PARTIAL DENSITY OF n-QUASI-PARTICLE EXCITATIONS AND EXPERIMENTAL ESTIMATION OF COEFFICIENT OF VIBRATION ENHANCEMENT OF NEUTRON RESONANCES DENSITY

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Abstract

Experimental level density below B_n in the mass region $40 \leq A \leq 200$ was approximated with a high precision by the B.M. Strutinsky model in combination with exponentially decreasing coefficient of collective enhancement of level density for given number of excited quasi-particles at increasing excitation energy. This combination of model notions allows one to reproduce not only general trend of change in level density obtained by the Dubna group but and its fine structure. For the first time it was obtained realistic experimental information on change in ratio of level densities of quasi-particle and vibration types practically up to the neutron binding energy B_n for nuclei of any type.

1 Introduction

Functional relation of the ρ and Γ in the case of two-step reaction permits [1] one to realize practically model-less method for simultaneous determination of region of their possible values [2, 3]. In the case of one-step reaction – determination of only level density [4], or in common ρ and Γ [5] is impossible without the use of unverified experimentally hypotheses. And, as a consequence, due to presence of not removable unknown systematical error. Estimation of reliability of its results in this situation requires obligatory test of all the set of the used hypotheses, methods of data analysis and realistic determination of total uncertainties $\delta\rho$ and $\delta\Gamma$.

Method [3] for extraction of the ρ and Γ parameters provides for partial accounting of dependence of the gamma-transition radiative strength functions on energy of excited by them level (its structure) and gives quite acceptable [6] at present accuracy in determination of ρ and Γ for maximum possible ordinary errors of experiment. Moreover, the results of analysis of intensities of the two-step gamma-cascades following thermal neutron capture [2, 3] are completely confirmed (owing to absolute presence of "step-like structure" in experimental values of ρ) below $\approx 0.5B_n$ by the data of reanalysis of intensities of the primary gamma-transitions after neutron capture in "averaged resonances" [7].

Registration of gamma-transitions to several final levels of product-nucleus as the second step of cascade sufficiently decreases real systematical error of the desired data obtained in Dubna. Moreover, obtaining of some information on number of parameters exceeding by a factor of ≈ 2 a number of intervals in experimental distributions of cascade intensity **does not contradict to mathematics in case of systems of non-linear**

equations. The key algorithms for the experimental data processing considerably decreasing systematical errors of the determined ρ and Γ values are presented in [8, 9, 10].

2 Dubna method

Systems of equations connecting the measured product spectra of nuclear reaction with desired values of ρ and Γ are non-linear. In some cases it is possible to determine maximum possible and final interval of the unknowns for system of non-linear equations even if the number of desired parameters is much more than the number of equations.

Intensity of cascades connecting compound-state λ and a group of low-lying levels f, as a function of energy E_1 of their primary gamma-transition is determined by the following relation:

$$I_{\gamma\gamma}(E_1) = \sum_{\lambda,f}\sum_{i}\frac{\Gamma_{\lambda i}}{\Gamma_{\lambda}}\frac{\Gamma_{if}}{\Gamma_i} = \sum_{\lambda,f}\frac{\Gamma_{\lambda i}}{<\Gamma_{\lambda i}> m_{\lambda i}}n_{\lambda i}\frac{\Gamma_{if}}{<\Gamma_{if}> m_{if}}. \quad (1)$$

Excitation energy of nucleus (energy of intermediate cascade level E_i) is unambiguously determined by the energy E_1: $E_i = B_n - E_1$.

Functional (1) depends on both ratio of partial and total radiative widths Γ of primary E_1 and secondaryE_2 cascade gamma-transitions between levels λ, i and f, and on number $n = \rho \times \Delta E (m = \sum n)$ of excited levels in different energy intervals.

The type of cascade transition (dipole electric or magnetic), spin and parity of the excited intermediate level i is unambiguously determined by known values J^{π} of levels λ and f. Practical absence of cascades between levels with $|J_{\lambda} - J_f| > 2$ excludes a necessity to take into account transitions of higher multipolarities in analysis like [2, 3]. Cascade intensity in equation (1) is directly proportional to derivative $d\Gamma/dE$ and, to the first approach, inversely proportional to ρ. Another form of relation between desired parameters as compared with usual evaporation and gamma-spectra provides and the least influence of correlation of parameters on their real uncertainty. Just this situation appears itself in experiment on determination of two-step cascade intensities. This allowed us to realize new of principle method for determination of main parameters of cascade gamma-decay from reaction $(n, 2\gamma)$ (Dubna metod). In practice, minimization of interval width of the ρ and Γ values in our case requires one additionally to connect (on base of experiment or theoretical notions) strength functions of the primary and secondary gamma-quanta by unambiguous relation and to involve in analysis information on level density near by ground state, information on neutron resonances and value of the total radiative width of compound-state. In this case, a given interval can have a width of, for example, some tens percents (and contain infinite number of possible solutions of equation system).

In practice, the total spectrum of random solutions of system of non-linear equations can be obtained by means of the Monte-Carlo method with strongly differing initial values of parameters [2, 3] or Gauss method with maximally wide of variations of initial set of desired values and different parameters of regularization of degenerated matrixes.

Experimental data of the highest quality on cascade intensities were obtained on the gamma-gamma-coincidence spectrometer in Řež (given below isotopes Ge, ^{125}Te, Lu, W and ^{191,193}Os) by J. Honzatko and I. Tomandl. A portion of data for other nuclei was analyzed by group from Prague. Unfortunately, their analysis contains 3 fatal errors:

1. There is not taken into account the fact that the experimental spectra at any energy of cascade gamma-quantum are the sum of unknown intensities I_p and I_s of cascades with the primary (p) and secondary (s) gamma-transitions of near energy. The proof that the used set of ρ and Γ does not reproduce the distorted by arbitrary error intensities $I_p + \delta I$ and $I_s - \delta I$ is impossible without additional information like [10].

2. There is not taken into account that the total intensity of two-step cascades equals 100%. Any significant error in reproduction of cascade intensity in some number of experimental points by means of the tested set of ρ and Γ undoubtedly means that the corresponding functions deviate to unknown degree from their values under determination.

3. As a consequence, the measured intensities (1) can be reproduced with the same χ^2 by infinite set of the ρ and Γ values. By this, belonging of ρ and Γ to corresponding interval of the χ^2 values is not a proof that the tested functions correspond to desirable ones because there is no method to exclude false variants.

Method [2, 3] is to the less extent distorted by errors of this kind and, therefore, provides obtaining of maximum reliable at present data on level density and emission probability of nuclear reaction products.

3 Model notions and approaches

Model approximation of the obtained in [2, 3] level densities was performed in [11, 12] in zero approach (coefficients of collective enhancement of level density do not depend on nuclear excitation energy) because of lack of experimental data on function $K_{\rm col} = f(E_{\rm ex})$ below B_n. Any known models [13] or hypotheses concerning this point need in test and making more precise because the developed by now theoretical notions of, for example, coefficient of vibration enhancement of level density $K_{\rm vib}$ are based mainly on the experimental data like [4] and therefore they can be rather mistaken.

Reanalysis [7, 14, 15] of the data of the $(\overline{n}, \gamma)$ reaction allowed one for the first time to get preliminary experimental data on energy dependence of $K_{\rm vib}$. The largest volume of information on these intensities was obtained from the experiment for three gadolinium isotopes [16, 17, 18]. As it is seen from Fig. 1, experimental data can be in the first approach described by dependence of the following type

$$K_{\rm vib} = Aexp(-(E - U_{\rm n})/E_\lambda) + 1 \qquad (2)$$

and then used as the next approach for parameterization of the data [2, 3]. In this

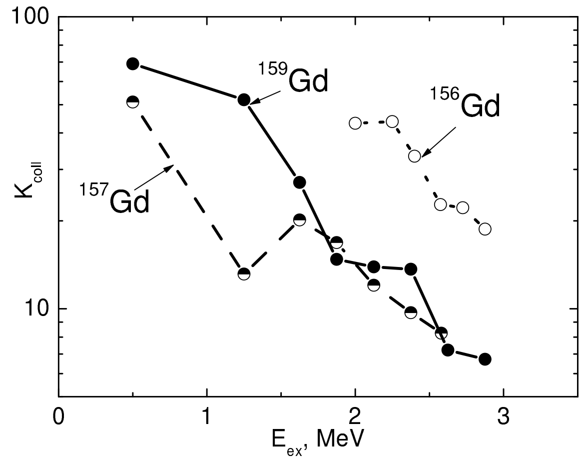

Figure 1: Coefficients of $K_{\rm coll}$ from the data of reaction $(\overline{n},\gamma)$

expression, A and E_λ are fitted parameters, and breakup threshold of the next Cooper pair $U_{\rm n}$ is simultaneously the parameter of Strutinsky model [20] for density ρ_n n-quasi-particle excitations:

$$\rho_n = K_{\rm coll}\frac{(J+1)exp(-(j+1/2)^2/(2\sigma^2))}{2\sqrt{(2\pi)}\sigma^3}\Omega_n(E)$$
$$\Omega_n(E) = \frac{g^n(E-U_n)^{n-1}}{((n/2)!)^2(n-1)!}. \qquad (3)$$

According to theoretical analysis [19] the values of parameter $g = 6a/\pi^2$ at different excitation energy U depend on shell inhomogeneties of single-particle spectrum and can be represented as functional dependence on asymptotic value $a_{\rm asim}$ of parameter of level density and Strutinsky shell correction δW. At present corresponding approximation is used

$$a = a_{\rm asim}(1 + (1 - exp(-0.062U)\delta W/E)) \qquad (4)$$

in the frameworks of generalized model of superfluid nucleus for nuclear temperature exceeding the adopted in this model value of temperature of phase transition superfluid – usual state of nucleus $T = 0.567\Delta_0$. The value of level density parameter $a_{\rm asim}$ at excitation energy of about 100 MeV for nuclear mass A equals $0.048A + 0.257A^{2/3}$ [19]. Correctness of the use of approximation (4) in interval from B_n to excitation enrgy of about 1-3 MeV is unknown. Nevertheless, this dependence was used further for estimation of influence of variation of parameter g in (3) on break-up of nucleon pairs and other parameters of model. The necessary for this data for g =const are given in [11, 12].

Energy dependence of parameter g is unambiguously determined by given above relations, therefore the total density of levels for n-quasi-particle excitations depends only on unknown parameters A, E_λ and $U_{\rm n}$. Very precise approximation of experimental values

ρ is achieved, as a rule, at accounting for break-up of maximum four Cooper pairs of nucleons.

Unfortunately, there was not found solution of problem of identification of type of breaking at different excitation energy Cooper pairs – neutron or proton. For the main part of the studied nuclei the number of neutrons is noticeably larger than number of protons. Correspondingly, the value of parameter g in expression (2) for pair of proton quasi-particles can differ by some tens percents (and more) from analogous parameter for neutron quasi-particles. This circumstance brings in additional error in the desired value $K_{\rm vib}$.

Naturally, the resulting accuracy of performed approximation depends not only on adopted assumptions on energy dependence of coefficient $K_{\rm coll} = f(E)$. So, experimental intensity of two-step cascades is determined by level density in narrow ($\delta J \leq 4$) spin window. As a consequence – on accuracy and reliability of model setting in (3) of spin cut-off parameter σ and real coefficient of rotational enhancement of level density in non-spherical nuclei. Because of smallness of J it is adopted below that the coefficient of collective enhancement of level density in deformed nuclei is approximately equal to the coefficient $K_{\rm vib}$ of only vibration enhancement of ρ. In the frameworks of existing theoretical notions [13, 19]

$$K_{\rm vib} = exp(\delta S - \delta U/t) \qquad (5)$$

it is determined by change in entropy and nucleus excitation energy caused by appearance of vibrations of nuclear surface at thermodynamics temperature t. But the theory [13] does not give at present suitable for practical fit of experimental data [2, 3] dependence $\delta S = f(E_{\rm ex})$ and $\delta U = \phi(E_{\rm ex})$.

Experimental data [2, 3] for even-even nuclei ^{74}Ge, ^{124}Te, ^{156}Gd, ^{168}Er, ^{190}Os and ^{196}Pt below excitation energy of about two energies of nucleon pairing δ give some grounds to assume that the parameter δS fluently increases with increase of energy $E_{\rm ex}$. Unfortunately, the use of this information on function $\delta S = f(E_{\rm ex})$ in region of break-up of the second and following pairs seems to be at present purposeless. This results from the fact that the approximation of ρ begins at high enough excitation energy and speed of increase of partial level density at $n \geq 4$ quickly grows. Therefore, the use of expression (3) allows one to expect obtaining most probably of low estimation of $K_{\rm vib}$. This conclusion follows from comparison between obtained here values and results of approximation [11, 12]. (Early approximations were performed under condition $K_{\rm vib}$ =const). That is why the use of expression (2) with parameters A and E_λ varied for each breaking pair can be considered as acceptable enough at given stage of analysis of experiment.

4 Results of approximation of level density

Results of approximation of ρ for 40 nuclei studied in Dubna are shown in figures 2-5.

The main conclusion of the performed analysis – noticeably increased in comparison with previous variant [12] precision of approximation of experimental data. The achieved

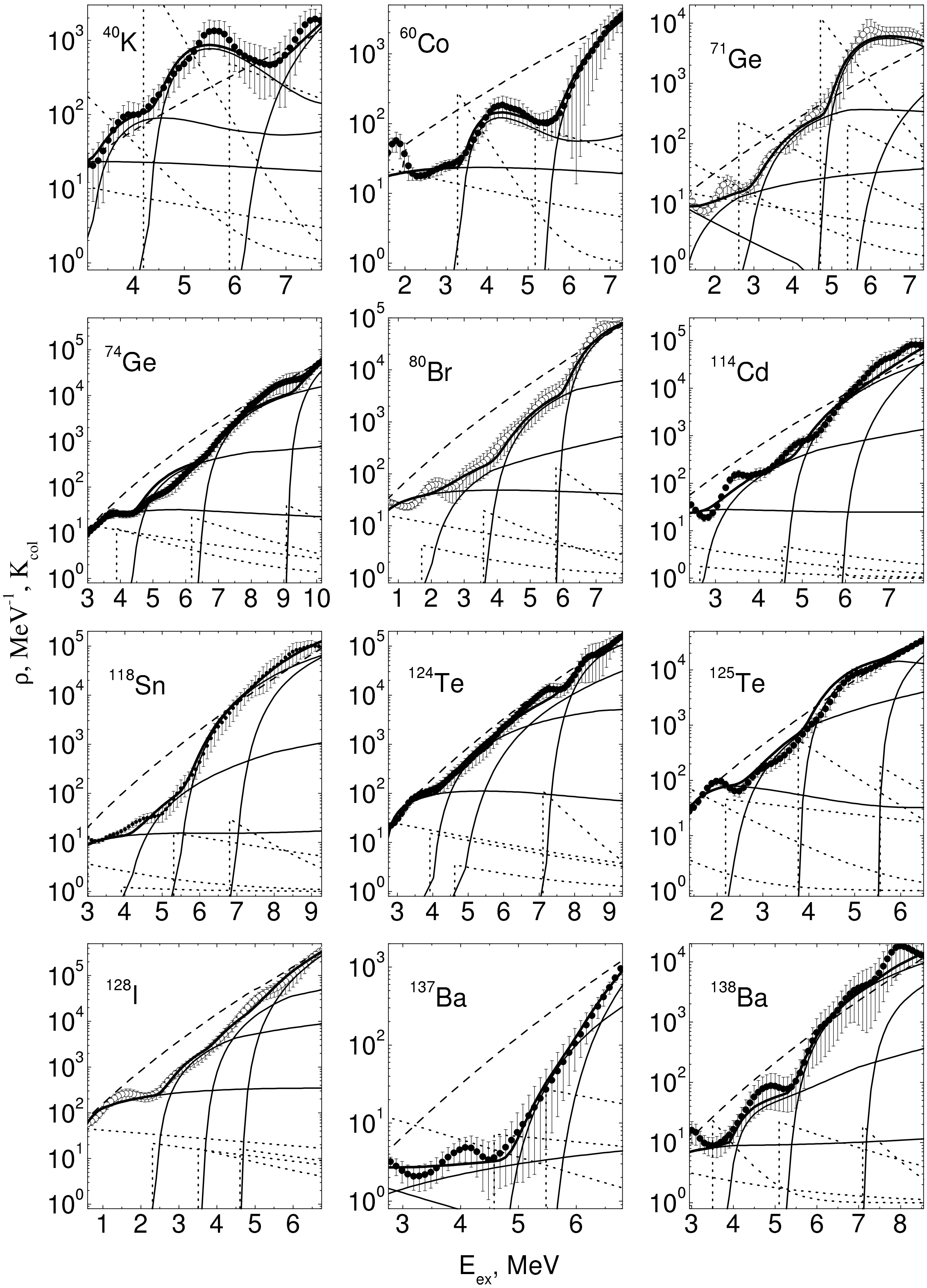

Figure 2: Results of approximation of the experimental data on level density ρ for nuclei ^{40}K, ^{60}Co, ^{71,74}Ge, ^{80}Br, ^{114}Cd, ^{118}Sn, ^{124,125}Te, ^{128}I, ^{137,138}Ba by partial level densities [20] under condition $g = f(E_{\text{ex}})$. Points with errors: $\circ$ – data [2], $\bullet$ – [3]. Thin lines – partial densities, thick lines – their sum. Dashed line – level density calculated within model [21]. Dotted lines – the best values of coefficients of collective enhancement of ρ.

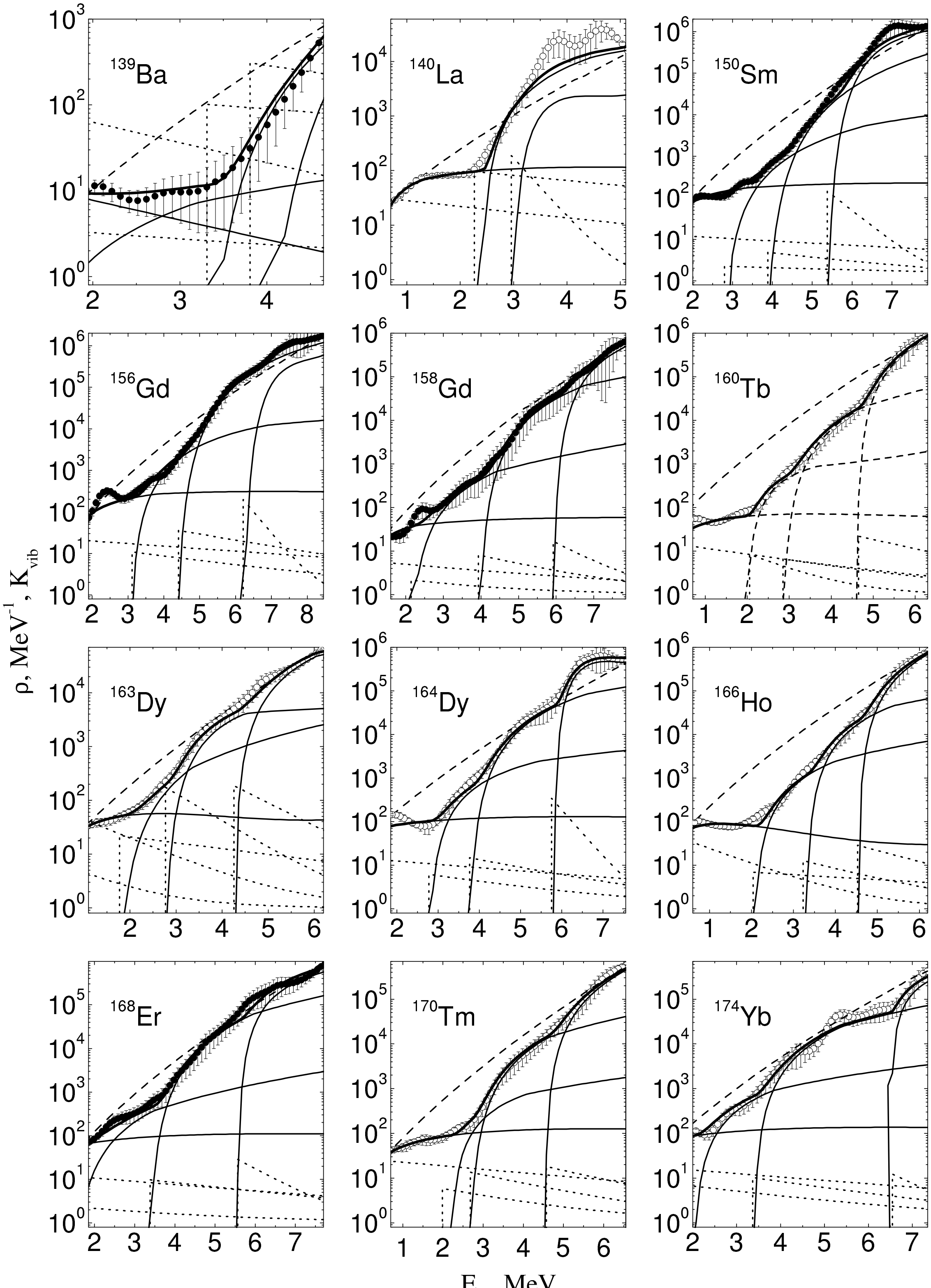

Figure 3: The same, as in Fig. 1, for ^{139}Ba, ^{140}La, ^{150}Sm, ^{156,158}Gd, ^{160}Tb, ^{163,164}Dy, ^{166}Ho, ^{168}Er, ^{170}Tm and ^{174}Yb.

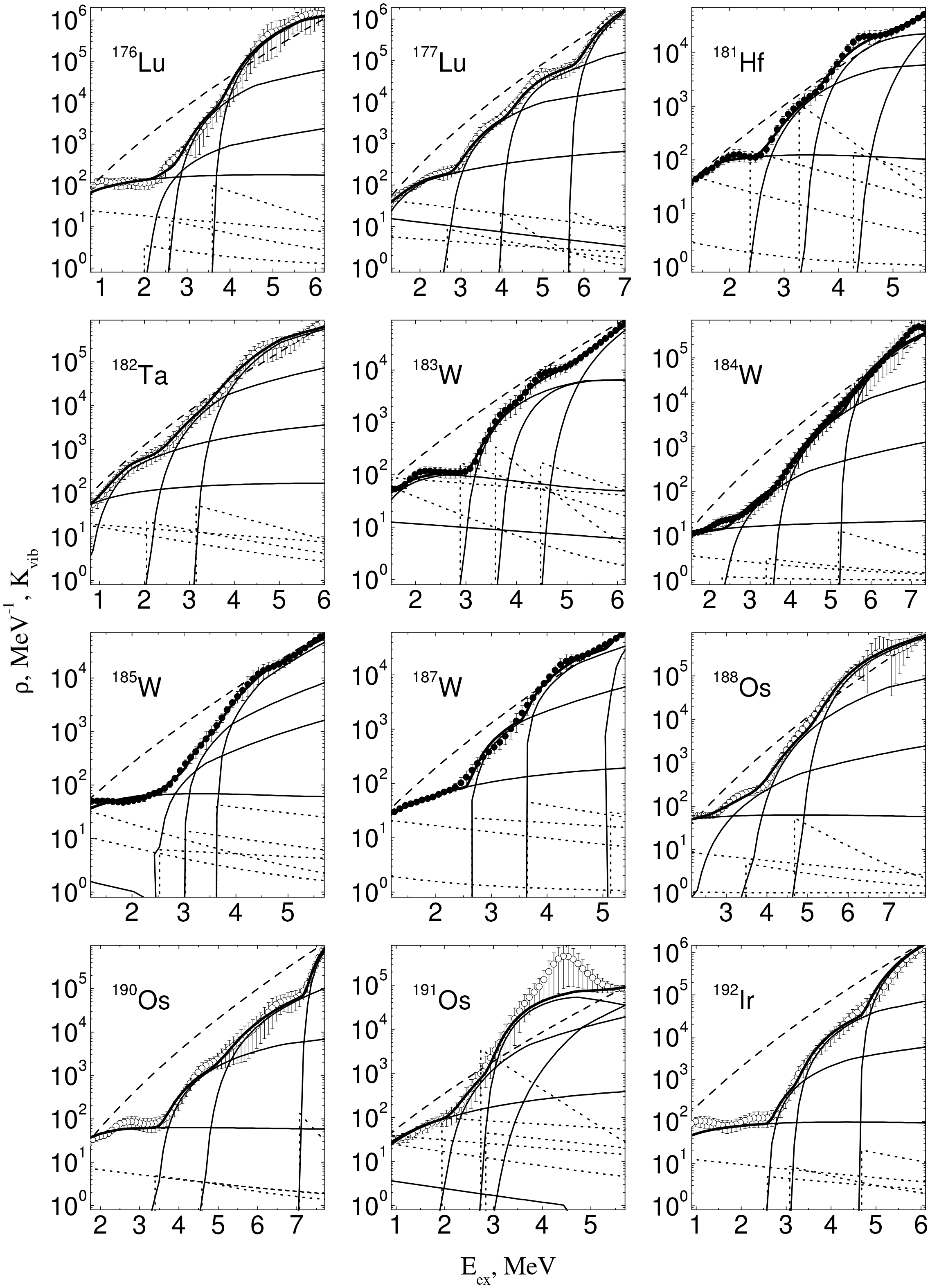

Figure 4: The same, as in Fig. 1, for ^{176,177}Lu, ^{181}Hf, ^{182}Ta, ^{183,184,185,187}W, ^{188,190,191}Os, and ^{192}Ir.

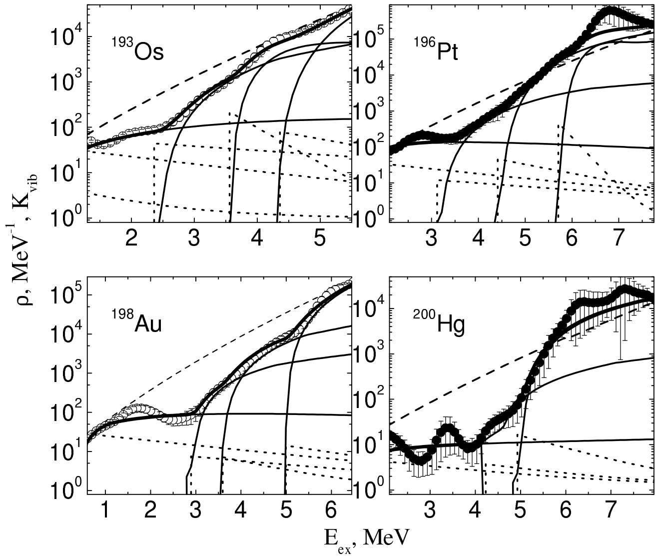

Figure 5: The same, as in Fig. 1 for ^{193}Os, ^{196}Pt, ^{198}Au and ^{200}Hg.

precision of fitting provided considerably better than in [12] reproduction of experimental value of ρ for nuclei ^{40}K and ^{60}Co. Some discrepancy between experiment and fitting rested for these and other nuclei can be partially compensated, probably, by involving in analysis of partial density of quasi-particles with another value of g (id est – proton quasi-particles in given case). It is not excluded that simultaneous taking into account of break-up of Cooper pairs of both neutrons and protons is to higher extent necessary for increase of precision of approximation for light nuclei than for heavy ones.

Comparison of divergence between experimental level density and the best approximation with width of interval of equal-probable values ρ ("errors" of experimental data) shows that the shape of dependence of ρ on excitation energy is reproduced with very good precision.

Potentially the experiment allows one to distinguish with good precision and study partial level density for any breaking Cooper pair of nucleons. It is rather essentially because it follows with rather high probability from figures 2-5 that the level density of given nucleus is determined by concrete values of correlation functions δ at its different excitation energy. As a consequence, one can assume that the precision of model-less methods [2, 3] for experimental determination of ρ already exceeds the precision of existing theoretical models.

The values of coefficients K_{vib} for nuclei with different parity of nucleon number are presented in Fig. 6. They correspond to values obtained by approximation at $E_{\text{ex}} = B_n$ for pair with maximal density $\rho_n(B_n)$. These results are compared with the data on K_{vib} determined in [1] for g which does not depend on nuclear excitation energy. An extent of reliability of (4) cannot be estimated at modern state of experiment, therefore general trend of change and extent of dispersion of the data presented in Fig. 6 give at the moment the most reliable experimental information on K_{vib} in spite of their considerable dispersion.

It is very difficult to explain very considerable discrepancy of the data for nuclei with different nucleon parity by systematical error of experiment. Therefore, the solution of

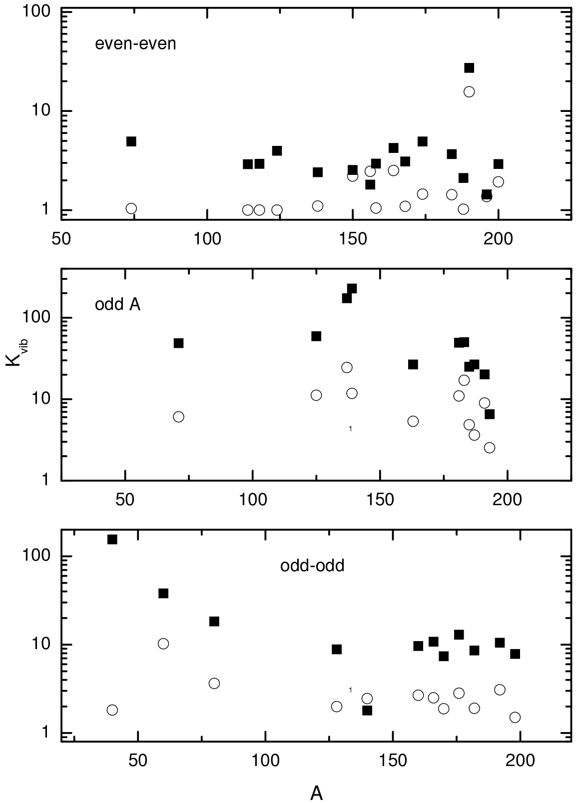

Figure 6: The most probable value of coefficient of collective enhancement of level density in nuclei with different parity of nucleons. Circles – the data from [1], squares – from present analysis.

this problem requires further investigation.

Variations of the U_n value from given analysis and [1] are maximal for threshold of appearance of the second pair of quasi-particles. But even here they do not exceed in the average 10%.

5 Results of analysis and conclusion

1. Even this simple, as it is presented in (2), practically semi-phenomenological accounting for energy dependence of coefficient of vibration enhancement of level density on nuclear excitation energy very considerably increases precision of the experimental data as compared with [11, 12].

2. The shape of general trend of the experimental level density of given nucleus is first of all determined by concrete values of correlation functions of the second and following Cooper pairs of nucleons and to the less extent – by their number. The position of the break-up threshold of these Cooper pairs weakly depends on the variant of the functional dependence of g used for given nucleus. Break-up of different pairs at adopted by analogy with [11, 12] notions of energy dependence of nucleon correlation functions in heated nucleus occurs in succession, with big enough difference of threshold.

3. For the main part of the studied nuclei, the variations of $K_{\rm vib}$ for different breaking Cooper pairs are small. But the possibility of sharp enhancement of level density of vibration type some below B_n cannot be excluded. This situation is to the maximum extend observed in nuclei ^{40}K, ^{60}Co, ^{71}Ge, ^{140}La, ^{191}Os and ^{196}Pt. Analogous assumption for higher than B_n excitation energy needs in experimental test.

4. Possible interpretation of different degree of local enhancement of level density in region $\sim 0.8B_n$ – the probability to transfer energy of excited quasi-particles to unexcited "frame" is different even for neighboring nuclei. This conclusion follows, first of all, from the data on cascade gamma-decay of isotopes ^{176,177}Lu, ^{183,184,185,187}W, ^{188,190}Os and ^{191,193}Os, two-step cascades to low-lying level of whose were measured in the same experiment for each element. Id est – ordinary systematical errors in these nuclei are maximally correlated. And existing discrepancies in the two-step gamma-cascade intensities are mainly caused by difference in shapes of energy dependence of level density and radiative strength functions in neighboring isotopes of the same element.

5. According to results of analysis, the expected portion of nuclear levels with mainly (or completely) phonon components of wave function in region of neutron resonances can exceed portion of levels of quasi-particle type by a factor of several times. Besides, it increases for the pair "neutron+proton" and, especially "odd single nucleon" with respect to even-even nucleus. Two last results are in qualitative agreement with conclusion [22] on influence of structure of decaying compound state on shape of the radiative strength function observed in the experiment. Id est – on the relation of excitation probabilities of levels of quasi-particle and vibration types by primary gamma-transitions following gamma-decay of neutron resonances with different structures of their wave functions.

So, one can conclude that the analysis of the experimental data on level density and radiative strength functions derived from the data of the $(n, 2\gamma)$ reaction allows one to obtain inaccessible up to now information on properties of a nucleus in region of its transition from excitations of the simplest type to compound states.

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