NEUTRON LIFETIME DEPENDENCE ON DECAY ASYMMETRY

V.V. Vasiliev
RF SSC Institute for theoretical and experimental physics (ITEP)
25, Bol. Cheremushkinskaya st., Moscow, Russian Federation

Abstract

Neutron decay events are considered as two different (and asymmetrical) flows of decay with different signs of electron momentum projection onto neutron spin. Determination of the decay constants separately on the different flows (neutron amounts) leads to two different values of the main decay constants corresponding to the different reduced decay flows. Appreciation of the asymmetry leads to the difference of the weighted average value for the total decay constant from its arithmetic average of two main decay constants. In terms of neutron lifetime this effect is expressed in the displacement of the weighted average value τ_Z of lifetime from the arithmetic average value τ'_0. It is shown that this displacement is described by the expression $\tau_Z = \tau'_0 \cdot \frac{1-\Delta^2}{1+\Delta^2}$, $\Delta = A \cdot \frac{\overline{v}_e}{c}$, where $\frac{\overline{v}_e}{c}$ is the mean electron helicity just at the decay time, A – neutron spin-electron correlation coefficient. The correspondence of this expression to the known experimental results is demonstrated.

As it is known, the probability of the neutron decay $n \to pe^- \overline{\nu}$ with electron emission in the neutron spin direction differs from the decay in the opposite direction [1]. For the case the following formula for the neutron decay probability W can be written

$$dW(E_e,\Omega_e) = W_0\, dE_e d\Omega_e \{1 + A(\boldsymbol{S_n} \cdot \boldsymbol{p_e})/E_e\}, \qquad (1)$$

where A – neutron spin-electron correlation coefficient, $\boldsymbol{p_e}$ is the electron momentum, $\boldsymbol{S_n}$ is the neutron spin; E_e is the total electron energy, Ω_e is the solid angle of electron emission, W_0 is a normalization constant. The A-coefficient averaged over experimental results is equal to $A = -0.1173 \pm 0.0013$ [2], this means that the decay probability with electron emission opposite to the spin is higher than the decay probability with emission in the spin direction.

Let us consider Christensen's scheme [3], in which two electron detectors were installed symmetrically on the both sides of the horizontal neutron beam. The uniform magnetic field perpendicular to the beam was used for collection of the decay electrons from the beam to the detectors. Let the detectors be installed now on the left and the right sides of the beam. Now the magnetic field is oriented horizontally. Lorenz's force compels every electron, produced at the neutron decay in the definite beam section, to drift along the magnetic field lines to the left or to the right side of the beam, moving along a helix. The drift direction and the electron helicity along the field correspond to the velocity component acquired by the electron just at the neutron decay time. So the magnetic field distorts the space velocity distribution of electrons, acquired exactly at the decay time, conserving only the velocity component along the field.

Let us add to the scheme the full (100%) polarization of the neutron beam and suppose that neutron spins are oriented from the left to the right. Let us consider the left electron detector as ideal one and denote it by the letter *L (left)*, and its counts by the letter *N* with subscript *L*, N_L. Let the second ideal detector be settled symmetrically on the right of the beam being denoted by *R* (right) and its counts by N_R

These *L*- and *R*-detectors observe the same segment of the beam from the both sides. In such a way the situation of intentional decay anisotropy, detected by the two ideal and identical in their properties detectors of electrons, is created. In this scheme every detector "sees" the specific decay picture. Just in the decay time neutrons are separated into two types (sets), *L* and *R*. *L*-detector observes only *L*-decays that are decays with emission of electron in opposite direction to spin of decaying *L*-neutrons. *R*-detector observes only decays with emission electrons in the direction of *R*-neutron spins. Before decay occurred it is impossible to predict which set the every neutron belongs to. Nevertheless, there is an obvious statement that the total number of *Z* neutrons in the given set-up consists of two neutron sets *L* and *R* being equal to their sum. This is trivial consequence of the fact that every neutron decays sometimes, emitting its electron to the left or to the right.

Now it seems to be important to consider the neutron decay from the point of the two decay sets and determine the influence of the asymmetry on the decay constant. Thus we start with the above mentioned equality

$$N_Z = N_L + N_R. \qquad (2)$$

The definition of decay constant is well known:

$$\lambda = \frac{1}{N} \cdot \frac{dN}{dt}. \qquad (3)$$

Let us emphasize, however, that in (3) the rate of change of neutron number *N*, $\frac{dN}{dt}$ is reduced just by this number *N*. Let us apply definition (3) to the sum (2). It is easy to obtain the following

$$\frac{1}{N_Z} \cdot \frac{dN_Z}{dt} = \frac{1}{N_L} \cdot \frac{dN_L}{dt} \cdot \frac{N_L}{N_L + N_R} + \frac{1}{N_R} \cdot \frac{dN_R}{dt} \cdot \frac{N_R}{N_L + N_R}. \qquad (4)$$

In correspondence with (3), we get the following expression for the total decay constant

$$\lambda_Z = \lambda_L \cdot \frac{N_L}{N_L + N_R} + \lambda_R \cdot \frac{N_R}{N_L + N_R}. \qquad (5)$$

Here the expressions $\lambda_L = \frac{1}{N_L} \cdot \frac{dN_L}{dt}$ (6a) and $$\lambda_R = \frac{1}{N_R} \cdot \frac{dN_R}{dt} \tag{6b}$$

define constants in sets of L- and R-decay events, introducing different decay constants in the sets of the left and right neutron numbers N_L and N_R. Let us introduce in (5) the notation $W_L = \frac{N_L}{N_L + N_R}$, where W_L is the decay probability with emission of electron to the left, and $W_R = \frac{N_R}{N_L + N_R}$, where W_R is the decay probability with emission of electron to the right. Due to (1) the decay probabilities to the left and to the right are different, that is $W_L \neq W_R$. Hence we have got $N_L \neq N_R$. And, as far as N_L and N_R differ from each other in magnitude and are statistically independent arguments, the decay constants λ_L and λ_R are different too. Thus, it was proved, that, from the fact of asymmetry of electron emission in the direction of and opposite to neutron spin it follows that $N_L \neq N_R$ and $\lambda_L \neq \lambda_R$.

It was shown as well in (5), that the decay constant in full quantity Z of neutrons is the weighted average of the partial decay constants λ_L and λ_R.

From the asymmetry $N_L \neq N_R$, it follows that λ_Z differs from the arithmetic average

$$\lambda_0 = \frac{\lambda_L + \lambda_R}{2}. \tag{7}$$

In accordance with definitions (3) and (6) the decay constant is flow of decay events into the detector acceptance, reduced to one decaying neutron. Briefly, the decay constant is the averaged decay flow of one neutron in the field of view of the detector.

Expression (5) is similar to the weighted average of any quantity X, having two values X_1 and X_2 in two different states 1 and 2 realized with two different probabilities W_1 and W_2

$$X_W = \sum_i X_i W_i.$$

Analogously, it can be stated that neutron decays from two different states with different probabilities and different specific flows of decay events.

Let us consider the simplest hydrodynamic analogue (hypothetical model) of neutron decay in magnetic field taking into account the decay asymmetry. Let it be settled that neutron spin is oriented from the left to the right opposite to the magnetic field. Let the decay of neutron be confined by some cylindrical container, coaxial to neutron spin. This neutron-container includes two holes, made through the butt-ends of its cylinder. Let it be that there are some wanderings (oscillations around the central point) of electron in the container or in the force field created it, confined by the wall of the container. The hole on the butt-end up to the spin direction has a diameter of σ_L. The hole on the butt-end down the spin has a diameter of σ_R. Let us introduce a conceptional quantity j_0 of density of electron interactions with the container wall (or decay current density). The act of decay to the left takes place when the electron of the initial *Z* neutron hits the *L*-hole thus turning it into the *L*-neutron. The act of decay to the right takes place when the electron of the initial *Z* neutron hits its *R*-hole thus turning it into the *R*-neutron. Then *L*-decay constant as the electron flow through *L*-hole is expressed as the following product $\lambda_L = j_0 \cdot \sigma_L$. The *R*-decay constant as the electron flow through *R* –hole is expressed as a product $\lambda_R = j_0 \cdot \sigma_R$. The question is: what is the decay constant for the initial *Z* neutron? It follows from the model that the left decay probability is equal to the ratio of *L*-hole cross-section to the sum of cross-sections of the container holes, that is

$$W_L = \frac{\sigma_L}{\sigma_L + \sigma_R}.$$

Analogously the decay probability to the right is equal to the ratio of *R*-hole cross-section to the sum of cross-sections of the container holes, that is

$$W_R = \frac{\sigma_R}{\sigma_L + \sigma_R}.$$

As far as the probability of confined oscillating electron to hit one or another hole is very little, it is possible to consider the decay current densities near the both holes as close and equal to j_0. So the ratios of cross-sections can be replaced by the ratios of *L*- and *R*-decay flows or decay constants. Thus we get for the probabilities above

$$W_L = \frac{\lambda_L}{\lambda_L + \lambda_R} \tag{8a}$$

and

$$W_R = \frac{\lambda_R}{\lambda_L + \lambda_R}. \tag{8b}$$

The same result could be obtained considering the decay probability as a ratio of one or another branch of the decay flow to their initial sum.
Substituting (8a) and (8b) into (5), we get for the total decay constant

$$\lambda_Z = \lambda_L \cdot \frac{\lambda_L}{\lambda_L + \lambda_R} + \lambda_R \cdot \frac{\lambda_R}{\lambda_L + \lambda_R} \tag{9}$$

In this approach λ_Z appears to be a weighted average composed by the initial partial decay constants λ_L and λ_R.

As a rule the neutron beam polarization is supported by a magnetic field, which in the detector region coincides with the field transporting electrons. That is the reason why character of electron movement in the magnetic field permits to be approximated by a simplified consideration of formula (1). The approximation consists in the consideration of only two cases of electron emission: one strictly opposite to the neutron spin and another strictly in the spin direction. When doing this let us use the average electron velocity $\overline{v}_e$ in the neutron decay spectrum. We admit that the specific flows of decay events are defined by the average probability (1) in the direction of and opposite to the spin, which leads to

$$\lambda_L = \lambda_0 \cdot [1 + \Delta], \tag{10a}$$

$$\lambda_R = \lambda_0 \cdot [1 - \Delta], \tag{10b}$$

where $\Delta = A\left(\frac{\bar{v}_e}{c}\right)$, λ_0 – arithmetic average (7) of the initial constants, $\left(\frac{\bar{v}_e}{c}\right) = \sqrt{\frac{\bar{T}_e}{\bar{T}_e + m_e c^2} \cdot \left(2 - \frac{\bar{T}_e}{\bar{T}_e + m_e c^2}\right)}$, $\bar{T}_e$ – kinetic energy averaged over the electron spectrum, $\bar{T}_e \sim 391$ keV [4], $m_e c^2 = 511$ keV, c is the velocity of light.

In (10) the absolute value of A is used, and sign of correlation is introduced explicitly. The decay angular distribution and the average projection of electron velocity in the decay instant are included in the concept of average velocity along the field.

So, expressions (10) simplify the decay by two initial constants or by two flows of neutron decays which have different rates.

The arithmetic average value λ_0 (7) can be considered as the decay constant of "unpolarized" neutrons or neutrons "without spin".

In the sum ($N_L + N_R$) one L-neutron weighs in decay events more than R-neutron. Writing the sum of the specific decay flows ($\lambda_L + \lambda_R$), we see, that the relative contribution of specific L-flow is $\frac{\lambda_L}{\lambda_L + \lambda_R}$, while the contribution of R-flow is equal to $\frac{\lambda_L}{\lambda_L + \lambda_R}$. Then in the real beam the decay numbers ΔN_L and ΔN_R for the time ΔT are $\Delta N_L = \lambda_L \cdot N_L \cdot \Delta T$ and $\Delta N_R = \lambda_R \cdot N_R \cdot \Delta T$. The neutrons are distributed between the decay channels accordingly to (8a) and (8b) $N_L = N_Z \cdot \frac{\lambda_L}{\lambda_L + \lambda_R}$, $N_R = N_Z \cdot \frac{\lambda_R}{\lambda_L + \lambda_R}$. Then in L- and R-sets of decays we obtain the result $\Delta N_L = \lambda_L \cdot N_Z \cdot \frac{\lambda_L}{\lambda_L + \lambda_R} \cdot \Delta T$ and $\Delta N_R = \lambda_R \cdot N_Z \cdot \frac{\lambda_R}{\lambda_L + \lambda_R} \cdot \Delta T$. Let us calculate the total decay constant for the unification Z (sum of sets) of two different types of decay. For the sum of decays let us write on Z-set $\Delta N_L + \Delta N_R = \lambda_Z \cdot N_Z \cdot \Delta T$, from which the total constant λ_Z is equal to

$$\lambda_Z = \lambda_L \cdot \frac{\lambda_L}{\lambda_L + \lambda_R} + \lambda_R \cdot \frac{\lambda_R}{\lambda_L + \lambda_R}.$$

That reproduces expression (9) for the total decay constant. Thus, after averaging over the sum of different sets of decay events the total specific decay flow λ_Z (total decay constant) is equal to the weighted average of initial decay flows λ_L and λ_R. The quantity λ_L is realized in the greater number of decays than λ_R. That is the reason why the weighted average (9) is displaced from the arithmetic average $\lambda_0 = \frac{\lambda_L + \lambda_R}{2}$, being closer to λ_L. Substituting expressions (10a, 10b) into (9), it is easy to see that using of the weights leads to the following connection of the weighted average λ_Z and the arithmetic average λ_0

$$\lambda_Z = \lambda_0 \cdot (1+\Delta^2). \tag{11}$$

Although the understanding of decay constant as (7) is generally accepted, it seems to be more adequate to adopt the average neutron decay constant realized in the nature as (9). The understanding of difference between average values (7) and (9) can give a positive result for determination of values λ_L and λ_R. From this point it becomes necessary to create a method which in a set of measured electron rates is able to determine the value of λ_0, using its exactly central position between λ_L and λ_R. Such method is suggested in [5, 6]. As for the determination of λ_Z, the method of storage of ultra cold neutrons till their beta-decay gives a result for the neutron lifetime corresponding just to the weighted average that is $\tau_Z=1/\lambda_Z$.

Let us express (11) in terms of the neutron lifetime. To simplify the following relations we again use the notification $\Delta= A\left(\frac{\overline{\mathrm{v}}_e}{c}\right)$ and write the commonly used form of lifetimes $\tau_L=1/\lambda_L$, $\tau_R=1/\lambda_R$, $\tau_0=1/\lambda_0$, $\tau_Z=1/\lambda_Z$. Then it follows from (10a, 10b) that

$$\tau_L = \frac{\tau_0}{1-\Delta^2}\cdot(1-\Delta), \tag{12}$$

$$\tau_R = \frac{\tau_0}{1-\Delta^2}\cdot(1+\Delta). \tag{13}$$

Denoting $\frac{\tau_0}{1-\Delta^2}$ as τ_0' in (12), (13), we obtain

$$\tau_0' = \frac{\tau_0}{1-\Delta^2} \tag{14}$$

and, using (10a, 10b) and (11), we get

$$\tau_Z = \tau_0' \cdot \frac{1-\Delta^2}{1+\Delta^2}. \tag{15}$$

Here, in accordance with (12) and (13), τ_0' is the middle point between values τ_L and τ_R. Hence, the quantity τ_0' is the arithmetic average for neutron lifetime, observed by counting of decay electrons. Thus, the decay asymmetry calibrates τ_0, transforming it (14) into τ_0'. It should be emphasized that τ_L and τ_R are physical quantities, which could be observed in a specific performed experiment. Formula (15) gives possibility to calculate the neutron lifetime till beta decay from the values of τ_0' and correlation coefficient A. This neutron lifetime coincides by definition with the value of τ_Z. On the contrary the measurements of two different average lifetime values included into (15) give possibility to determine A, involving the data on decay electron spectrum.

Let us note that τ_0 could be classified as lifetime of «unpolarized» neutron. This quantity could be called true neutron lifetime τ_β, defined here as

$\tau_\beta \equiv \tau_0 = 1/\lambda_0$ at the assumption that the true neutron lifetime does not depend on angular correlations. The neutron storage time τ_Z differs from the lifetime of «unpolarized» neutron τ_β.

Novelty of the proposed approach gives a chance to fix the displacement of decay constant (11) from its value (7) by the quantity, determined by the decay asymmetry. Here we refer to papers [5, 6], in which the experimental value for τ'_0, the arithmetic average for the neutron lifetime, was determined to be equal to $\tau'_0 = 900.0 \pm 0.1$ s. Using this value in (15), we have done a plot of the dependence of the neutron lifetime weighted average on spin-electron correlation coefficient *A* (fig.1). It is clear that the neutron lifetime value τ_Z=883.3 corresponds to |*A*|=0.1174, which agrees very well with "*A* world average value" mentioned above.

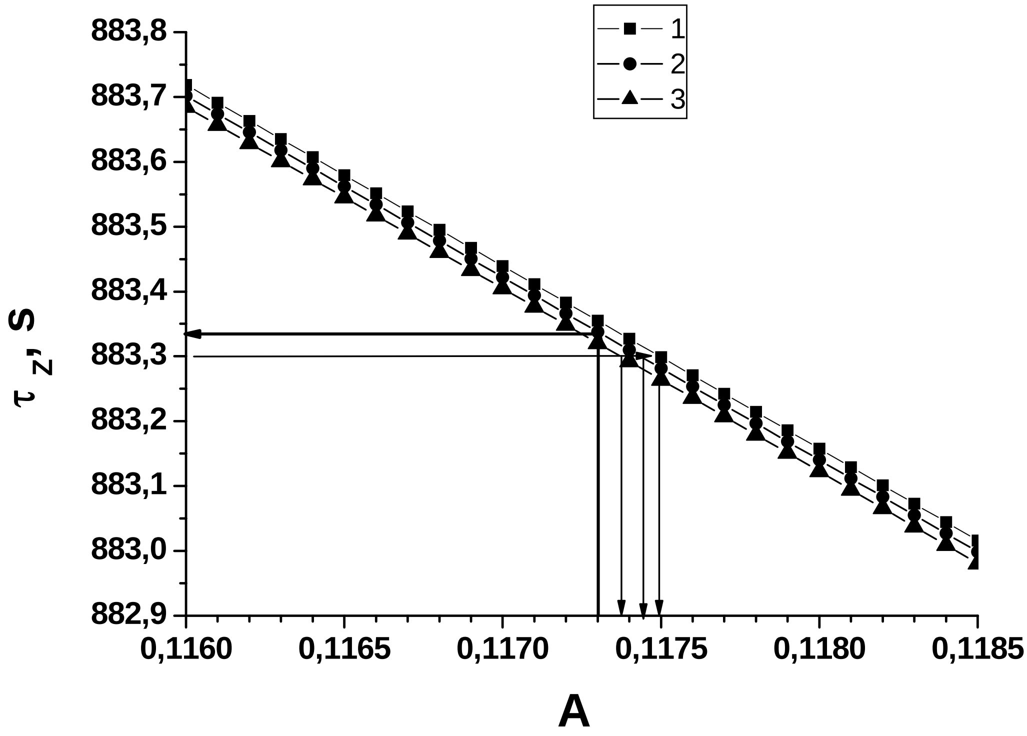

Fig.1. Dependence of neutron lifetime in a perfect storage on asymmetry coefficient *A* for the mean kinetic energies of electron $\overline{E}_e$=390 keV (1), 391 keV (2) и 392 keV (3) with τ'_0=900 s.

So the simplest calculations using integration over the angle of electron emission with respect to the neutron spin by taking into account weights of the neutron decay spin modes lead to the effect of displacement of arithmetic average relative to the weighted average of the neutron lifetime. The value of this displacement corresponds to the known measurements of the neutron beta-decay asymmetry.

As far as the neutron decay and its correlations do not depend on the magnetic field, the named relations (9) and (15) remain valid for the neutron storage experiment in every type of a trap – magnetic or nonmagnetic. It means that in any neutron trap where polarized or non-polarized neutrons are stored the decays with electron emission in the neutron spin direction and in the opposite direction always take place. Thus we always have different rates of electron production depending on neutron spin-electron correlation. Thus expressions (11-15) are valid in every case without any exception.

Conclusion

This paper demonstrates that existence of the neutron decay asymmetry with adequate and consistent application of well known definition of the decay constant automatically lead to the fact of existence of two different neutron lifetimes, weighted average and arithmetical average, with appreciable displacement between them. This displacement is determined by the neutron spin-electron correlation and the mean electron helicity in the neutron decay. The obtained estimation of this displacement is in a good agreement with known experimental results for the neutron lifetime and neutron spin-electron correlation coefficient.

The obtained results could be useful to perform new experiments to observe the existence of two main neutron lifetimes τ_L and τ_R which are contributed into the averaged values with different weights.

The effect of the neutron lifetimes displacement (or spin shift of the neutron lifetimes) opens new possibilities in the neutron decay research.

It becomes clear that this effect being unknown and unexplored could be a possible source of systematic errors in many years experimental activity on the neutron lifetime measurements.

Thus we have got new reasons to reach a new stage of precision in the neutron decay measurements.

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