13–18]. Neutrons could also probe spin-dependent interactions in a wider distance range [19], or spin-independent interactions in the range of several micrometers [18, 20, 21].

In this contribution we give the quantitative constraints on the parameters of the additional interaction, λ and g using the existing data on neutron scattering at nuclei. A detailed analysis is presented in [22].

II. SLOW NEUTRON / NUCLEI INTERACTION WITH EXTRA-SHORT-RANGE INTERACTIONS

The scattering of slow neutrons on atoms is described by the scattering amplitude $f(\mathbf{q})$; this can be represented by a sum of a few terms [23]:

$$f(\mathbf{q}) = f_{\text{nucl}}(\mathbf{q}) + f_{ne}(\mathbf{q}) + f_V(\mathbf{q}) \tag{2}$$

The first and the most important term represents the scattering due to the nuclear neutron-nucleus interaction. At low energies discussed in this article, it is isotropic and

*Electronic address: nesvizhevsky@ill.eu
†Electronic address: pignol@lpsc.in2p3.fr
‡Electronic address: protasov@lpsc.in2p3.fr

energy-independant, because the nuclear radius is much smaller than the wavelegth of slow neutrons:

$$f_{\rm nucl}(\mathbf{q}) = -b. \quad (3)$$

The coherent scattering lenght b is the fundamental parameter describing the interaction of slow neutrons with a nucleus [24].

The second term is the amplitude of so-called electron-neutron scattering due to the interaction of the neutron charge distribution with the nucleus charge and the electron cloud. This amplitude can be written as

$$f_{ne}(\mathbf{q}) = -b_{ne}(Z - f(Z, \mathbf{q})), \quad (4)$$

where $f(Z, \mathbf{q})$ is the atomic form-factor measured in the X-rays experiments and b_{ne} is a constant called the electron-neutron scattering length, which is directly related to the neutron charge radius [23] and to the neutron electromagnetic form-factor $G_E(\mathbf{q}^2)$ by

$$b_{ne} = -\frac{2}{a_0}\frac{m}{m_e}\frac{dG_E(\mathbf{q}^2)}{d\mathbf{q}^2}\bigg|_{\mathbf{q}^2=0}, \quad (5)$$

m and m_e being the neutron and electron masses, a_0 the Bohr radius. This contribution to the total scattering amplitude is as small as a per cent for heavy nuclei.

In the presence of a new interaction (1), the scattering for a center of mass momentum $\hbar k$ due to the extra interaction, within the Born approximation, is given by

$$f_V(\theta) = -A\frac{g^2}{4\pi}\hbar c\frac{2m\lambda^2/\hbar^2}{1+(q\lambda)^2} \quad (6)$$

where $q = 2k\sin(\theta/2)$, θ is the scattering angle.

Any other possible contributions to the scattering amplitude $f(\mathbf{q})$, due to non zero nuclear radius, nucleon polarizability, etc. are very small in the energy range discussed here [23].

The nuclear scattering lengths are measured for almost all stable nuclei, using a variety of methods. A review of the different methods and a complete table of the measured scattering lengths can be found in [25]. We can distinguish two classes of method, with different sensitivities to a new interaction.

The first class – including the interference method, the total reflection method, the gravity refractometer method – measures the forward scattering amplitude $f(\mathbf{q} = 0)$. These methods actually measure the mean optical potential of a given material, called the Fermi potential, due to the coherent scattering of neutrons at many nuclei. The Fermi potential is related to the *forward* scattering amplitude.

In the presence of the new force, the measured scattering lenght can be separated into a nuclear and an additional term

$$b_{\rm opt} = -f(\mathbf{q} = 0) = b + A\frac{mc^2}{2\pi\hbar c}\, g^2\lambda^2 \quad (7)$$

The second class of method – including the Bragg diffraction method and the transmission method – uses non-zero transferred momentum. In the Bragg diffraction method, the scattering amplitude for a momentum transfer of $q_{\rm BD} = 10\ {\rm nm}^{-1}$ is measured. One actually extracts, besides the nuclear term, an extra contribution according to (6)

$$b_{\rm BD} = b + A\frac{mc^2}{2\pi\hbar c}\, g^2\frac{\lambda^2}{1+(q_{\rm BD}\lambda)^2} \quad (8)$$

In the case of the transmission method, the total cross-section is measured. Generally, neutrons with energies of about 1 eV are used; they are much faster than slow neutrons, and no coherent scattering can be observed. An additional interaction would manifest itself by an energy dependance of the extracted scattering length

$$b_{\rm TR}(k^2) = \sqrt{\frac{\sigma_{\rm tot}}{4\pi}} = b + A\frac{mc^2}{2\pi\hbar c}\, g^2\lambda^2\frac{\ln(1+4(k\lambda)^2)}{4(k\lambda)^2} \quad (9)$$

Finally, we should also mention the very popular Christiansen filter technique; this measures *relative* scattering lengths, so we do not consider this data.

III. RANDOM POTENTIAL NUCLEAR MODEL

A simple and robust limit on the additional Yukawa forces can be easily obtained by neglecting the small term due to the neutron-electron scattering and by studing the general A-dependence of the scattering amplitude. In the domain of $\lambda \leq 1/q_{\rm BD}$, the optical and Bragg diffraction methods are sensitive to the same amplitude

$$b_{\rm Meas} = -f(\mathbf{q} = 0) = b + A\frac{mc^2}{2\pi\hbar c}\, g^2\lambda^2 \quad (10)$$

as clear from (7) and (8). The presence of additional forces would be apparent from the linear increase of the measured scattering length as a function of A in addition to the A-dependence of the nuclear scattering length.

There exists a simple and elegant semi-phenomenological approach that describes the nuclear dependence [27]. It assumes that a nucleus can be presented as an attractive "square well" potential, with radius $RA^{1/3}$ and depth V_0 for slow neutrons. The scattering length would then be equal to

$$b(A) = RA^{1/3}\left(1 - \frac{\tan(X)}{X}\right), \quad (11)$$

where $X = \frac{RA^{1/3}}{\hbar}\sqrt{2mV_0}$ is supposed to be a random variable distributed uniformly over the range $[\pi/2, 5\pi/2]$; the lower value corresponds to the appearance of a bound state and the upper limit is set sufficiently large not to influence the results of the present analysis; more details can be found in [27].

This model describes well the distribution of all experimental data; the value of the only free parameter in this model is estimated to be $R = 1.44 \pm 0.05$ fm at the 68 % C.L. The likelihood function at its maximum satisfies $\ln(L) = -254$ for 216 degrees of freedom.

With a short-range new interaction included in the analysis we have to consider the random variable

$$b_{\rm Meas} = RA^{1/3}\left(1 - \frac{\tan(X)}{X}\right) + b_{\rm Extra}\, A. \qquad (12)$$

where the effect of the extra interaction is the slope $b_{\rm Extra} = \frac{mc^2}{2\pi\hbar c}\, g^2\lambda^2$ of the linear term. The linear term is compatible with zero, as expected. We thus obtain a quantitative constraint for the coupling $g(\lambda)$ [22]:

$$g^2\lambda^2 \leq 0.016\ {\rm fm}^2 \quad \text{at 95\% C.L.} \qquad (13)$$

This result is presented in fig. 1 for the distance range of interest, $10^{-12} - 10^{-10}$ m.

IV. CONSTRAINT FROM COMPARAISON OF FORWARD AND BACKWARD SCATTERING OF NEUTRONS

Another way to constrain on aditional Yukawa forces consists in comparing the scattering lengths measured by different methods.

As explained above, the scattering lengths measured using the Bragg diffraction method $b_{\rm BD}$ and the interference method $b_{\rm opt}$ do not show the same sensitivity to a new short-range interaction. According to (7) and (8), the ratio of the two values should deviate from unity in the presence of an additional interaction

$$\frac{b_{\rm opt}}{b_{\rm BD}} \approx 1 + \frac{A}{b}\,\frac{mc^2}{2\pi\hbar c}\, g^2\,\lambda^2\,\frac{(q\lambda)^2}{1+(q\lambda)^2} \qquad (14)$$

We found a set of 13 nuclei for which both measurements exist. Taking into account systematic errors in those experiments as described in [22], we obtain the constraint

$$g^2\lambda^2 \frac{(q\lambda)^2}{1+(q\lambda)^2} \leq 0.0013\ {\rm fm}^2 \quad \text{at 95\% C.L.} \qquad (15)$$

corresponding to the bold limit in fig. 1.

V. ELECTROMAGNETIC EFFECTS

Up to now, the amplitude due to a new additional interaction $f_V(\mathbf{q})$ has been compared to the nuclear one $f_{\rm nucl}(\mathbf{q})$ (see (2)). One could compare it to a smaller amplitude due to an electromagnetic interaction ($f_{ne}(\mathbf{q})$). This idea was first proposed in ref. [14].

One could repeat the previous analysis using measurements of the total cross-section at energies of ≈ 1 eV ($1/k = 5$ pm) instead of the Bragg diffraction. If the range of a new interaction is larger than 1 pm, the scattering length extracted would be free of any extra contribution. However, the residual electromagnetic effects due to the neutron square charge radius can mimick in this case an extra-force contribution in the quantity $b(1\ {\rm eV}) - b_{\rm opt}$, as this contribution is energy-dependent and proportional to the charge number of the atoms. The extracted difference $b(1\ {\rm eV}) - b_{\rm opt}$ therefore contains the two contributions:

$$\begin{aligned} b(1\ {\rm eV}) - b(0) &= Zb_{ne} \qquad (16)\\ &- A\frac{mc^2}{2\pi\hbar c}\, g^2\lambda^2 \left(1 - \frac{\ln(1+4(\frac{\lambda}{5\ {\rm pm}})^2)}{4(\frac{\lambda}{5\ {\rm pm}})^2}\right) \end{aligned}$$

Unfortunately, there is very clear disagreement between the two groups of values for $b_{ne}^{\rm exp} = \frac{b(1\ {\rm eV})-b(0)}{Z}$ known as the Garching-Argonne and Dubna values [28]

$$\begin{aligned} b_{ne}^{\rm exp} &= (-1.31 \pm 0.03)\times 10^{-3}\ {\rm fm}\ [\text{Gartching-Argonne}]\\ b_{ne}^{\rm exp} &= (-1.59 \pm 0.04)\times 10^{-3}\ {\rm fm}\ [\text{Dubna}] \qquad (17) \end{aligned}$$

The discrepancy is much greater than the quoted uncertainties of the experiments and there evidently an unaccounted for systematic error in at least one of the experiments.

In order to overcome this difficulty we could determine b_{ne} from the experimental data on the neutron form factor (5). The simplest way to do this consists in using a commonly accepted general parametrization of the neutron form factor [29]:

$$G_E(\mathbf{q}^2) = -a\mu_n \frac{\tau}{1+b\tau} G_D, \qquad (18)$$

where $\mu_n = -1.91\mu_B$ is the neutron anomalous magnetic moment, $\tau = \mathbf{q}^2/4m^2$ and

$$G_D(\mathbf{q}^2) = \frac{1}{(1+\mathbf{q}^2/0.71\ ({\rm GeV/c})^2)^2}, \qquad (19)$$

is so-called dipole form factor ; a and b being fitting parameters.

A fit of an existing set of the neutron form factor experimental data [30] yields the following values for the parameters:

$$\begin{aligned} a &= (0.77 \pm 0.06)\\ b &= (2.18 \pm 0.58) \end{aligned}$$

with $\chi^2/{\rm NDF} = 15.3/27$. The b_{ne} determined in this way is

$$b_{ne} = (-1.13 \pm 0.08)\times 10^{-3}\ {\rm fm}. \qquad (20)$$

Our principal conclusion consists in the observation of (underestimated) systematical uncertainties in the presented experiments. Therefore a single experiment/method can not be used for any reliable constraint. A conservative estimate of the precision of the b_{ne} value could be obtained from analysing the discrepancies in the results obtained by different methods; it is equal to

$\Delta b_{ne} \leqslant 6 \times 10^{-4}$ fm. The corresponding contraint at the 2σ level [22]

$$\frac{mc^2}{2\pi\hbar c}\, g^2\lambda^2 \left(1 - \frac{\ln(1+4(\frac{\lambda}{5\ \mathrm{pm}})^2)}{4(\frac{\lambda}{5\ \mathrm{pm}})^2}\right) \leqslant \Delta b_{ne} \qquad (21)$$

is represented by the dot-dashed line in fig. 1 and 2.

VI. ASYMMETRY OF SCATTERING

As is clear from fig. 1, the best constraint was obtained from the analysis of the energy dependence of the neutron scattering lengths in the b_{ne} measurements inspite of systematic errors in these experiments. However, the precision here is limited by the correction for the b_{ne} value itself. An obvious proposal for improving this constraint would be to set up experimental conditions free of the b_{ne} contribution. This is indeed possible, because neutron-electron scattering is essential for fast neutrons only, and is absent for slow neutrons.

We propose improving the experiment [26] and measuring the forward-backward asymmetry of the scattering of neutrons at atoms of noble gases, in the following way: the initial velocity of the neutrons should correspond to the range of very cold neutrons (VCN); the double differential measurement of neutron velocity before/after scattering should be used to calculate the transferred momentum for every collision.

The measurement described above could provide an accuracy of at least 10^{-3} for the ratio of forward to backward scattering probabilities and a corresponding constraint for the additional short-range interaction shown in fig. 1. The relative drop in sensitivity at a few times 10^{-11} m is due to the appearance of neutron electron scattering; the range of interest for this possible constraint is $10^{-11} - 10^{-8}$ m.

VII. CONCLUSION

We analysed the constraints for extra short-range interactions on the basis of the existing data on neutron scattering. These constraints are several orders of magnitude better than those usually cited in the range between 1 pm and 5 nm. The reliability of these constraints was supported by the application of several independant methods with comparable accuracy, as well as by the use of a major fraction of known neutron scattering lengths and treatment of the data in a most conservative way. One constraint obtained within the random potential nuclear model was based on the absence of an additional linear term in the mass dependance of the neutron scattering lengths. It would be difficult to improve this constraint in either experimental or theoretical terms. Another constraint was derived by comparing two types of neutron scattering experiments with different sensitivities to the extra short-range interactions. These

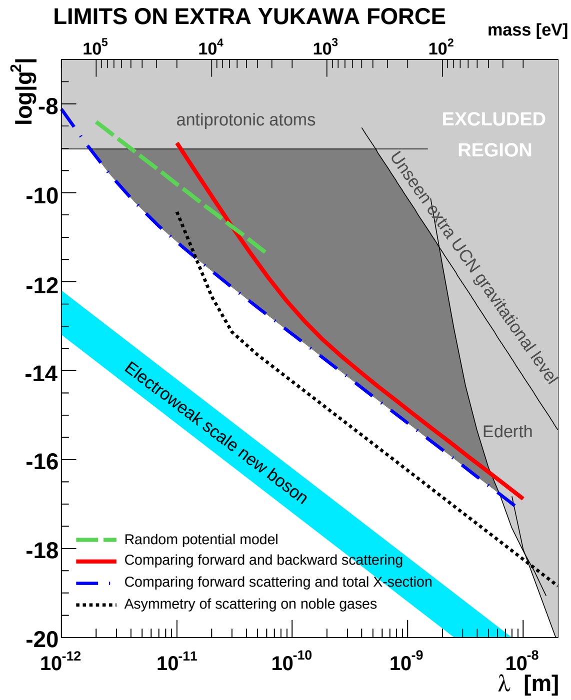

FIG. 1: The shaded regions correspond to current experimental limits on extra Yukawa interaction. It includes constraint at 95 % C.L. (dashed, dot-dashed and bold lines) obtained in this article, and the existing constraints [11, 13]. The dotted line is an estimation of the sensitivity of proposed experiment.

are interference experiments measuring forward neutron scattering and the Bragg diffraction. The accuracy here is limited by the relatively poor precision of the Bragg scattering technique. Significant improvements in the accuracy of such experiments would be particularly interesting. Further constraints were estimated using the energy-dependence of the neutron scattering lengths at heavy nuclei. They are limited by the precision of our knowledge of the neutron-electron scattering length. An elegant method for further improving such constraints would consist in achieving experimental conditions free of b_{ne} contribution. This is indeed possible, given that neutron-electron scattering is essential for fast neutrons only. The experiment would consist in scattering very cold neutrons at rare noble gases and in measuring precisely the differential asymmetry of such scattering as a function of the transferred momentum.

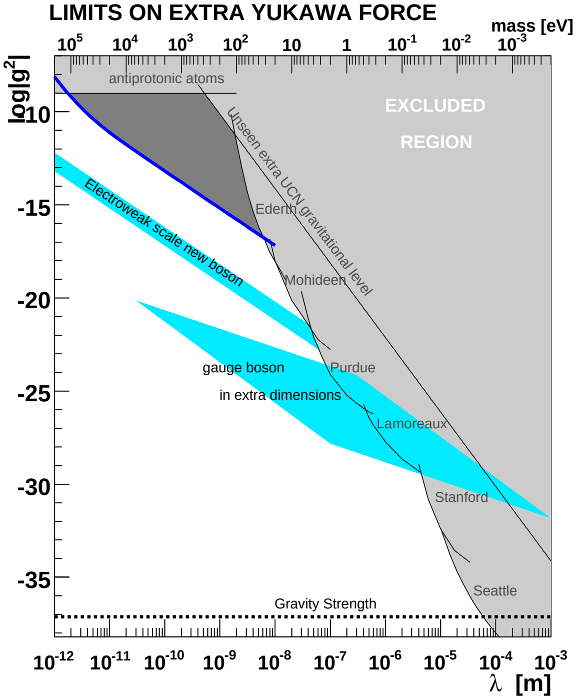

FIG. 2: Experimental limits on extra interactions including the best neutron constraint obtained in this article (bold line). Two theoretical regions of interrest are shown: new boson with mass induced by electroweak symmetry breaking [10], and new boson in extra large dimensions [4].

[1] W.-M. Yao *et al.*, J. Phys. G **33**, 1 (2006).
[2] T. D. Lee and C. N. Yang, Phys. Rev. **98**, 1501 (1955).
[3] C. D. Hoyle *et al.*, Phys. Rev. D **70**, 042004 (2004); D. J. Kapner *et al.*, Phys. Rev. Lett. **98**, 021101 (2007).
[4] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999).
[5] I. Antoniadis *et al.*, Phys. Lett. B **436**, 257 (1998).
[6] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125**, 136 (1983); Phys. Lett. B **125**, 139 (1983).
[7] M. Visser, Phys. Lett. B **159**, 22 (1985).
[8] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).
[9] J. D. Lykken, Phys. Rev. D **54**, 3693 (1996).
[10] P. Fayet, Phys. Rev. D **75** 115017 (2007); P. Fayet, arXiv:hep-ph/0111282.
[11] M. Bordag, U. Mohideen and V. M. Mostepanenko, Phys. Rept. **353**, 1 (2001).
[12] M. Hori *et al.*, Phys. Rev. Lett. **91**, 123401 (2003).
[13] V. V. Nesvizhevsky and K. V. Protasov, Class. Quant. Grav. **21**, 4557 (2004).
[14] H. Leeb and J. Schmiedmayer, Phys. Rev. Lett. **68**, 1472 (1992).
[15] A. Frank, P. van Isacker and J. Gomez-Camacho, Phys. Lett. B **582**, 15 (2004).
[16] P. J. S. Watson, arXiv:hep-ph/0406308.
[17] G. L. Greene and V. Gudkov, Phys. Rev. C **75**, 015501 (2007).
[18] V. V. Nesvizhevsky, G. Pignol and K. V. Protasov, Proceedings of the 42th Rencontres de Moriond (2007).
[19] S. Baeßler, V. V. Nesvizhevsky, K. V. Protasov and A. Y. Voronin, Phys. Rev. D **75**, 075006 (2007).
[20] O. Bertolami and F. M. Nunes, Class. Quant. Grav. **20**, L61 (2003).
[21] V. V. Nesvizhevsky *et al.*, Nature **415** 297 (2002).
[22] V. V. Nesvizhevsky, G. Pignol and K. V. Protassov, Phys. Rev. D **77**, 034020 (2008).
[23] V.F. Sears, Phys. Reports **141**, 281 (1986).
[24] E. Fermi, L. Marshall, Phys. Rev. **71**, 666 (1947).
[25] L. Koester, H. Rauch and E. Seymann, Atomic Data and Nuclear Data Tables **49**, 65 (1991).
[26] V. E. Krohn and G. R. Ringo, Phys. Rev. **148**, 1303 (1966).
[27] M. Peskhin and G. R. Ringo, Am. J. Phys. **39**, 324 (1971).
[28] S. Kopecky, J. A. Harvey, N. W. Hill, M. Krenn, M. Pernicka, P. Riehs, and S. Steiner, Phys. Rev. **C56**, 2229 (1997).
[29] S. Glaster *et al.*, Nucl. Phys. B **32**, 221 (1971); A. F. Krutov and V. E. Troitsky, Eur. Phys. J. A **16**, 285 (2003).
[30] `http://www.jlab.org/~cseely/formfactor_data.txt`