RADIATIVE CAPTURE OF THERMAL AND RESONANCE NEUTRONS, MAIN PARAMETERS OF THE GAMMA-DECAY PROCESS AND PROPERTIES OF THE $^{174}$Yb NUCLEUS

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Abstract

The re-analysis of the data published on the primary gamma-transition intensities following capture of 2 keV neutrons in $^{173}$Yb have been performed. Distribution of dispersion of these intensities with respect to mean value was approximated over different energy intervals of the primary gamma-transitions. This allowed one to estimate independently on other experimental methods the expected numbers of levels of both parities for spins $J=1-4$ and possible total sum of partial widths for primary electric and magnetic dipole gamma-transitions to the levels with excitation energy up to 4 MeV. The determined level densities and summed radiative strength functions confirm peculiarities of analogous data derived from the two-step $\gamma$-cascade intensities following thermal neutron radiative capture in nuclei from the mass region $40 \leq A \leq 200$ and permits one to estimate sign and magnitude of their systematical uncertainties. The latter can be due only to very strong dependence of radiative strength functions of cascade gamma-transitions to lower-lying levels on their structure.

1. Introduction

Analysis of the two-quantum cascade intensities following thermal neutron capture [1] in large set of nuclei showed a presence of strongly appeared itself step-like structure with the width of 2 MeV in density of nuclear levels $\rho$ below $\approx 0.5B_p$. The further development of this method [2] resulted in additional confirmation of the fact of strong dependence of partial widths $\Gamma_f$ of not only primary but also following $\gamma$-transitions on excitation energy of nuclear levels $f$ in the region of mentioned above structure. Accounting for this circumstance considerably decreased level density determined according [2] with respect to results [1].

The methods [1,2] belong to the class of inverse tasks (determination of unknown parameters of functions measured experimentally) and, therefore, require maximum possible test and revealing all the sources of systematical errors). There must be solved the problem of discovering probable dependence of the obtained according to [1,2] parameters of $\gamma$-decay of highly-excited levels on energies of neutron resonances $\lambda$ and possible influence of their structures on parameters of the reaction under study.

Intensities $I_{\gamma\gamma}$ of two-step cascades to any group of low-lying levels in arbitrary nucleus are determined by radiative strength functions $k$ of their primary and secondary transitions in combination with level densities of both parities for fixed spin window. Functional relation between them is non-linear. Therefore, the region of multitude of the possible $\rho$ and $k = f / A^{2/3} = \Gamma_f / E_{\gamma}^{3} A^{2/3} D_{\lambda}$ values precisely reproducing experimental spectra is always limited [1] by finite intervals of values for both parameters. This interval is narrow enough under condition that the ratio between partial widths $\Gamma$ of primary and secondary $\gamma$-transitions of the same multipolarity was set for total interval of their possible energies and nucleus excitation energy on the grounds of additional experimental information [2] or some hypotheses [3,4]. Just nonlinear relation between $\rho$ and $k$ with $I_{\gamma\gamma}$ allows one to get the mentioned parameters of $\gamma$-decay from the only experiment with acceptable uncertainty. Only this approach provides one to get the $\rho$ and $k$ values with the least (as compared with the other existing methods) systematical errors [5].
By now, experimental measurements of $I_{\gamma\gamma}$ were performed only for thermal neutron capture. The accumulated experimental data allowed determination of the $\rho$ and $k$ values for 42 nuclei [1] (only in the frameworks of the usually used hypothesis [3,4] on independency of radiative strength functions on nuclear excitation energy). Besides, these data were obtained for two tens of these nuclei with relatively realistic (but partial) accounting for function $k(E_\gamma, E_f)$, experimentally estimated below excitation energy $E_f \approx 4$ MeV [2]. This circumstance and obviously observed dependence of cascade intensities on structures of all three levels (including initial compound-state) [6] caused necessity to obtain new experimental data on $\rho$ and $k$ from other experiments on investigation of gamma-decay.

At present, search for influence of the initial compound state structure on gamma-decay process is the first-turn task of experiment. Really this can be done in analysis of the experimentally measured primary gamma-transitions following capture of “filtered” neutrons with energy 2 MeV. The authors of corresponding experiments [7] practically used their data only for determining spin and parities of excited levels on the base of notions of the limit “statistical” theory of gamma-decay. I. e., in the frameworks of hypotheses of independency of $k(E1)$ and $k(M1)$ on structure of any decaying ($\lambda$) and excited ($f$) levels nuclear levels and applicability of the Porter-Thomas distribution [8] for describing random deviations of gamma-transition partial widths in arbitrary interval of their mean values. There are no experimental grounds for hypotheses [3,4,8] for the data like [7] (concrete nucleus, a given set of primary gamma-transition intensities). Therefore, analysis method must take into account possibility of complete non-execution of assumptions mentioned above.

Quite enough basis for the analyses suggested below are the following statements:

the experimental values of $\rho \Delta E$ and sum $\Gamma_{\lambda, f}$ can be determined with acceptable errors by extrapolation of the measured experimentally distribution of random intensities of gamma-transitions to zero threshold of their registration;

averaging of fluctuations of the primary gamma-transition widths over initial compound-states decreases their dispersion independently on extent of truth of hypothesis [8]. I. e., any set of gamma-transition intensities from $(n, \gamma)$ reaction can be described by $\chi^2$–distribution with unknown number $\nu$ of degrees of freedom. (In the other words – it is close to normal distribution with dispersion $\sigma^2 \approx 2/\nu$). Corresponding method was developed and tested on large set of data on two-step cascade intensities in [9].

The Porter-Thomas distribution correctly describes random fluctuations of partial widths of tested gamma-transitions only when their amplitudes are described by normal distribution with zero mean. That is why, they must be a sum of a large number of items with different signs and the same order of magnitudes. This condition is to be fulfilled if wave functions of levels connected by gamma-transition contain a big number of items of different signs and equal order of magnitude. Matrix element for amplitude of gamma-transition must have only these components.

2. **Main aspects of modern theoretical notions of the compound-state gamma-decay**

On the whole, existing theoretical concepts, for example, Quasiparticle-Phonon Nuclear Model (QPNM) [10,11] call doubts in applicability of enumerated above so primitive ideas of gamma-decay. In particular, the studied in frameworks of QPNM regularities of fragmentation of states with different complexity [12] directly point to presence of items with large components of wave functions in the primary gamma-transition amplitudes. First of all, this concerns wave functions of excited levels [13, 14], but it is not excluded that wave functions of decaying compound-states (neutron resonances) also have large phonon, in particular, components. This directly points to potential possibility of
rather considerable violations of the Porter-Thomas distribution. These violations can appear themselves in limited [12] energy intervals of final levels and change dispersion of real distribution with respect to [8] in any side. Ratio between absolute values of items in amplitude of any gamma-transition and their signs for the data like [7] are unknown. Therefore, the analysis of experimental data suggested below must take into account a possibility of strong dependence of the primary transition partial widths on structure and, correspondingly, energy of excited levels and cover all the possible spectrum of their random deviations from the mean value.

3. Specific of experimental data from \((\bar{n},\gamma)\) reaction

In the experiments performed in BNL and later in Kiev were investigated different nuclear-targets. But for the analysis presented below was chosen even-odd nuclear-target \(^{173}\)Yb. This choice is caused by both the maximum primary gamma-transitions energy interval [7] and presence of determined in [1] level densities and primary gamma-transition strength functions. This nucleus is also very good for both search for discrepancy of general trend for energy dependencies of \(\rho\) and \(k(E1)+k(M1)\) revealed earlier at thermal neutron capture in neighboring nuclei [1, 2] and estimating systematical uncertainty of method [1]. The latter is completely determined by errors in the performed by now experiments on measuring thermal neutron capture spectra and insufficient set of the data obtained.

Even-even (odd-odd) nucleus has two possible spins of resonances. Therefore, analysis of intensities in this case requires one to introduce and then to determine maximal number of parameters. Even-odd compound nuclei represent particular case of the problem under consideration.

The width FWHM=850 eV of filtered neutron beam with 2 keV energy in performed experiments is determined by interference minimum in total cross-section of scandium. The mean spacing between neutron resonances in \(^{173}\)Yb equals 7.8 keV, and dispersion \(\sigma^2=2/\nu\) of their expected distribution in very rough approach can be estimated by value of \(-0.05\) (\(\nu \approx 40\)).

It is assumed in analysis that all the distributions of the primary gamma-transition intensities from reaction \((\bar{n},\gamma)\) have only the following unknown parameters:

(a) the mean reduced intensity \(\left< I_{\gamma}^{\text{max}} / E_{\gamma}^3 \right>\) of gamma-transitions exciting levels \(J=2,3\);
(b) the portion \(B = \left< I_{\gamma}^{\text{min}} / I_{\gamma}^{\text{max}} \right>\) of the reduced gamma-transition intensity to levels \(J=1,4\) with respect to their intensity to levels \(J=2,3\);
(c) the independent on spins of the levels excited by the primary gamma-transitions ratio \(R_k=k(M1)/k(E1)\);
(d) the expected and equal numbers \(N_{\gamma}\) of gamma-transitions to levels to \(J=2,3\) and \(J=1,4\);
(e) as well as the dispersion \(\sigma^2\), measured in units of number of degree of freedom \(\nu\).

Naturally, these parameters are to be determined independently for each energy interval of the primary gamma-transitions. Statistical uncertainties in determination of the experimental \(\left< I_{\gamma} / E_{\gamma}^3 \right>\) values increase experimental dispersion \(\sigma^2\) of distribution and decrease the \(\nu\) value. It is assumed that their relative systematical errors in each energy interval are practically equal. Of course, this notion assumes that, in limits of the excitation energy intervals \(\Delta E \approx 200-300\) keV, the structures of compound-state and all excited levels
connected by gamma-transition weakly influence the $< I_\gamma / E_\gamma^3 >$ values of the gamma-transitions of the same type populating these levels.

Both approximation and interpretation [6] of the experimental data on the $k(E1)+k(M1)$ values and results of experimental determination of structures of low-lying nuclear levels show that this assumption can contain considerable uncertainty (especially for wide energy intervals of the primary gamma-transitions under study). But, maximum precision in determination of the most probable $N_\gamma$, $B$, $R_k$, $\nu$ and $< I_\gamma^{\text{max}} >$ values can be achieved, in principle, by recurrent optimization of energy intervals of the primary gamma-transitions where these parameters are determined.

One more problem is related to small size of set and difference in number of electric and magnetic dipole primary gamma-transitions in given intervals $\Delta E$. Therefore, it is necessary to introduce and to fix in analysis some assumption on numbers of levels of positive and negative parities in given excitation energy interval of a nucleus. Below is used hypothesis of equality of numbers of electric and magnetic gamma-transitions. In practice, there is possible variation of their ratio for any given excitation energy interval. The problem of difference of level densities with different parities disappears in the case of $R_k \approx 1$, maximum error of determination of $N_\gamma$ for $R_k \approx 0$ corresponds to transitions with the least intensities and insignificantly distorts the desirable sum $\sum < I_\gamma^{\text{max}} > / E_\gamma$. Error of approximation in intermediate variant will be determined, first of all, by difference in level densities of positive and negative parities, i.e., it will decrease when increases excitation energy (as it is predicted in the whole by modern theoretical calculation of this nuclear parameter [15]).

Approximation of a mixture of two types of random values with different mean parameters by any distribution cannot determine their belonging to one or another type without the use of additional information. But accounting for known fact that the magnetic gamma-transitions to the lowest levels are by order of magnitude less than electrical transitions, one can extrapolate inequality $R_k = k(M1)/k(E1) < 1$ up to excitation energy of studied nuclei where $R_k = 1$. But it is not excluded that at higher excitation energy $k(M1)/k(E1) > 1$.

Strength functions of $p$-neutrons in ytterbium isotopes are many times less than strength functions of $s$-neutrons. The former in actinides exceeds [16] strength function of $s$-neutrons by a factor of 1.5-2.0. Authors [17] estimated, that portion of capture of 2 keV $p$-neutrons equals approximately 15%. If one does not account for possible appearance of some number of the primary dipole gamma-transitions to levels $J=0$ and $J=5$ following capture of $p$-neutrons, then this capture leads, most probably, to change in the $R_k$ values for different energies of excited levels and corresponding increase in $\nu$. That is why, small number of $p$-neutron captures in $^{173}$Yb must not anywise strongly influence precision of determination of the expected $N_\gamma$ and sums $k(E1)+k(M1)$.

Distributions of the random $< I_\gamma^{\text{max}} > / E_\gamma$ were approximated by analogy with [9] for cumulative sums in function of increasing values of intensities.

4. Results of analysis

Experimental distributions of cumulative sums of reduced intensities of the primary gamma-transitions $\sum I_\gamma / E_\gamma^3 = F(< I_\gamma / E_\gamma^3 >, N_\gamma, \nu, R_k)$ calculated for different values of concrete parameters are presented in Fig. 1 (only for one from two possible for given nucleus
sets of spins of final levels). As it is seen from the figure, one can hope for obtaining quite reliable estimations of parameters $N_\gamma$, $R_k$, $\nu$, $\langle I_\gamma \rangle$ with acceptable errors of about 10% or some bigger.

Fig. 1. Variations of dependence of cumulative sum of intensity on cascade intensity for: (a) – number of gamma-transitions, (b) – ratio $R_k$, (c) - value of $\nu$, (d) – registration threshold of gamma-transition. The values of varied parameters are given in figures. The values of other parameters correspond to the set: $N_\gamma=10$, $R_k=0.5$, $\nu=10$ and zero registration threshold of gamma-quantum.

It can be see from Fig. 1(a) that cumulative sum of intensities must be determined for excitation energy interval of even-odd nucleus containing $N_\gamma \sim 5$-10 gamma-transitions and two times more – for even-even nucleus. Their bigger quantity cannot provide for good sensitivity at determining the most probable expected number of gamma-transitions for zero threshold of experiment. In the case of their less quantity, a discreteness of experimental cumulative sum becomes essential.

It is seen from Fig. 1(b) that there is possible to determine the $R_k=k(M1)/k(E1)$ values in interval from 0 to ~0.5 with acceptable reliability and reveal a fact of closeness of the $k(M1)$ and $k(E1)$ values. Fig. 1 demonstrates possibility of precise enough determination of expected dispersion for given set of experimental intensities for $\nu \approx 10$ or less.

One can expect that the precision in determination of the registration threshold of gamma-transition intensity is high enough and error of its determination can be ignored. But, it can be done only at correspondence between experiment and adopted hypotheses of distribution shape of random intensities of the primary gamma-transitions. At presence of functional dependence of the primary gamma-transition intensities on some “hidden” parameter, the maximum errors, most probably, are possible for approximated values of the most probable number of gamma-transitions $N_\gamma$ and their expected deviation $\delta N_\gamma$ from the mean value. Modern nuclear theory does not consider this possibility. There are no experimental data on existence of “hidden” dependence, as well. Below it is not taking into account.

Experimental relative intensities $<I_\gamma^{\max}/E_\gamma^1>$ together with their best approximation are given in Fig. 2. The data are presented so that the expected sum of intensity of gamma-
cascades lying below detection threshold corresponds to the most probable value of cumulative sum for \( <I_{\gamma}^{\max}/E_{\gamma}^{3}> = 0 \).

Precision of determining parameters of approximating curve at small excitation energy of final levels \( E_f \) must additionally get worse due to inequality of level densities of different parity. Most probably, this increases error of extrapolation to zero intensity of gamma-transition. In practice, this can result in overestimation of \( N_\gamma \). Comparison between approximated value of this parameter and known number of levels with spins 1-4 points [18,19] to overestimation of the \( \sum N_\gamma \) value below \( E_{\gamma} = 2.6 \) MeV by a factor of 1.5-1.6.

The best values of fitting parameters \( \nu \) and \( R_k \) are given in figs. 3 and 4. Noticeable change in these approximation parameters for \( E_i > 2.5 \) MeV points to considerable change in structure of even-odd isotope under consideration in given excitation energy region.

The best values of level density \( \rho = \sum_{J,\pi} N_{\gamma} / \Delta E \) and sums of radiative strength functions \( \sum <I_{\gamma}> / E_{\gamma}^{3} N_{\gamma} \) are shown in figs. 5 and 6. Intensities and strength functions in both [1] and [7] were normalized to absolute values.

Fig. 2. Experimental value of cumulative sum of reduced experimental intensities \( <I_{\gamma}/E_{\gamma}> \) for \(^{174}\text{Yb}\) - histogram. Smooth curve represents the best approximation. Excitation energy intervals \( E_i \) of final levels are shown in figures.
Fig. 3. The ratio $k(M1)/k(E1)$ for different energy of levels excited by dipole primary gamma-transitions – histogram. Points with errors show interval of possible values of ratios $R_k=0.5$ from the data [1].

Fig. 4. The best values for dispersion of random intensities of the primary gamma-transitions to levels with different spins. Solid histogram represents values of parameter $\nu$ for gamma-transitions to levels with $J=2,3$, dashed histogram – to final levels with $J=1,4$.

Fig. 5. Experimental and model values of level density in $^{174}$Yb. Curve 1 represents level density calculated within model [20], curve 2 shows expected density of two-quasiparticle levels for their appearance threshold 1 MeV, curve 3 – density of four-quasiparticle levels for threshold 3.1 MeV. Solid histogram corresponds to the best approximation of data [7], dotted histogram shows data [9], points with errors represent data [1].

Fig. 6. The sums of radiative strength functions for the best approximation of the data [7] – histogram. Dashed curves represent predictions of models [3,21] in sum with $k(M1)=$const. Points with errors represent data [1].
Physically important information on structure of levels in the interval of their excitation from ~2 to ~4 MeV can be extracted from the coefficient of collective enhancement of level density:

\[ \rho(U,J,\pi) = \rho_{qp}(U,J,\pi)K_{coll}(U,J,\pi). \]  

According to modern notions, \( K_{coll} \) determines degree of enhancement of density of purely quasiparticle excitations \( \rho_{qp}(U,J,\pi) \) of deformed nucleus due to its rotation and vibrations. One can accept in the first approach that in narrow excitation energy interval and very narrow spin window considered here, parameter \( K_{coll} \) is approximately equal to coefficient \( K_{vibr} \) of vibrational enhancement of level density. In general form, the latter is determined by change in nuclear entropy \( \delta S \) and re-distribution of nuclear excitation energy \( \delta U \) between quasiparticles and phonons at temperature of a nucleus \( T \):

\[ K_{vibr}=\exp(\delta S-\delta U/T). \]  

Available experimental data on level density and model notions of it do not allow unambiguous and reliable determination of the \( K_{vibr} \) value for arbitrary nuclear excitation energy \( U \) even for zero systematical error in determination of function \( \rho(U,J,\pi) \).

Unfortunately, there was not found possibility for unambiguous experimental determination [23] of breaking threshold \( E_N \) of the first, second and following Cooper pairs, value and form of correlation functions \( \delta N \) of nucleons pair number \( N \) in heated nuclei. The main uncertainty of \( E_N \) is caused by lack of experimental data on function \( \delta N=f(U) \), the secondary – by ambiguity of density of two-quasiparticle levels in model [24]. So, according to three different model ideas, the threshold \( E_2 \) of appearance of four-quasiparticle excitations was found to be equal to 1.7, 3.2 [23] and 3.4 MeV [25]. It is most likely [22] that the breaking threshold of the secondary Cooper pair \( \approx 3.3 \) MeV has the least systematical uncertainty. In this case, good estimation for \( \rho_{qp}(U,J,\rho) \) will be density of two-quasiparticle excitations calculated in accordance with [24]. The \( K_{coll}^{-1} \) value obtained is shown in Fig. 7. There is observed significant correlation of this coefficient with the value of \( \delta_1 \) from [25] and from the second variant of analysis [22] in the excitation energy interval \( \approx 2.0 - 3.1 \) MeV. Decrease in correlation coefficient at higher excitation energy can be related to both significant contribution of four-quasiparticle excitations in function \( \rho(U,J,\pi) \) and less than it is adopted in [22, 25] velocity of decrease of function \( \delta_1 \) at \( U > 3 \) MeV.

Graphical representation of the relationship between the coefficient of collective enhancement of level density and correlation function of nucleon pair in \(^{174}\text{Yb}\). Circles show coefficient of collective enhancement of level density, curve – values of the \( \delta_1 \) parameter used in [25] for calculation of partial density of two-quasiparticle levels.

The data presented allow the following conclusions:
1. In the $^{174}$Yb nucleus excited in $(\bar{n},\gamma)$ reaction with neutron energy of $\approx 2$ keV are observed the same properties which were revealed for approximately four tens of nuclei from the mass region $40 \leq A \leq 200$: step-like structure in level density and significant local enhancement of radiative strength functions of the primary gamma-transitions to corresponding low-lying levels. The data on these parameters of cascade gamma-decay obtained in [1], most probably, contain significant errors: level density in the region 2.5 to 4.0 MeV (or wider) is overestimated, as minimum, by several times.

Strength functions are accordingly underestimated. Most probable, these errors are caused by significant increase in strength functions of the secondary cascade gamma-transitions to the levels of step-like structure. This effect was revealed and reproduced in calculation [2] in some neighboring even-even nuclei in form of significant increase in cascade population of levels lying above 3-4 MeV (or lower by the $\delta$-value – in odd nuclei).

2. In the excitation energy region about 2.0-2.5 MeV occurs abrupt change in level structure. It appears itself in considerable increase of the $k(M1)/k(E1)$ values and in strong difference of distribution of random gamma-transition amplitudes from normal one.

3. Experimental ratios $k(M1)/k(E1)$ can be used for obtaining more unambiguous values of radiative strength functions for $E1$- and $M1$-transitions and data on relation between level densities of different parity in the frameworks of methods [1,2].

4. The main part of the primary gamma-transitions observed in $(\bar{n},\gamma)$ reaction corresponds, probably, to population of levels with large and weakly fragmented phonon components of wave functions.

5. Conclusion

Analysis of available experimental data on intensities of the primary gamma-transitions following 2 keV neutron capture in compound nucleus $^{174}$Yb demonstrated presence of step-like structure in level density and increase in radiative strength functions of transitions to levels in region of this structure, at least, for the primary dipole transitions. I. e., it confirmed main conclusions of [1,2] and simultaneously showed necessity to reveal and remove systematical experimental errors in alternative methods of determination only of level density and all other parameters of cascade gamma-decay. That is why, two problems become the most important - correct accounting for influence of level structure on probability of nuclear evaporation and emission of cascade gamma-quanta in investigation of nuclear reactions on beams of accelerators as well as considerable decrease of systematical errors of experiment. Comparison of the data presented in figs. 5 and 6 with the obtained intensities of two-step gamma-cascades allows preliminary conclusion that sharp change in structure of decaying neutron resonances, at least, in interval of their energies of $\approx 2$ keV is not observed.

REFERENCES